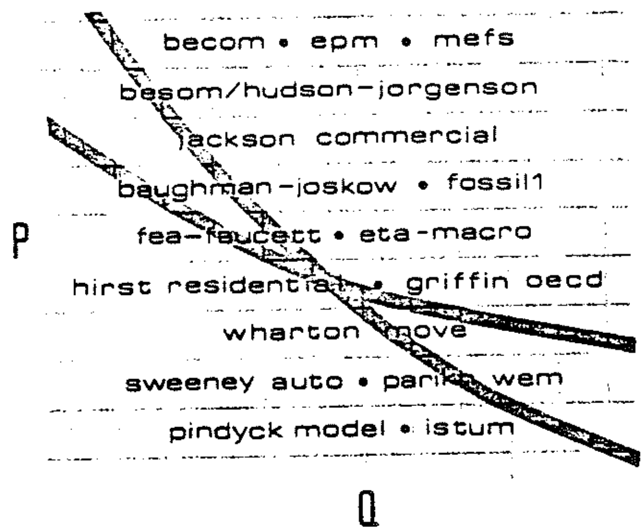
AGGREGATE ELASTICITY OF ENERGY DEMAND



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EXECUTIVE SUMMARY

The response of energy demand to changes in energy prices is central to the evaluation of energy policies. To study this response, analysts and modelers have developed many sophisticated models of energy demand. These models, differing in their structure and degree of detail, do not lend themselves to easy comparisons of the relationship between demand and prices.

Simplified to a single number, the response of demand to changes in price can be described as an elasticity. The aggregate price elasticity of energy demand is equal to the percent reduction in energy demand produced by a 1 percent increase in energy price, with all else held constant. This definition of elasticity is presented graphically in Figure 1. By convention, aggregate elasticities are positive whenever price increases lead to a decrease in demand.

Straightforward in concept, the aggregate elasticity of demand for energy is elusive in practice. However, there is an appeal to this simple single parameter as an indicator of important underlying relationships. There is little doubt that analysts will continue to use this single elasticity to describe aggregate changes in future energy demand. If it is to be used correctly, there must be an improvement in its definition and measurement. The present report works towards this end by summarizing results from the Energy Modeling Forum (EMF) comparison of energy demand models.

The goal of this study is the description of the aggregate price elasticity of demand implicit in energy demand models. An EMF working group conducted experiments with 16 detailed models of the energy sector. The group developed consistent estimates of the 15-, 25-, and 35-year energy demand elasticities implicit in each of the models. The comparison of results is descriptive; there was no attempt to produce a single best estimate of the demand elasticity.

The aggregate elasticity or the set of fuel-specific elasticities is critically important for many analyses. The higher the elasticity of demand for energy, the smaller will be the impact on GNP (gross national product) of a given reduction in the quantity of energy available, and the smaller will be the impact of a given increase in the cost of imported energy. The higher the elasticity, the lower will be the forecasted future consumption of energy in high price situations, the less urgent will be the perceived need for energy supply technologies, and the lower will be projected world oil prices.

Since energy includes a number of heterogeneous commodities, each with a separate price, these prices and quantities must be aggregated in order to calculate a single elasticity. The choice of aggregation rules can influence the resultant estimations. The group examined several alternative indexes for the aggregation: Paasche, Laspeyres, Tornquist, and Btu-weighted. Similarly, the elasticity can vary by a factor of nearly two over different stages from production to consumption. The group compared two standardized points of measurement:

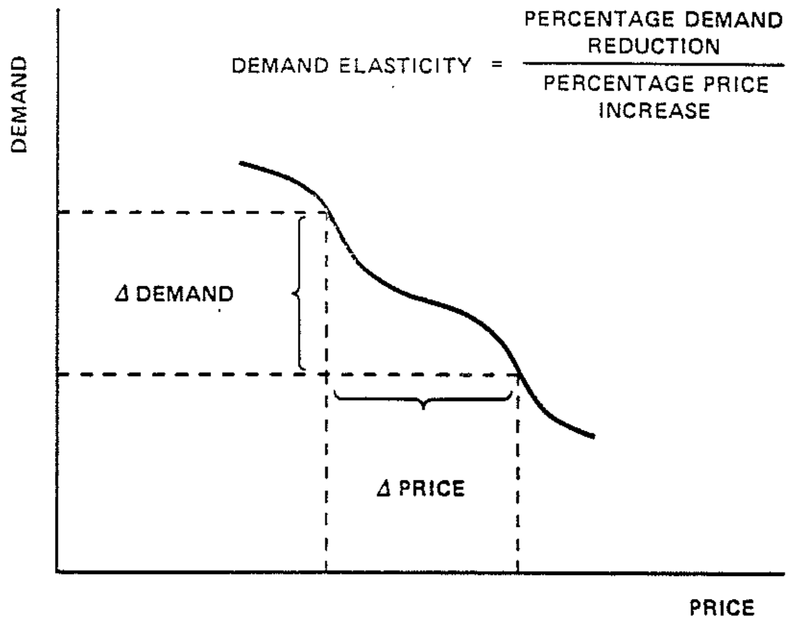

Figure 1 Elasticity Definition

- primary energy, measured directly before refining, electricity generation, and synthetic fuels conversion losses; and
- secondary energy, measured directly after conversion and refining losses.

Primary energy prices are generally lower and quantities are higher than for the corresponding secondary values; consequently, the primary elasticities will be generally lower than the secondary elasticities.

Total demand for energy can adjust in many ways. The detailed energy demand models focus on different system elements, possibly fixing some demands while calculating others. Some of the models include restrictions which reduce the ability of the economy to adjust to higher prices or which reduce the measured elasticity. Additionally, disaggregated elasticities may be estimated statistically or by other approaches, i.e., engineering or judgmental.

The working group addressed the above and other issues to conclude:

- The more comprehensive models, covering all energy-using sectors, incorporating the full range of potential flexibility, and directly utilizing historical data to statistically estimate parameters, were generally characterized by the highest implicit aggregate secondary demand elasticities. Long-run aggregate secondary demand elasticity estimates for these five models ranged between 0.3 and 0.7.
- Other models produced lower estimates either because they incorporated lower subjectively determined component elasticities or because of the limited scope of energy-use substitutions addressed in the models. Four out of the five comprehensive models in this category produced elasticity estimates in the range from 0.1 to 0.2; a fifth estimate was 0.6.

- As shown in Table 1, demand elasticities vary significantly across sectors, among models, and among assumed policy regimes. Automobile efficiency standards, for example, may lower the elasticity for gasoline by a factor of 2 to 3.

- Aggregate elasticity estimates were not sensitive to the choice of Paasche, Laspeyres, or Tornquist indexes but differed when the Btu-weighted index was chosen. Because the Btu-weighted index is theoretically less attractive, the Paasche index is used in this summary.

- The aggregate demand changes projected with many models were sensitive to the specific composition of price changes. Since an aggregate analysis may be inadequate or misleading for these models, a method was developed to illustrate the implications of different compositions of price changes.

- The study did not examine the adjustment dynamics inherent in the models. However, energy demand adjustments occur slowly since demand is linked to the stock of energy-using equipment. Analysis of the post-1973 experience indicated that the conservation actually experienced to date could be consistent with any of the estimated long-run primary energy elasticities implicit in the models.

- There is a range of uncertainty associated with any demand elasticity estimate; the actual elasticity could be greater than or less than any of the point estimates presented here. The range of uncertainty is not the same as the range of elasticity estimates. Several sources of uncertainty exist in each model: measurement error in the data,

Table 1

25-YEAR SECONDARY DEMAND ELASTICITY ESTIMATES

Sectors	Parameter Estimation Methodology	
	Statistical	Other Approaches (Engineering or Judgmental)
All Sectors	0.3-0.7	0.1-0.6
Residential	0.5-1.0	0.4
Residential/Commercial	0.5-0.8	0.5
Commercial	0.5	0.3-0.4
Commercial/Industrial	0.3-0.7	0.1
Industrial	0.2-0.5	0.2-0.7
All Transportation	0.2-0.5	0.4
Automobile Gasoline		
with Efficiency Standards	0.1-0.2	--
without Efficiency Standards	0.1-0.5	--

parameter estimation uncertainty, and model specification errors. However, limitations in the current state of the art either preclude calculation of explicit uncertainty measures or make calculation extremely costly.

- Measurement of prices and quantities at the secondary energy level was more useful for computing dependable aggregate elasticities than measurement at the primary level. The conditions necessary to insure the theoretical consistency of aggregate economic indexes are more nearly satisfied at the secondary level. Additionally, most demand models are structured in terms of delivered energy, which can be adjusted easily to the secondary energy level; more arbitrary procedures were required to adjust data to the primary energy level.

- The elasticities differ due in part to differences in modeling approaches, techniques, or assumptions. Detailed models often represent some components of energy demand as independent of price, thereby biasing downward the calculated aggregate elasticity. Engineering process models may exclude unrecognized technological options and may underestimate the substitution flexibility of the economy.

Based upon its findings, the working group recommends the following:

- The Energy Information Administration (EIA) of the U.S. Department of Energy, working in close coordination with modelers, should develop consistent accounting conventions and standardized data for demand analysis. The working group spent a great deal of time on standardizing the data without fully succeeding.

- Modelers should improve their practice in publishing assumptions, error statistics, robustness tests, validity tests, descriptive information, and historical data supporting their models. Agencies funding model development should insist on and support this effort.

- Modelers should make aggregate elasticities, whether explicit or implicit, a standard component of the documentation of demand models. The EIA should publish a set of definitions and computational procedures for calculating these elasticities and the associated adjustment time lags.

- Modelers should develop and then consistently utilize techniques for describing the uncertainties in their models. This will require basic research to develop the methodology. Funding organizations and modelers, realizing that analysis of uncertainties is costly but important, should budget sufficient funds for this activity.

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ACKNOWLEDGMENTS

The fourth Energy Modeling Forum study, "Aggregate Elasticity of Energy Demand," was initiated as an experiment at the request of the Energy Modeling Forum Senior Advisory Panel at its 1977 annual meeting. The objective was to obtain improved information about aggregate elasticities of substitution between energy and other inputs in the economy. During the first EMF study, "Energy and the Economy," this parameter was identified as being fundamental in estimating the relationship between reduced availability of energy and economic growth. A study design was developed and modelers were asked to exercise their energy demand models along the study specifications.

During the course of the study, it became apparent that issues of interpretation were fundamentally important and that the initial plan to conduct the study as an experiment without any working group meetings would not be feasible. Therefore, two meetings were held, the first at the John F. Kennedy School of Government at Harvard University and the second at the University of Chicago, hosted respectively by William Hogan and Thomas Long. Working group members participating in the study and these meetings are listed on page xvii. Most of the issues dealt with in this report were debated during those meetings. Most subsequent work was conducted by members of the working group and the Energy Modeling Forum staff. This work is reflected in both the summary report (Volume 1) and the complete report (Volume 2).

Many people made important contributions throughout the study. John Weyant served as study coordinator and in this regard played major roles in drafting the study design and the summary report, held primary responsibility for the writing of several chapters, and supervised the EMF staff effort. Tom Wilson held initial responsibility for developing the software that made the model comparison possible and also was responsible for some of the chapter writing. The final computer graphics work was conducted by Steve Duvall.

An extensive amount of work was conducted by the various researchers whose models were exercised in the study. Individuals responsible for running models included Martin Baughman, Phillip Budzik, Ronald Cooper, Richard Goettle, James Griffin, Eric Hirst, Jerry Jackson, Dilip Kamat, Damian Kulash, Colin Loxley, Alan Manne, Shirish Mulherkar, Roger Naill, Shailendra Parikh, Robert Pindyck, and Geoffrey Ward.

Extensive work also was conducted by the various chapter authors, listed in the table of contents.

Besides working group members, Senior Advisory Panel members contributed significantly through their initial identification of the study topic, their assistance in identifying working group members, and through their reviews of the report. We would like to particularly thank Harvey Brooks, Floyd Culler, Herman Dieckamp, Joseph Fisher, Henry Linden, Chauncey Starr, and Robert Wycoff for their comments on the final report as it went through the review process.

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Primary financial support for this study was provided by the Electric Power Research Institute. Several participating institutions assisted further by underwriting the cost of running the models under the various scenarios and/or by underwriting transportation costs of members to the meetings. These include the Brookhaven National Laboratory, Congressional Budget Office, Energy and Environmental Analysis, Inc., Lawrence Livermore Laboratory, Massachusetts Institute of Technology, Oak Ridge National Laboratory, Stanford University, U.S. Department of Energy, University of Chicago, University of Houston, and University of Texas.

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SENIOR ADVISORY PANEL

The Energy Modeling Forum seeks to improve the usefulness of energy models by conducting comparative tests of models in the study of key energy issues. The success of the Forum depends upon the selection of important study topics, the broad involvement of policymakers, and the persistent attention to the goal of improved communication. The EMF is assisted in these matters by a Senior Advisory Panel that recommends topics for investigations, critiques the studies, guides the operations of the project, and helps communicate the results to the energy policymaking community. The role of the Panel is strictly advisory. The Panel is not responsible for the results of the individual EMF working group studies.

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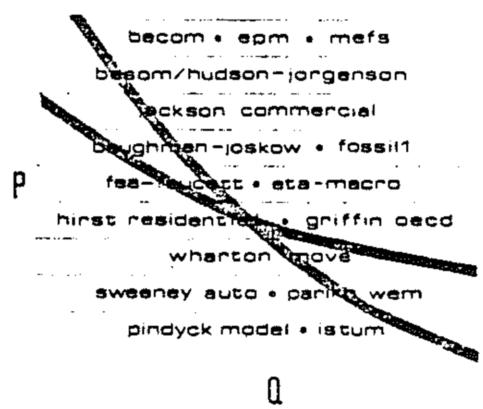
Chapter 1

SUMMARY REPORT*



*This chapter formerly was EMF Report 4, Volume 1.

Chapter 1

SUMMARY REPORT

INTRODUCTION

EMF 1 identified the importance of the aggregate elasticity of energy demand:

The first EMF study, "Energy and the Economy," produced estimates of the aggregate elasticity of substitution for primary energy implicit in six models of energy and the economy.[1] In that study, the working group identified the importance of the aggregate elasticity and called for an examination of more detailed demand models. During its review of the study, the EMF Senior Advisory Panel cited the importance of a careful investigation of energy demand models to clarify estimates of the aggregate elasticity. The present study is a response to those suggestions.

the percent decrease in demand produced by a 1 percent increase in price.

The aggregate price elasticity of energy demand is the percent reduction in energy demand produced by a 1 percent increase in energy price, with all else held constant. By convention, aggregate elasticities are positive whenever price increases lead to decreases in demand.[2]

Aggregate demand elasticities were estimated for 16 energy models but no single best estimate was identified.

The working group conducted experiments with 16 detailed models of the energy sector. Table 1-1 provides a complete listing of the models. The uncertainty in the current state of the art is emphasized. This group made no attempt to produce a single best estimate of the demand elasticity; at a minimum, the study provides simple summary statistics for comparing energy demand models. In addition, the results suggest the limitations of an aggregate model of energy demand for some applications and, therefore, reinforce the need for more detailed models of energy use.

Table 1-1

MODELS USED IN THE AGGREGATE ELASTICITY OF
ENERGY DEMAND STUDY

<u>Energy-Economy Models</u>

- Brookhaven Energy System Optimization Model/Hudson-Jorgenson (BESOM/H-J), Brookhaven National Laboratory and Dale Jorgenson Associates
- Energy Technology Assessment-MACRO (ETA-MACRO), Alan Manne, Stanford University
- Parikh Welfare Equilibrium Model (Parikh WEM), Shailendra Parikh, Stanford University

<u>Energy System Models</u>

- Baughman-Joskow (Baughman-Joskow), Martin Baughman and Paul Joskow, University of Texas
- Energy Policy Model (EPM), Lawrence Livermore Laboratory
- FOSSIL1 (FOSSIL1), Dartmouth System Dynamics Group, Dartmouth College
- Griffin Organization for Economic Cooperation and Development (Griffin OECD), James Griffin, University of Houston
- Mid-Range Energy Forecasting System (MEFS), U.S. Department of Energy
- Pindyck International Study (Pindyck), Robert Pindyck, Massachusetts Institute of Technology

<u>Sectoral Models</u>

- Buildings Energy Conservation Optimization Model (BECOM), Brookhaven National Laboratory
- Federal Energy Administration-Faucett (FEA-Faucett), Carmen Difiglio and Damian Kulash, Federal Energy Administration
- Industrial Sector Technology Use Model (ISTUM), Energy and Environmental Analysis, Inc.
- Jackson Commercial (Jackson Commercial), Jerry Jackson, Oak Ridge National Laboratory
- The ORNL Residential Energy-Use Model (Hirst Residential), Eric Hirst and Janet Carney, Oak Ridge National Laboratory
- Sweeney Automobile Model (Sweeney Auto), James Sweeney, Stanford University
- Wharton Motor Vehicle Model (Wharton MOVE), Wharton Econometric Forecasting Associates

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The remainder of this report

- establishes the significance of elasticities in energy policy and planning,
- identifies some key issues to be addressed in computing aggregate elasticities,
- gives an overview of the models employed in the study,
- describes the experimental design for estimating the aggregate elasticity implicit in each model,
- reports the study results, and
- makes recommendations.

SIGNIFICANCE OF ELASTICITIES

Demand elasticity estimates are crucial to energy policy analyses.

The aggregate elasticity of energy demand or the set of fuel-specific elasticities is critically important for many analyses, such as forecasts of future energy consumption, evaluations of the appropriate timing of energy technology development, assessments of energy tax policies, and predictions of OPEC (Organization of Petroleum Exporting Countries) pricing strategies. The higher the price elasticity of demand for energy, the greater will be the impact on GNP of a given tax on energy, the smaller will be the impact of a given reduction in the quantity of energy available, and the smaller will be the impact of a given increase in the cost of imported energy.[3] The higher the assumed elasticity, the lower will be the forecasted future consumption of energy in high price situations, the less urgent will be the perceived need for new energy supply technologies, and the lower will be projected world oil prices.

Aggregate elasticities are more easily understood and communicated than are detailed elasticities.

Detailed elasticities by fuel and sector are important for many of these same analyses, and a full-scale analysis of a particular policy may depend upon the use of detailed models. But detailed model differences and their implications may be difficult to perceive and communicate. Aggregate elasticity calculations provide simple summary parameters that are easily understood and used. And since many energy prices tend to increase together, aggregate elasticities give a rough estimate of the magnitude of the energy consumption changes resulting from pervasive changes in the energy situation, such as those caused by increases in world oil prices. These rough estimates may be adequate for many purposes.

The projected ratio of energy use to GNP depends upon the aggregate demand elasticity.

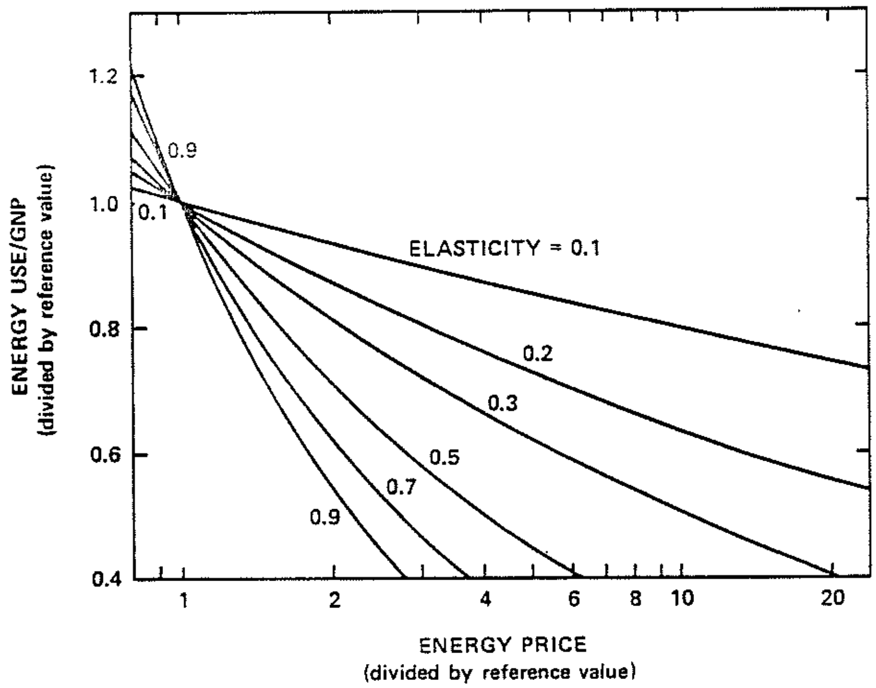
One simple use of aggregate elasticity estimates is the calculation of the extent to which energy consumption can be reduced in response to increasing energy prices or costs. The consumption reduction depends upon the reduction in aggregate economic activity and the ability of actors in the economy to substitute other inputs for energy. However, the ratio of energy use to GNP depends primarily upon the ability of the economy to substitute capital and labor inputs for energy, as measured by the aggregate elasticity of demand.

The higher the elasticity, the smaller the price increase required to reduce the energy/GNP ratio a given amount.

Figure 1-1 illustrates the ratio of future energy use to GNP as a function of energy price for demand elasticities ranging from 0.1 to 0.9. The coordinates are standardized at unity by dividing by Reference values in order to facilitate examination of fractional changes in the ratio. If the long-run elasticity of demand for energy is as low as 0.1, a 10-fold increase in energy prices is required to ultimately reduce the energy/GNP ratio by 20 percent. However, if the elasticity is as high as 0.9, the same 20 percent reduction in the energy/GNP ratio can be achieved by a 28 percent increase in energy price. Thus, forecasts of future energy consumption and analyses of the efficacy of conservation programs in reducing energy demand depend upon the aggregate price elasticity.

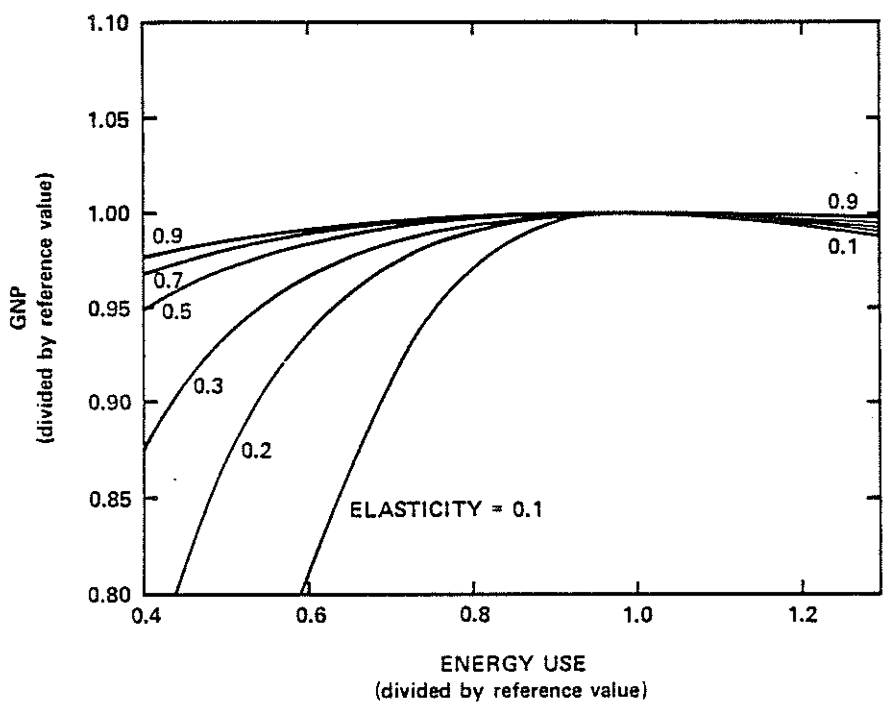
The aggregate elasticity is important in estimating the economic costs of reductions in energy use.

Results from analysis with a simple aggregate model of energy-economy interactions demonstrate the importance of the elasticity of energy demand or the closely related elasticity of substitution between aggregate energy and other inputs, i.e., capital and labor.[4,5] Figure 1-2 illustrates, for various values of the aggregate elasticity, the relationship between the GNP and energy use in this model.[6] The results demonstrate the importance of the elasticity of energy demand in determining the magnitude of the macroeconomic impacts of reductions in energy use. If the



Note: Energy use and price measured at secondary level.

