AMENDED 7/24/2022 TO DELETE A BRIEF REMARK NOT PERTINENT TO THE OVERALL ARGUMENT, THAT SHOULD HAVE BEEN REMOVED PRIOR TO SUBMISSION, AND TO CORRECT TYPO ON PAGE 7.

EXPERT TESTIMONY OF KEITH DEVLIN, PH.D.

In connection with
SCOTT R. BERNARD vs. PUBLIC POWER, LLC.

1. QUALIFICATIONS

I am an emeritus mathematician at Stanford University, having retired at the end of 2018.

Born in the UK in 1947, I obtained a BSc Degree in Mathematics (First Class) at Kings College London in 1968 and completed all the requirements for a PhD in Mathematics (specializing in Mathematical Logic) at the University of Bristol in 1971 (the degree was awarded early in 1972). I have lived in the United States since 1987, becoming a naturalized citizen in 2000.

Much of my more recent past and current research is focused on the use of different media to teach and communicate mathematics to diverse audiences. In this connection, I am a co-founder and President of an educational technology company, BrainQuake, that creates mathematics learning video games.

For my doctoral dissertation and for the first twenty years of my career as a professional mathematician, I focused on Mathematical Logic and Axiomatic Set Theory, both highly abstract branches of pure mathematics. During the 1980s, my interest shifted to applications of mathematics, originally motivated by an attempt to develop new mathematical techniques (based on Logic and Set Theory) to study communication using everyday language. That led to my working on the Theory of Information, Models of Reasoning, applications of mathematical techniques in the study of communication, Mathematical Cognition, and (after the terrorist attack on New York on 9/11/2001) the design of information systems for intelligence analysis.

I have written 33 scholarly books on mathematics and related fields, and published over 80 research articles. I am a recipient of the Pythagoras Prize, the Peano Prize, the Carl Sagan Award, and the Joint Policy Board for Mathematics Communications Award. In 2003, I was recognized by the California State Assembly for my "innovative work and longtime service in the field of mathematics and its relation to logic and linguistics." I am a Fellow of the American Association for the Advancement of Science and a Fellow of the American Mathematical Society. For many years I was "the Math Guy" on National Public Radio.

At the time of writing, the academic ranking site academicinfluence.com lists me in #1 place in their global list of "Top Influential Mathematicians of 2010–2020" [1]

In the early part of my career, I had a number of university research/teaching positions at universities in Scotland, Norway, Germany, and Canada, and from 1977 to 1987 I was a Lecturer and then Reader in Mathematics at Lancaster University in the UK. My research focus at that time was a continuation of my doctoral research, in Mathematical Logic and Axiomatic Set Theory.

The shift in my research interests in the 1980s, from pure, abstract Logic towards applying mathematics (in particular the methods of Mathematical Logic and Set Theory) to problems of communication led to an invitation to spend the 1987-88 year at Stanford University. I took a one-year leave from Lancaster, but in the event I stayed at Stanford a second year, and then never returned to my position in the UK. My positions at Stanford from 1987 to 1989 were Visiting Professor of Mathematics and Senior Research Scholar at the University's Center for the Study of Language and Information (CSLI).

CSLI is a highly interdisciplinary joint research venture of Stanford, SRI International, and Xerox PARC, established in 1983 to study linguistic communication from a variety of disciplinary perspectives, including mathematics. Stanford's original invitation to spend a year at the university was for me to join the large and growing international, interdisciplinary group of researchers at CSLI.

From 1989 to 1993, I was the Carter Professor of Mathematics and Chair of the Mathematics Department at Colby College in Waterville, Maine. From 1993 to 2001 I was Dean of the School of Science and Professor of Mathematics at Saint Mary's College of California in Moraga, California.

Throughout the 1990s, much of my research was done in collaboration with Dr. Duska Rosenberg, a UK-based professional linguist who specialized in the social aspects of language use. We worked together on two applied research projects, one for a UK computer manufacturer, the other for a large European Union construction project, writing a number of papers and eventually publishing a joint research monograph, *Language at Work*, about the use of language in the workplace and how seemingly routine, stylized communication could result in misunderstanding, and sometimes major losses of productivity, due to misalignment of the participants' contexts.

In 2001, the year after I became a naturalized US citizen, I returned to Stanford as the Executive Director of CSLI. Soon after, I co-founded and became Executive Director of Stanford's new mediaX program, that engaged in research for, and funded by, industry. In 2016, I co-founded and was Executive Director of the Human Sciences and Technologies Advanced Research Institute (H-STAR), an outgrowth of CSLI that broadened the scope from communication using language, to the other modes of communication that arose on the heels of the World Wide Web. H-STAR became the parent organization of the mediaX program.

In the post-9/11 decade, I was invited to participate in three research projects on intelligence issues, the first for a subcontractor to the National Security Agency, the second for a subcontractor to the US Navy, and the third directly for the US Army. In all those projects, a major focus was on the precision of communication—both direct- and computer-mediated-human-human, and human-computer—and how the context in which the communication took place influenced and impacted how the communication was understood.

I wrote three books during that decade that were closely related to my subsequent Defense Department work, and were in fact the reason I was asked to work on those projects:

Logic and Information. Cambridge University Press (1991), declared by the American Association of Publishers to be "Most Outstanding Book in Computer Science and Data Processing for 1991." This book developed mathematically-based methods for understanding how context influences communication, and formed the basis for my post-9/11 research for the US defense intelligence community.

