Unraveling Peers and Peer Effects: Comments on Goldsmith-Pinkham and Imbens' "Social Networks and the Identification of Peer Effects"

Matthew O. Jackson *

Draft: March 2013.

Abstract

Keywords:

1 Introduction

Understanding peer effects is of first order importance in a range of settings including education, labor markets, crime, voting, and consumer behavior. However, although casual introspection and some field experiments (e.g., Duflo and Saez (2003); Centola (2010)) suggest that the influence of friends and acquaintances on behavior can be substantial, there are formidable challenges in establishing peer effects using (increasingly available) purely observational data that couple behaviors with social relationships.

The challenges in establishing that peer effects are truly present include:

- Identification: a model of peer effects must be specified in a manner such that the channels through which peers influence one's behavior can be identified as well as distinguished from other sources of influence (**?**Bramoullé et al., 2009; **?**).

- Endogenous networks and homophily: linked individuals are likely to be similar not only in terms of observed characteristics but also in terms of unobserved characteristics that could influence behavior. By failing to account for similarities in (unobserved) characteristics, similar behaviors might be mistakenly be attributed to peer influence when they simply result due to similar characteristics (**?**Jackson, 2008). This is complicated by the richness of the set of possible characteristics that could matter, which not only involve innate or exogenous ones that might influence preferences, but also correlated ones that include exposure to common stimuli (**?**).

*Department of Economics, Stanford University, Stanford, California 94305-6072 USA, and external faculty member at the Santa Fe Institute, and a member of CIFAR. Email: jacksonm@stanford.edu.

- Computation: the possible set of networks of peer relationships is generally exponential in the size of the population and so having a model that allows for endogenous peer relationships can face significant computational challenges (Chandrasekhar and Jackson, 2012).

- Measurement error: relationships can be difficult to observe and quantify. This can reduce the power of a test, especially with self-reported relationships that are easily unobserved (**?**). In particular, this applies to data with caps on the numbers of friends that can be reported as in the Ad Health.

- Misspecification: specifying the appropriate set of peers, which can be time and context dependent, is difficult, as is correctly specifying the ways in which they influence each other, including possible heterogeneity among a given individual's friends.

The difficulties of convincingly demonstrating peer effects are complicated by the fact that the biases introduced by the various issues mentioned above can push in different directions. For example, not accounting for the potential similarity of friends along unobserved dimensions could lead one to over-estimate peer effects, while some misspecifications could lead to one to under-estimate peer effects. Thus, without some confidence that we have adequately addressed the long list of issues above, we cannot be confident in estimated peer effects. Does this mean that we should simply give up? Of course not, given that understanding these effects are essential to designing effective policies in so many applications, in both the developed and developing world.

In an ambitious and important paper, Goldsmith-Pinkham and Imbens (2013) wrestle with each of these issues. This leads to a somewhat sprawling analysis, but the sprawl is inevitable given that these challenges are intertwined. Goldsmith-Pinkham and Imbens take a useful tack by grounding their analysis in the context of a specific setting: peer effects on grade point averages in U.S. high schools using the Ad Health data set. It is not their intention that this be taken seriously as an empirical analysis, as they trim down the data to make for a very clear methodological discussion. Thus, the empirics are illustrative rather than informative. But the grounding of the problem is quite useful as addressing the many intertwined issues in estimating peer effects is much easier to discuss in a concrete context than in the abstract.

Goldsmith-Pinkham and Imbens' main contribution is careful attention to the endogeneity of the network, and pointing out some consequences of failure to appropriately account for endogeneity. They also develop implications of the failure to account for an endogenous network and methods of testing for whether omitted endogenous network effects could be biasing peer influence measurements.

The concern that the network could be endogenous and strongly correlated with individual characteristics is well-founded. Fitting a network formation model with race and other characteristics in preferences in the Ad Health data shows that individuals act as if they have a significant and strong preference to interact with others who have similar characteristics (Currarini et al., 2009, 2010). This together with the fact that characteristics correlate with GPA means that not accounting for potential similarities in unobserved characteristics among peers could substantially bias the estimation of peer effects.