Figure 1-1 Relationship between Energy Price and the Energy/GNP Ratio as a Function of Aggregate Elasticity



Note: Energy use and price measured at secondary level.

Figure 1-2 Relationship between Energy Use and GNP as a Function of Aggregate Elasticity for the Excise Tax Case long-run demand elasticity were 0.2, a 50 percent reduction in secondary energy inputs, motivated by measures which increase consumers' energy prices but not resource costs, such as a Btu (British thermal unit) tax, would ultimately lead to a 13 percent reduction in GNP each year. If the elasticity were 0.7, this same 50 percent reduction in secondary energy inputs would lead to only a 3 percent reduction in GNP.

The larger the elasticity, the smaller the long-run economic impacts of reduced energy availability.

These results have important implications for both energy policy and the validity of energy sector models. For example, if the demand elasticity were low, say 0.1 to 0.2, changes in energy availability could have major long-run economic effects. With such large feedbacks, one may question the logical consistency of models which take the GNP as given (partial equilibrium models). On the other hand, if the demand elasticity were high, say 0.6 to 0.7, changes in energy availability could be substantially decoupled from long-run economic growth, lending credibility to the partial equilibrium approach for analyses of conservation programs or tax measures.

ISSUES IN COMPUTING AGGREGATE ELASTICITIES

Many methodological problems were addressed.

The design and implementation of the study presented a number of methodological and definitional problems. The following issues were found to be particularly significant:

- distinctions between aggregate and single-fuel elasticities,
- aggregation of heterogeneous fuels,
- choice of index,
- composition of price change,
- standardization of aggregate economic activity,
- selection of measurement point,
- examination of dynamics, and
- characterization of uncertainty.

Aggregate Elasticities Versus Elasticities for Single Fuels

Energy price increases motivate interfuel and factor substitutions.

In response to an increase in the price of a single fuel, the demand for that fuel decreases as firms and individuals substitute other nonenergy products (factor substitution) and other energy forms (interfuel substitution) in place of the fuel whose price has increased. The interfuel substitution leads not only to decreases in the demand for the more costly fuel but also to increases in the demands for competing fuels. As a result, aggregate energy demand is reduced by less than the demand for a single fuel.

The elasticity of demand for specific fuels normally exceeds the aggregate demand elasticity.

The percentage decrease in the demand for a single fuel as its price increases by 1 percent, holding other prices constant, is its "own elasticity" of demand. The "cross elasticity" of demand is defined as the percentage increase in one fuel's demand when the price of a competing fuel is increased. The aggregate demand elasticity for energy is defined as the percentage reduction in aggregate demand in response to a 1 percent increase in aggregate price. For individual fuels, the aggregate elasticity of demand is generally smaller than the own elasticity of demand.

These points can be illustrated by a simple numerical example. Assume two types of energy with demands represented by E1 and E2. Their prices, P1 and P2, are related by the following linear inverse demand functions:

$$E1 = 1.2 - P1 + 0.8\ P2, \text{ and}$$

$$E2 = 1.2 - 0.8\ P1 - P2 \quad .$$

Total energy demand (ET), the sum of E1 and E2, is

$$ET = 2.4 - 0.2\ P1 - 0.2\ P2 \quad .$$

Suppose that P1 = P2 = 1, then E1 = E2 = 1 and ET = 2.

Failure to distinguish between the two elasticity concepts can lead to misapplication.

In the above example, a 10 percent increase in P1, from 1 to 1.1, reduces the demand for energy type 1 by 0.1 units or 10 percent: the fuel-specific own elasticity of demand is 1.0. However, the demand for energy type 2 is simultaneously increased by 8 percent, a cross elasticity of 0.8, leading to a net decrease in total energy demand of 0.2 units or 1 percent. Since the average price of energy increases by 5 percent, the aggregate demand elasticity equals 0.2.[7] Similarly, a 10 percent increase in both prices implies a 10 percent increase in average price and a decrease in aggregate demand by 0.04 units or 2 percent. The aggregate demand elasticity is again calculated to be 0.2. This aggregate elasticity of 0.2 can be compared with the fuel-specific own elasticity of 1.0. Failure to distinguish clearly between the two concepts can easily lead to misapplication of estimated demand elasticities.

The Aggregation Problem

Prices and quantities of individual fuels must be aggregated.

Aggregate elasticity estimations require alternative projections of aggregate energy and its price. However, aggregate energy is neither marketed in the real world nor represented directly in the detailed energy demand models. Consequently, any scheme for measuring the aggregate price elasticities implicit in the models depends upon a rule for aggregating prices and quantities of individual energy products.

The choice of a particular aggregation rule can affect the results.

In the above example, aggregation consisted of adding total energy use and averaging prices. This natural scheme is one of many possible aggregation rules. Furthermore, adding quantities and averaging prices leaves open the question of what units to use: physical weight, economic value, Btus. If economic value is the unit to be used in aggregating, one must choose the prices to use in calculating economic value. If prices are changing, the choice is not obvious. If Btus are used, one must specify how to measure Btus: heat value under perfect combustion, heat value under combustion with normal utilizing equipment, or heat value of the raw materials used to produce the final energy commodity.[8]

The aggregation problem can be illustrated by an example.

The importance of the aggregation problem is illustrated by further use of the example. Assume energy type 1 is electricity, type 2 is natural gas, and the units used above are measured in terms of delivered Btus, output or delivered energy. Alternatively, one may aggregate by the Btus required to generate the final product (primary energy). To aggregate in terms of primary energy, all coefficients in the first equation must be multiplied by three.[9]

The new equations are

$$E1' = 3.6 - 3.0\ (3P) + 2.4\ P2$$
$$E2 = 1.2 + 0.8\ (3P) - P2\ ,$$

where P, the price of electricity inputs, equals 1/3 P1 and

$$ET' = 4.8 - 2.2\ (3P) + 1.4\ P2\ .$$

Now, if P1 = P2 = 1, then E1' = 3, E2 = 1, and ET' = 4.

Two methods of aggregation were considered.

For this example, a 10 percent increase in electricity price increases the average price of output energy by 5 percent and reduces total demand for primary energy by 5.5 percent, yielding an aggregate elasticity equal to 1.1. A 10 percent increase in the price of natural gas increases average delivered price by 5 percent and increases total energy input demand by 3.5 percent, yielding an aggregate demand elasticity of -0.7. Hence, a seemingly simple change of the units used in the aggregation can change the magnitude and even the sign of the implicit aggregate demand elasticity. Two approaches to the aggregation problem were considered: (1) estimating a production or demand function for aggregate energy and (2) using price and quantity indexes for energy.

Estimating a production function for aggregate energy required too much data.

One way to approach the aggregation problem is to postulate the existence of a production function for energy with individual fuels being the inputs or, equivalently, to postulate a unit-cost function with the price of aggregate energy determined by component fuel prices. The production function formulation of the aggregation problem has a strong theoretical foundation, for example, the description in Diewert.[10] The equivalent unit-cost function approach is prevalent in energy-economy models, for example, in the work of Hudson and Jorgenson.[11] However, the production/unit-cost function approach to the problem of computing the price and quantity of aggregate energy requires far more information than needed to determine the price elasticity for aggregate energy. A more economical and direct approach was needed.

Therefore, index numbers were used.

The use of index numbers provides a short cut for determining the aggregates directly from their components, without having to statistically estimate the full aggregation rules. Index numbers provide an approximation to the production function and in some special cases can be exact. The index number approach is developed more fully in Chapters 2 and 4.

The Choice of Index

Most indexes, except the Btu-weighted, are consistent with optimizing behavior.

Many different index formulas can be used to aggregate the data provided by the models, and different aggregators can result in different aggregate elasticities from the same data, as shown above. Several of the most commonly used indexes, Paasche, Laspeyres, Ideal, Tornquist, and Btu-weighted, were considered in this study. Except for the Btu-weighted, these indexes are consistent with the assumption of optimizing behavior on the part of producers and consumers.

The Laspeyres, Paasche, and Ideal indexes use prices and quantities as weights for aggregation.

The familiar Laspeyres and Paasche indexes correspond precisely to the examples above if the units correspond to economic value. Both quantity indexes are constructed by summing economic values (price times quantity) of energy consumed; both price indexes are constructed by averaging prices of energy consumed, using quantities as weights. The Laspeyres quantity index uses price weights corresponding to the Reference case or base year; the Paasche quantity index uses price weights corresponding to the specific scenario or the current year. In a similar manner, the Laspeyres price index uses Reference case or base year quantities for averaging, while the Paasche price index uses specific scenario or current year quantities. The "Ideal" index is the geometric mean of the Laspeyres and Paasche indexes.

These indexes produce approximations to the aggregate production function.

Each of the three indexes, Laspeyres, Paasche, and Ideal, provides an approximation to the quantity and average prices obtainable from an aggregate production function or cost function. The approximation from the Reference case levels is very close for infinitesimal changes in price and quantity. For larger changes, however, the approximation becomes less exact.

And for small price changes, four indexes yield identical results.

The more complex Tornquist index has been developed to provide an exact calculation for large price and quantity changes. The Tornquist index is exact for all possible prices under the special assumption that the unit-cost function is quadratic in logarithms of energy input prices (a "transcendental logarithmic function").[12] For small price and quantity changes, all four indexes are identical to one another. Thus, the aggregate elasticities calculated with the four indexes will be similar.

Although used pervasively, the Btu-weighted index has serious theoretical shortcomings.

The last index considered, Btu-weighted, is not based on the assumption of optimizing behavior by economic agents but, rather, on an implicit assumption that all energy forms are perfect substitutes on a Btu basis. Despite its theoretical shortcomings, this index was included here because of its pervasive use in aggregate energy accounting. This quantity index is simply the quantity sum over the various fuels, with units corresponding to the heat content (Btu) of the fuels. The price index is the average of the prices, weighted by Reference case quantities.

The Composition of Price Change

The aggregate elasticity can depend on the composition of price change.

Whenever heterogeneous commodities are aggregated, calculated aggregate demand elasticities may depend upon specific price changes. The elasticity can be positive or negative depending upon the mix of price changes: in the earlier example, aggregate price increases led to aggregate quantity decreases or increases, depending upon the price change composition. The example can be expanded to examine a 10 percent increase in the price of commodity

1 coupled with a 10 percent decrease in the price of commodity 2. In this case, the average price remains unchanged while the aggregate demand for primary energy is reduced by 9 percent. Since aggregate demand changes while aggregate price remains constant, even the concept of a demand function is questioned for aggregated commodities.

Average aggregate elasticity estimates may be sample-dependent.

When several different price changes are considered, an average elasticity can be calculated. However, this does not avoid the problem. If elasticities depend upon the composition of price changes, the average elasticity will depend upon the sample of price changes selected for the calculation.

The extent to which the price change composition affects the aggregate elasticity is important.

For some cases, the choice of an appropriate index will eliminate the problem. In general, however, there is no assurance that any index will allow the measured elasticity to be independent of the precise configuration of price changes. The relevant issue is not whether the price change composition matters but how significantly the composition problem influences measured results. Equivalently, the issue may be posed as one of determining the range of price-change directions over which a particular aggregate elasticity estimate approximately describes the aggregate behavior of the energy demand model.

The composition of price changes greatly affects the elasticity calculated for some models but not for others.

Sweeney, in Chapter 9, has developed a decomposition approach in the context of the present study. The decomposition method is based upon a simple but important observation: when an aggregated class of commodities is considered, a single-valued demand function for the aggregate may not even exist. The approach can be used to estimate an average elasticity for each model and index while preserving the notion that the calculated elasticity will, in general, depend upon the precise price change composition, as illustrated above. In such a situation, it is important to determine how nearly the relationship between quantity and price aggregates can be approximated by a single-valued demand function. This, then, provides the motivation for the decomposition method discussed more fully in Chapter 9. Because the decomposition technique is still experimental, it does not form the basis of the summary tables or graphs in this report. Rather, a conventional averaging of elasticities has been utilized except where indicated.

Standardizing Aggregate Economic Activity

Increasing energy prices implies reductions in aggregate economic activity.

In response to increasing energy prices, one can expect pervasive changes in economic activity. Interfuel and factor substitutions imply shifts in the composition of economic output. Since energy is a factor of production in the economy, a reduction in energy use without corresponding increases in the availability of other productive factors implies a net reduction in the aggregate real value of goods and services produced in the economy. In fact, increases in the costs of importing or producing energy reduces the GNP even if the aggregate value of goods and services produced by the rest of the economy remains unchanged; as energy costs increase, more claims must be made against the output of the rest of the economy in order to import or produce a given quantity of energy. On the other hand, energy conservation strategies, such as energy-use taxes, may stimulate changes which lessen the economic impact of higher energy prices.

Higher energy prices affect energy demand both directly and indirectly through reductions in economic output.

For analytical purposes, it is useful to separate these various energy demand impacts into two components: (1) a demand response to higher price with aggregate economic activity held constant plus (2) a demand response to changed income or aggregate economic activity stemming from energy price or cost increases. The first component corresponds to the aggregate price elasticity of demand addressed in this study. The second component depends upon the income or economic activity elasticity of demand and the influence of energy system changes on aggregate economic activity. This last issue was the topic of the first EMF study and is not addressed further here.[1]

The separation of price effects into two components typically varies by sector.

The precise manner of separating price effects into the two components typically varies among sectors being represented. For industrial sectors, the price elasticity is usually defined by holding constant the aggregate output from the particular sector. For the residential sector, disposable personal income is normally held constant, while for the transportation sector either GNP or disposable income may be standardized.

The price elasticity of demand is defined holding constant either GNP or aggregate output from the energy-consuming sectors.

In order to define an aggregate price elasticity of demand, one must specify the measure of aggregate economic activity to be standardized. Two choices are natural: GNP or aggregate output of the energy-consuming sectors of the economy, referred to as the rest of the economy.[13] This choice will influence the measured price elasticity of demand for energy. If GNP is held constant in defining the price elasticity, energy demand can be modeled as depending on GNP and a pure price effect. If aggregate output of the rest of the economy is held constant, energy is modeled as depending upon aggregate output and a pure price effect.

The calculated elasticity depends on this choice.

These two pure price effects generally differ in magnitude from one another. When demand reductions are motivated by price changes alone, e.g., taxes not coupled to cost changes, the price elasticity is generally larger if calculated by standardizing GNP than if calculated by standardizing the aggregate output of the rest of the economy. This occurs primarily because GNP declines less than aggregate output in response to energy price changes. Since that portion of the demand reduction modeled as working through economic activity is greater under the latter definition, the calculated pure price effect must be smaller if the sum of the two effects is to remain the same. In contrast, when the demand reduction is motivated by cost increases, the converse is true. In response to cost increases, GNP declines more than aggregate output. Therefore, the measured price elasticity based upon standardizing GNP is smaller than that based upon standardizing aggregate output.

This study standardized GNP.

This study examined responses to changes in the cost of producing or importing energy. The elasticities using both definitions were calculated, but the definition based upon standardizing GNP was adopted.

The Point of Measurement

Inconsistencies in the point of measurement are pervasive in the use of elasticities.

Since energy prices and quantities can be measured at many points in the energy system, inconsistencies in the point of measurement are pervasive in comparisons of the models. It is desirable for many purposes to measure aggregate energy demand or aggregate energy demand elasticities as close to the point of consumption as possible.[14] This suggests measurement of elasticities at the retail level. However, it was felt that a standardized comparison at the retail level would be impractical because there are many different retail products sold to many consumers in many localities at many different prices.

In this study, energy is measured at the primary level and at the secondary level.

Lack of appropriately disaggregated data at the retail level has also prompted many of the modelers to look elsewhere in the system for a point of measurement where the aggregation assumptions can be satisfied. The U.S. Bureau of Mines has historically referred to energy measured directly before electricity generation and other conversion losses as gross energy inputs and energy measured directly after these losses as net energy inputs. In this study, these quantities are referred to as primary and secondary energy, respectively.

For a single fuel, the relationship between elasticities measured at different points is straightforward.

In general, it is difficult to separate problems of aggregation from problems associated with point of measurement. However, for models which consider only one fuel, the importance of the point of measurement problem can be easily demonstrated. For example, the Sweeney Auto model considers only automobile gasoline use and, therefore, problems of aggregation do not appear in presenting the results of the model. Sweeney, of course, used data aggregated from individual gasoline sales. Calculation of retail price elasticity and secondary price elasticity are straightforward; calculation of primary elasticity requires a modeling of cost and quantity changes associated with refinery operations.

In this case two types of markups are common.

For the simple case in which there are price markups and energy conversion losses from point to point in the energy production/consumption chain, differences in elasticities are easy to calculate. Two types of markups between levels are common. A proportional markup multiplies the lower level price by some constant greater than unity, while an additive markup adds some fixed cost to the price. The proportional markup guarantees that the same percentage changes in prices and quantities occur at each level in the system and, therefore, elasticities are the same at each point of measurement. For additive markups, an increase in price at one level produces a different percentage change in price at another level. This results in different elasticities at different points of measurement.

With additive markups, retail elasticities exceed secondary elasticities, which exceed primary elasticities.

Suppose prices at the retail, secondary, and primary levels are denoted by PR, PS, and PP, respectively, and that there are additive markups, TR and TS, between the secondary and retail levels and between the primary and secondary levels, respectively. The secondary elasticity will be smaller than the retail elasticity by a factor of (PR - TR)/PR, while primary elasticity will be smaller than secondary elasticity by a factor of (PS - TS)/PS. The elasticity at a point in the supply chain, e.g., primary energy, will be smaller than the elasticity at a point farther downstream, e.g., retail energy, whenever there are additive markups between levels.

As an example, elasticities are calculated at three levels for the Sweeney Auto model. At the retail level, the elasticity is 0.76, while at the secondary and primary levels the corresponding elasticities are 0.46 and 0.40, respectively.

Similar calculations have been performed for the aggregate of all energy in the economy, under an assumption that all price increases between the secondary and retail levels can be modeled as additive markups. Figure 1-3 illustrates the relationship between the secondary and retail elasticities, assuming additive markups and normalizing the elasticity at the retail level to unity.[15] Under these assumptions, the ratio between the two elasticities increases sharply as energy price increases.

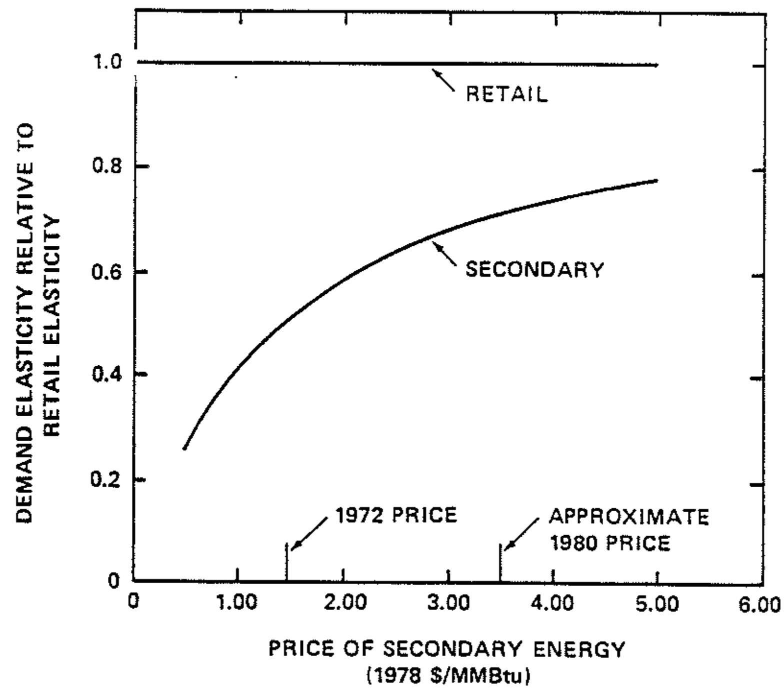


Figure 1-3 Ratio of Retail and Secondary Elasticities as a Function of Secondary Energy Price

A general method for translating elasticities from point to point is proposed.

Nissen, in Chapter 5, has proposed a general methodology for translating elasticities from point to point in the presence of interfuel competition. This method promises to provide a comprehensive approach for estimating aggregate functional approximations of detailed models.

Aggregate primary and secondary elasticities are computed for each model.

A majority of researchers report total energy before conversion losses, and this point of measurement is employed in many discussions of aggregate energy demand. Therefore, the study was designed to produce estimates of primary elasticities implicit in the models. However, since enough data were available to also estimate secondary energy price elasticities implicit in the models, the working group calculated and reported elasticities at both points of measurement.

Dynamics

Dynamic responses of the models to price increases are not examined.

In general, since the capital stock of energy-using equipment adjusts only slowly, short-run elasticities (one to five years) may be expected to be considerably smaller than long-run elasticities. Furthermore, calculated elasticities may depend upon the timing of price changes. The working group calculated 15-, 25-, and 35-year aggregate elasticities in response to price changes initiated in 1975. Thus, while the estimates are not precisely long-run equilibrium elasticities, the 25- and 35-year estimates closely approximate long-run elasticities. The experiment does not examine the dynamics of the models, although several models explicitly account for delays in consumer responses to changes in energy prices.

Long-run elasticities greatly exceed short-run elasticities.

Slow adjustment creates another empirical problem. When data used for econometric testing are derived from a short time period, only short-run elasticities can be easily observed. Therefore, elasticities estimated using only a short time series of data, for example, only post-1973 data, are heavily dependent upon assumptions about the rate of adjustment of the capital stock. The timing of consumer responses and the analysis of post-1973 data are discussed at length in Chapters 12 and 13.

Characterizing Uncertainty

It is desirable to examine the uncertainty associated with elasticity estimates;

Assumptions about price responsiveness are central to conclusions obtained in energy policy studies. Consequently, it is desirable to characterize the uncertainty associated with aggregate elasticity estimates. This is especially true since error costs may be asymmetric and highly nonlinear. The costs of taking actions based upon overestimates of energy demand elasticities may be very different from those based upon underestimates.

however, this goal could not be achieved.

Unfortunately, the theory and data are not available to develop a satisfactory description of the uncertainty associated with the aggregate elasticity estimates. The lack of a formal description of the processes by which the energy system structure can change may be the most important source of error in elasticity estimates. The working group could not meet the data and computational requirements for characterizing uncertainty even for the conceptually simple problem of measuring the propagation of parameter error in the models. This troublesome problem, which is discussed in Chapter 11, remains as a caution to model users and a challenge to model developers.

OVERVIEW OF THE MODELS

Sixteen models were analyzed.

Sixteen models were included in this study, as shown in Table 1-1. The models are described in detail in Chapter 3. While the models emphasize different aspects of energy demand, they overlap in their estimation of the relationships between energy demands and energy prices. These models are representative of the spectrum of available analysis tools and share many important features.

The models vary in scope.

The models vary in scope, including sectoral models, energy system models, and energy-economy models.[16] The energy system models represent the entire energy system, while the sectoral models focus on specific sectors of the energy system, thus resembling components of the energy system models. The energy-economy models explicitly represent the influence of changes in the energy system upon the rest of the economy, while the energy system models do not.

The energy-economy models include energy-economy interactions.

The energy-economy models include consideration of energy demand within a general equilibrium framework representing the economy. These models typically use an economic growth model to integrate (1) an interindustry input-output description of the economy's production sector, (2) a detailed process description of the energy sector, and (3) a consumer demand module. However, the models differ substantially in level of aggregation, representation of substitution possibilities, and representation of energy supply and demand dynamics. The energy-economy models included in the study are (1) BESOM/H-J, a simplified version of the Brookhaven system/Hudson-Jorgenson, (2) ETA-MACRO, and (3) Parikh WEM. All of these models are nationally aggregated.

Energy system models represent all energy demands.

The energy system models represent all energy demands but not interactions between the energy sector and the rest of the economy. These models also differ substantially in level of aggregation, substitution possibilities, and representations of energy supply and demand dynamics. The energy system models included in the study are (1) Baughman-Joskow, (2) EPM, (3) FOSSIL1, (4) Griffin OECD, (5) MEFS, and (6) Pindyck.[17,18]

Sectoral models include detailed representations of specific energy-consuming sectors.