Language at Work: Analyzing Communication Breakdown in the Workplace to Inform Systems Design, joint with the UK-based linguist Duska Rosenberg, Stanford University: CSLI Publications and Cambridge University Press (1996), which was based on research on communication we did for a UK computer manufacturer and a large European construction project.

InfoSense: Turning Information into Knowledge, W. H. Freeman (1999). This was in some ways a simplified version of *Language at Work*, and was written for a general business-leaders audience. I used examples of human communication in warfare, air-traffic control, and large corporations to explain and analyze how and why communication can break down due to misunderstandings caused by context (e.g. battlefield failures and airline disasters).

All have relevance to the legal case under consideration, as does my more recent fourth book:

Introduction to Mathematical Thinking, Amazon/Keith Devlin (2012). This was written to accompany the Stanford Online Course I created and launched on Coursera in 2012. The course is still being offered, and is regularly listed as one of the most popular online math courses offered by that platform. Many of the examples and exercises in the book and in the course are about how people use language, both within mathematics and in everyday life (including some that touch on our legal system).

2. WHAT I HAVE BEEN RETAINED TO DO

In late 2021, I received an email from Mr. Paul Markoff, of Markoff Leinberger LLC in Chicago, in which he wrote:

> *Dr. Devlin, I am an attorney in Chicago that handles consumer rights cases. I have one involving math and linguistics (specifically, the meaning of words that effectively create a mathematical formula in a sales pitch). Would you have any interest in discussing the case further and potentially providing expert testimony in it? The phrase at issue is the following: "guaranteed not to exceed 15% of your introductory rate."*

Based on my extensive research experience since moving to the USA in 1987, which I outlined above, I fairly quickly agreed to do as he requested.

I have never been asked before to provide expert testimony. My only connection with the legal system was in 2005, when I wrote, in response to a request from a Public Defender, an amicus brief for a case in a D.C. court case, explaining and providing commentary on the mathematical calculations involved in a DNA Cold Hit identification in a capital case.

Based on previous remuneration rates I charged for my participation in research projects for the US Department of Defense in the period 2000–2010, and consulting work since then, and after consulting with a colleague who does a considerable amount of mathematics-related expert testimony, I settled on a fee of $750/hour, which Mr. Markoff accepted. I began work in January of this year.

3. DOCUMENTS REVIEWED

A. PP00002-19: Transcript of the actual telephone exchange between Plaintiff and Defendant's human sales agent and the automated sales system.

B. PP009414/5/6: IL Script for the automated sales transaction in dispute.

C. PP000084/5/6: Telemarketing script for the human agent involved in the transaction.
D. PP000051: The verification audio recording of the transaction between Plaintiff and Defendant's human & automated sales system, lasting 7min 53 secs.
E. Expert Report from Sarah Butler, dated October 17, 2018.

I also consulted a number of websites providing advice on the preparation of expert testimony. In preparing my written testimony I found this passage on the website of Cornell University's Legal Information Institute particularly helpful, given the particular features of the task I was faced with.[1]

"Notes of Advisory Committee on Proposed Rules An intelligent evaluation of facts is often difficult or impossible without the application of some scientific, technical, or other specialized knowledge. The most common source of this knowledge is the expert witness, although there are other techniques for supplying it.

Most of the literature assumes that experts testify only in the form of opinions. The assumption is logically unfounded. The rule accordingly recognizes that an expert on the stand may give a dissertation or exposition of scientific or other principles relevant to the case, leaving the trier of fact to apply them to the facts. Since much of the criticism of expert testimony has centered upon the hypothetical question, it seems wise to recognize that opinions are not indispensable and to encourage the use of expert testimony in non-opinion form when counsel believes the trier can itself draw the requisite inference. The use of opinions is not abolished by the rule, however. It will continue to be permissible for the experts to take the further step of suggesting the inference which should be drawn from applying the specialized knowledge to the facts. See Rules 703 to 705."

After some deliberation of the issue, including consulting the documents and audio-file listed above, I decided to present the body of my testimony in two parts. In PART 1, I will provide my own expert opinion on the core utterance in the Case. In PART 2, I will present relevant background information showing that my opinion accords with, and is supported by, a well-established, widely accepted, federally supported framework for the use of mathematics in society.

4. PART 1 (OPINION)

I will examine, as a professional mathematician with demonstrated expertise in the use of language in the business world, the use of the following term in a telemarketing transaction:

"Customer, you understand that you have chosen the variable rate plan with price protection for one year, guaranteed not to exceed 15% of your introductory rate."

To commence, what does the above statement mean? Specifically, if the introductory rate is R cents per unit ("R¢/unit" in common abbreviated form), then what is the amount in cents that the rate is guaranteed not to exceed?

[1] https://www.law.cornell.edu/rules/fre/rule_702

The answer to that question is straightforward. The price is guaranteed not to exceed 0.15 x R¢ per unit. For example, if the introductory rate is 6.99¢ per unit, the statement would say that the price for the year should be no more than 0.15 x 6.99¢ = 1.0485¢ per unit.

That is, in my expert view, beyond question.