To tackle this issue, Goldsmith-Pinkham and Imbens model friendship formation in a manner that explicitly allows for similarity along an unobserved dimension. The important insight that they make clear from the network formation model is the following. If two individuals are friends, and hence peers, but differ in their observed characteristics, then it is more likely that the pair are similar on the unobserved characteristics. That is, given that friendships form based largely on similarities, if the observables are not similar then it must be (or is likely to be) that the unobserved characteristics are similar. If those unobserved characteristics relate to behavior, then that means that the pair's behaviors should be more similar than one would estimate simply based on their observed characteristics. Thus, the errors in predicted behaviors based on observables should be stronger when friends' observed characteristics diverge. This also means that if one does not correctly model friendships and account for potentially unobserved characteristics then one could inflate the peer effects.

Let us examine the details to be clear. In their model an individual i's GPA, Y_i, is a function of the student's observed characteristics, X_i, the average peer GPA, $\overline{Y}_{(i)}$, and the average peer characteristics, $\overline{X}_{(i)}$, and the unobserved characteristic, ξ_i:

$$Y_i = \beta_0 + \beta_X X_i + \beta_{\overline{Y}} \overline{Y}_{(i)} + \beta_{\overline{X}} \overline{X}_{(i)} + \beta_\xi \xi_i + \eta_i. \tag{1}$$

The utility of i for the relationship with j is proportional to the difference in their observed and unobserved characteristics, $|X_i - X_j|$ and $|\xi_i - \xi_j|$, as well as whether they had a relationship and/or a friend in common in the previous period, D_{0ij} and $F_{0,ij}$:

$$U_i(j) = \alpha_0 + \alpha_X |X_i - X_j| + \alpha_\xi |\xi_i - \xi_j| + \alpha_d D_{0ij} + \alpha_f F_{0,ij} + \epsilon_{ij}.$$

The probability of i and j being linked is then given by:

$$\Pr[link_{ij}] = \left(\frac{\exp(U_i(j))}{1 + \exp(U_i(j))} \right) \left(\frac{\exp(U_j(i))}{1 + \exp(U_j(i))} \right).$$

This specification of 'Goldsmith-Pinkham and Imbens network formation model circumvents the central computational issues since an individual's preference for a relationship with another person does not depend on any of the rest of the network structure except whether they had a friend in common in an earlier period. This means that the formation of the current network can be estimated link-by-link and instead of facing an exponential number of calculations instead it is a quadratic one in the number of nodes. While this independence of links is violated in many settings (including Ad Health), it allows Goldsmith-Pinkham and Imbens to concentrate on other issues for their methodological illustrations. Nonetheless, this means that this model of network formation is most likely (substantially) misspecified and so the endogeneity effects are likely to be biased and any inference has to be interpreted accordingly. Even with this computational simplification, likelihood calculations are still demanding because of the unobserved characteristics that have to be inferred for each student,

ξ_i (so an n dimensional vector, reintroducing the exponential calculations), and Goldsmith-Pinkham and Imbens use MCMC sampling methods coupled with Bayesian estimation.

This specification makes it simple to see how failing to account for the endogenous network would affect a peer effects analysis. If α_ξ is negative, then utilities for a relationship are higher if the unobserved characteristics ξ_i and ξ_j are similar. If one then estimates the relationship (1) without accounting for the unobserved characteristics (so setting $\beta_\xi = 0$), when i and j have similar ξ_i and ξ_j they are not only likely to be peers, but they are then also likely to have similar η's since those will absorb the missing ξ's. Indeed, as Goldsmith-Pinkham and Imbens find, the residuals from estimations of (1), when not including the unobserved characteristics, correlate with whether i and j are linked. From their estimations it also appears that the missing characteristics play a substantial and significant role in network formation. Very interestingly, however, despite this large role of unobserved similarities in link formation, Goldsmith-Pinkham and Imbens also find that accounting for these characteristics does not substantially impact the estimation of peer effects.

