



Statistical Theory and Modelling for Turbulent flow

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1 Preface

This book evolved out of lecture notes for a course taught in the Mechanical Engineering department at Stanford University. The students were at M.S. and Ph.D. level. The course served as an introduction to turbulence and to turbulence modeling. Its scope was single point statistical theory, phenomenology, and Reynolds averaged closure. In preparing the present book the purview was extended to include two-point, homogeneous turbulence theory. This has been done to provide sufficient breadth for a complete introductory course on turbulence.

Further topics in modeling also have been added to the scope of the original notes; these include both practical aspects, and more advanced mathematical analyses of models. The advanced material was placed into a separate chapter so that it can be circumvented if desired. Similarly, two-point, homogeneous turbulence theory is contained in part III and could be avoided in an M.S. level engineering course, for instance.

No attempt has been made at an encyclopedic survey of turbulence closure models. The particular models discussed are those that today seem to have proved effective in computational fluid dynamics applications. Certainly, there are others that could be cited, and many more in the making. By reviewing the motives and methods of those selected, we hope to have laid a groundwork for the reader to understand these others. A number of examples of Reynolds averaged computation are included.

It is inevitable in a book of the present nature that authors will put their own slant on the contents. The large number of papers on closure schemes and their applications demands that we exercise judgement. To boil them down to a text requires that boundaries on the scope be set and adhered to. Our ambition has been to expound the subject, not to survey the literature. Many researchers will be disappointed that their work has not been included. We hope they will understand our desire to make the subject accessible to students, and to make it attractive to new researchers.

An attempt has been made to allow a lecturer to use this book as a guideline, while putting his or her personal slant on the material. While single point modeling is decidedly the main theme, it occupies less than half of the pages. Considerable scope exists to choose where emphasis is placed.

1.1 Motivation

It is unquestionably the case that closure models for turbulence transport are finding an increasing number of applications, in increasingly complex flows. Computerised fluid dynamical analysis is becoming an integral part of the design process in a growing number of industries: increasing computer speeds are fueling that growth. For instance, computer analysis has reduced the development costs in the aerospace industry by decreasing the number of wind tunnel tests needed in the conceptual and design phases.

As the utility of turbulence models for computational fluid dynamics (CFD)

has increased, more sophisticated models have been needed to simulate the range of phenomena that arise. Increasingly complex closure schemes raise a need for computationalists to understand the origins of the models. Their mathematical properties and predictive accuracy must be assessed to determine whether a particular model is suited to computing given flow phenomena. Experimenters are being called on increasingly to provide data for testing turbulence models and CFD codes. A text that provides a solid background for those working in the field seems timely.

The problems that arise in turbulence closure modeling are as fundamental as those in any area of fluid dynamics. A grounding is needed in physical concepts and mathematical techniques. A student, first confronted with the literature on turbulence modeling, is bound to be baffled by equations seemingly pulled from thin air; to wonder whether constants are derived from principles, or obtained from data; to question what is fundamental and what is peculiar to a given model. We learned this subject by ferreting around the literature, pondering just such questions. Some of that experience motivated this book.

1.2 Epitome

The prerequisite for this text is a basic knowledge of fluid mechanics, including viscous flow. The book is divided into three major parts.

Part I provides background on turbulence phenomenology, Reynolds averaged equations and mathematical methods. The focus is on material pertinent to single point, statistical analysis, but a chapter on eddy structures is also included.

Part II is on turbulence modeling. It starts with the basics of engineering closure modeling, then proceeds to increasingly advanced topics. The scope ranges from integrated equations to second moment transport. The nature of this subject is such that even the most advanced topics are not rarefied; they should pique the interest of the applied mathematician, but should also make the R & D engineer ponder the potential impact of this material on her or his work.

Part III introduces Fourier spectral representations for homogeneous turbulence theory. It covers energy transfer in spectral space and the formalities of the energy cascade. Finally rapid distortion theory is described in the last section. Part III is intended to round out the scope of a basic turbulence course. It does not address the intricacies of two-point closure, or include advanced topics.

A first course on turbulence for engineering students might cover part I, excluding the section on tensor representations, most of part II, excluding chapter 8, and a brief mention of selected material from part III. A first course for more mathematical students might place greater emphasis on the latter part of chapter 2 in part I, cover a limited portion of part II — emphasizing chapter 7 and some of chapter 8 — and include most of part III. Advanced material is intended for prospective researchers.

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