That is what anyone who understands percentages would take the offer to mean, since understanding such an expression in that way is part of what constitutes understanding percentages. Indeed, that very statement is very reminiscent of test questions presented to students in multiple-choice tests to assess the degree to which they understand percentages, and could readily be used as such. The Web is replete with examples of test questions about percentages just like that.

I understand that Defendant maintains the phrase could mean a price cap of 1.15 x R¢ per unit. In which case, if the introductory rate is 6.99¢ per unit, the price for the first year should be no more than 1.15 x 6.99¢ = 8.0385¢ per unit.

If 1.15 x R¢ per unit is the price cap the company wanted to base their offer on, then they could (and in my expert view should) have used different terminology. There are several possible formulations that would convey exactly that cap to anyone who understands percentages. For example,

“Customer, you understand that you have chosen the variable rate plan with price protection for one year, guaranteed not to exceed 115% of your introductory rate.”

Or perhaps

“Customer, you understand that you have chosen the variable rate plan with price protection for one year, guaranteed not to exceed your introductory rate by more than 15%.”

In my expert opinion, this too is all beyond question.

But this case exists because it *has* been questioned.

I am aware that the Defendant commissioned a market survey to determine what a sample of the population understands Defendant’s offer to mean. Such surveys can and do provide useful information about society. That is their purpose.

What polls cannot do is determine the correct value of a numerical expression that specifies the unit price for electricity, any more than they could determine the value of 6 x 9.

Taking figures from that report as examples, when presented with the Defendant’s actual offer sentence, 63.3 percent indicated that they expected their rate to increase, and 25.6 percent took it as a commitment that their rate would go down. (Butler report, pp.5-6.)

What those results show is that 63.3% of the population is wrong. The results also show that 25.6% of the population is correct.

25.6% is a substantial proportion. For context, I note that it’s almost identical to the proportion of the population of Illinois that, according to testing data from the Federal National Assessment of Educational Progress (NAEP, a.k.a. “The Nation’s Report Card”), were found to have demonstrated “Proficiency”[2] in mathematics in Grades 8 and 12 between 1990 and 2019. [2]

Specifically, for Grade 8, the percentage at “proficient” ranged between 22% and 27%, with a mean of 24.2% over ten readings; for Grade 12, sampled less frequently, the percentage at or above “proficient” ranged from 26% to 27%, with a mean of 26.5% over just two readings (for 2009 and 2013). [op. cit.]

I note also that understanding the utterance in question to mean the rate will go down is exactly the kind of mental recognition that is part-and-parcel of what mathematicians and mathematics educators would describe as “understanding percentages,” as I indicated earlier.

In my professional expert opinion, it is a plausible hypothesis that the customers who took the Defendant’s offer to mean the rate would be capped 0.15 x R₵ were predominantly, and perhaps even exclusively, customers who are above Grade 12 level in mathematics. This seems entirely reasonable to me, given that the offer asks the consumer to make a decision that could be used effectively and meaningfully in any multiple-choice test for mathematical proficiency with fractions and percentages.

To repeat, my expert opinion is that the offer Defendant made, guaranteed, as a matter of fact, that the rate for the first year would not exceed 0.15 x R₵.

Moreover, let me stress again that my expertise is not solely in the inner mechanics of mathematics. As I summarized earlier, the second half of my fifty years as a professional mathematician were spent largely on very applied projects dealing with human communication, and how/why it can go wrong. As a result, I am knowledgeable of the way language is used in society, and believe that, as a mathematician, I am well qualified to opine on the case in hand.

But in addition to providing my expert opinion, I am also well placed to provide relevant background information that US society (in fact, any modern, industrial society) puts enormous weight on importance of mathematics and on how it is used, and goes to considerable effort to ensure that the population is equipped to use mathematics responsibly. And, as I will show, that background information strongly supports the expert opinion I expressed above.

In particular, I understand Defendant’s argument to be based on the fact that with natural language, usage determines meaning. Having collaborated closely with a professional sociolinguist for many years and directed a Stanford University research center for several years where a number of world class linguists conducted their research, I am very familiar with that argument. However, the nature of mathematics and its role in any organized society is such that, when natural language is used to talk about mathematics, that argument does not apply.

The background information I shall present in Part 2 demonstrates why an attempt to use the “language use determines meaning” argument does not apply when it comes to mathematics.

[2] NAEP uses this as a technical term. It is significantly higher than “Basic” but lower than what they call “Advanced”. https://nces.ed.gov/nationsreportcard/guides/scores_achv.aspx

Indeed, modern societies typically mandate their education systems to ensure that common practice for using mathematics would prevent such a drift occurring. [See BACKGROUND INFORMATION D on page 13 below.]

For context, I note that some (maybe much) of Part 2 will be familiar, or at least not come as a surprise, to a scientist or engineer, or to anyone else who makes regular use of mathematics in their work. According to the most recent Federal data [3], such individuals constitute around 23% of the US workforce.

5. PART 2 (RELEVANT BACKGROUND)

Defendant's argument attempts to leverage (at least implicitly) the fact that, whereas mathematics can be applied with precision when used in physics, astronomy, and the natural sciences in general, there is always an unavoidable degree of slack when mathematics is applied in (and to) the everyday world. After all, people are not predictable the way physical systems are.