The significant effect of unobserved similarity on link formation but the absence of any real impact the estimation of peer effects is surprising given that, for instance, race, wealth and gender play significant roles in friendship formation in the Ad Health data and also correlates with GPA. Thus, not properly accounting for such characteristics should presumably impact the peer influence estimates substantially and significantly. It is also surprising given that Goldsmith-Pinkham and Imbens found that the residuals in the peer equation estimation correlate significantly with friendship. One possible explanation for this pattern of findings lies the functional specification of the unobserved characteristics and how they might influence both link formation and behavior. Here, the specification of the unobserved variable ξ very stripped down and just a binary variable. This could be enough to capture whether linked individuals are "similar" or "different" along unobserved characteristics, and hence enough to detect whether unobserved characteristics matter in link formation. However, it could be that the manner in which unobserved characteristics matter in peer effects is much more complicated and not at all captured by such a formulation.

In particular, it could be that the sorts of missing characteristics that matter are things like study habits, whether or not students are involved in extracurricular activities, their socio-economic status, and so forth. Two people are more likely to be friends if they are "similar" rather than "different" on these characteristics, so the estimation picks up whether two students are similar or different on these dimensions. Thus, the specification can detect whether similarity on unobserved characteristics matters significantly in forming a peer relationship. However, that does not encode the information about how many hours the students spend studying, or whether their parents attended college, and so forth. These are the sorts of missing characteristics that are likely to matter in GPA. Thus, the takeaway from this paper by Goldsmith-Pinkham and Imbens should be the overarching methodological points, but not the particular specifications nor the particular findings when these are applied to the data.

There are several other interesting questions that Goldsmith-Pinkham and Imbens also explore regarding peer interactions: Do former friends matter? Do friends of friends matter? Does it matter whether both friends name each other or just one names the other? The answers to these questions paint an interesting picture. They find that the current GPAs of former friends matter almost as much as those of current friends. Friends of friends who are not current friends also provide a significant peer effect. Yet they find little difference between the impact of friendships as a function of whether friendship nominations are reciprocated or not.

It seems that there is a common explanation to all of these observations: unobserved friendships and thus measurement error. Past friendships could still be friendships, but ones that are simply not named on the second survey. Given that the Ad Health data have students typically only having five to ten friends, many peers go unnamed. Similarly, it could be that pairs of individuals who are at distance two in the measured network are actually friends, but the link is not mistakenly unobserved. Indeed, as estimated by Jackson et al. (2012) in a different setting, pairs of individuals who have friends in common are roughly five times more likely to be friends than not. In fact, they estimate that small amounts of measurement error could account for many of the pairs who have friends in common but appear not to be friends. Finally, failures in reciprocation can be due mistaken observations in the data. Indeed, as found by Banerjee et al. (2012) reciprocation rates can be low even when individuals are asked to name relatives; and so many relationships are simply not named by the subjects in surveys. Thus, it is clear that there are contexts with high rates of failure of reciprocation purely from measurement error and that many relationships go unreported in survey data. In summary, the mis-measuring of current friendships makes it appear as if past friendships that are no longer friendships matter, or having friends in common matter, when it could be that neither of these are really causal, but instead just correlating with measurement error in observing current friendships.

It is important to emphasize one thing in closing. There is a temptation to write down simple econometric models in the abstract that a researcher could take off-the-shelf and adapt to a particular setting. What should be taken away from the analysis here, and developed further, is an understanding of how to address issues of specification, endogeneity, omitted variables, measurement errors, etc., but *not* the particular functional forms for doing this. There are many channels for peer effects even restricting ourselves to an education setting: students study together, they may exert pressures on each other not to study, they might engage in illicit activities together, they may serve as role models for each other, they may feel pressures to conform, they may be searching for an identity and sense of belonging, they may experience parental influences, they may experience complementarities in their payoffs from the educational decisions, they may be exposed to stimuli that they react to communally, and so forth. Clearly this long and still incomplete list of potential peer effects are unlikely to all be captured in a simple, e.g. linear, manner based on some measurement of a grade or other outcome. While this requires adding complexity to the modeling, being very explicit about the specific type of effect that one is seeking to identify is much more likely to yield a useful model. This is illustrated by Banerjee et al. (2012) who dissect peer effects in the context of word-of-mouth spread of a new product (microfinance in rural Indian villages). By explicitly modeling information passing in the network and fitting that model to data, they find that what appears to be a strongly positive peer/endorsement effect becomes insignificant (and even slightly negative) once one models information passing. An interpretation of their findings is that a given individual is more likely to participate when more friends do, is not due to some complementarity, peer influence, or endorsement effect, but rather due to the fact that having more friends take up the product simply makes one more likely to be aware of the product. While the particulars of the finding are relative to the specifics of the setting, the broader message is that more explicit modeling of the details of peer interaction can lead to very different conclusions from what one might infer based on standard linear-in-means style regressions. More generally, such an approach will require models that account for the wide multitude of different forms of learning, peer influences, and peer interactions.