The sectoral models address energy use in specific demand sectors. Typically, these models have little detail on energy supply, and they do not consider energy conversion processes or energy-economy feedbacks. On the other hand, concentration on one demand sector allows a more detailed modeling of various energy uses. The study included seven sectoral models: (1) BECOM, (2) FEA-Faucett, (3) Hirst Residential, (4) ISTUM, (5) Jackson Commercial, (6) Sweeney Auto, and (7) Wharton MOVE.

The models differ systematically in data and accounting systems.

The models systematically differ in the data and accounting systems used. The energy-economy models typically utilize U.S. Bureau of Economic Analysis (BEA) industrial classifications. This allows commercial energy demands and transportation demands, except gasoline for private automobiles, to be treated in an interindustry input-output framework. In the energy system models, demands are typically disaggregated by U.S. Bureau of Mines (BOM) consuming sectors: residential, transportation, commercial, and industrial.

The models focus on different system elements contributing to energy demand flexibility.

Each model includes some mechanism for projecting future energy demands, and each examines some aspects of the role of prices in determining demands. Total demand for energy can adjust in many ways, and the models focus on different system elements that contribute to the flexibility in energy demand, fixing some quantities while determining others. At one extreme, the BECOM and ISTUM models hold constant all demands for energy services, solving for the specific equipment and fuels required to provide those energy services at the lowest cost. At the other extreme, the Griffin OECD and Pindyck models implicitly allow all system adjustments to be captured in their parameter estimation.

Some models explicitly include substitution processes; others leave these processes implicit.

Some models, such as BECOM, BESOM/H-J, or Hirst Residential, explicitly identify the precise channels through which the substitution occurs, using a detailed process representation or an aggregate production function representation of labor, capital, energy, and materials substitution. Other models, such as Baughman-Joskow, Griffin OECD, and Pindyck, leave substitution processes implicit.

The responsiveness of demand includes interfuel and factor substitution.

The responsiveness of demand to price changes can be thought of as being composed of two components: interfuel substitution and factor substitution. Inclusion of interfuel substitution permits examination of those interactions where the choice of one form of energy versus another is important. Inclusion of factor substitution permits representation of switches from energy to other factors of production. The degree of interfuel and factor substitutions represented varies significantly among the models employed in the study.

The degree of product and geographic aggregation varies significantly among the models.

Also varying among the models is the degree of product and spatial aggregation. Geographic aggregation is important because many economic decisions can vary between regions, and the ability to transport energy between regions may be limited in some cases. The level of product aggregation can be important to the extent that alternative energy products are not perfect substitutes. The significance of both issues is that the aggregation level implicitly defines some of the substitution possibilities included in the models. Specifically, commodities within a particular product or regional aggregate are implicitly considered to be perfect substitutes or perfect complements, i.e., used only in fixed proportions, for each other.

Representation of energy conversion activities influences a model's aggregate primary elasticity.

Because the study required specification of exogenous price trajectories for primary energy fuels, the resource supply dynamics built into the models are irrelevant for comparison of demand behavior. However, representations of energy conversion activities are important for determining aggregate elasticity of demand for primary energy. Three energy conversion sectors are typically included in the models: (1) an electric utility sector,

(2) an oil refining sector, and (3) a coal synthetics conversion sector. These are represented quite differently among the various models.

Some models represent the dynamics of the demand response; others are static.

Slow adjustment of capital stock, location patterns, and behavioral choices typify the energy demand response to changes in economic conditions. Energy models can be either static or dynamic. The static models, such as BESOM/H-J and Pindyck, do not explicitly incorporate either foresight or slow adjustment processes. In contrast, dynamic models like ETA-MACRO incorporate temporal considerations within a multiperiod framework.

Some dynamic models assume perfect foresight; others assume myopic behavior.

Two attributes may be used to describe the differences across dynamic models: treatment of expectations and speed of adjustment. Expectations may be myopic, with current decisions determined entirely by current and past conditions, or clairvoyant, with current decisions based upon complete knowledge of all past, present, and future prices. The statistical models implicitly embody myopic expectations, while intertemporal optimization models, such as BECOM, assume clairvoyant behavior. A model might permit instantaneous adjustment in variables resulting from exogenous shocks or the adjustment may be gradual.

In all the dynamic models, a key element is the treatment of adjustments in the stock of capital goods. Some models, such as FEA-Faucett, Hirst Residential, and Wharton MOVE, treat these capital adjustment processes explicitly, while others, such as Baughman-Joskow or Griffin OECD, leave these processes implicit in the model equations and parameters.

Statistical, engineering, and judgmental parameter estimation approaches are employed.

Besides differing in sectoral coverage, treatment of the relationship between the energy sector and the rest of the economy, and in several important structural characteristics, the models differ fundamentally in approaches to parameter measurement. The model developers employed one or more of three basic approaches: (1) statistical estimation of fuel and/or sector price response, (2) detailed engineering specifications of alternative energy-using technologies, and (3) judgmental estimation of fuel and/or sector price response. Table 1-2 indicates the principal methods used for parameter estimation. Some models include a combination of approaches to parameter estimation. Of these, BESOM/H-J and ETA-MACRO are classified according to the parameter estimation methods used to develop the demand representation of the model. However, such a classification was not possible for the Hirst Residential and Jackson Commercial models since the statistical and engineering approaches were intertwined in these models. The comparison of estimation methods can be found in Chapter 3.

THE EXPERIMENTAL DESIGN

Primary energy fuel prices were varied; other assumptions were standardized across scenarios and models.

Except for primary energy fuel prices, which were varied during the experiment, all energy sector assumptions were those used by the Modeling Resource Group (MRG) of the Committee on Nuclear and Alternative Energy Systems (CONAES) study.[19] The constant dollar price levels of oil and gas, coal, and nuclear fuel for

Table 1-2

PARAMETER ESTIMATION APPROACHES

	Statistical	Engineering	Judgmental
Energy-Economy Models	BESOM/H-J		ETA-MACRO Parikh WEM
Energy System Models	Baughman-Joskow[a] Griffin OECD MEFS Pindyck		EPM FOSSIL1
Sectoral Models	FEA-Faucett Sweeney Auto Wharton MOVE[b]	BECOM ISTUM	
	Hirst Residential[c] Jackson Commercial[c]	Hirst Residential[c] Jackson Commercial[c]	

[a]Excludes the transportation sector and industrial feedstocks.

[b]Only results from the automobile gasoline demand component were reported to the EMF.

[c]Combines both the statistical and engineering approach.

the Reference price case were chosen to increase gradually over time. Oil and gas were assumed to cost $2.00 per million Btu in 1975 and to increase 2 percent per year in real terms through 2010;[20] coal and nuclear fuel were assumed to cost $0.75 and $0.40 per million Btu in 1975, respectively, and to increase 1 percent per year through 2010. The reference price levels are not especially important: the elasticities are calculated based on the changes in prices from those levels and the resulting changes in quantities.

Changes in aggregate price level and individual fuel prices were considered.

The alternative cases explored changes in the composition of the aggregate price around the reference price levels as well as changes in the aggregate price level (Chapter 2). The primary energy input prices were increased or decreased 25 percent for all years, yielding three price levels for each primary energy input. The price levels were varied from their reference levels individually and collectively, defining the nine primary energy price cases shown in Table 1-3.

Each price case was run for each model. The resulting demand projections by fuel and sector were reported to the working group by all modeling teams except for Wharton Econometric Forecasting Associates, whose representative reported results only from their automobile gasoline demand model (MOVE).[21]

A constant elasticity demand function was estimated for each model for several years.

The EMF staff used the reported data to calculate price and quantity indexes for each model in each price case for 1990, 2000, and 2010. The Reference case prices and demands for the particular year of interest were used as weights in the indexes. A simple constant elasticity aggregate energy demand function was statistically fit to the aggregate data for all cases available from a given model in a particular year.[22] Thus, a single best aggregate elasticity estimate was obtained for each model and each year.

Table 1-3

PRIMARY ENERGY PRICE CASES

Case		Price Level		
		Oil & Gas	Coal	Other
1.	Reference	Reference	Reference	Reference
2.		High	Reference	Reference
3.		Reference	High	Reference
4.		Reference	Reference	High
5.		High	High	High
6.		Low	Reference	Reference
7.		Reference	Low	Reference
8.		Reference	Reference	Low
9.		Low	Low	Low

CONCLUSIONS

The magnitude of the demand elasticity depends on a number of factors.

Contrary to popular usage, the energy demand elasticity cannot even be defined consistently without explicit specification of several factors. The point of measurement, method of aggregation, price change composition, time frame, taxes, and regulations assumed can significantly affect the calculated value of the aggregate elasticity. Even if these factors are standardized, differences in parameter estimation approaches and structural characteristics lead to a range of elasticity estimates.

The Point of Measurement

Results showed no consistent relationship between secondary and primary elasticity estimates.

The results in Table 1-4 show no consistent relationship between primary and secondary aggregate elasticity estimates generated within the study. The difficulty seems to reside chiefly with primary elasticity estimates. In calculating primary elasticities for sectoral models, a somewhat arbitrary exogenous mix of fuels for electricity generation was assumed. In addition, aggregation problems seemed to be more severe when primary energy consumption estimates rather than secondary estimates were utilized.

Table 1-4

COMPARISON OF 25-YEAR PRIMARY AND SECONDARY ELASTICITY ESTIMATES
(Paasche Index)

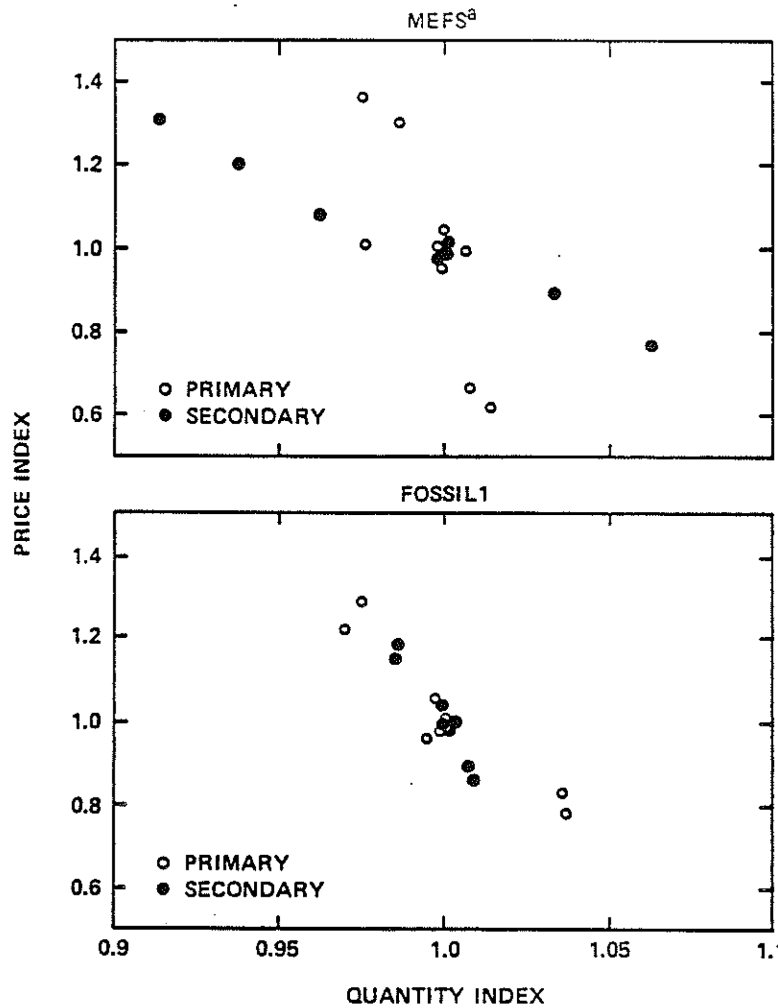
Model	Demand Sector	Elasticity of Demand	
		Primary	Secondary
Baughman-Joskow	Total	0.49	0.61
BECOM[a]	Residential/ Commercial	--	0.51
BESOM/H-J	Total	0.48	0.42
EPM	Total	0.79	0.57
ETA-MACRO	Total	0.30	0.18
FEA-Faucett[a,b]	Auto	0.13	0.15
FOSSIL1	Total	0.11	0.08
FOSSIL1 Conservation	Total	0.17	0.16
Griffin OECD	Total	0.45	0.55
Hirst Residential[b]	Residential	0.24	0.44
ISTUM	Industrial	--	0.24
Jackson Commercial[b]	Commercial	0.17	0.37
MEFS[a,b]	Total	0.04	0.31
Parikh WEM	Total	0.11	0.10
Pindyck[b]	Total	0.44	0.70
Sweeney Auto[a,b]	Auto	0.40	0.46
Wharton MOVE[a]	Auto	0.19	0.21

[a]15-year estimates reported.

[b]No electricity generation sector.

Primary elasticity estimates were too unreliable; the study focused on secondary elasticities.

Figure 1-4 displays primary and secondary price and quantity indexes generated for the various scenarios for two models. For the MEFS model, the aggregation problem is signaled by the irregular scatter of aggregate primary energy price and quantity observations. While both the primary and secondary data are scattered over a region, it is easier to estimate a functional relationship between secondary energy price and quantity. The differences were not nearly as striking for the FOSSIL1 model. However,



[a]15-year estimates only.

Figure 1-4 25-Year Total Demand Primary and Secondary Energy Price Responses for the MEFS and FOSSIL1 Models

decomposition results, discussed at a later point, confirmed the judgment that primary energy elasticities were too undependable. Therefore, the working group decided to concentrate on the secondary energy elasticities.

The Choice of Index

Except for the Btu-weighted, elasticity estimates were not sensitive to choice of index.

Secondary aggregate elasticity estimates were not sensitive to the choice of Paasche, Laspeyres, Ideal, or Tornquist indexes. But for some models the use of a Btu-weighted index led to different calculated elasticities, as shown in Table 1-5. These results suggest the importance of specifying the class of price index used whenever aggregate elasticities are reported. Because the Btu-weighted index is theoretically less attractive, the Paasche index is used in reporting results in the present report.

Table 1-5

COMPARISON OF 25-YEAR SECONDARY DEMAND ELASTICITY ESTIMATES BASED ON SELECTED INDEXES[a]

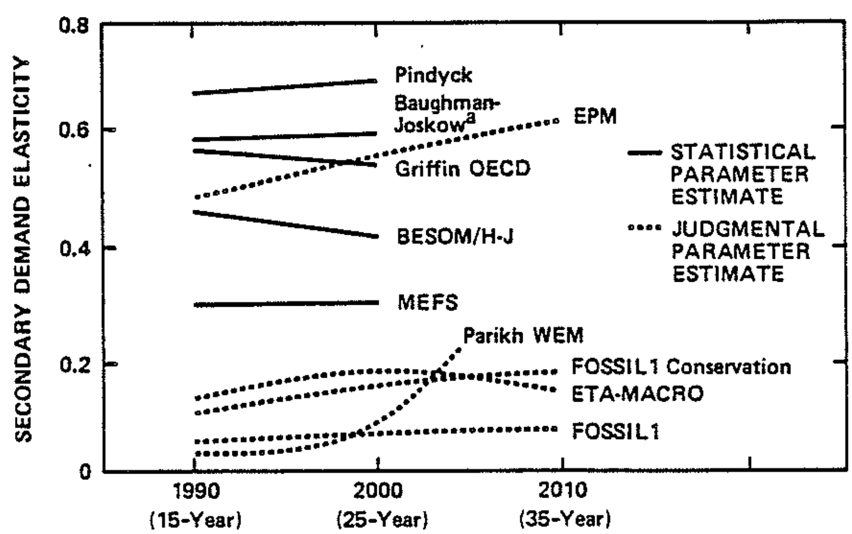
Model	Index			
	Tornquist	Paasche	Laspeyres	Btu-weighted
Baughman-Joskow	0.61	0.61	0.61	0.59
BECOM	0.54	0.51	0.57	1.00
BESOM/H-J	0.43	0.42	0.44	0.46
EPM	0.57	0.57	0.57	0.59
ETA-MACRO	0.19	0.18	0.20	0.35
FEA-Faucett[b]	0.15	0.15	0.15	0.15
FOSSIL1	0.08	0.08	0.08	0.14
FOSSIL1 Conservation	0.16	0.16	0.16	0.23
Griffin OECD	0.56	0.55	0.57	0.68
Hirst Residential	0.45	0.44	0.45	0.47
ISTUM	0.24	0.24	0.24	0.01
Jackson Commercial	0.38	0.37	0.38	0.33
MEFS[b]	0.31	0.31	0.32	0.33
Parikh WEM	0.09	0.10	0.09	0.10
Pindyck	0.71	0.70	0.71	0.66
Sweeney Auto[b]	0.46	0.46	0.46	0.46
Wharton MOVE[b]	0.21	0.21	0.21	0.21

[a]Same demand sectors as reported in Table 1-4.

[b]15-year estimates reported.

The comprehensive statistically estimated models produced higher elasticity estimates than did those models with judgmental parameter estimates.

The aggregate secondary demand elasticity estimates are shown in Figure 1-5. The more comprehensive models, covering all energy-using sectors, incorporating the full range of potential substitutions, and directly utilizing historical data to estimate statistically the demand parameters, were generally characterized by higher implicit long-run aggregate secondary demand elasticities than were the other models. The statistically estimated models are represented with solid lines in Figure 1-5 and include Baughman-Joskow (0.6), BESOM/H-J (0.4), Griffin OECD (0.5), MEFS (0.3), and Pindyck (0.7). The models that consider all consuming sectors but employ judgmental parameter estimates generally exhibit lower elasticities. These models are represented with broken lines in Figure 1-5 and include EPM (0.6), ETA-MACRO (0.2), FOSSIL1 (0.1), FOSSIL1 Conservation (0.2), and Parikh WEM (0.1).[23]



Note: The Paasche index was used to calculate these estimates.

[a]Does not include transportation demands.

Figure 1-5 Aggregate Total Demand Elasticity Estimates

Statistical models could be biased in either direction.

The statistical approach is based on the analysis of historical data and consequently presumes that technology accommodating new combinations of productive factors will be available. Thus, these methods may tend to overestimate or underestimate the flexibility in the system, depending on whether the technological response to changing prices turns out to be greater or less than the historical response.

However, the statistically estimated models produced consistently high elasticities,

Table 1-6 presents 25-year elasticities by sector for all models included in the study. These numerical results demonstrate consistently high elasticities among the statistically estimated models. If these models most correctly describe the future behavior of energy demand, energy growth can be substantially decoupled from economic growth, energy conservation programs can effectively lower energy consumption, and energy supply development programs are less critically needed.

while judgmental models demonstrated lower elasticities.

The judgmental models also show a surprising degree of consistency in projected responses to higher energy prices, exhibiting relatively low demand elasticities. If these models most correctly describe the energy system, energy growth will be tightly linked to economic growth, energy conservation programs cannot be expected to greatly lower energy consumption, and supply development programs are more critically needed.

A consensus on the most likely elasticity was not achieved.

After much debate, the working group was unable to reach a consensus as to which class of models most accurately describes the world. Aggregate demand elasticity estimates from the various models are provided to sharply delineate the range of disagreement on the probable values of this critical summary statistic.

Differences in Model Structure

Structural characteristics represented in a model can influence its elasticity.

Structural characteristics represented in a model can also influence the calculated elasticity. Since there are many ways total energy demand can change in response to changing incentives, failure to implicitly or explicitly represent each of the basic adjustment mechanisms will bias elasticities downward.

Table 1-6

25-YEAR SECONDARY DEMAND
ELASTICITIES, BY SECTOR
(Paasche Index)

Sector	Statistical		Engineering		Judgmental	
Residential	Hirst Residential[a]			0.4		
	Griffin OECD	0.9	BECOM	0.6		
	MEFS	0.5				
	Pindyck	1.0				
Residential/ Commercial	Baughman-Joskow	0.8	BECOM	0.5	EPM	0.5
	BESOM/H-J	0.7				
	MEFS	0.5				
Commercial	MEFS	0.5	BECOM	0.3		
	Jackson Commercial[a]			0.4		
Commercial/ Industrial	Griffin OECD	0.3				
	Pindyck	0.7				
Industrial	Baughman-Joskow	0.4	ISTUM	0.2	EPM	0.7
	BESOM/H-J	0.5				
	MEFS	0.2				
Transportation[b]	BESOM/H-J	0.2			EPM	0.4
	FEA-Faucett	0.1				
	Griffin OECD	0.5				
	MEFS	0.3				
	Pindyck	0.5				
	Sweeney Auto	0.5				
	Wharton MOVE	0.2				
All Sectors	Baughman-Joskow[c]	0.6			EPM	0.6
					ETA-MACRO	0.2
	BESOM/H-J	0.4			FOSSIL1	0.1
	Griffin OECD	0.5			FOSSIL1[d]	0.2
	MEFS	0.3			Parikh WEM	0.1
	Pindyck	0.7				

[a]Combines both the engineering and statistical approach.

[b]The FEA-Faucett, Sweeney, and Wharton MOVE results are for automobile gasoline only. These are 15-year elasticities. All runs exclude the new car fuel efficiency standards.

[c]Excludes the transportation sector.

[d]FOSSIL1 Conservation.

Models representing components of energy demand as independent of price bias elasticity estimates downward.

Several of the sectoral energy models represent components of energy demand as independent of price, thereby biasing downward the calculated aggregate elasticity. For example, BECOM and ISTUM assume end-use service demands to be independent of price. This constraint within the model ignores the possibility of selecting alternative processes for the production of a given commodity; it even precludes the possibility of modifying thermostat settings in response to higher fuel prices. This modeling approach, therefore, biases elasticity estimates downward.

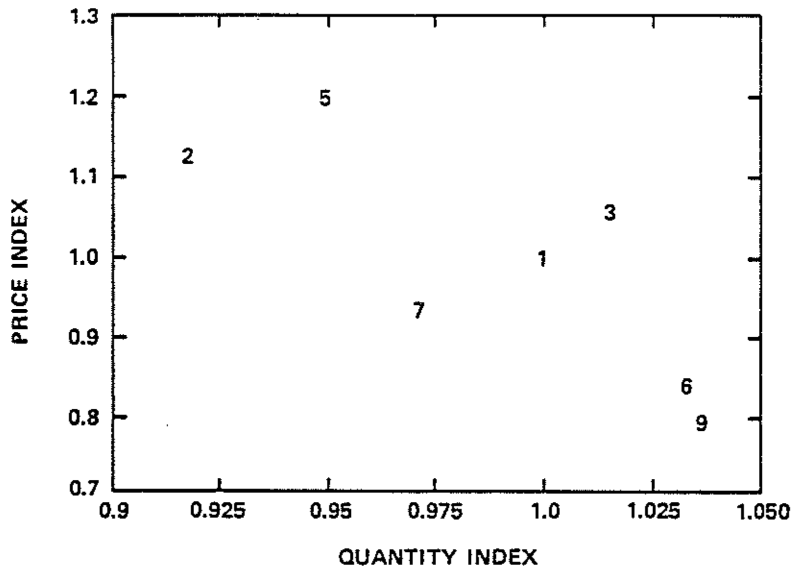
Elasticity estimates from sectoral models were lower than estimates from comprehensive models.

The range of estimates from the sectoral models, as shown in Table 1-6, was from 0.1 for FEA-Faucett to 0.6 for BECOM, and included 0.4 for Hirst Residential, 0.2 for ISTUM, 0.4 for Jackson Commercial, 0.5 for Sweeney Auto, and 0.2 for Wharton MOVE. In general, sectoral models are characterized by lower elasticities than those characterizing corresponding sectors of the more comprehensive models. This difference may occur partially because many sectoral models use engineering process data including a limited range of technological options. The effects of differences in model structure on the elasticity estimates are discussed in more depth by Griffin and Wood in Chapter 8.

The Composition of Price Change

Some model results were very sensitive to the price change composition; for these models, an aggregate elasticity can be misleading.

The results from many models were sensitive to the specific composition of price changes. For these models, an aggregate analysis may be inadequate or misleading. For example, Figure 1-6 shows price and quantity indexes generated by applying the ISTUM model to seven of the price cases. If the secondary energy price and quantity indexes were available only for cases 1, 3, and 7, the ISTUM aggregate elasticity estimate would be negative: aggregate price increases would be expected to lead to aggregate demand increases, not decreases! This anomalous result occurs primarily because oil and gas price and demand play crucial roles in aggregate index computation and because oil and gas prices remain unchanged over these three scenarios. Conversely, if only cases 1, 5, and 9 or only cases 1, 2, and 6 were available, positive elasticities would be calculated, although the magnitudes would differ in these two situations.