In fact, there is some intrinsic slack on *all* occasions when mathematics is used, even in physics. The critical issue when mathematics is used, in any domain, is the degree of slack that can be tolerated. That degree of tolerance always depends on the application.

Part 2 will be longer than Part 1 since supplying the requisite background information requires some more general background knowledge of how mathematics is used in the world, which I will provide in summary form.

I'll start off by introducing a simple picture of how mathematics works in society that will be useful to draw upon to present the specific background knowledge I believe is relevant to deciding this case. It reflects a common perception of mathematics acquired in the school math class, though I have only rarely seen it articulated explicitly, as I am doing here.

The "Black Box" view of Mathematics

There is a perception of how mathematics is used in the world that I will refer to as the "Mathematical Black Box." Inside the box, all the calculations are performed, all the formulas are evaluated, all the equations solved, etc. Using mathematics in the world then consists of hooking up the black box's input channel and output channel to the world, feeding in data (say numbers) through the input channel, and extracting the answers (maybe new numbers) from the output channel.

The Black Box can be a useful metaphor, and for this testimony it is particularly appropriate, so I will use it throughout. It is, however, historically inaccurate. It does not apply to the first few millennia of mathematics' growth and use, when people just thought and reasoned about the world in what we would now call a mathematical way. But it is possible to think of mathematics as a black box by the time of the famous Ancient Greek mathematicians around 600–300bce, the era of Pythagoras and Euclid, and it is certainly a useful metaphor for the way mathematics is used today.

The question the black box metaphor raises—and in my view this is at the heart of the legal case this testimony focuses on—is how the black box connects to the world. Do the input and output ports restrict data to conform to certain constraints? Or is it left to the user to input whatever data they want, in whatever format they want, and feel free to modify the output data however they want? Are they permitted to modify the input and output ports to make them more to their liking?

To put it another way, is connection to the world an integral part of mathematics, or are users of mathematics free to feed in and extract data any way they want? Where are the boundaries of the enterprise we call mathematics? Is “the mathematics” purely the inner workings of the box, or does it include the formatting of the input and output ports? [Note: the formatting of the ports might, and in fact does, depend on the particular application domain, as I will explain.]

“Who controls the ports?” is actually a very significant question for an organized society, since the answer dictates who get to call the shots as to what is legitimate and what is not, whenever mathematics is used.

That question, “In an organized society, who gets to call the shots when it comes to mathematics?” is answered for us by the world we live in: the mathematical community does. It’s a tautological answer. For, given the nature of mathematics and its role in society, *the very fact that we are in an organized society depends* on mathematics itself “calling those shots.” As I will describe later, since the very beginnings of the discipline, mathematics has developed *in conjunction* with a structured society, and indeed was an integral, and crucial, part of the original emergence of organized society, and it remains so today.

Let me repeat for emphasis: In terms of the box metaphor, in a modern organized society, the formatting of the input and output ports of the mathematical black box are necessarily an integral part of the device.

People do have *some* freedom of how they input and output data—the ports have some degree of tolerance—but it is limited. If those limits are ignored, then you are no longer in an organized society; you have an anarchy. Remove those limits on use of the black box, and you lose your monetary system, your legal system, your science, engineering, technology, medicine, transportation systems, communication systems, everything.

Of course, one person ignoring the limits on one particular occasion will, in most cases, not cause society to break down. But an organized society maintains its order by following the rules and norms associated with its various component parts. Someone could choose to use the mathematical black box incorrectly. And sometimes, incorrect use arises through human error, sometimes with negative consequences. (I’ll give a famous example later.) Likewise, each of us is free to ignore the laws of physics and step off the roof of a high rise building if we so choose, and in that case the consequences are inevitably negative.

In my view, in terms of the black box metaphor, the Defendant’s formulation of their key offer sentence break the rules that govern input and output ports of the mathematics black box. In this particular instance, doing so is unlikely to result in death or massive destruction. But an organized society depends upon its members following the norms that govern the way all its inter-connected parts interact, and commerce is no exception. If one party to a contract that involves use of the black box (even in a minimal way) breaks those norms, it is up to the society's legal system to adjudicate. The remainder of this testimony can be viewed as my providing that legal system with the background knowledge necessary to do so in an informed manner.

Misunderstanding the black box

In my experience, many people assume that when we use the word "mathematics" or the term "doing the math", we are referring exclusively to the inner workings of the black box. That is almost certainly based on imperfect memory of their school math classes, or maybe they never really understood what the teacher was trying to help them learn. After all, much of the time in the math class is devoted to mastering some of the machinery inside the box (how to add, how to divide, how to compute percentages, etc.).

It is then not surprising that many people graduate from high school thinking that mathematics *is* just the activities of "performing calculations and solving equations". But that flies in the face of the entire history of the subject. In the early days of mathematics, there was no "black box". People dealt directly with objects in the world, as I mentioned earlier. Over the centuries, mathematics gradually developed a black-box structure, but specifying the input and output ports was always, and is today, an integral part of doing/using mathematics.

BACKGROUND INFORMATION A: Word problems are ubiquitous in mathematics education.