These challenges become even more substantial when building models of peer effects that also incorporate network formation, as there can be complex feedbacks between behavior and friendship choices. While strategic network formation models have been studied in some detail in the theory literature (e.g., (Jackson and Wolinsky, 1996; Bala and Goyal, 2000; Jackson, 2008)), models designed for empirical implementation of strategic network formation are still in their infancy (Christakis et al., 2010; Mele, 2011; Chandrasekhar and Jackson, 2012; Leung, 2013). The very important lesson that we should take from the analysis of Goldsmith-Pinkham and Imbens is that accounting for the endogeneity of relationships in analyses of peer effects is feasible, and can provide substantial new insights, for example into unobserved characteristics that might correlate both with behavior and friendship formation. The specifications that are needed to properly model both network formation and peer effects require careful additional analysis in context, and provide us with a rich agenda going forward.

References

Bala, V. and S. Goyal (2000): "A noncooperative model of network formation," *Econometrica*, 68, 1181–1229.

Banerjee, A., A. Chandrasekhar, E. Duflo, and M. Jackson (2012): "Diffusion of Microfinance," *NBER Working Paper 17743 http://www.stanford.edu/ jacksonm/diffusionofmf.pdf.*

Bramoullé, Y., H. Djebbari, and B. Fortin (2009): "Identification of peer effects through social networks," *Journal of Econometrics*, 150, 41–55.

Centola, D. (2010): "The Spread of Behavior in an Online Social Network Experiment," *Science*, 329: 5996, 1194–1197, DOI: 10.1126/science.1185231.

Chandrasekhar, A. and M. Jackson (2012): "Tractable and Consistent Random Graph Models," *SSRN Working Paper: http://ssrn.com/abstract=2150428*.

Christakis, N., J. Fowler, G. Imbens, and K. Kalyanaraman (2010): "An Empirical Model for Strategic Network Formation," *NBER Working Paper*.

Currarini, S., M. Jackson, and P. Pin (2009): "An economic model of friendship: Homophily, minorities, and segregation," *Econometrica*, 77, 1003–1045.

——— (2010): "Identifying the roles of race-based choice and chance in high school friendship network formation," *Proceedings of the National Academy of Sciences*, 107, 4857–4861.

Duflo, E. and E. Saez (2003): "The Role of Information and Social Interactions in Retirement Plan Decisions: Evidence from a Randomized Experiment," *Quarterly Journal of Economics*, August, 815–842.

Goldsmith-Pinkham, P. and G. Imbens (2013): "Social Networks and the Identification of Peer Effects," *Journal of Business and Economic Statistics*.

Jackson, M. (2008): *Social and economic networks*, Princeton: Princeton University Press.

Jackson, M., T. Barraquer, and X. Tan (2012): "Social Capital and Social Quilts: Network Patterns of Favor Exchange," *American Economic Review*, 102, 1857–1897.

Jackson, M. and A. Wolinsky (1996): "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory*, 71, 44–74.

Leung, M. (2013): "Two-Step Estimation of Network-Formation Models with Incomplete Information," *working paper*.

Mele, A. (2011): "A Structural Model of Segregation in Social Networks," *working paper*.