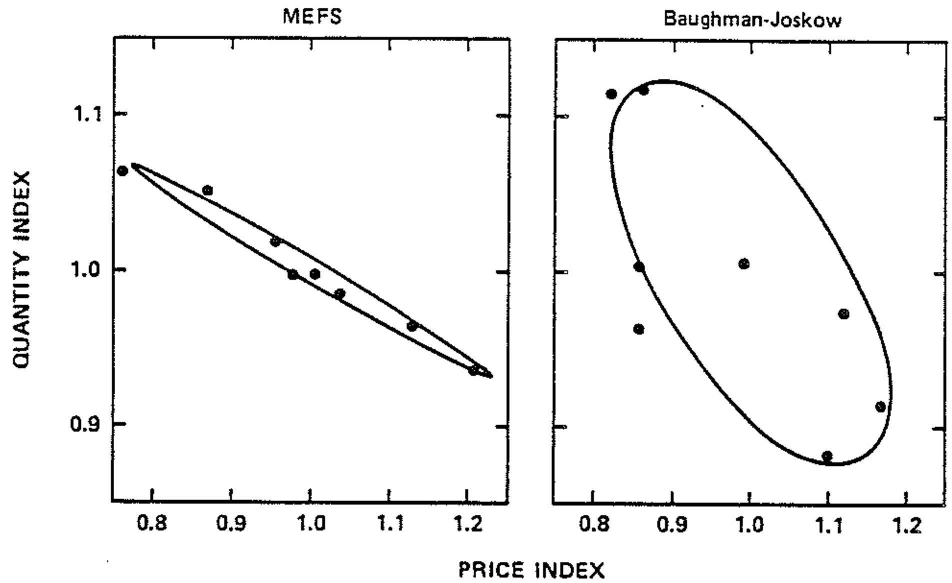
Note: The numbers refer to price cases. The Paasche index was used to calculate these estimates.

Figure 1-6 25-Year Industrial Secondary Energy Index Values for the ISTUM Model

A single-valued demand function for an aggregated class of commodities may not exist, as has been illustrated above. In this case, the notion of an aggregate demand function must be replaced with the notion of a demand region: a set of price and quantity index combinations which could occur in response to price changes of fixed magnitude but varying directions. The decomposition approach provides an approximate description of the shape of such a demand region and includes a method for fitting the region to data generated by the various models.

The decomposition results from two models are contrasted.

Figure 1-7 illustrates the decomposition results for the MEFS and Baughman-Joskow models. The ellipse in each graph provides an estimate of the demand region outer limits generated in response to price changes of unit magnitude but varying direction. Each graph includes individual observations from the models, scaled to unit magnitude price changes. Aggregate elasticities from the MEFS model are much less sensitive to the direction of the



Note: The Paasche index was used to calculate these estimates.

Figure 1-7 Comparison of 25-Year Secondary Energy Decomposition Results from the MEFS and Baughman-Joskow Models

price change than are those from the Baughman-Joskow model. Consequently, greater care must be exercised in calculating and using aggregate elasticity estimates from the Baughman-Joskow model.

Decomposition parameter estimates for all models are presented.

Table 1-7 presents results obtained by applying the decomposition technique to those models that ran sufficient numbers of scenarios. The limited number of independent observations from the other models precluded the use of this technique. For each index, three numbers characterize the demand region approximation for each model:

- σ, the average elasticity, represents the elasticity averaged over all possible price directions;
- τ, the directional elasticity, represents the demand region width (the maximum quantity index change occurring at zero price index change, divided by the maximum price index change); and

Table 1-7

25-YEAR SECONDARY FUELS DECOMPOSITION RESULTS
(Paasche Index)

Model	Sector	Average Elasticity σ	Directional Elasticity τ	Shape Factor ν
Baughman-Joskow	Total	0.46	0.53	1.14
BECOM[a]	Residential/ Commercial	-4.08	-3.86	0.94
BESOM/H-J	Total	0.52	0.42	0.80
EPM	Total	0.45	3.22	7.17
ETA-MACRO	Total	0.22	0.05	0.25
FOSSIL1	Total	0.07	0.05	0.68
FOSSIL1 Conservation	Total	0.15	0.06	0.36
Griffin OECD	Total	0.67	1.58	2.34
Hirst Residential[a,b]	Residential	0.31	0.24	0.78
ISTUM[b]	Industrial	-3.74	7.76	-2.07
Jackson Commercial[a,b]	Commercial	0.32	0.09	0.28
MEFS	Total	0.29	0.03	0.11
Parikh WEM[b]	Total	0.15	0.92	6.23
Pindyck	Total	0.68	0.04	0.06

[a]Less than three fuels were reported.

[b]Less than eight scenarios were reported.

- ν, the shape factor, represents the width relative to the average elasticity.

These parameters are described more fully in Chapter 9.

The shape parameter summarizes sensitivity to price change composition. Low shape parameter values imply the aggregate elasticity is a good summary descriptor.

The shape factor (ν) can be used to indicate the range of price change directions for which the average demand elasticity (σ) closely approximates a model's behavior. Large values of the shape factor imply that the actual elasticity is highly dependent upon the direction of price change while small values imply a relative insensitivity to direction. For example, if all directions of price change are viewed as possible, absolute values of the shape factor less than 0.4 imply actual model elasticity is within plus-or-minus 50 percent of the average elasticity for 71 percent of all possible directions and within 20 percent for 55 percent of directions. Absolute values of the shape factor greater than 2 imply the actual model elasticity differs from the average elasticity by more than plus-or-minus 50 percent for 69 percent of the directions and by more than 100 percent for 53 percent of the directions.

Using this criterion, the average secondary elasticity provides a poor descriptor of the Baughman-Joskow, EPM, Griffin OECD, and Parikh WEM models and a relatively weak descriptor of the BECOM and FOSSIL1 models. For these models, the aggregate secondary demand elasticity is highly sensitive to directions of price change. Conversely, the average secondary elasticity provides a good summary descriptor of the ETA-MACRO, Jackson Commercial, MEFS, and Pindyck models.

For most models, the aggregate secondary elasticity provides a better descriptor than aggregate primary elasticity.

As shown in Table 1-8, the average primary elasticity provides a good summary descriptor for fewer models, BESOM/H-J and Pindyck, and a poor descriptor for many models. For many of the models, the shape factor is larger when estimates are based upon primary data rather than secondary data. This confirms the judgment that the aggregate elasticity of primary energy is less reliable as a model descriptor than is the aggregate elasticity of secondary energy.

Response Dynamics

Many models include rudimentary descriptions of adjustment processes.

Although energy demands can be expected to change gradually in response to price changes, many models include only rudimentary descriptions of the adjustment processes and thus cast little light on the response dynamics. Yet, energy policy and planning debates generally require information not only about the 25-year responses to actions taken now but also about responses occurring over the first weeks, months, or years after the price change. Failure of modelers to examine the short-run demand responses to price changes in conjunction with the long-run responses or, worse yet, a failure to properly differentiate among the various time horizons will tend to seriously limit model usefulness.

Table 1-8

25-YEAR PRIMARY FUELS DECOMPOSITION RESULTS
(Paasche Index)

Model	Sector	Average Elasticity σ	Directional Elasticity τ	Shape Factor ν
Baughman-Joskow	Total	-0.95	1.15	-1.21
BESOM/H-J	Total	0.57	0.17	0.29
EPM[a]	Total	-0.56	1.23	-2.22
ETA-MACRO	Total	0.57	0.95	1.66
FOSSIL1	Total	0.03	0.13	4.12
FOSSIL1 Conservation	Total	0.07	0.15	2.15
Griffin OECD	Total	0.32	0.37	1.15
Hirst Residential[a]	Residential	0.23	-0.05	-0.21
Jackson Commercial[a]	Commercial	0.17	-0.02	-0.14
MEFS	Total	0.07	0.24	3.45
Parikh WEM[a]	Total	-6.48	26.70	-4.12
Pindyck	Total	0.47	0.03	0.07
Wharton MOVE	Total	0.27	0.47	1.75

[a]Less than eight price cases were reported.

25-year elasticities were used as long-run equilibrium values.

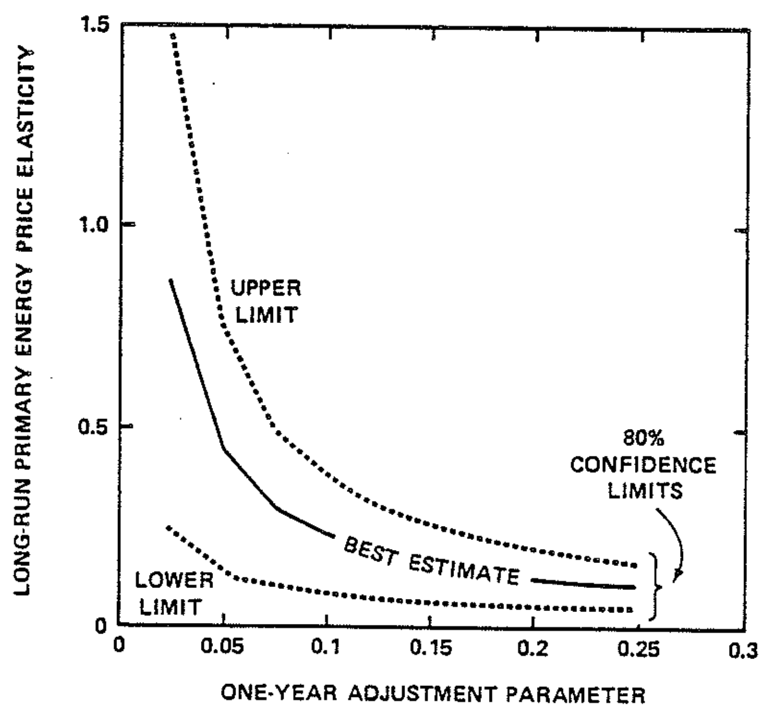
The model comparisons performed in the present study did not allow examination of the adjustment processes to higher prices inherent in the models. Nor were long-run equilibrium elasticities directly estimated. Rather, 25-year elasticities were taken as estimates of the long-run equilibrium values. Since Fromholzer and Lau (Chapter 11) have shown that the particular price paths employed in the study probably lead to only a very small--less than 10 percent--divergence between long-run equilibrium elasticity estimates and 25-year elasticities, the failure to directly estimate long-run equilibrium values should not be of great consequence.

The post-1973 experience could be consistent with the entire range of primary elasticities.

Only Manne and Wilson (Chapter 12) and Hogan (Chapter 13) attempted to directly calculate the aggregate elasticity from historical data rather than developing disaggregated models. These analyses have value as pedagogical devices and as mechanisms for performing several simple, but important, consistency checks. Incorporating recent historical data within the framework of a dynamic adjustment model allows comparison of recent historical trends with long-run aggregate elasticity estimates obtained from the models included in the present study.

In a particular year, the response of demand to higher prices depends on the long-run elasticity and the lag parameter.

For example, the solid line in Figure 1-8 plots the mean primary energy demand elasticity estimates from the Manne-Wilson analysis as a function of the lag parameter. The lag parameter represents the ratio of the one-year elasticity to the long-run equilibrium elasticity. It can be very roughly interpreted as the fraction of energy-consuming capital stock newly installed each year. Since the fraction of capital stock newly installed each year is normally small--certainly less than 0.25, and probably less than 0.1--the one-year adjustment parameter can be expected to be correspondingly low. Thus, the Manne-Wilson analysis, which incorporates data from the post-1973 experience, indicates that the primary energy consumption changes actually experienced through 1978 could be consistent with the entire range of estimated long-run elasticities implicit in any of the models participating in



Note: These are direct estimates from 1950-1978 aggregate data.

Figure 1-8 Dependence of Long-Run Elasticity Estimates on Assumed One-Year Adjustment Parameter the experiment. Taken alone, the post-embargo primary energy data do not provide sufficient evidence to reject either higher elasticity estimates from the statistical models or lower elasticity estimates from the other models.

Taxes and Regulations

Taxes and regulations can alter the demand elasticity.

Taxes and regulations can also significantly influence the demand elasticities. For example, gasoline taxes lead to an additive markup on gasoline price, causing the retail demand elasticity to differ from the secondary elasticity. The larger the gasoline tax, the greater the wedge between these elasticities. Thus, increases in gasoline taxes can be expected to reduce the secondary elasticity if the retail elasticity is left unchanged.

Fuel-use efficiency standards tend to reduce demand elasticities while reducing energy consumption.

Regulations which mandate the characteristics of energy-consuming equipment tend to reduce the demand elasticity while presumably reducing energy consumption. For example, the imposition under the Energy Policy and Conservation Act (EPCA) of binding corporate average fuel efficiency (CAFE) standards on automobile manufacturers implies that, over the range of prices for which the standards are binding, the average new car fuel efficiency will cease to be greatly dependent upon gasoline price. The penalties motivate manufacturers to just meet the standards, but provide no motivation for exceeding them. The standards preempt the consumers' shifts toward more efficient automobiles as gasoline prices increase. Therefore, in the presence of binding standards, automobile fleet characteristics will depend less upon gasoline price and the demand elasticity for gasoline will decrease. As shown in Table 1-9, this phenomenon appears striking in results from the Sweeney Auto model and less dramatic in results from the FEA-Faucett and Wharton MOVE models.

This effect is stronger in some models than in others.

The secondary elasticity for the Sweeney Auto model is slightly less than 0.5 without the CAFE standards and slightly less than 0.2 with these standards. The Wharton MOVE and FEA-Faucett model elasticities are also somewhat less in the presence of the standards.

Table 1-9

15-YEAR SECONDARY ELASTICITY ESTIMATES WITH AND WITHOUT CAFE[a] STANDARDS

Automobile Gasoline Model	Secondary Elasticity Estimates	
	Without CAFE Standards	With CAFE Standards
FEA-Faucett	0.15	0.11
Sweeney Auto	0.46	0.17
Wharton MOVE	0.21	0.19

[a]Corporate average fuel efficiency.

Additional fuel efficiency standards are expected.

Other efficiency standards are being or will be promulgated in the energy area. Building energy performance standards (BEPS) and appliance efficiency standards preempt the role of prices in motivating fuel efficiency increases. The standards produce a shift in the demand curve and make demand less price sensitive. Thus in the residential and commercial sectors, these standards can be expected to reduce demand elasticities while reducing consumption.

Characterizing Uncertainty

There is uncertainty associated with the elasticity estimates.

There is a range of uncertainty associated with any demand elasticity estimate; the actual elasticity could be greater than or less than any of the point estimates presented here. While there was general agreement that the range of uncertainty is not the same as the range of elasticity estimates, measures of the uncertainties implicit in the estimates were not explicitly calculated.

An exception was the Manne-Wilson analysis. With this specification, the dotted lines in Figure 1-8 represent 80 percent confidence bounds--based upon the recent data--around the mean long-run elasticity estimates for various assumed adjustment speeds. One cannot, however, use the Manne-Wilson analysis to infer quantitative measures of uncertainty associated with other aggregate elasticity estimates.

Measures of uncertainty in elasticity were not calculated.

Several sources of uncertainty exist in each model--measurement error in historical data, parameter estimation uncertainty, specification errors in formulating the model, and errors in engineering judgments. However, limitations in the current state of the art either preclude calculation of explicit uncertainty measures or make calculation extremely costly. Furthermore, the objective measures of uncertainty which could be conceptually calculated can never be expected to capture the subjective uncertainties in the minds of many potential model users or modelers. For these reasons, the goal of developing a satisfactory measure of uncertainty has not been reached within this study. Lau proposes such a measure and describes an appropriate estimation methodology in Chapter 10.

RECOMMENDATIONS

Based upon its findings, the working group makes the following recommendations.

The EIA should develop standard definitions for energy demand data and elasticities.

The Energy Information Administration (EIA) should undertake the development of consistent accounting conventions and standardized data designed in close coordination with modelers. Reconciliation of disparate energy data sources among the various federal agencies should be a part of this activity. Model comparisons and interpretations are made difficult by the diversity of accounting conventions. Sectoral definitions vary among models. Historical data purportedly measuring the same concepts vary among federal agencies, e.g., gasoline data reported by the Federal Highway Administration and the EIA. The working group spent a great deal of time on standardizing the data without fully succeeding.

Modelers should publish more complete descriptions of their models.

Modelers should improve their practice in publishing assumptions, error statistics, robustness tests, validity tests, descriptive information, and historical data supporting their models. Agencies funding model development should insist on and support this effort. Some such improvements in disclosure practices are costly to implement; this increased cost must be taken into account when planning a modeling project. Others, however, are less costly. For example, an econometrician estimating alternative specifications of one relationship should publish parameter estimates for all or a range of the specifications tried rather than simply the one or two "preferred" equations. While simple disclosure would not be very costly, it would greatly help users evaluate the robustness of crucial parameters. Similarly, publication of historical data supporting the model would allow other researchers to attempt duplication of the results, test alternative specifications not reported, and detect differences stemming from different sets of historical data used by different modelers.

Implicit aggregate elasticity estimates should become a standard part of model documentation.

Modelers should make aggregate elasticities a standard component of demand model documentation. The EIA should publish a set of definitions and computational procedures for calculating these elasticities. It is recommended that aggregate demand elasticities at three levels--primary, secondary, and, if possible, delivered--be disclosed, based upon model simulations. These elasticities should be reported for total demand and the various sectors modeled. Since aggregate elasticities depend upon the composition of price changes, tables of fuel-specific own and cross elasticities of demand should also be reported as standard components of model documentation. Since elasticities vary over time, these statistics should be reported for short, medium, and long time horizons.

Modelers should develop and utilize techniques for describing uncertainties in their models.

Recognizing the difficulty of adequately dealing with uncertainty in projections, the working group recommended that modelers develop and then consistently utilize techniques for describing the uncertainties in their models. This will require basic research to develop the methodology. Funding organizations and modelers, realizing that analysis of uncertainties is costly but important, should budget sufficient funds for this activity.

NOTES

1. EMF Report 1, Energy and the Economy, Volume 1, Energy Modeling Forum, Stanford University, Stanford, California, September 1977.

2. This differs from some definitions where own-price elasticities are negative, reflecting the inverse relationship between demand and price.

3. The decrease in consumers' surplus resulting from an energy tax is greater for larger elasticities. In the case of an increase in imported energy price, the additional wealth transfer to oil exporting nations is much smaller for a larger elasticity, offsetting the larger decrease in consumers' surplus.

4. Sweeney, James L., "Energy and Economic Growth: A Conceptual Framework," in Directions in Energy Policy: A Comprehensive Approach to Energy Resource Decision-Making, B. Kursunoglu and A. Perlmutter (eds.), pp. 115-140, Cambridge, Mass.: Ballinger Publishing Company, 1979.

5. The elasticity of substitution is defined as the rate of change of the ratio of energy inputs to other inputs, e.g., capital and labor, with respect to changes in the relative prices of those input factors. For value shares of energy in the economy less than 0.1, the two elasticities differ by less than 10 percent. The elasticity of substitution is the key parameter in a seminal model of energy-economy interactions. See Hogan, W. W., and A. S. Manne, "Energy-Economy Interactions: The Fable of the Elephant and the Rabbit ?" Energy and the Economy, Volume 2, Appendix B, Energy Modeling Forum, Stanford University, Stanford, California, September 1977.

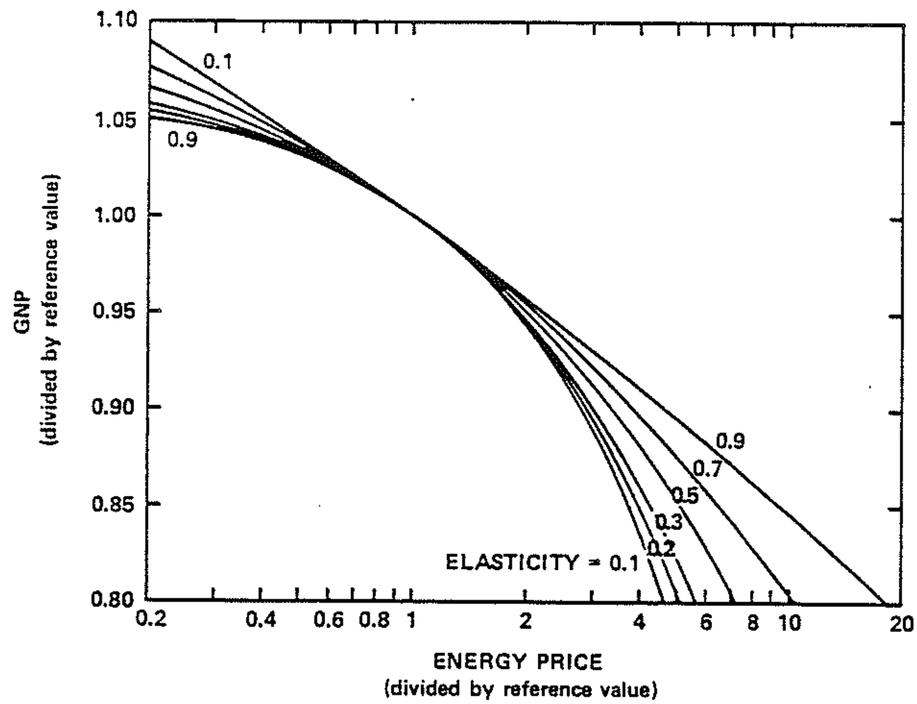
6. For this graph, it is assumed that the levels of capital and labor inputs to the economy and the state of technological progress in any year are independent of the price of energy. These assumptions focus on the aggregate economic costs of restricting energy use below the GNP maximizing level, not on the wealth redistributional aspect of changing energy prices. These results are based on a model (see note 4) that assumes the elasticity of energy demand is constant for a wide range of price changes, including prices far outside the historical experience. This is a convenient assumption but may be erroneous for large price changes like the 10- to 20-fold increases shown at the extremes of Figures 1-1 and 1-2. Similarly, this model assumes optimizing behavior on the part of energy consumers.

   It is also assumed in Figure 1-2 that the decrease in secondary energy use is the result of an increase in the price of energy not related to an increase in its cost, such as an excise tax on secondary energy use. Conversely, if the price increase is the result of an increase in the cost of obtaining energy, such as an increase in the price of oil imports or in the cost of producing domestic supplies, Figure 1-9 is appropriate. In this latter case, for a given cutback and value for the aggregate elasticity, the GNP decrease is much greater than in the previous case; more of society's nonenergy inputs (capital and labor) must be directly utilized to produce/purchase the remaining energy supplies.

   Details required on all these calculations are included in Appendix A, p. 1-50.

7. The averaging is based upon fixed quantity weights equal to their base case values. This is generally referred to as a Laspeyres price index.

8. In electricity generation, the heat value of fossil fuel inputs is roughly three times the heat value of the electricity produced.



Note: Energy use and price measured at secondary level.

Figure 1-9 Relationship between Energy Price and GNP as a Function of Aggregate Elasticity for the Cost Increase Case

9. This example used energy input quantities and output prices. The price index, therefore, requires averaging output prices based upon output quantities--here all one. Alternatively, input prices can be used. In such case, the magnitude of the price change is reduced by a factor of three while its coefficient is increased by an identical factor. All resultant elasticities will be unchanged.

10. Diewert, W. E., "Application of Duality Theory," in M. E. Intrilligator, Frontiers of Mathematical Economics, Volume 3, 1978.

11. Hudson, E. A., and D. W. Jorgenson, "U.S. Energy Policy and Economic Growth, 1975-2000," Bell Journal of Economics, Volume 5, No. 2, Autumn 1974.

12. Lau, L. J., "A Note on Exact Index Numbers," Department of Economics, Stanford University, Stanford, California, March 1977.

13. Here GNP is defined as that portion of total output value of the rest of the economy not used to produce or purchase energy. Hence, GNP is the output available for final consumption and investment.

14. It is at this point that consumer choices are made and, with appropriate disaggregation, the products and choices of consumers are most nearly homogeneous.