Discussion Because it is important to society for people to understand that responsible use of mathematics includes making sure that the input and output data conform to the black box's specification, teachers typically present students with word problems like this one:

"A school needs to order some mini-buses to take 200 students on a field trip. Each bus can take 16 students. How many buses must be ordered?"

A student who takes an "inside the black box" approach to this problem, may say "200/16 = 12.5, so order twelve and a half buses" (or sometimes "twelve buses and half a bus left over").

Indeed, a great many students do answer that way. Every year. In terms of the (inside-the-box) calculation, it's correct. But the question is not about numbers, it's about buses. You cannot have half a bus. The correct answer to the problem as posed is "13 buses." Used in the context of this word problem, the ports on the mathematical black box are restricted to input and output of positive whole numbers.

Teachers hope that by giving their students problems like this, when they get them wrong on their first attempt, that may impact them sufficiently to realize that it is important to take account of the way any calculation they perform comports with the world situation in which it is being used.

Given the simplicity of this word problem, this might seem to be a fine point of little importance. In fact, it is so important that the United States government and the nation's education system have put significant ongoing resources into making sure people do not fall into the "don't worry about the data going into and out of the black box" trap. Expensive equipment and lives have been lost because of black box errors, as I will illustrate.

The point is, except in the educational case where the goal is *solely* to help students master specific mathematical techniques, using mathematics to solve "real-world problems" should always involve consideration of relevant real-world factors. Mathematics teachers are trained to ensure students understand and appreciate this. That's why they give students word problems. (A math exercise designed solely to provide practice with, say, the arithmetic would not require consideration of any "real-world" factors, even the simplistic ones you get in word problems.)

Why is this relevant here? The fact that word problems are routinely given in math education is evidence that society puts value on its citizens learning to appreciate that using math responsibly requires taking due care with the input and output data.

BACKGROUND INFORMATION B: The historical development of mathematics in society.

Discussion To the best of our knowledge, based on archeological evidence, mathematics began around 10,000 years ago in Sumeria, motivated by legal and financial needs to keep records of property ownership and trade as their society started to become more complex and organized. [4] [5] Their efforts to introduce order to, and regulation of, property ownership and trade went along with what we could now call a monetary system. (So numbers, money, and the regulatory systems around them, all grew together in tandem—along with written language, it appears.)

Ever since then, as mathematics has grown and developed, a main driving force was always to obtain more precision. The introduction of modern (i.e., "Hindu-Arabic") arithmetic into northern Italy in the early 13th Century gave rise, *in that one region and in a matter of a few decades*, to banks, modern accounting methods, the insurance industry, and the formation of large, international trading organizations, all of which rapidly spread to all of Northern Europe. Such is the power for society of the precision and accuracy provided by mathematics when it is put in the hands of the People. (I wrote a book that describes that historical arithmetical and commercial revolution. [6]) Mathematics always promised, and has repeatedly delivered, accurate information that could be trusted.

Why is this relevant here? The entire framework of mathematics arose, and was developed and refined, to provide ever greater precision and accuracy in human activities, in the first instance to support trade and commerce and a financial system, and in due course to support the development of modern science and several generations of technologies.

Achieving that accuracy is hard; even today many people find math difficult to learn, let alone master. Yet the payoff, to society and the individual, is undoubtedly worth it.

Today, the precision and accuracy that mathematics can provide was illustrated recently by NASA being able to design and build the James Webb Space Telescope and send it to a precise location in space one million miles from Earth (the L2 point), where it is currently being "unpacked" and remotely deployed, ready to study our surrounding universe. [7]

Back home on earth, we all of us benefit from the precision and accuracy of mathematics every minute of every day, since the phenomenal accuracy of mathematics underlies all of our science, technology, engineering, medicine, and our financial systems.

We are so used to benefiting from that accuracy, that most people rarely reflect on it. We just know that mathematics gives us secure, accurate, trustworthy information. No other human framework gives us that degree of exactitude and reliability. Provided, aways, that those of us who develop and certify that mathematics do our jobs correctly, and those who make use of that precious, powerful tool use it wisely and faithfully. When utilizing a tool that, for 10,000 years, Humankind has come to regard as providing true, precise, accurate, and unambiguous information, careless application (and on occasion nefarious application) can mislead, sometimes with damaging or even lethal consequences.

In making any use of mathematics, just as important as making sure that any statements of mathematical values are accurate and that any and all calculations are mathematically correct, is making sure the mathematics correctly connects to the real-world situation it is being applied to.

For example, in 1999, NASA lost its $125-million Mars Climate Orbiter due to a simple math error made by the engineers. [8]

The error was not in any of the calculations. The advanced mathematical computations were perfect. It was in the units being used. In short, the error was that using the wrong units meant that the mathematics did not connect accurately to the world; a middle-school-level error. The engineers should, of course, have spotted the error, but they didn't. Scientists and engineers, in particular, are so comfortable with mathematics that, provided they trust the source, they tend to believe mathematical results. In this case, their trusted source, namely, other members of the distributed team—were using different units.

BACKGROUND INFORMATION C: The US has historically put massive emphasis on the importance of mathematics and its accuracy, and directed significant resources into achieving its benefits in terms of a prosperous economy and our national defense, and continues to do so.

Discussion Much of our nation's current prosperity can be traced back to the foundation of the National Science Foundation in 1950. [9]

The preamble to *National Science Foundation Act of 1950* that Congress passed that year, states that the purpose of the NSF was to be: "To promote the progress of science; to advance the national health, prosperity, and welfare; to secure the national defense; and for other purposes." [10]

Throughout the lifetime of almost all living American citizens, the NSF has promoted and supported the creation and operation of a massive science and engineering framework, including a major emphasis on mathematics and science education at all levels from Kindergarten though university graduate school, a crucial enterprise that continues to this day. The 2022 budget for the NSF stands at $8.5BN.

Mathematics is central to that scientific enterprise. It is the bedrock of all science and engineering, that depend, not only on the sure, reliable accuracy of the results mathematics produces, but also on the fact that mathematics provides a reliable backbone to human communication. The NASA disaster cited above indicates, by virtue of how unusual it is, the degree to which our society depends on using mathematics accurately in our communications. In a society where accurate use of mathematics is the norm, and hence is relied upon, misuse of math can have damaging, expensive, and sometimes dangerous or lethal consequences.

Why is this relevant here? Since its early beginning 10,000 years ago, the big payoff of mathematics has always been greater precision and accuracy. That's arguably the most significant benefit it gives us. Since the early 1950s in particular, the US has recognized that our future prosperity and security depend on our making good use of the precision and accuracy provided by mathematics whenever and wherever possible. Precision and accuracy are central.

BACKGROUND INFORMATION D: In industrialized nations all over the world (including the US), there are just two school subjects that are obligatory, and operate under some form of national guidance: mathematics and the nation's language.

Discussion Industrialized nations recognize that math and language are special cases, and take steps to ensure that all citizens have mastery of those two human skillsets. Mastery of language is essential for us to communicate with one another, to collaborate, to be productive members of society, to live in a secure environment, and on a personal level, to get the greatest satisfaction from our lives. Mathematics gives us a mechanism for precision and accuracy in our daily activities, whenever required—and to the degree that is required—be those activities at work or in our non-work lives.

The degree of precision we bring to our lives with mathematics depends on the circumstances. Scientists and engineers may use 10 or more decimal places. Banks typically use 4 decimal places (hundredths of cents) in calculations, and 2-decimal places (cents) in reporting. When people are doing their daily grocery shopping, they rarely worry about the cents, and frequently pay little attention to single dollars. When purchasing a new car, individuals may make decisions based on the price in thousands, or possibly down to $500 differences. When we decide which house to purchase, a difference of $1,000 may have little significance to us. The degree of precision we use (usually) depends on the specific context.

In connection with the case in hand, I note that the difference between the offer the Defendant actually made, where the rate would be capped at 0.15 x R¢ and the offer they claim they intended to make, where the cap would be 1.15 x R¢, is almost 800%, which is an enormous difference. Since the average residential electricity bill in Illinois is almost $100/month [11], an eight-fold difference in unit cost would be highly significant to many customers.

Why is this relevant here? The one thing mathematics gives us that no other subject does is a tool to achieve whatever degree of precision a particular activity or domain requires. With mathematics, we can achieve far greater precision than with everyday language. Every nation in the world singles out *both* mathematics and language as special cases for education because they know, as developed societies have known for 10,000 years since math began, that language alone does not have sufficient precision and accuracy for the functioning of such a society, and that you need mastery of both language and math for the society to function in a proper and orderly manner.

It is implicit in this global exceptionalizing of both language and mathematics in national education systems, that both are of supreme importance to a well-functioning society. That is is why neither can over-ride the other; they both stand as equals. In particular, when it comes to mathematics, common use of language cannot determine what a mathematical statement means in society.

Of course, not every individual achieves mastery of language and/or mathematics. Individuals with inadequate mastery of one or both have to rely, implicitly if not explicitly, on others to exercise the required mastery. In the case of a financial transaction between an individual consumer and a supplier who seeks to secure customers from all sectors of society, the onus is surely on the supplier to use those two tools well, because it can be expected that a customer may not have the requisite mastery. The NAEP data cited earlier implies that as many as 80% of American customers can be assumed to have inadequate mastery of percentages. Far fewer have a diminished functional mastery of their native language.

One particular feature of this case that I noticed when I read a transcript of the original sales transaction between Plaintiff and Defendant [EXHIBIT A], which became even more stark when I listened to the 7min 53sec verification audio recording (PP000051), is the high degree of care Defendant took to ensure that the Defendant's English language statements—delivered automatically from a carefully crafted script—were correctly and accurately understood by the Plaintiff. I understand that is normal in telephone sales. Yet their formulation of the key mathematical information about the price (also delivered automatically) was not precise (indeed, in my expert view it was incorrect). Yet, mathematics is the very communications tools developed over 10,000 years to achieve precision and accuracy and minimize the likelihood of error.

I have studied that script [EXHIBIT B]. There are 31 prompts to ensure the Customer has fully understood the proposed agreement. A common scripted exchange takes the form

> "Customer, you understand that … Correct?"

In the verification audio recording of the originating transaction involving Plaintiff (PP000051), a transaction of 7min 53sec, that particular "Correct?" prompt is used a total of 5 times (at 3:45, 4:14, 4:36*, 6:25*, 6:40; the two asterisked instances are where the crucial 15% rate-change cap is stated; that part of the transaction was repeated after interjection by the human agent).