15. These curves were generated under the assumption that markups between the secondary point and the retail point are additive. To the extent that some of the markups are multiplicative, the difference between secondary and delivered elasticities will be smaller than indicated in the graph. The curves were calibrated as follows:

$$\eta_R = \frac{\partial q}{\partial P} \cdot \frac{P_R}{q}$$

$$\eta_S = \frac{\partial q}{\partial P} \cdot \frac{P_S}{q} = \eta_R \frac{P_S}{P_R} ,$$

where η_R = retail elasticity

η_S = secondary elasticity .

From Chapter 4, the 1972 price of secondary energy (P_S) was equal to \$1.43 per million Btu (1978 dollars). Taking retail as the aggregate price delivered to all consumers--from Hogan, Chapter 13--the 1972 price of retail energy ($P_R = P_S + 1.39$) was \$2.82 per million Btu (1978 dollars). Therefore,

$$\frac{\eta_S}{\eta_R} = \frac{P_S}{P_S + 1.39} .$$

The 1980 secondary price of \$3.50 per million Btu (in 1978 dollars) was obtained by inserting industrial fuel prices for January 1980 from the May 1980 issue of the U.S. Department of Energy Monthly Energy Review into the industrial sector aggregation formula given in Hogan, Chapter 13.

16. This taxonomy of energy models was first suggested in "Energy System Modeling and Forecasting," by K. C. Hoffman and D. O. Wood in Annual Review of Energy, Volume 1, J. J. Hollander (ed.), Palo Alto, Calif.: Annual Reviews, Inc., 1976.

17. Results from a variant of the FOSSIL1 model, denoted the FOSSIL1 Conservation model, were also examined in the study. It employed disaggregated elasticities twice as large as in the baseline FOSSIL1 system.

18. The Griffin OECD and Pindyck models were estimated with OECD (Organization for Economic Cooperation and Development) cross-sectional data. However, only the U.S. portions of these models were utilized in the present study.

19. Modeling Resource Group of the Committee on Nuclear and Alternative Energy Systems, "Energy Modeling for an Uncertain Future," National Research Council, Washington, D.C., 1978.

20. The price of oil is measured at the refinery input, the price of gas at the city gate, and the price of coal delivered to electric utilities. The price of nuclear fuel is for enough fuel to provide 10^6Btu (293 kWh) of electricity at the busbar times a nuclear fuel-to-electricity conversion efficiency of 0.36.

21. In reporting the results of the experiment, the modelers were asked to use the following primary and secondary points of measurement.

Primary Energy

- Oil and Gas: Includes shale oil and biomass. Quantity of oil measured at the refinery input; quantity of natural gas measured at the city gate. Composite price measured at the refinery input.

- Coal: Quantity of coal measured after cleaning; price measured delivered to electric utilities.

- Nuclear: Includes nuclear, solar, geothermal, and hydroelectric generation, as well as geothermal and solar heat. Primary energy equivalents (quantities) for nonfossil energy sources that are converted to electricity are computed as the amount of electric energy generated using the resource divided by the approximate thermal efficiency for electricity generation from fossil fuels equal to 0.36. Primary energy equivalents for nonfossil energy sources that are used directly as thermal energy are computed as the amount of fossil fuel energy replaced. This is assumed to be at the rate of (1/0.36)(1.09) = 3.03 input Btu/end-use Btu. This assumes 100 percent efficiency for replacement of delivered electricity, accounts for a 9 percent transmission loss, and converts to fossil input equivalent. In both cases, the price for the primary energy input is computed as the price of enough nuclear fuel to provide 10^6 Btu (293 kWh) of electricity at the busbar times a nuclear fuel-to-electricity conversion efficiency of 0.36.

Secondary Energy

- Oil and Gas: Quantity of oil measured at the refinery output; quantity of natural gas measured at the city gate; composite price measured at the refinery output.

- Coal: Net of coal used for electricity generation and synthetic fuels production; price measured delivered to industry.

- Electricity: Includes geothermal and solar heat. Quantity and price of electricity measured at the busbar. Primary energy equivalents for nonfossil energy sources that are used directly as thermal energy computed at 1.09 input Btu/end-use Btu. This assumes 100 percent efficiency for replacement of delivered electricity and accounts for a 9 percent transmission loss.

22. The following equation was estimated by least squares:

$$\ln \frac{E}{GNP} = C - \varepsilon \ln P_E ,$$

where E = aggregate energy demand,

GNP = gross national product,

C = constant,

ε = aggregate elasticity, and

P_E = aggregate energy price.

23. The ETA-MACRO model explicitly includes an aggregate elasticity parameter. That parameter is defined in terms of a Btu-weighted index with proportional price increases. Electricity price and demand are measured at the busbar, but the price and demand for oil and gas are measured at the refinery input. Thus, in the parlance of the current study its point of measurement is somewhere between the primary and secondary levels. The value for this parameter assumed in the present study is 0.25. The Paasche index aggregate secondary elasticity estimate as calculated here is 0.18 and the Btu-weighted estimate as calculated here is 0.35.

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Appendix A

A BRIEF OVERVIEW OF THE SWEENEY ENERGY-ECONOMY MODEL

by

James L. Sweeney

This appendix provides a brief description of the Sweeney energy-economy model which was used to generate Figures 1-1, 1-2, and 1-9. It can include the aggregate elasticity of energy demand as an explicit parameter. In addition, it highlights the difference in the economic impacts of energy price changes produced by higher taxes and those resulting from increases in the cost of importing or producing domestic energy.

The basic model is that of a three-sector economy, consisting of an energy-importing sector, a domestic energy-producing sector, and the rest of the economy (ROE). Energy is an intermediate good; output from the ROE may be either a final or an intermediate good.

The first sector imports some quantity of energy (E_I) at a fixed price (P_I), which includes the price paid to the exporting source plus any cost of the import activity. The net resource cost to acquire this energy is simply $P_I E_I$, which becomes one claim on the gross output from the rest of the economy.

The second sector uses output from the rest of the economy in order to produce domestic energy. The quantity of ROE outputs used to produce E_D units of energy will be represented as $H(E_D)$. This is simply the cost function for producing domestic energy.

The total energy used in the economy is the sum of the domestically produced energy plus the imported energy, as represented in Eq. 1A-2 below.

The rest of the economy uses as inputs a quantity of capital services (K), a quantity of labor services (L), and a quantity of energy (E), and produces as output an aggregate quantity of goods and services (which depends on K, L, and E). This aggregate quantity will be referred to as the gross output of the economy. The relationship between K, L, E, and the resulting gross output can be expressed in terms of an aggregate production function of the economy, F(K, L, E). The net national product (Y) is the difference between the gross output and the costs of importing plus domestically producing energy. This is expressed in Eq. 1A-1 below. Equation 1A-1 follows directly since, by definition, net national product (NNP) equals total value of final products produced in the economy or equals the sum over all sectors of the economy of value added.

In a competitive economy* several marginal conditions must also hold.† The price of energy (P_E) will equal its marginal productivity, since competitive firms are assumed to be price-taking profit maximizers. And in a competitive economy the price of energy facing the domestic supplier (P_D) will equal the marginal cost of producing energy. Supply price will equal the energy price plus any tax on domestic production (T_D). These relationships are expressed in Eq. 1A-3. Similar relationships hold for imported energy: the energy price in competitive equilibrium will equal the import price (P_I) plus the tax on imported energy (T_I). This is represented by Eq. 1A-4. Finally, there are necessary marginal conditions for market clearing in the capital and labor markets. The price of labor services (P_L) or of capital services (P_K) is simply equal to the appropriate marginal productivity. These conditions appear as Eqs. 1A-5 and 1A-6.

Six equations thus represent the economy. Equation 1A-1 defines net national product; 1A-2 equates energy consumed to domestic plus imported energy; and 1A-3 through 1A-6 represent the marginal conditions describing a competitive economy in equilibrium.‡ Note that supply conditions for capital and labor have not been written. These will be discussed at a later point.

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*The results derived here are also applicable to an efficient centrally planned economy.

†The implicit assumption here is either there are no market failures in the economy other than those explicitly discussed or the specific market failures in the economy remain unchanged as energy system changes occur. In particular, this assumption may be violated if any energy conservation program that eliminates market failures is implemented. In that case, the function F(K, L, E) or the marginal conditions, Eqs. 1A-3 and 1A-4, may change as a result of the program.

‡While this paper discusses E as a scalar variable under the implicit assumption that energy can be viewed as a single aggregate quantity, E can also be viewed as a vector, thereby allowing a disaggregation by energy type. The interpretations in each equation will be straightforward.

$$Y = F(K, L, E) - H(E_D) - P_I E_I \quad \text{(1A-1)}$$

$$E = E_D + E_I \quad \text{(1A-2)}$$

$$P_E = \partial F/\partial E = dH/dE_D + T_D \quad \text{(1A-3)}$$

$$P_E = \partial F/\partial E = P_I + T_I \quad \text{(1A-4)}$$

$$P_K = \partial F/\partial K \quad \text{(1A-5)}$$

$$P_L = \partial F/\partial L \quad \text{(1A-6)}$$

Impacts of energy system changes on NNP can be evaluated by use of Eqs. 1A-1 through 1A-6, since the impacts of each of the four analytical representations can be evaluated by means of these equations.

For small changes in any analytical representation, the impact on NNP can be calculated by taking the total differentials of Eqs. 1A-1 and 1A-2 and by using Eqs. 1A-3 through 1A-6. This allows one to derive Eq. 1A-7, below, which expresses changes in net national product as a sum of weighted changes in capital, labor, domestic energy production, imported energy production, import price, and domestic price. In Eq. 1A-7, the change in the domestic cost function is expressed as a change in the average cost function (equal to δAC_D) times the quantity of domestic energy (E_D).

$$\Delta Y = -E_I \Delta P_I - E_D \delta AC_D + T_I \Delta E_I + T_D \Delta E_D + P_L \Delta L + P_K \Delta K \quad \text{(1A-7)}$$

Equation 1A-7 will be fundamental to all subsequent analysis of the relationships between energy sector changes and net national product changes. The various terms on the right-hand side of Eq. 1A-7 are interpreted as components of NNP change. These components are: direct effects of cost changes, consisting of changes in the resource cost function; "welfare costs" of price changes stemming from divergences between prices and costs; and induced effects via labor quantity and capital quantity. The change in the NNP equals the sum of the components.

To use Eq. 1A-7, it is necessary to specify a complete model that describes capital, labor, and energy quantity changes in response to changes in the underlying environment. This can be done by more completely specifying the production functions and cost functions described previously and modeling the supply functions for capital and labor, or else simpler models can be postulated in order to illustrate the fundamental interactions. The latter procedure will be followed in this paper.

Taxes vs. Cost Increases

As a first illustration, Eq. 1A-7 can be used to examine the differences in NNP impacts stemming from resource cost increases and those motivated by tax increases. It will be assumed that capital and labor are supplied perfectly inelastically and that there are no changes in the cost function for domestically produced energy. Therefore, the first two terms and the last term on the right-hand side of Eq. 1A-7 will be identically zero.

If all taxes remain zero while imported energy prices change, then Eq. 1A-7 is reduced to the simple differential equation

$$\frac{dY}{dP_I} = -E_I \ . \qquad (1A\text{-}8)$$

If the import price remains constant while the tariff on all energy (imported plus domestic) changes, Eq. 1A-7 is reduced to the differential equation

$$\frac{dY}{dE} = T \ , \qquad (1A\text{-}9)$$

where T is the tax rate on all energy.

Note that in the case of import price changes, the impact on NNP of a price change is proportional to energy imports. However, in the case of a tariff on all energy, the impact is independent of the quantities imported, but depends on the change in the quantity of energy consumed.

In order to use Eqs. 1A-8 and 1A-9 to obtain more precise estimates of NNP change than are provided by the above inequalities, it is necessary to describe the model more fully. In particular, the aggregate production function must be specified more completely in order to relate energy demand or energy imports to energy price and NNP. For this discussion it will be assumed that all energy is imported, so that $E_I = E$. And it will be assumed that the aggregate production function is such that in a competitive equilibrium the quantity of energy demanded is the following constant-elasticity function of NNP and energy prices: Equation 1A-10 will be somewhat loosely referred to as a demand function for energy in this economy, and ε will be referred to as the price elasticity of demand for energy.

$$E = A\,Y\,P_E^{-\varepsilon} . \qquad (1A\text{-}10)$$

What then are the impacts on NNP of an import price change? Equation 1A-10 can be used to eliminate energy consumption in Eq. 1A-8, giving a simple differential equation relating Y and P_I:

$$\frac{dY}{dP_I} = -A\,Y\,P_I^{-\varepsilon} . \qquad (1A\text{-}11)$$

This differential equation can be solved to give the following equation relating NNP to imported energy price:

$$\frac{Y}{Y_0} = \exp\left\{\frac{S_0}{1-\varepsilon}\left[1-\left(\frac{P_I}{P_{I0}}\right)^{1-\varepsilon}\right]\right\} , \qquad (1A\text{-}12)$$

where

$$S_0 = \frac{P_{I0}E_0}{Y_0} .$$

S_0 is expenditures on energy in the base case as a fraction of base case NNP, and will be referred to as the value share of energy in the economy. The symbols E_0, Y_0, and P_{I0} are the base case values of E, Y, and P_I, as indicated previously. Equation 1A-12 shows that the impact of increasing energy import prices on NNP is increasing in S_0: the greater the base case expenditure on energy (as a fraction of NNP), the greater will be the economic impact of an import price change. Also, the greater the elasticity of demand, the smaller the impact of a given import price increase on NNP.

Equation 1A-12 can be used directly to calculate the GNP impacts of increases in the cost of secondary energy inputs. To calibrate the system, it is assumed that in the year 2010, if the OPEC price increase had not occurred, the secondary energy price would be $\$1.20/10^6$ Btu (1975\$), secondary energy demand would be 164 quads, and NNP would be \$4,400 in 1975\$. Thus, $S_0 = (\$1.20)(164)/(4400) = 0.045$. Given these values for Y_0, S_0, and P_{I0}, Eq. 2A-12 was used to produce Figure 1-9.

The key issues for evaluating NNP impacts of cost increases are (1) the magnitude of the cost increase, (2) the expenditure on energy as a fraction of NNP in the base case, and (3) the elasticity of demand for energy. Whether the cost increase stems from domestic cost function increases or import price increases is irrelevant for evaluating impacts on NNP.

What are the impacts on NNP of a tax change? Under an assumption that a tax T is imposed on all energy, whether imported or domestically produced, Eq. 1A-9 is the fundamental differential equation. It will be assumed that the domestic supply price of energy is determined by the fixed import price of P_I. Then, from Eq. 1A-11, demand for energy can be expressed as follows:

$$E = A\,Y(P_I + T)^{-\varepsilon} . \qquad (1A\text{-}13)$$

Equations 1A-9 and 1A-13 are sufficient to determine NNP and energy consumption as functions of the tariff. These equations are independent of the fraction of energy imported and thus the impact of a tariff on NNP will be independent as well.

No closed-form solutions to Eqs. 1A-9 and 1A-13 have been found. However, solutions can be obtained numerically. Taking the total differential of Eq. 1A-13 and substituting into Eq. 1A-9 and simplifying produces

$$dY = \frac{-\varepsilon(TE)}{(P_I + T)(L - TE/Y)}\, dT .$$

This equation can be used to calculate the incremental GNP loss for an incremental increase in the tariff (or tax). For large tariffs, the GNP effects are accumulated by calculating losses attributable to successive tariff increments with E continually adjusted via Eq. 1A-13. Given the benchmarking assumptions discussed previously, this system was used to generate Figure 1-2.

Interestingly, in this simple framework the response of the energy-GNP ratio to price does not depend on whether the price increase results from an increase in costs or taxes. For this purpose, Eq. 1A-10 can be rewritten as:

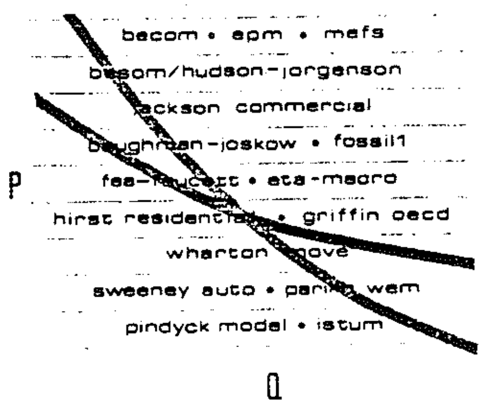
$$\frac{E}{Y} = AP_E^{-\varepsilon},$$

where $P_E = P_I + T$. This relationship was employed to produce Figure 1-1.

Chapter 2

THE DESIGN OF THE EXPERIMENT*

John P. Weyant†
William W. Hogan
Dennis R. Fromholzer



*This chapter formerly was Working Paper EMF 4.0.

†The authors would like to thank Lawrence Lau, Alan Manne, James Sweeney, David Behling, and Ernie Berndt for many helpful ideas and suggestions.

PREFACE

This chapter is the original description of the EMF "Aggregate Elasticity of Energy Demand" study. It has not been revised since the inception of the study. Consequently, it serves as a record of: (1) the motivation for the study, (2) the design of the experiment, and (3) the original proposal for estimating the aggregate elasticity from the model results that were to be collected.

The motivation for the study did not change significantly as the study progressed. However, an energy-economy model in which the aggregate elasticity of energy demand can be included as an explicit parameter was developed by James Sweeney. Results from that model are included in Chapter 1 (Figures 1-1, 1-2, and 1-9) and an appendix to that chapter.

It would have been difficult to revise the design of the experiment once it was in progress. Fortunately revisions were not necessary; enough data were generated to support a number of interesting analyses.

However, as the study progressed, the analysis of the results of the experiment did develop considerably from what is suggested here. First, based on the work of Borison and Sweeney in Chapter 4, "The Theory and Practice of Energy Aggregation" and Sweeney in Chapter 9, "Price and Quantity Change Decomposition for Aggregated Commodities," aggregate energy demand and price were calculated using four alternative index sets: (1) Paasche, (2) Laspeyres, (3) Tornquist, and (4) Btu-weighted. The first three indexes are all based on the assumption of optimizing behavior by energy consumers and, hence, will produce approximately identical results for small price changes. The Btu-weighted index was included simply because it is used so pervasively: it has no basis in economic theory. For similar reasons, prices and quantities from each model for the Reference price case in the year of the aggregation were used as a base in the index calculations rather than the common 1972 price/quantity base suggested here.

Chapter 2

THE DESIGN OF THE EXPERIMENT

INTRODUCTION

The responsiveness of energy demand to changes in the real price of energy is a central topic in energy policy discussions. Aggregated to a single parameter, discussion of this topic focuses on the measurement of the elasticity of energy demand. This parameter is known to be a key indicator of the economic impact of changes in energy availability. Additionally, this simple elasticity is often used as a summary descriptor of detailed energy models.

Straightforward in concept, the elasticity of demand for energy is elusive in practice. Implementation of the aggregate model on which it is based is problematical. Data limitations, measurement problems, vagaries in classification schemes, even rigorous definitions of the concept of primary energy confound the estimation and precise use of this natural analytical framework. Still, reference to the aggregate elasticity remains pervasive in energy analyses. The appeal of this parameter as an indicator of important underlying relationships cannot be overlooked. If it is to be used meaningfully, there must be an improvement in its definition and measurement. The present paper works towards this end by summarizing one approach to the problem. This approach calls for experiments with detailed models of the energy sector to characterize the aggregate elasticity. It is argued that this will provide an improved definition and estimate of the underlying parameter. The improved definition and estimate should have a number of benefits, such as sharpening the analysis of energy policies, pointing out the limitations of the aggregate model, giving a simple summary statistic and standard test for comparing across energy models, and leading to further interest in the development and implementation of better accounting schemes for energy supply and demand analysis.

Results From a Simple Model of Energy-Economy Interactions

One point of departure for the proposal described in this paper is the analysis of an aggregate model of energy-economy interactions developed by Hogan and Manne [1]. This elementary model was formulated to illustrate the extent to which changes in energy availability could impact the growth of the economy. The results highlight the central importance of the energy substitution potential.

In this simple model, the gross output of the economy Y is equal to the value of the two inputs: primary energy, E, and all other factors, R. Thus,

$$Y = P_E E + P_R R , \qquad (2\text{-}1)$$

where P_E and P_R are the prices of the two inputs. Further, treating energy as an intermediate good, GNP is equal to the payments to R. It is assumed that there exists a production function, $Y = F(E,R)$, by which the economy produces its gross output from the two inputs.

Figure 2-1 is a plot of gross output as a function of primary energy inputs with all other inputs held constant. GNP is the difference between the gross output and the value of the primary energy inputs. To achieve a reduction in energy use from some reference level E^0 to a lower level E^1, a tax equal to the slope of the production function at E^1 less the price of primary energy inputs might be imposed. If the tax is not returned to the economy (as would be the case if it was imposed by OPEC), GNP^1 would be obtained, while $\overline{GNP}^1$ would result if the tax were returned to the economy.

Equivalently, the economy can be viewed as maximizing the value of gross output less the costs of inputs:

$$\text{Max} \quad F(E,R) - P_E E - P_R R . \qquad (2\text{-}2)$$

A first order condition for this maximization is

$$\frac{\partial F}{\partial E} = P_E . \qquad (2\text{-}3)$$

In order to implement this conceptual model, a specific functional form for F must be selected. A production function that provides a minimal approximation of the substitution potential of any production function with adequate flexibility for the analysis of the feedback issue is the constant elasticity of substitution production function:

$$Y^{\frac{\sigma-1}{\sigma}} = aE^{\frac{\sigma-1}{\sigma}} + bR^{\frac{\sigma-1}{\sigma}} . \qquad (2\text{-}4)$$

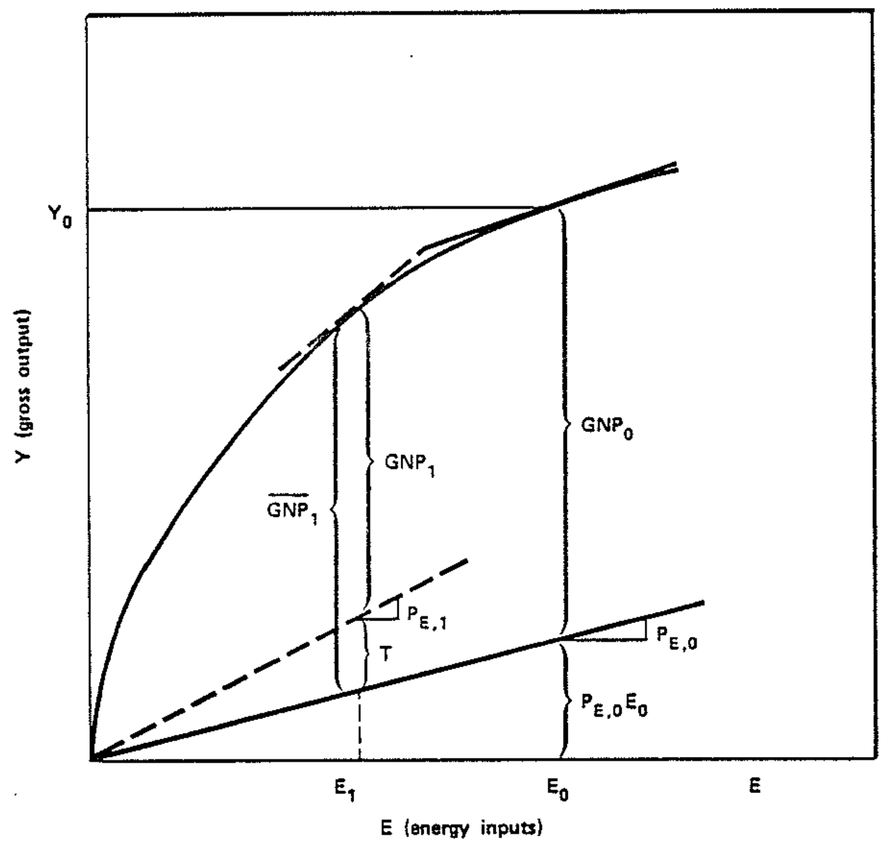


Figure 2-1 Output as a Function of Energy Input

This production function has the property that the elasticity of substitution σ, defined as

$$\sigma = -\frac{\partial \ \ell n(E/R)}{\partial \ \ell n \left(\frac{\partial F/\partial E}{\partial F/\partial R}\right)} \tag{2-5}$$

is a constant. This parameter represents, on a percentage basis, how the ratio of the input factors varies with changes in the marginal rate of substitution between them. (In perfect competition, the marginal rate of substitution will equal the ratio of the factor prices.) Figure 2-2 shows the shape of the isoquants (curves of constant output) for three different values of the elasticity of substitution. If the elasticity of substitution is zero, energy and other inputs are complements; if it is ∞, they are perfect substitutes; and if it is one, the increase in the

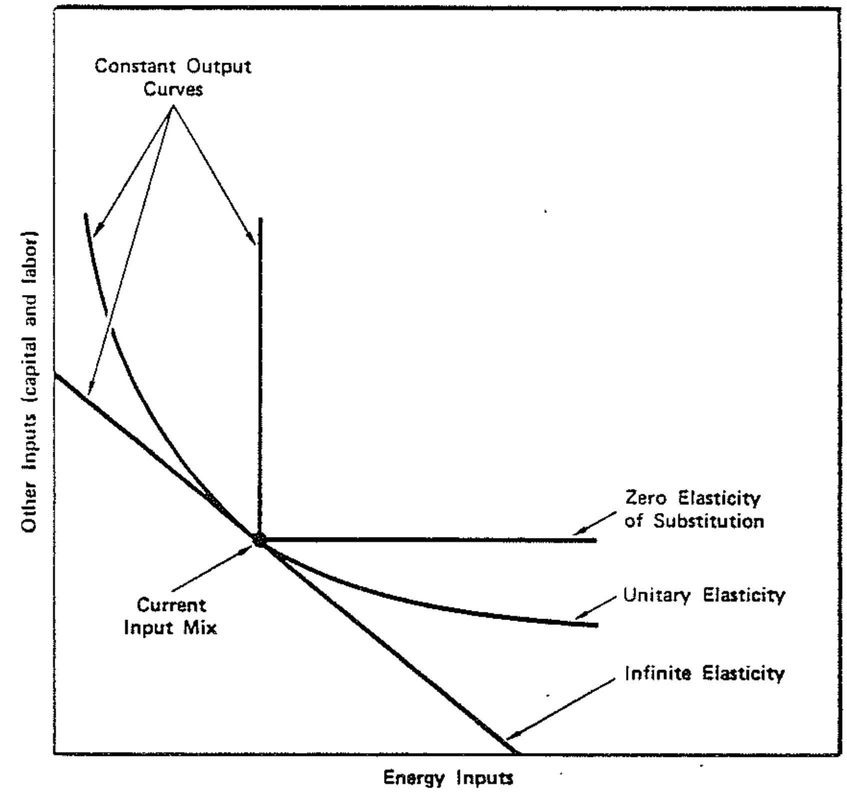


Figure 2-2 The Elasticity of Substitution Concept

marginal rate of substitution is matched by an equal but opposite percentage change in the ratio of the input factors.* With Eq. 2-4 as the form for the aggregate production function, the first order optimality condition, Eq. 2-3, may be written as

$$\frac{\partial F}{\partial E} = a\left(\frac{Y}{E}\right)^{\frac{1}{\sigma}} = P_E \, . \qquad (2\text{-}6)$$

*The functional form given in Eq. 2-4 does not hold for $\sigma = 0, 1,$ or ∞. However, appropriate limits can be calculated to derive the functional forms of the isoquants shown in Figure 2-2.

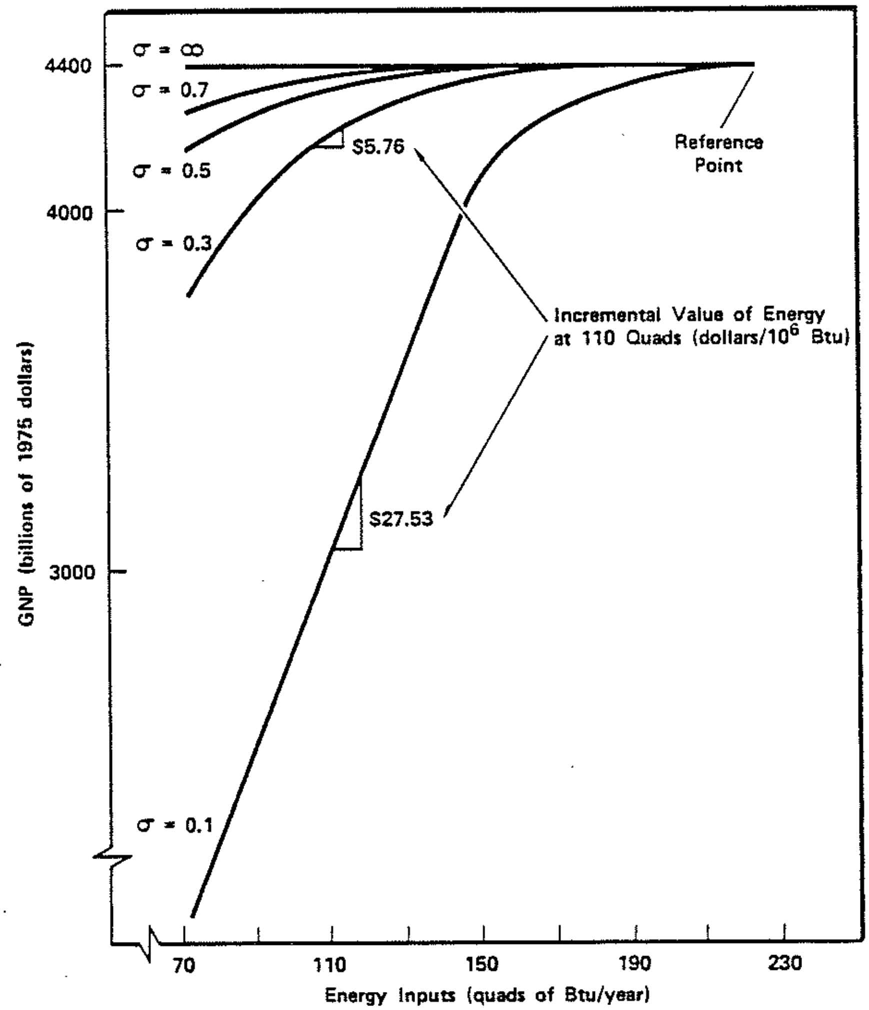


Figure 2-3 Economic Impacts of Energy Reductions in the Year 2010 for Various Elasticities of Substitution (σ)

Figure 2-3 shows the calculations of the GNP in this model under one set of assumptions and for various values of σ. These results demonstrate the importance of the elasticity of substitution between energy and other goods in determining the magnitude of the impact of cutbacks in energy inputs on the macroeconomy. If $\sigma = 0.1$, a 50% reduction in primary energy inputs, holding R constant, leads to a 28% reduction in GNP, while if $\sigma = 0.7$, a 50% reduction in primary energy inputs would lead to only a 1% reduction in GNP.

These results have important implications for both energy policy and the validity of models of the energy sector. If the elasticity of substitution is low, say 0.1 or 0.2, changes in energy availability can have major long-run economic effects. With such large feedbacks, the consistency of partial equilibrium models of the energy sector is called into question. On the other hand, if the elasticity of substitution is high, say 0.6 or 0.7, changes in energy availability may be substantially decoupled from the long-run economic condition. Then, the deletion of the feedback from energy use to GNP in energy sector models can be justified. It seems essential, therefore, to understand the energy substitution potential and to develop more reliable estimates of the aggregate substitution parameter.

Results from the CONAES Modeling Resource Group

One approach to the evaluation of the substitution potential has been pursued in the Committee on Nuclear and Alternative Energy Systems (CONAES) Modeling Resource Group (MRG). The MRG utilized six energy sector models in its study of long-run energy alternatives [2]. Common assumptions effectively standardized the supply sides of the models, but the difficulty of standardizing the demand sides was not overcome. Differing aggregations, differences in the points of measurement for the energy product demands, and disparities in the corresponding demand price elasticities produced significant differences in the resource consumption projections and impact estimates generated by the various models.

This result led to substantial interest within the MRG in comparing the demand-side assumptions built into the participating models. By the halfway point of the study, the models began to be characterized frequently in terms of their degree of price responsiveness. Subsequently, the idea developed to summarize the demand-side behavior of the models in terms of a single price elasticity for primary energy. This resulted in several attempts to measure and compare the aggregate price elasticities by making analytical transformations on the disaggregated elasticities built into the models. Differing points of measurement and aggregations for the energy product demands complicate the direct analytical approach. Near the completion of the work of the MRG the idea of obtaining aggregate elasticity estimates via simulation was addressed. However, the data available from scenarios that had already been run was not enough to be used to make aggregate elasticity estimates and there was not enough time to design the proper experiment. It is the purpose of the present paper to continue this discussion and develop a proposal to conduct such an experment.

Interest in a summary statistic for comparing models was caused by concern over the MRG's assumption of no energy-economic feedback. The MRG models were predicting much lower energy GNP/ratios than found in previous studies. If the energy sector has a large impact on the remainder of the economy, then the MRG models would be invalidated. This concern motivated the development of the simple model of energy-economic impacts described in the previous section. Thus, the determination of the aggregate elasticity of substitution, which is the key parameter in determining the economic impact, became crucial to the credibility and explanation of the MRG results.

The Relationship Between the Elasticity of Substitution and the Price Elasticity for Primary Energy

The interpretation of the elasticity of substitution is aided by recognition of its virtual equivalence with the more familiar price elasticity of energy demand.

Equation 2-6 of the simple model of energy-economy interactions may be rewritten as

$$E = Ya^{\sigma} (P_E)^{-\sigma} . \qquad (2\text{-}7)$$

Interestingly the form of the demand functions for energy products approximated in many models of the energy sector,

$$E = GNP\ b(P_E)^{\eta} , \qquad (2\text{-}8)$$

is almost identical to Eq. 2-7 [3,4,5]. In fact, if gross output, Y, is approximately equal to GNP, as is the case when the value share, s, of primary energy is small, the parameter, $-\sigma$, is essentially equivalent to the constant price elasticity, η.

Accounting for the effects of E on output in Eq. 2-7, we obtain

$$\frac{d\ \ell n\ E}{d\ \ell n\ P_E} = -\frac{\sigma}{1-s} . \qquad (2\text{-}9)$$

Hence, these two estimates of the responsiveness of energy demand to changes in prices differ by the factor $-\frac{1}{1-s} \simeq -1$. Consequently, the constant income price elasticity for primary energy implicit in the energy sector models is approximately equal in magnitude to the constant output elasticity of substitution between primary energy and other goods as used in the simple aggregate production function.

APPROACHES TO THE MEASUREMENT OF THE AGGREGATE ENERGY SUBSTITUTION ELASTICITY

The elasticity of substitution (σ) between energy and other goods is important in energy policy analysis and in determining energy sector model validity. This section explores alternative approaches to the determination of σ. One approach is to measure the price elasticity for primary energy implicit in models of the energy sector. Since, as summarized below, the other approaches have not produced conclusive results, it seems desirable to pursue this alternative.

The Traditional Statistical Approach

The most natural approach to the determination of σ is the direct statistical estimation based on historical data. A theoretical framework is required to implement this approach. There is a certain arbitrariness in the selection of any one framework but some simplification must be made for the analysis to proceed. To implement such an estimation, Fromholzer and Hogan [6], following Nordhaus and Tobin [7], represent the operation of the economy in terms of an aggregate production function with capital, labor, and primary energy inputs:

$$Y = \left[a \left\{ (A_K K)^{\varepsilon} (A_L L)^{1-\varepsilon} \right\}^{-\rho} + b(A_E E)^{-\rho} \right]^{-1/\rho} , \qquad (2\text{-}10)$$

with

$$(a,b > 0; \quad \rho > -1) ,$$

where

Y = gross output,

K = capital inputs,

L = labor inputs,

E = primary energy inputs,

t = time,

A_K = the augmentation level of capital induced by technological change ($A_K = e^{g_K t}$),

A_L = the augmentation level of labor induced by technological change ($A_L = e^{g_L t}$),

A_E = the augmentation level of energy induced by technological change ($A_E = e^{g_E t}$),

ε = the share of value-added payments to capital,

ρ = a parameter determining the elasticity of substitution

$\sigma = \frac{1}{1+\rho}$ ($\sigma \neq 0,1,\infty$), and

a,b = scale parameters.

They argue that this functional form exhibits sufficient flexibility to encompass the most important of the diverse views about the economy's potential to substitute other inputs for primary energy. Note that this framework splits the "other inputs" category of the simple model of energy-economy interactions into capital and labor components. This is necessary because the constant capital and labor inputs assumption of the simple static model does not hold over the time period from which the data are drawn, but data on the separate capital and labor inputs are available. The aggregation and separability assumptions tacitly made in this approach presume the existence of a consistent index of primary energy allowing the representation of the economy in terms of the aggregate production function. Fromholzer and Hogan employ coal-equivalent Btus as the aggregate index of primary energy consumption.

Manipulation of the optimality conditions for the production function leads to

$$\ell n\ Z = C + \rho\ \ell n(E/R) + \rho\lambda t \tag{2-11}$$

where

Z = the ratio of the value share of "other inputs" to that of energy,

C = a constant, and

$\lambda = g_E + \varepsilon g_K + (1 - \varepsilon)\ g_L$.

Following Nordhaus and Tobin, Fromholzer and Hogan used capital and labor values from Denison [8] to construct the index R. The prices for primary energy are measured at the earliest point possible in the distribution system to avoid as much as possible the inclusion of the cost of the fungible capital and labor needed to extract and transform the energy. Two series for energy prices, one based on expenditure data and the other based on fixed weights, were extracted from Schurr and Netshert [9]; and these two series were expanded to four by alternatively including and excluding fuel wood as a primary energy resource. Autocorrelation in the data was reduced by aggregating the data into five-year intervals using the arithmetic mean.

Unfortunately, the results for the parameters of the model fail to conform to the theoretical assumptions underlying the assumed aggregate production function. In particular, the estimates of ρ are uniformly less than -1 which implies negative values for σ. Either the model, the data, or the estimation procedure must be rejected as unsatisfactory and no usable estimate of the elasticity of substitution is obtained.

The difficulty in estimating the elasticity of substitution from aggregate time-series data may be attributable to a number of factors including a failure of the theory adopted in the construction of the aggregate production function. Of course, the simplification of the aggregate production function must be wrong in detail but it may be expected to perform well as a first approximation. It may be unreasonable, however, to expect it to perform well for all types of energy sector changes, particularly those which may violate the implicit assumptions of the simple model. There is a need to define the tests to which the model should be applied.

The representation of the total energy sector in terms of the inputs of primary energy, represented as a single aggregate commodity, fails to recognize the diversity of energy goods and the changes in the composition of total energy that have been underway. Not all properties of energy can be captured in the simple measure of coal-equivalent Btus. Over this century there have been dramatic shifts from coal to oil and gas plus greater electrification, changing the uses and value of energy. The regional distribution of energy consumption and relative energy prices is changing and this may confound the aggregate national data. Further, it is delivered energy that ultimately contributes to the production process or the satisfaction of consumer demands. Changes in the prices or quantities of delivered energy may not be reflected in the primary energy data, and, therefore, would elude the aggregate analysis. A disaggregated model with product differentiation, regional detail, and a careful integration of consumer behavior is needed to separate these and related effects in the historical data. This elementary fact is recognized in the variety of energy demand models which go to great and expensive lengths to include the detail needed to sort out the historical experience.

An additional problem with relying solely on historical data to estimate σ is that in this approach no account is taken of new technologies that are being developed to produce and utilize energy. Projected costs and efficiencies for these technologies can, however, be included in the structure of energy sector models giving explicit representation to the possibility of deviations from historical behavioral response trends.

The Use of More Disaggregated Models of Energy and the Economy*

The move to a detailed model sacrifices the simplicity of interpretation found with the aggregate production function. This increased detail disguises the debate about the aggregate elasticity of substitution; a debate that will continue whatever the validity of any particular aggregate model. From one perspective, the detailed model skirts the hard question by attacking its theoretical basis. But from another perspective, the detailed model may provide the resolution of the issues at the aggregate level. Suppose that the aggregate model is a good approximation to the economy but only for the proper experiment: a simple variation in the price or availability of all primary energy. History has not provided us with this clean experiment. A detailed model is needed to track the real data and then this detailed model becomes a better representation of reality, albeit one that is difficult to interpret. However, if this detailed system is a good representation of reality we may interpret the aggregate production function as an approximation to the detailed model as well as to the true economy. Unlike the case with the true economy, we can now employ the detailed model to conduct the controlled test needed to estimate the parameters of the aggregate production function. We can vary the price of all primary energy without requiring major changes in the composition or regional distribution of its use. Long-run adjustments can be worked out without requiring many years to develop the data. The detailed model captures the structure of the energy demand relationships and the aggregate production function can summarize some of the important characteristics of the detailed model.

Many argue that shifts in the composition of GNP in response to higher energy prices and/or energy shortages will be a central element in the overall response of the macroeconomy. Thus, the idea of using models with a sectoral breakdown of the aggregate energy and nonenergy sectors, employed in the simple model of energy-economy interactions, to estimate σ by simulation has much appeal. The parameter σ would, thus, be an output from experiments using these models rather than an input to them. This is precisely what was done in the first EMF study, "Energy and the Economy" [10]. In that study, six models were used to estimate σ. That resulted in a range for σ between 0.2 and 0.6. These results provide reasonable bounds on the macroeconomic impacts of reduced energy availability, but unfortunately include values for σ which lead to both significant and insignificant levels of feedback from energy availability to GNP. Thus, the value of using detailed models to estimate σ has been indicated but not fully realized. It can be

*Like the last section, this section borrows heavily from Fromholzer and Hogan [6].

expected that more disaggregated energy sector models provide a richness of detail that will improve the direct estimation of the substitution potential. And the aggregate statistic will serve as a useful comparative indicator of the demand side behavior of the energy models.

The Use of Models of the Energy Sector

The models employed in the first EMF study included explicit representation of both the energy and nonenergy sectors of the economy, albeit at a very aggregate level. In partial equilibrium models of the energy sector, explicit representation of the nonenergy sectors is dropped, which allows a more disaggregated representation of the energy sectors to become computationally feasible. Further, according to the simple model of energy-economy interactions there is an approximate equivalence (in absolute value) between the elasticity of substitution and the price elasticity for primary energy in the partial equilibrium models. Thus, it may be possible to exploit the greater detail of the energy sector models to reduce the uncertainty about σ embodied in the results of the first EMF study. The alternative methods for obtaining estimates of the price elasticity for primary energy from partial equilibrium models of the energy sector are explored in the following section.

APPROACHES TO THE MEASUREMENT OF THE AGGREGATE ENERGY PRICE ELASTICITY

Many large scale energy sector demand models [e.g., 3, 4, and 5] contain constant* price elasticity representations of the demand for energy products. This suggests the possibility of deriving aggregate price elasticity estimates by direct analytical transformations and aggregations of the disaggregated elasticities built into the detailed models. This method is not without its shortcomings. An alternative approach calls for the direct numerical simulation of the detailed models to compute the substitution parameters. Both methods, analytical and simulation, are discussed below.

The Analytical Approach

The analytical approach is based on the observation that given the prices and corresponding quantities demanded for all the energy products represented in an energy sector model, as well as any price elasticities built into the model, it is possible in principle to calculate the price elasticity for any energy product represented in that model. The analytical approach will first be described for the simple case of the flow of a single energy product through the energy system and then for the multiproduct case.

*In these models, the elasticities are assumed to be constant for ease of estimation using sparse data.

Consider the flow of a single energy product through the energy system. Suppose that in one model the demand for this product is measured at two different points, points 1 and 2, respectively, in the sequence of processing, refining, and transportation.* The price calculated at point 2, p_2, will differ from the price calculated at point 1, p_1, by the cost, c, of supplying one unit of energy to that point from the required number of units at point 1. Further, if we let τ represent the thermal energy content of the fuel delivered to point 2 per unit of thermal energy content supplied at point 1, the prices and quantities at the two points can be related as

$$p_1 = (p_2 - c)\tau$$

and

$$q_1 = q_2/\tau .$$

Then, if one assumed a constant own-price elasticity of demand, η_2, measured at point 2, this will induce a nonconstant own-price elasticity of demand, η_1, at point 1 as follows:

$$\eta_1 = \frac{p_1}{q_1}\frac{\partial q_1}{\partial p_1} = \frac{(p_2 - c)\tau}{q_2/\tau}\frac{\partial(q_2/\tau)}{\partial(p_2 - c)\tau}$$

$$= \frac{(p_2 - c)}{q_2}\frac{\partial q_2}{\partial p_2} = \left(\frac{p_2 - c}{p_2}\;\frac{p_2}{q_2}\frac{\partial q_2}{\partial p_2}\right) = \left(\frac{p_2 - c}{p_2}\right)\eta_2 . \qquad (2\text{-}12)$$

Thus, when the conversion/transportation cost, c, represents a significant part of the price p_2, there is a large change in the elasticities. For example, the Nordhaus [4] model includes an own-price elasticity of -1.28 for energy used in the transportation sector measured at the point of end-use (i.e., axle of the vehicle). This seems quite high, but when transformed to the refinery gate,† an own-price elasticity for transportation energy in the form of gasoline at the refinery gate of only -0.64 is obtained.

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*For example, point 1 might be at wholesale before distribution to energy consumers, while point 2 might be at retail after distribution to energy consumers.

†Using the actual prices that are projected by that model for the MRG base case scenario in 2010.

In the general case, where the model includes multiple products at each point in the energy transportation/conversion system, the task of computing aggregate elasticities at a specified point is considerably more difficult. Not only will there be an own-price elasticity for each energy product, but also cross-price elasticities between each pair of energy products to consider. Even if the matrix of own- and cross-price elasticities, N, is specified at the point in the energy system where the aggregate elasticity estimate is desired, an aggregation rule for the individual energy product elasticities must be specified. For example, the aggregation rule at point i can be expressed as

$$\eta_i = \eta_i(N_i^{11}, N_i^{12} \ldots, N^{M_i,M_i}, p_i^1 \ldots, p_i^{M_i}) = \eta_i(N_i;p_i) , \qquad (2\text{-}13)$$

where

η_i = the aggregate price elasticity at point i;

N_i^{jk} = the price elasticity of the demand for the jth energy product measured at point i with respect to the price of the kth energy product measured at point i in the energy system;

N_i = the matrix of energy product price elasticities measured at point i in the energy system;

p_i^j = the price of the jth energy product measured at point i in the energy system;

p_i = the vector of energy product prices measured at point i in the energy system; and

M_i = the number of energy products measured at point i in the energy system.

Now, if in addition the individual energy product price elasticities are measured at a different point in the energy system than where the aggregate price elasticity estimate is desired, the whole vector of price elasticities must first be transformed to that point. The multiproduct point of measurement formula analogous to the single product transformation formula Eq. 2-12 can be written as

$$N_1(C, p_2) = A(C, p_2)N_2 , \qquad (2\text{-}14)$$

where

$C = (C^{mn})$ = the matrix of costs for providing one unit of the nth energy product measured at point 2 from the required number of units of the mth energy product measured at point 1 and

$A(C, p_2)$ = a matrix of functions that transforms the matrix of elasticities measured at point 2, N_2, to a matrix of elasticities, N_1, measured at point 1.

Thus, it is theoretically possible to implement the analytical approach in the multiproduct case. However, as argued below, the data and computational requirements for this method can exceed those required to solve the model itself. The matrix of transformation functions, A, is likely to be quite nonlinear and discontinuous.