More generally, I counted a total of 29 information exchanges where the purpose was to ensure that all the information being gathered by the system was correct (00:37, 00:54, 1:17, 1:36, 1:57, 2:06, 2:52* 3:33?*, 3:45, 4:03, 4:14, 4:29, 4:36, 4:50, 5:01*, 5:15*, 5:27*, 5:38*, 5:51*, 5:57*, 6:20, 6:38, 6:40, 6:51, 7:17, 7:27, 7:35, 7:40, 7:45). The 8 asterisked instances are exchanges involving the human agent, not the automated system; the remaining 21 are all part of the automated script.

As I say, that may be standard procedure, but the implementation of the natural language components of the transaction stands in stark contrast to an incorrect use of society's tried-and-tested, tailor-built tool for handling numerical and financial precision.

BACKGROUND INFORMATION E: The US Education System (and the State of Illinois) establishes standards for mathematics and language education that, in the case of mathematics, put considerable emphasis on educational goals that ensure, *inter alia*, that when people use mathematics, they pay particular attention to precision, including precision with how the mathematics connects to the context in which it is being used. In terms of the black-box metaphor, US mathematics education is tasked to ensure that when students learn math, part of what they learn is that the input and output ports cannot be tampered with and must be used with care.

Discussion Like many nations, the United States sets educational standards for mathematics proficiency, at both a national and state levels. We do so because, as noted already, we recognize the crucial importance of the precision and accuracy that comes from mathematical proficiency. In the US, the most recent standards, the Common Core State Standards (CCSS), were established by an Act of Congress in 2010, following an initiative by a bipartisan group of governors, education experts and philanthropists. [12]

The CCSS lay out yearly benchmarks for the nation's schools from Kindergarten through Grade 12, in mathematics and language arts. The fact that mathematics is the only discipline outside of language to be covered shows the importance America attaches to mathematical proficiency. The nation quite literally depends upon having a mathematically proficient workforce and citizenry, and it knows it.

Currently, 41 states have adopted the CCSS, including Illinois, where the State Board of Education formally adopted them on June 10, 2014, and fully implemented them in the 2014-15 school year. [13] [14]

The CCSS built on and updated a framework for K-12 mathematics education laid out by the Mathematical Sciences Education Board (MSEB) of the National Academy of Sciences in 2001. [15] I was serving on that Board at the time the report was released.

While there is ongoing debate about curricular details that stem from the CCSS, that debate focuses on curricula that have been, and are being, developed to meet the standards. The mathematics education community is, however, in full agreement on what are called the Mathematical Practices, a collection of just eight very basic practices that are essential for students to master in order to be equipped for life and work in the 21st Century. [16] I list them below.

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP4 Model with mathematics.

MP5 Use appropriate tools strategically.

MP6 Attend to precision.

MP7 Look for and make use of structure.

MP8 Look for and express regularity in repeated reasoning.

Note that these are *practices*; not curriculum topics that apply only in the school math classroom. They lay out the basic principles of human reasoning that are important when using mathematics to make decisions and act in today's world.

The Mathematical Practices are viewed as so fundamental, that they are presented right at the top of the first page of the CCSS-Mathematics website. [16]

The Illinois State Board of Education likewise lists the Mathematical Practices in priority position on the Illinois Learning Standards & Instruction website. [13] [14] In doing so, both the nation and the State of Illinois are acknowledging that not only is mastery of mathematics itself (i.e., inside the black box) important, but so too is skill in making practical use of mathematics in various aspects of our lives (i.e., connecting the black box to the world).

Mathematical Practices MP4 and MP6 are of particular relevance to the present case.

MP4 refers to the way mathematics is used in practice out in the world. In terms of the black box metaphor, it's how the black box is hooked to the world. If that step is executed with insufficient care, disaster can result, as in the case of the NASA satellite I described earlier. The black box worked fine on the inside, the error was in the data input, which resulted in a data output that destroyed a $125M spacecraft.

Mathematical Practice MP6 speaks for itself, and its inclusion as one of just 8 practices listed as forming the heart of modern mathematics education indicates the importance the nation and the states that have adopted the standards attach to precision in using mathematics. (Illinois is one of those states. Incidentally, all states provide mathematics education guided by the Mathematical Practices, even if they do not use that terminology; the practices really are, in the 21st Century, as American as apple pie.)

Why is this relevant here? Taken together, Mathematical Practices 4 and 6 imply that the input and output ports of the mathematical black box must be hooked up with care, depending on the individual circumstances. While the Common Core as a whole is a lengthy list of mathematical standards for future citizens to meet, there are just eight the top-level Mathematical Practices, and two of the eight make it clear that using mathematics properly includes taking care to ensure all the numbers, percentages, formulas, equations, and computations handled when using mathematics connect properly and accurately to the world.

BACKGROUND INFORMATION F: Mathematics achieves its precision not by human agreement but by its inner logic.

Discussion A major part of what makes mathematics such a secure foundation for society, especially its power to give all the precision and accuracy you need, whatever the circumstances, is that mathematical truth, uniquely, is not an issue of human opinion or choice or everyday practice.

We do not vote to decide the value of 2 x 3.

We do not change its value when the economy goes up or down.