Problems with the Analytical Approach

There are complexities caused by definitional differences among energy models. Energy models differ in the collection of energy commodities a particular elasticity refers to and where the quantities demanded and prices for these products are measured. For example, nonfossil primary energy fuel inputs are measured alternatively (1) at their fossil fuel replacement values, (2) at their natural energy flow values, and (3) not at all. Additionally, in some energy models, demands are specified by geographic region [11], while in others they are aggregated nationally [12]. Furthermore, in some models, the elasticities are specified in terms of average prices, while in others, they are specified in terms of marginal prices. Also, different aggregating schemes are sometimes used in defining the aggregate elasticities built into the models. Finally, in some models, short-run elasticities are specified, while in others, only long-run elasticities are included. For those models that include both short-run and long-run price elasticity effects, the lag structure built into the model complicates the application of the analytical approach still further. In this case, the individual energy product elasticities depend on previous prices for energy products and both the aggregate elasticity calculation, Eq. 2-13, and the elasticity transformation relation, Eq. 2-14, depend on previous as well as current energy prices and elasticities. The analytical approach is still conceptually feasible in this case, but the data and computational requirements are even larger than for the case with no lag structure. Further, differences in the way energy product demands and their elasticities are specified in the various energy sector models results in the need for more data (for instance the transportation/conversion markups, C) to implement the analytical approach than is typically contained in the model documentation.

A major problem one encounters in attempting to compare elasticities arises when the energy products they refer to are aggregated differently. In this case, the aggregation rule, Eq. 2-13, will have a different set of arguments for each model. Thus, the need arises to insure the consistency of the individual aggregation rules.

When the energy products are measured at different points in the energy system as well as aggregated differently, analytical elasticity comparisons become even harder to make. Not only do the elasticities require aggregation as per Eq. 2-13, but also transformation as per Eq. 2-14. The data requirements for these calculations can be substantial. Further, the computational burden could easily exceed that required to solve the original model. For example, in order to construct the matrix A in Eq. 2-14, the prices and elasticities at point 2 would first have to be used to compute demands at that point. Then the conversion/transportation costs would also have to be considered in computing how much of each product at point 1 gets converted to each of the products at point 2. Implemented in this manner the calculations could be difficult to manage. The energy models, in fact, solve the problem of how much of each product gets sent where as the key output of their overall solution methodologies. This ordinarily involves an algorithm or simulation approach which exploits the problem structure to simplify the calculations. Thus, the analytical approach to the computation of aggregate elasticities could involve more computational effort than required to solve the detailed model itself. It seems clear then that the same computational approach used for the solution of the model might be followed in estimating the aggregate elasticities. The model's own structure performs the transformations and aggregations, often implicitly. Only results required for the measurement of the total embedded substitution need be reported. The design of such an experiment is the subject of the following section.

An Alternative Approach to Measuring/Comparing Elasticities

An alternative approach to the problem of measuring/comparing elasticities implicit in energy models that circumvents some, but not all, of the difficulties with the analytical approach is the use of simulation and numerical differentiation. As in the case of the analytical approach, the principal motivation for this approach is to "model the models" in order to more fully understand their demand-side differences and the implications of these differences for model validity and energy policy analysis. Although, in general, the elasticities built into the various energy sector models are applied at different points in their representations of the energy system, prices and corresponding quantities demanded are, or could be, reported at any point within the system. This eliminates the need to perform the difficult point of measurement transformations represented in the relations, Eq. 2-14, of the analytical approach. Additionally, one can then compute an aggregate price and quantity for primary energy directly using aggregation rules for these entities rather than using aggregation rules for the component elasticities as per Eq. 2-13.* Then by varying the component primary energy input fuel prices and observing the corresponding changes in the aggregate demand, an estimate of the aggregate price-elasticity for primary energy can be obtained. Users of these models invariably think of them in these aggregate terms. Some try to approximate aggregate elasticities, but from very incomplete scenario results and not generally in a manner consistent with similar calculations by other researchers. The proposal here is to formalize this process and put it on a consistent basis.

In the simulation approach, a consistent set of definitions for the individual primary energy input fuels and their prices is provided. Further, since the time paths for the input fuel prices are inputs to the experiment, a stable long-run price trajectory can be selected, and prices and quantities requested far enough into the future so that only long-run elasticity estimates are obtained. Thus, many of the problems with the analytical approach caused solely by definitional differences can be avoided in the simulation approach. Finally, in the simulation approach, consistent aggregation rules for, and definitions of, the primary energy input fuels and their prices are specified. This insures the comparability of the aggregate elasticity estimates generated by the various models.

A schematic of the process of generating aggregate primary energy price elasticity estimates via simulation is shown in Figure 2-4. An energy sector model is first run for some base case scenario. The primary energy fuel inputs and prices for that scenario are used to compute an aggregate price and quantity for primary energy using a prespecified aggregation rule. Then, the primary energy input fuel prices are changed and the aggregation calculation is repeated. This generates a set of aggregate prices and quantities that can be used to estimate the price elasticity

*Technically, one form for the aggregation rule in the analytical approach would be to compute a base aggregate price and quantity for primary energy and then change the individual primary energy input fuel prices and compute the resulting primary energy input fuel demands using the individual energy input fuel price elasticities. The input fuel prices and quantities could be aggregated to allow the aggregate price elasticity to be estimated from the two aggregate price and quantity pairs. This is quite similar to the simulation approach, and could eliminate the need to associate a particular composition for the price increase with each aggregate price elasticity estimate.

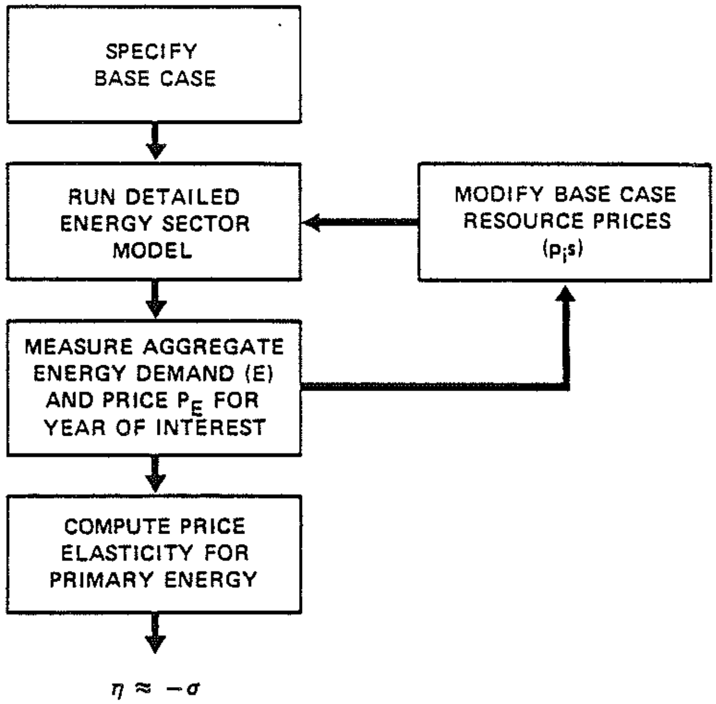

Figure 2-4 Overview of Proposed Methodology

for aggregate primary energy.* This process, using models to generate data to be used in elasticity estimation, is quite similar to the methodology used by Griffin [13] to estimate a production function for electricity generation using a process model.

Historical changes in the contribution of the individual primary input fuel prices and quantities to their aggregates may make it difficult to attempt to estimate elasticities statistically directly on the basis of aggregate data. But the observed aggregate elasticities for the models are the result of perfect experiments involving no measurement error and in which the modeler controls the composition of the aggregate price increases. The generated data may propagate errors of real world measurement embedded in a model's coefficients, but they measure precisely

*Actually, according to the simple model of energy-economy interactions, the elasticity of substitution can be estimated directly using Eq. 2-7 and the aggregate price elasticity can be estimated directly using Eq. 2-8.

what the model says about the real world. The proposal is to measure what the models say about the real world, but in the aggregate. To the extent that the more detailed structures of the models approximate the real world, so will these aggregate elasticity estimates.

APPROACHES TO THE MEASUREMENT OF AGGREGATE PRIMARY ENERGY AND ITS PRICE

The goal of this proposed study is the estimation of two aggregate parameters: the aggregate elasticity of substitution (σ) and the aggregate price elasticity (η). These estimations require data in the form of alternative projections of aggregate primary energy (E) and its price (P_E). However, aggregate primary energy is neither marketed in the real world nor treated explicitly in the models. Consequently, in order to implement any scheme for measuring the price elasticity for primary energy implicit in energy sector models via simulation, it is necessary to have some scheme for aggregating the prices and quantities of the individual primary energy inputs projected by the models. This section first develops a set of definitions for aggregate primary energy input. This requires aggregation of several primary energy inputs defined in the models. Next, desirable properties of aggregation rules for the more heterogeneous primary energy input fuels are identified. Then two approaches to this aggregation problem are considered: (1) estimating a production function for aggregate primary energy, and (2) the use of price and quantity indexes for primary energy. Finally, the proper points of measurement for the individual products that make up the aggregate are considered.

Defining the Primary Energy Input Fuels

The energy sector models differ significantly in the level of aggregation of the primary energy inputs included. Especially bothersome are the inclusion in the models of a variety of new technologies (i.e., geothermal, solar, etc.) and diverse regional disaggregation schemes.

The energy sector models employ various levels of aggregation for primary energy inputs. Thus, standardized definitions of primary energy input fuels must involve some partial aggregation of the primary energy inputs specified in many of the models. It is technically possible to check for the separability of the primary energy inputs for the case of consumer preferences, within a proposed input fuel aggregate from those in the other proposed aggregates, by using tests analogous to those described in Jorgenson and Lau [14]. One could determine aggregation rules for the inputs within each aggregate as well. In practice, however, the form of the required tests would vary among models and require a substantial number of simulation runs from each of the modeling systems. Furthermore, since many of the primary energy inputs represented in the models are relatively homogeneous, this detailed aggregation may be unnecessary. For example, since natural gas and oil are nearly perfect substitutes in most end uses, their quantities should be approximately additive in a production function for aggregate primary energy.

Thus, although any aggregation rule will be somewhat arbitrary, it is proposed here that the fuels be defined such that the primary energy inputs included within the definitions of each main fuel are relatively homogeneous. Aggregation rules are then required to combine the more heterogeneous set of partially aggregated primary energy inputs.

Despite the growing interest in new energy technologies, coal, oil, natural gas, and nuclear fuels will be the major primary energy inputs between now and the end of the century. Here, it is assumed that federal price controls (if any) will be imposed such that oil and natural gas sell at approximately Btu-equivalent prices. Additionally, following the Bureau of Mines, nuclear fuel is considered a primary energy input fuel. Further, because of their small likely impact between now and the end of the century and their similar capital requirements, the contribution of new technologies (e.g., solar, geothermal, etc.) will be lumped in with the nuclear fuel inputs. Thus, only three primary energy input fuels--coal, oil and natural gas, and nuclear fuel--are defined here and each of these is obtained by addition of the coal-equivalent Btus. The next subsection identifies some desirable properties for the formulation of aggregation rules to transform the prices and quantities of these three primary energy input fuels into a single price and single quantity for aggregate primary energy.

Desirable Properties for Aggregation Rules for Primary Energy and Its Price

Desirable properties for aggregation rules for primary energy and its price include invariance, homogeneity, and value equality. Specific aggregation rules may have additional properties, but at least this minimum set seems necessary for consistency.

Let $e = (e_1, \ldots, e_N)$ be a vector of quantities of individual primary energy fuel inputs and $p = (p_1, \ldots, p_N)$ a corresponding vector of primary energy fuel prices. The aggregation rule for E is said to exhibit invariance if the aggregate quantity E in a particular year can be expressed as a function of the vector of the quantities of individual primary energy input fuels demanded in that year. That is, $E = E(e)$. Similarly P_E is invariant if $P_E = P_E(p)$. Without this property the aggregation rules would not be unique.

Linear homogeneity means that if all components of the vectors of individual quantities or prices are multiplied by some constant, say λ, the resulting aggregate quantity or price will also be multiplied by the same constant λ. That is

$$E = \frac{1}{\lambda}(E\lambda e)$$

and

$$P_E = \frac{1}{\lambda} P_E(\lambda p) \ .$$

This implies that the energy sector as a whole exhibits neither increasing nor decreasing returns to scale in primary energy fuel inputs.

The value equivalence condition is an accounting relationship that requires that the sum of the value of the component primary energy input fuels equal the value of aggregate primary energy. That is

$$\sum_{i=1}^{m} p_i e_i = P_E \cdot E \ .$$

In the following sectors, two broad approaches to the problem of computing an aggregate quantity and price for primary energy are explored. One determines general aggregation rules for all possible component prices and quantities, while the other uses only observed values.

The Production Function Approach

One way to approach the primary energy aggregation problem is to postulate the existence of a production function for primary energy with the individual primary energy fuels being the inputs. Alternatively one can postulate a dual unit cost function with the price of aggregate primary energy represented as a function of the prices of the individual primary energy fuel input prices. This approach is used, for example, by Hudson and Jorgenson [15] as part of their energy-economy model of the U.S. The elegant theory of production functions and dual cost functions is collected in Diewert [16].

The approach described here employs the framework of a partial equilibrium analysis of an aggregate production function. The relationship between the primary energy fuel inputs $e_1, \ldots, e_N$ and the aggregate primary energy output E is assumed to be determined by a production function, $E(e)$, defined on the positive orthant to be positive, concave, and homogeneous of degree one, i.e.,

$$E = E(e), \quad e = e_1, \ldots, e_N ;$$

$$E(e) > 0 \quad \text{for} \quad e > 0 ;$$

$$E(\lambda e^1 + (1 - \lambda)e^2) \geq \lambda E(e^1) + (1 - \lambda)E(e^2) \quad \text{for} \quad \lambda \, \varepsilon[0,1] ;$$

and

$$E(\alpha e) = \alpha E(e) \quad \text{for} \quad \alpha \, \varepsilon[0,\infty] .$$

If p_i is the price of e_i and the producer is in competitive equilibrium, then there exists a corresponding cost function

$$C(E, p) = \text{Min}[pe | e > 0, E(e) = E] .$$

The cost function is positive on the positive orthant, concave and homogeneous of degree one in p, and homogeneous of degree one in E. Interpreting the unit cost $C(1, p)$ as the price of aggregate energy, P_E, we get

$$P_E(p) = \text{Min}[pe | e > 0, E(e) = 1] .$$

To implement the production function approach, an appropriate functional form for the production function for primary energy or the unit cost function must be selected.

For the current application, the transcendental logarithmic functional form [17,18, 19] has many attractive properties. Neither aggregate quantities nor prices are available directly from the model runs. However, the share of each primary energy input fuel in the total value of all primary energy input fuels, is observable. For a transcendental logarithmic production function, equations which represent the observable value shares as logarithmic functions of the input fuel quantities (prices) can be derived. Further, these equations are linear in the parameters of the production function to be estimated. Additionally, the translog can provide a local second order approximation to any production function.

Transcendental logarithmic functions are quadratic in the logarithms of the independent variables. Such functions have many interesting statistical and theoretical properties [16,17,18,19]. Either a translog production function or a translog unit cost function for aggregate primary energy can be estimated; one can not, however, be inferred from the other because the translog functional form is not self-dual [16]. However, by the value equivalence property, the remaining aggregate can be calculated as the total value of the inputs divided by the aggregate for which the estimation has been made.

The translog unit cost function for primary energy may be written as:

$$\ell n\ P_E = \alpha_0 + \sum_{i=1}^{N} \alpha_i\ \ell n\ p_i + 1/2 \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij}\ \ell n\ p_i\ \ell n\ p_j\ , \qquad (2\text{-}15)$$

where we require that

$$\sum_{i=1}^{N} \alpha_i = 1\ ,\ \gamma_{ij} = \gamma_{ji}\ ,\ \sum_{j=1}^{N} \gamma_{ij} = 0\ ,\ \forall_i\ .$$

The conditions on the parameters are equivalent to imposing linear homogeneity and value equivalence on the unit cost function. The optimality conditions for this functional form for the unit cost function imply the following set of value share equations:

$$\frac{p_i e_i}{\sum_{j=1}^{N} p_j e_j} = \alpha_i + \sum_j \gamma_{ij}\ \ell n\ p_j\ ,\quad i = 1, \ldots, N-1\ . \qquad (2\text{-}16)$$

Once the parameters of these equations have been estimated (including α_0) Eq. 2-15 can be used to compute a price for aggregate primary energy based on the prices of the individual primary energy input fuels. The quantity of aggregate primary energy can then be computed as

$$E(e) = \frac{\sum_{j=1}^{N} p_j e_j}{P_E(p)} \qquad (2\text{-}17)$$

by the value equivalence property.

The restrictions on the parameters in Eq. 2-15 reduce the statistical estimation problem considerably. In the absence of these restrictions for the case of three primary energy input fuels, 12 parameters would have to be estimated, whereas taking account of the restrictions only 5 independent parameters need be estimated.

The production function approach has the advantage that several goodness of fit tests of the translog approximation are possible (for details see [13]). These include testing the symmetry conditions, which is equivalent to testing for linear homogeneity, evaluating the Hessian matrix for the base case set of prices, and checking it for negative semidefiniteness, which is equivalent to checking for concavity. Finally, elasticities of substitution and cross-price elasticities between the individual primary energy input fuels and own-price elasticities for these fuels can be computed and checked for consistency. However, to adequately estimate even the five parameters required in the three inputs case, as many as 15 or 20 simulation runs with the energy sector models could be required.

The Index Number Approach

The production/unit cost function approach to the problem of computing the price and quantity of aggregate primary energy requires and generates far more information than required to determine the price elasticity for aggregate primary energy. A more economical and direct approach is sought. The theory of economic index numbers provides a shortcut for determining the aggregates directly from their components without having to statistically estimate the full aggregation rules. Index numbers provide an approximation to the production function and, in the case of the translog function, this approximation can be exact. The use of index numbers provides a quick estimate of the aggregate values, but sacrifices the information used in displaying cross-price effects.

Redefining the economic price and quantity indexes for the cost of living problem [20] to the case of a production function for primary energy yields the following:

> Quantity index—$\hat{e}(e^1, e^0; p^\alpha)$. This measures for two presented primary energy input situations, e^0 and e^1, the ratio of the minimum costs required to produce their respective levels of aggregate primary energy in the face of a reference primary energy price situation, p^α.

Price index--$\hat{p}(p^1, p^0; e^a)$. This measures for two presented primary energy price situations, p^0 and p^1, the ratio of the minimum costs required to produce the level of aggregate energy corresponding to a given primary energy input situation, e^a.

Further, an expenditure function for primary energy can be defined as the minimum expenditure required in primary energy price situation p^j to provide the amount of aggregate primary energy that can be produced from a vector e^k of primary energy input fuels as follows:

$$C(p^j, e^k) = \min_e \ p^j e$$

$$\text{s.t.} \quad E(e^k) = E(e) \ .$$

Then, we can express the price and quantity indexes for aggregate primary energy as ratios of these expenditure functions. For example,

$$\hat{e}(e^1, e^0; p^1) = \frac{C(p^1; e^1)}{C(p^1; e^0)}$$

and

$$\hat{p}(p^1, p^0; e^1) = \frac{C(p^1; e^1)}{C(p^0; e^1)} \ .$$

Samuelson and Swamy [20] have shown that a necessary and sufficient condition for these indexes to be invariant is for the function E to be homothetic (this is equivalent to assuming P_E is homothetic). In this case,

$$\hat{e}(e^1, e^0; p^\alpha) = \hat{e}(e^1, e^0) \quad \forall\alpha$$

and

$$\hat{p}(p^1, p^0; e^a) = \hat{p}(p^1, p^0) \quad \forall a \ .$$

In fact, in this case, we may relate the indexes defined in this subsection to the production and unit cost functions defined in the previous subsection as follows:

$$\hat{e}(e^1, e^0) = \frac{E(e^1)}{E(e^0)} ,$$

and

$$\hat{p}(e^1; e^0) = \frac{P_E(e^1)}{P_E(e^0)} .$$

As an aid to practical applications of this theory, Samuelson and Swamy derive relations for the precision with which certain index formulae approximate the true index numbers. Additionally, Diewert [21] has shown that certain index number formulae are exact for certain classes of functional forms. Lau [22] has demonstrated that the Tornquist [23] index number is exact for the class of homogeneous of degree one transcendental logarithmic functions. This is the functional form suggested for P_E in the approach described in the previous section. Consequently, if it is believed a priori that the translog gives a good fit to the true production function, aggregate prices for primary energy can be computed by using the following Tornquist formula for an aggregate primary energy index directly without first having to estimate a translog unit cost function

$$\ln \frac{P_E(p^1)}{P_E(p^0)} = \frac{1}{2} \sum_{i=1}^{N} \left[\frac{p_i^0 e_i^0}{\sum_j p_j^0 e_j^0} + \frac{p_i^1 e_i^1}{\sum_j p_j^1 e_j^1} \right] (\ln p_i^1 - \ln p_i^0) . \qquad (2\text{-}18)$$

Thus, the price index for any primary energy price situation, say p^1, relative to some base year primary energy price-quantity situation, say that which prevailed in 1972, can be computed.

By using the index number approach, we avoid having to estimate the translog unit cost function for primary energy, but we lose the capability of testing for the goodness of fit of the translog approximation. About the only test we can perform with the index number data is to test for homotheticity by checking for transitivity. For example, for three primary energy input situations 0, 1, and 2, we could check to see whether

$$\frac{P_E(p^2)}{P_E(p^0)} = \frac{P_E(p^2)}{P_E(p^1)} \frac{P_E(p^1)}{P_E(p^0)} .$$

The Point of Measurement Question

There are two generic points within the energy system where total energy has typically been measured: (1) before electric and synfuels conversion losses, and (2) after these losses have been netted out. The Bureau of Mines [24] has historically referred to energy measured at (1) as gross energy inputs and at (2) as net energy inputs. Hence, to be consistent with the analytical structure developed in this proposal, they are referred to as primary and secondary energy, respectively.

Since a majority of researchers do, in fact, report total energy before conversion losses and because this is the point of measurement employed in the simple model of energy-economy interactions described above, the "Aggregate Elasticity of Energy Demand" experiment is designed to produce estimates of the price elasticity for primary energy implicit in the energy sector models. As a by-product of this experiment, however, enough data to estimate the secondary energy price elasticity implicit in the models can probably be collected. For many reasons, primarily proximity to the point of actual choice of use, this secondary measure may in the long run be more valuable. In any event, it will come as a free by-product. A comparison of elasticity estimates measured at the two points and post hoc checks for consistency of the two estimates can be pursued.

MEASURING THE AGGREGATE ELASTICITIES

This section describes how the aggregate price (P_E) and quantity (E) data generated by inputting the models outputs into the aggregation rules described in the previous section can be used to estimate the aggregate elasticity parameters. Equation 2-7 is the basis for the estimation of the elasticity of substitution (σ) using the aggregate price and quantity data generated by the energy models. For estimation purposes this equation can be rewritten as

$$\ln \frac{E}{Y} = B + \sigma \ln P_E , \qquad (2\text{-}19)$$

where B is a constant.

Similarly, the equation used to estimate the price elasticity for primary energy implicit in the energy models is Eq. 2-8, which can be rewritten as

$$\ln E = A - \eta \ln P_E , \qquad (2\text{-}20)$$

where A is a constant that includes predominantly income effects for the year in question.

For economy of exposition, only the proposed estimation for the price elasticity for primary energy (η) using Eq. 2-20 is described here. Exactly the same procedures, but with E/Y replacing E, will be employed to estimate the elasticity of substitution (σ) directly using Eq. 2-19. The elasticity estimates should be virtually equivalent in magnitude.

Only two basic approaches to the measurement of the price elasticity for primary energy seem reasonable: (1) computing arc elasticities between pairs of P_E and E observations, and (2) estimating a constant price elasticity demand function for primary energy using all the P_E and E data. A simple plot of all the P_E and E observations may help determine which approach to try.

If the experiment is well designed, a simple plot of all the P_E and E data should also give some idea of whether differences in the composition of the aggregate price are important. That is, it is possible that different sets of primary energy input fuel prices that lead to the same P_E lead to different values of total primary energy $P_E \cdot E$ and, thus, different E's for a given P_E. This would mean that the aggregate unit cost function is not an adequate approximation over the range of primary energy input fuel prices considered and that, consequently, each elasticity estimate would have to have associated with it a statement about the composition of the associated price increase. This result would still be useful since most models project fairly consistent patterns of price increases for all but the most extreme scenarios, but it would, of course, limit the generality of the results of the study. If, on the contrary, the unit cost function for aggregate primary energy is exact, it will capture all of these compositional effects.