Nor should we conduct a survey to determine what proportion of the population thinks the answer is (A) less than 6, (B) greater than 6, or (C) equal to 6.

Within the discipline of mathematics itself (i.e., inside the mathematical black box), the method to establish truth was established by the ancient Greeks around 350bce, namely truth is established by rigorous proof from accepted, carefully crafted basic assumptions (“axioms”). Opinion or preference, either individual or social, plays no role.

In this regard, mathematics is different from the natural sciences. In physics, for example, “truth” amounts to “based on the best observational or experimental evidence we have available today.” Fortunately for society, we have today a considerable amount of evidence to support physics, but the natural sciences cannot, and hence never will, provide the certitude of mathematical truth we get within the mathematical black box.

Why is this relevant here? That incredibly powerful, inside-the-black-box certitude is all to naught if the box is not correctly hooked-up to the world. That’s why the enterprise of doing and using mathematics, when it is being used in the world, regards those input and output ports as an integral part of “doing/using mathematics.” Any tool is only as good as the person who uses it. A tool that offers unlimited precision and accuracy in regards to its inner machinery has to be used with appropriate care. Even with the best intentions, mistakes arise when using the input and output ports to get data in and out, as occurred with the NASA satellite. Humans make errors. When they do, society rightly holds responsible the individual(s) who choose to use the tool and then, for whatever reason, misuse it when they do so.

Note that the black box is just a metaphor. It serves to separate (in our minds) the absolute certainly and correctness of mathematics itself, as a body of definitions, facts and procedures, from the activity of people using it in the world.

To be sure, much of the time when someone is engaged in doing or using mathematics, they are working “inside that box” (writing down formulas, solving equations, and the like). And in so doing, they are liable to make mistakes. But that is a mistake in execution of the mathematics, not the mathematics itself. (We say “that person got it wrong;” not “mathematics is wrong.”) The black box picture of using mathematics implicitly assumes all the work done inside the box *can* be done correctly, even when done by a human (as is often the case—though today we do have machines that can do any and all procedural math, not just the arithmetic that electronic calculators do).

The point of the black box metaphor, however, is to separate the part of mathematics that *can* be carried out free of any error, imprecision, or inaccuracy, from the part where that process connects to the surrounding world. That connection part can never be perfectly accurate and error free, since that is not true of the real world the box is being connected to, even when it is being connected to a highly precise real world such as physics. That's why so much emphasis, by governments and education authorities, is put on ensuring all citizens *know how* to make safe, effective, and proper use of a tool so accurate and precise we can conduct complex engineering construction projects in the depths of outer space.

6. CONCLUDING SUMMARY

In Part 1 of this testimony, I expressed my expert opinion that Plaintiff's understanding of the offer made to him by Defendant is correct. I did so as a professional mathematician with demonstrable expertise and considerable experience both in abstract pure mathematics (indeed Logic, the science of rigorous human reasoning) and in the use of mathematics in human transactions, including transactions conducted using everyday language.

I also said that it is my expert opinion that the results of a survey of a population sample cannot decide the matter. If a survey shows that 63% percent of those surveyed believe X, and X is, as a matter of pure fact, wrong, then what the survey shows is that 63% of the population is wrong.

In Part 2, I presented and discussed 6 pieces of background that, I claim, show that my opinion is in accord with, and supported by (A) educational practice, (B) 10,000 years of the historical development of mathematics, (C) US national policy regarding the use of mathematics, (D) the structure of US national education, (E) the standard the US sets for mathematics education, and (F) the very nature of mathematic truth.

BACKGROUND INFORMATION A: The ubiquity of word problems in mathematics education.

BACKGROUND INFORMATION B: The historical development of mathematics.

BACKGROUND INFORMATION C: The US has historically put massive emphasis on the importance of mathematics and its accuracy and significant resources into achieving its benefits in terms of a prosperous economy and our national defense.

BACKGROUND INFORMATION D: In every nation in the world, there are just two school subjects that are obligatory and almost always subject to come national control: mathematics and the nation's language.

BACKGROUND INFORMATION E: The US Education System (and the State of Illinois) establishes standards for mathematics (and language) education that put considerable emphasis on educational goals that ensure that when people use mathematics they pay particular attention to precision, including precision with how the mathematics connects to the context in which it is being used.

BACKGROUND INFORMATION F: Mathematics achieves its precision not by human agreement but by its inner logic.

I maintain that, when taken together, these six pieces of background information about mathematics and its use in a modern society like the US show (in particular) two things:

i. People in the society are entitled to assume that, when a mathematical formula is evoked in a natural language transaction, particularly in a context that makes it clear that considerable care has gone into the formulation of the delivery, then (at least) equal care has gone into the formulation of the term that specifies the formula; there should be no possibility that a sufficiently numerate individual is misled. (The input port of the black box.)
ii. An argument that a population survey can render acceptable (in, say, a financial transaction), a mathematical interpretation of a natural language expression that mathematically proficient individuals declare to be otherwise, cannot be sustained.

7. CITATIONS

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8. DECLARATION & SIGNATURE

My opinions expressed in this Expert Testimony are made on the basis of my long career as a professional mathematician. All statements I present as facts are to the best of my knowledge true. I declare under penalty of perjury that the foregoing is true and correct.

Keith Devlin

February 12, 2022