Arc Elasticity Estimates

The arc elasticity between any two observations (P_E^1, E^1) and (P_E^2, E^2) can be expressed as

$$\eta_{arc} = \frac{E^2 - E^1}{P_E^2 - P_E^1} \frac{P_E^1 + P_E^2}{E^1 + E^2} . \qquad (2\text{-}21)$$

Thus, one could track a variable elasticity demand curve for aggregate primary energy given enough observations.

Constant Elasticity Estimations

It is also possible to use all the data to fit a constant elasticity demand curve like Eq. 2-20

$$\ln E = A - \eta \ln P_E \; . \qquad (2\text{-}20)$$

Here use will be made of all the P_E and E observations.

PROPOSED EXPERIMENT FOR MEASURING THE PRICE ELASTICITY FOR AGGREGATE PRIMARY ENERGY

This section describes an experiment designed to provide the raw data required to measure the price elasticity for aggregate primary energy implicit in models of the energy sector. Given sufficient time and effort, enough scenarios could be run with the energy models to enable all the calculations described thus far in this chapter to be pursued individually. As a practical matter, however, implementing the scenarios is a time consuming activity. Therefore, based partly on past experience [2,10], this proposal is limited to a request for nine scenarios. Consequently, the precise sequence of calculations to be performed upon the completion of the experiment will depend to some extent on the raw results obtained.

The Experiment

Except for primary energy input fuel prices, which are manipulated during the experiment, all energy sector assumptions are suggested from the base case used by the MRG [2]. These assumptions are shown here in Table 2-1. The price levels of oil and gas, coal, and nuclear fuel for the Reference case (the nominal price levels) are set in accordance with anticipated depletion effects. Oil and gas are assumed to cost \$2.00 per million Btu in 1975 and to increase 2% per year through 2010; coal is assumed to cost $\$0.75/10^6$ Btu and to increase 1% per year through 2010; and nuclear fuel is assumed to cost $\$0.40/10^6$ Btu in 1975 and to increase 1% per year through 2010.

The alternatives to the Reference case are designed to explore both changes in the composition of the aggregate price around the nominal price levels and changes in the aggregate price level. The primary energy input prices are varied up and down 25% in each year; this yields three price levels for each primary energy

Table 2-1

REFERENCE CASE ASSUMPTIONS
(1975$)

		Reference Case
1. Growth rate of real GNP, % per annum at constant 1970 energy prices:		
	1975–1990	4.08
	1990–2000	2.84
	2000–2010	2.48
	2010–	1.80
2. Capital costs of electricity generating technologies ($/KW electric, in year 2000; all capital costs reported at standardized 65% capacity factors):		
a. coal-fired plant, including equipment for sulfur removal, at 36% efficiency		$ 520
b. light water reactor (LWR)		650
c. representative advanced converter reactor (ACR), 0.82 conversion ratio		715
d. fast breeder reactor (FBR)		810
e. solar produced electricity		1730
3. Differential technical change in the energy sector (% per annum) over that in the rest of the economy:		0
4. Total average cost of synthetic fuels, delivered ($/$10^6$Btu; "availability" defined as production capacity of 0.5 x 10^{15} Btu/yr):		
a. coal gasification, available in 1990		$2.95[a]
b. coal liquefaction, available in 1995		2.95[a]
5. Discount rates (% per annum):		
a. pre-tax (for use in projecting pricing and investment decisions in the energy sector)		13.0
b. post-tax (for discounting benefits and costs of alternative technology options)		6.0
6. Year of commercial availability, at cumulative installed capacity of 6 GW electric:		
a. ACR		2000
b. FBR		2000
c. central station solar		2000

[a]Net of coal costs; thermal efficiency for coal conversion assumed to be 0.65.

Table 2-1 (continued)

	Reference Case
7. Ceiling on the growth rate in capacity of new technologies (% per annum), after commercialization (cumulative installed capacity of 6 GW electric, or its equivalent)	
a. first five years after initial availability	40
b. second five years after initial availability	30
c. third five years after initial availability	20
d. beyond 15 years after initial availability	10

Table 2-2

PRIMARY ENERGY PRICE CASES

Case	Price Level		
	Oil & Gas	Coal	Other
1. Reference	Nominal	Nominal	Nominal
2.	High	Nominal	Nominal
3.	Nominal	High	Nominal
4.	Nominal	Nominal	High
5.	High	High	High
6.	Low	Nominal	Nominal
7.	Nominal	Low	Nominal
8.	Nominal	Nominal	Low
9.	Low	Low	Low

input. The price levels are varied from their nominal levels individually and collectively; this yields a total of nine primary energy price cases as shown in Table 2-2.

Ex Post Calculations

The first calculation performed with each models' raw results will be to calculate the Tornquist price index, Eq. 2-18, for primary energy and corresponding quantity index for each reported year and price case. These will then be plotted for a particular year, e.g., year 2000. The experiment was designed specifically to investigate the impact of the composition of the price change on changes in these aggregate indexes. Thus, at this point we should be able to decide on the plausibility of the index number approach. If the translog approximation to the unit cost function is exact, the index numbers will capture the compositional shifts. If not, the production function itself may have to be estimated.

However, even if the index number approach is exact, the orthogonal nature of the price variations in the experimental design may also allow for a rough estimation of a translog unit cost function for primary energy. Griffin [13] has pointed out that since share equations, Eq. 2-16, are used to estimate the coefficients for the unit cost function, Eq. 2-15, if the translog approximation is exact, prices can be varied one at a time in the estimation without any loss of generality.

Finally, although the experiment described above was designed in terms of price increases for primary energy, it should be possible to obtain enough information to repeat the entire sequence of calculations for secondary energy. Thus, the reporting forms in the appendix include requests for data on secondary energy demands and their prices.

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Appendix A

OUTPUT REPORTING FORMS

It is proposed that two output forms be filled out by each modeler to report the results of the experiment. In Output Form #1, primary energy demands and prices, measured before electric conversion and synthetic fuels production conversion losses, are requested. In Output Form #2, secondary energy demands and prices, where these conversion losses are netted out, are requested. Additionally, where possible, it is requested that the quantities for primary energy input fuels be disaggregated by ultimate consuming sector. It may be of interest to perform some of the calculations described in the last section on a sector by sector basis. The footnotes to the forms describe the accounting conventions to be used in measuring and aggregating primary energy input fuels.

Output Form #1

Primary Energy Input Fuel Price and Quantity Data

Model ______________________________

Price Case	Input Fuel[a]	1990 (15-year)					2000 (25-year)					2010 (35-year)				
		Price ($/$10^6$Btu)	R/C Demand (quads)	Ind. Demand (quads)	Trans. Demand (quads)	Total Demand (quads)	Price ($/$10^6$Btu)	R/C Demand (quads)	Ind. Demand (quads)	Trans. Demand (quads)	Total Demand (quads)	Price ($/$10^6$Btu)	R/C Demand (quads)	Ind. Demand (quads)	Trans. Demand (quads)	Total Demand (quads)
1.	oil & gas coal electricity															
2.	oil & gas coal electricity															
3.	oil & gas coal electricity															
4.	oil & gas coal electricity															
5.	oil & gas coal electricity															
6.	oil & gas coal electricity															
7.	oil & gas coal electricity															
8.	oil & gas coal electricity															
9.	oil & gas coal electricity															

[a] Quantity of oil measured at the refinery output; quantity of natural gas measured at the city gate; composite price measured at the refinery output. Net of coal used for electricity generation and synthetic fuels production; price measured delivered to industry. Electricity includes geothermal and solar heat. Quantity and price of electricity measured at the busbar. Primary energy equivalents for nonfossil energy sources that are used directly as thermal energy computed at 1.09 input Btu/end-use Btu. This assumes 100% efficiency for replacement of delivered electricity and accounts for 9% transmission loss.

Output Form #2

Secondary Energy Input Fuel Price and Quantity Data

Model____

Price Case	Input Fuel[a]	1990 (15-year)					2000 (25-year)					2010 (35-year)				
		Price ($/$10^6$Btu)	R/C Demand (quads)	Ind. Demand (quads)	Trans. Demand (quads)	Total Demand (quads)	Price ($/$10^6$Btu)	R/C Demand (quads)	Ind. Demand (quads)	Trans. Demand (quads)	Total Demand (quads)	Price ($/$10^6$Btu)	R/C Demand (quads)	Ind. Demand (quads)	Trans. Demand (quads)	Total Demand (quads)
1.	oil & gas coal other															
2.	oil & gas coal other															
3.	oil & gas coal other															
4.	oil & gas coal other															
5.	oil & gas coal other															
6.	oil & gas coal other															
7.	oil & gas coal other															
8.	oil & gas coal other															
9.	oil & gas coal other															

[a] Includes shale oil and biomass. Quantity of oil measured at the refinery input; quantity of natural gas measured at the city gate. Composite price measured at the refinery input. Quantity of coal measured after cleaning; price measured delivered to electric utilities. The "other" category includes nuclear, solar, geothermal and hydro electricity generation, as well as geothermal and solar heat. Primary energy equivalents (that is quantities) for nonfossil energy sources that are converted to electricity are computed as the amount of electric energy generated using the resource divided by the approximate thermal efficiency for electricity generation from fossil fuels equal to 0.36. Primary energy equivalents for nonfossil energy sources that are used directly as thermal energy are computed as the amount of fossil fuel energy replaced. This is assumed to be at the rate of (1/0.36)(1.09) = 3.03 input Btu/end-use Btu. This assumed 100% efficiency for replacement of delivered electricity, accounts for a 9% transmission loss, and converts to fossil input equivalent. In both cases, the price for the primary energy input is computed as the price of enough nuclear fuel to provide 10^6 Btu (293 kWh) of electricity at the busbar times a nuclear fuel-to-electricity conversion efficiency of 0.36.

Output Form #3

Gross National Product Data from the Energy-Economy Models

Model ______________________

GNP (1975\$ x 10^9)

Primary Energy Price Case		2000 (25-year)	
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			

Note: 1975 actual GNP was \$1,500 x 10^9.

Chapter 3

MODEL COMPARISON*

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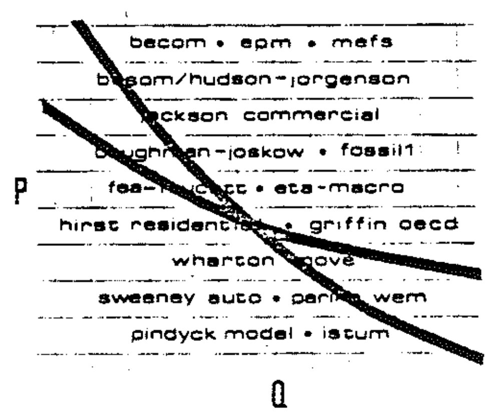


*This chapter formerly was Working Paper EMF 4.2.

†The authors would like to thank Adam Borison, Nancy Cimina, and Elizabeth MacRae for many helpful suggestions.

Chapter 3

MODEL COMPARISON

This chapter describes the models included in the EMF "Aggregate Elasticity of Energy Demand" study. A framework for comparison is constructed and used to describe each model. This chapter should provide sufficient knowledge of each model's structure to facilitate understanding the study's findings.

The following models were employed in the study:

<u>Energy-Economy Models</u>

Brookhaven Energy System Optimization Model/Hudson-Jorgenson (BESOM/H-J), Brookhaven National Laboratory and Dale Jorgenson Associates

Energy Technology Assessment-MACRO (ETA-MACRO), Alan Manne, Stanford University

Parikh Welfare Equilibrium Model (Parikh WEM), Shailendra Parikh, Stanford University

<u>Energy System Models</u>

Baughman-Joskow (Baughman-Joskow), Martin Baughman, University of Texas and Paul Joskow, Massachusetts Institute of Technology

Energy Policy Model (EPM), Lawrence Livermore Laboratory

FOSSIL1 (FOSSIL1), Dartmouth System Dynamics Group, Dartmouth College

Griffin Organization for Economic Cooperation and Development (Griffin OECD), James Griffin, University of Houston

Mid-range Energy Forecasting System (MEFS), U.S. Department of Energy

Pindyck International Study (Pindyck), Robert Pindyck, Massachusetts Institute of Technology

<u>Sectoral Models</u>

Buildings Energy Conservation Optimization Model (BECOM), Brookhaven National Laboratory

Federal Energy Administration-Faucett (FEA-Faucett), Carmen Difiglio and Damian Kulash, Federal Energy Administration

Industrial Sector Technology Use Model (ISTUM), Energy and Environmental Analysis, Inc.

Jackson Commercial (Jackson Commercial), Jerry Jackson, Oak Ridge National Laboratory

The ORNL Residential Energy-Use Model (Hirst Residential), Eric Hirst and Janet Carney, Oak Ridge National Laboratory

Sweeney Automobile Model (Sweeney Auto), James Sweeney, Stanford University

Wharton Motor Vehicle Model (Wharton MOVE), Wharton Econometric Forecasting Associates

Section 1

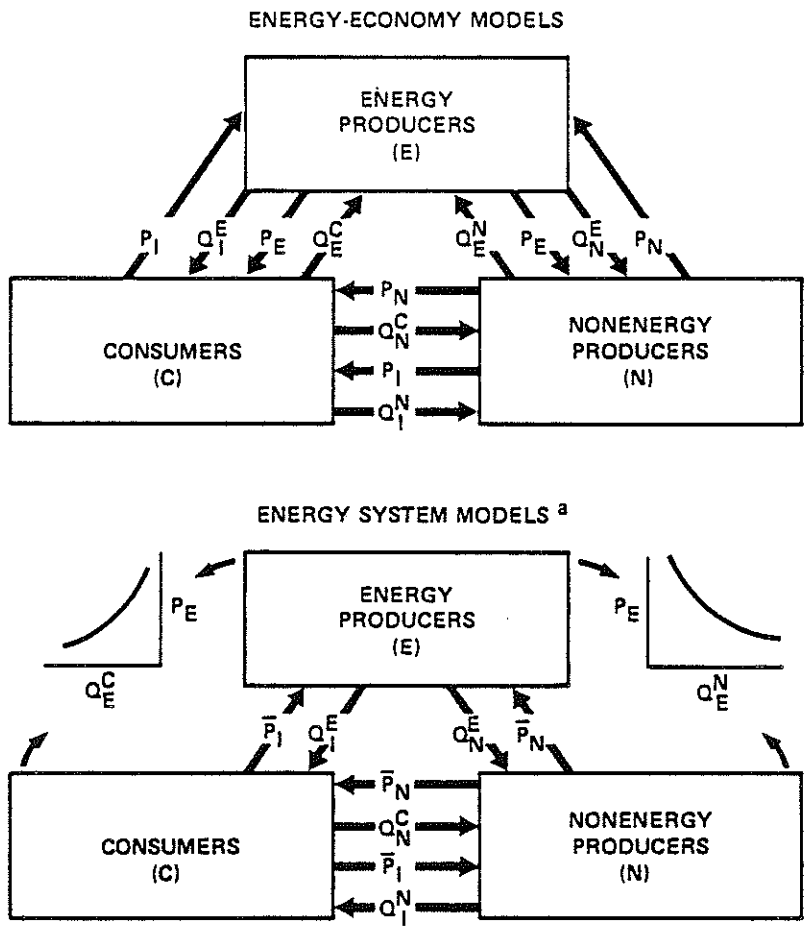
INTRODUCTION TO THE MODELS

MODEL SIMILARITIES AND DIFFERENCES

As shown above, the models vary in scope and include energy-economy, energy system, and sectoral models. While the models emphasize different aspects of energy demand, they overlap in their estimation of the relationships between energy demands and energy prices. These models are representative of the spectrum of available energy demand analysis tools and share many important features.

The energy system models represent the entire energy system, while the sectoral models focus on specific sectors of the energy system and, thus, resemble parts of the energy system models. The energy-economy models explicitly represent the influence of changes in the energy system upon the rest of the economy, while the energy system models do not.

Figure 3-1 contrasts the information flows in the energy-economy and energy system models. In the energy-economy models, all price factor input demands and product demands are solved for endogenously. In the energy system models, the prices of nonenergy products (P_N) and factor inputs (P_I) are assumed to be constant. Consequently, demand curve representations of the response of energy demands by consumers (Q_E^C) and nonenergy producers (Q_E^N) to changes in energy price (P_E) can be included. Furthermore, the energy-economy and energy system models typically employ different aggregate sectoral breakdowns in their energy demand projections; see Figure 3-2. The energy-economy models typically use Bureau of Economic Analysis (BEA) industrial classifications so that commercial energy and transportation demands, except gasoline for private automobiles, are all treated in an interindustry input-output framework. The energy system models, on the other hand, disaggregate demands according to Bureau of Mines (BOM) consuming sectors: residential, transportation, commercial, and industrial. However, many of those models aggregate some of the sectors; a residential/commercial combination is the most frequent aggregation.



[a] Barred variables are specified exogenously at constant level.

Key: P_i = Price set by i (i = E,C,N)

Q_i^j = Demand for output of i by j (j = E,C,N)

Figure 3-1 Information Flows in Energy-Economy and Energy System Models

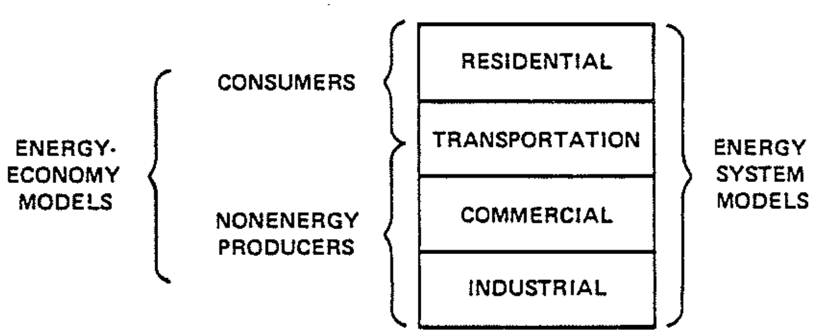


Figure 3-2 Comparison of Aggregate Sectoral Breakdowns in Energy-Economy and Energy System Models

Each model includes some mechanism for projecting future energy demands, and each examines some aspects of the role of prices in determining demands. Total energy demand can adjust in many ways, and the models focus on different system elements that contribute to the flexibility in energy demand, fixing some quantities, while determining others. At one extreme, the ISTUM and BECOM models hold constant all demands for energy services, solving for the specific equipment and fuels required to provide those energy services at the lowest cost. At the other extreme the Griffin OECD and Pindyck models implicitly allow all system adjustments to be captured in their parameter estimation.

Some models explicitly identify the precise channels through which the substitution occurs, using a detailed process representation (e.g., BECOM, Hirst Residential, and Jackson Commercial) or an aggregate production function representation of labor, capital, energy, and materials substitution (e.g., BESOM/H-J). Other models, such as Baughman-Joskow, Griffin OECD, and Pindyck, leave the substitution processes implicit.

The responsiveness of demand to price changes can be thought of as being composed of two components: interfuel substitution and factor substitution. Inclusion of interfuel substitution permits examination of those interactions where the choice of one form of energy or another is important. Inclusion of factor substitution permits representation of switches from energy to other factors of production. The degree of interfuel and interfactor substitution varies significantly among the participating models.

The interfuel and factor substitution possibilities are implemented with three mechanisms: (1) own-price elasticities, (2) cross-price elasticities, and (3) energy-GNP feedbacks. Demand curves with own-price elasticities represent society's willingness to pay for energy products in terms of nonenergy goods and services. They will lead to accurate predictions of energy product demands only if prices elsewhere in the economy are independent of prices in the energy sector. The energy-economy models explicitly consider these energy-GNP feedbacks. Inclusion of interfuel substitution via cross-price elasticities permits examination of those interactions where a conversion from one form of energy to another is important.

Varying also among the models is the degree of product and spatial aggregation. The level of spatial aggregation is important because many economic decisions can vary between regions, and the ability to transport energy between regions may be limited in some cases. The level of product aggregation can be important to the extent that the alternative energy products are not perfect substitutes. The significance of both issues is that the level of aggregation implicitly defines some of the substitution possibilities included in the models. Specifically, the commodities within a particular product or regional aggregate are implicitly considered to be perfect substitutes or perfect complements (i.e., used only in fixed proportions) for each other.

Since the study design specifies prices for primary energy and requires reporting of primary energy demand, the energy conversion activities included in a model are important determinants of its aggregate price elasticity for primary energy. Three energy conversion sectors are typically included in the models: (1) an electric utility sector, (2) an oil refining sector, and (3) a coal synthetics conversion sector. These are represented quite differently in the various models.

The models differ considerably in where they measure energy demands. They vary from ETA-MACRO, where demands for electricity are measured at the busbar and for nonelectric energy at the refinery gate, to BESOM/H-J where demands are measured at the point of end-use (e.g., the dwelling in the case of space heat and the axle in the case of automotive demands).

Slow adjustment of the capital stock of energy-using equipment, locational patterns, and behavioral patterns typifies energy demand. But some degree of foresight also is in evidence. Energy models can be either static or dynamic. Static models such as BESOM/H-J or Pindyck, do not explicitly incorporate either foresight or the slow adjustment processes. In contrast, dynamic models incorporate intertemporal effects within a multiperiod framework.

Two attributes may be used to describe the differences across the dynamic models: the treatment of expectations and the speed of adjustment. The expectations may be myopic, with current decisions determined entirely by current and past conditions, or clairvoyant, with current decisions based upon implicit complete knowledge of all past, present, and future prices. Most of the statistical models implicitly postulate myopic expectations, while intertemporal optimization models, such as BECOM, assume clairvoyant behavior. A model might permit instantaneous adjustment in variables resulting from exogenous shocks, or the adjustment may be gradual. In all the dynamic models, a key issue is the treatment of the stock of capital goods carried from one period to the next. Some models, such as FEA-Faucett, Hirst Residential, or Wharton MOVE, treat these capital adjustment processes explicitly, while others, such as Baughman-Joskow or Griffin OECD, leave these processes implicit in the model equations.

Besides differing in sectoral coverage, treatment of the relationship between the energy sector and the rest of the economy, and in several important structural characteristics, the models differ fundamentally in their approaches to parameter measurement. One or more of three basic approaches to parameter measurement are employed: (1) statistical estimation of the aggregate fuel and/or sector price response, (2) detailed engineering specifications of alternative energy-using technologies, and (3) judgmental estimation of the aggregate fuel and/or sector price response. Table 3-1 indicates the primary methods used for parameter estimation. Some models include a combination of approaches to parameter estimation. Of these, BESOM/H-J and ETA-MACRO are classified according to the parameter estimation methods used to develop the demand representation of the model. However, such a classification was not possible for the Hirst Residential and Jackson Commercial models since in these models, the statistical and engineering approaches were intertwined.

Table 3-1

PARAMETER ESTIMATION APPROACHES

<table>
<tr><th></th><th>Statistical</th><th>Engineering</th><th>Judgmental</th></tr>
<tr><td>Energy-Economy Models</td><td>BESOM/H-J</td><td></td><td>ETA-MACRO
Parikh WEM</td></tr>
<tr><td>Energy System Models</td><td>Baughman-Joskow[a]
Griffin OECD
MEFS
Pindyck</td><td></td><td>EPM
FOSSIL1</td></tr>
<tr><td rowspan="2">Sectoral Models</td><td>FEA-Faucett
Sweeney Auto
Wharton MOVE[b]</td><td>BECOM
ISTUM</td><td></td></tr>
<tr><td colspan="2">Hirst Residential[c]
Jackson Commercial[c]</td><td></td></tr>
</table>

[a]Excludes the transportation sector and industrial feedstocks.

[b]Only results from the automobile gasoline demand component were reported to the EMF.

[c]Combines both the statistical and engineering approach.

FRAMEWORK FOR COMPARISON

As stated above, the models all include some mechanism for projecting future energy demands. However, the level of product and spatial disaggregation varies significantly among the various models. In the following sections, the models are described according to their supply, conversion, and demand activities. Within each of these groups, characteristics, such as spatial and product aggregation, substitution possibilities, dynamics, and point of measurement are discussed.

Section 2

ENERGY-ECONOMY MODELS

The energy-economy models include consideration of energy demands within a general equilibrium framework. Thus, interactions between the energy sector and the remainder of the economy are explicitly represented. These models typically use an optimal growth model to integrate: (1) an interindustry input/output organization of the economy's production sector, (2) a detailed process description of the energy sector, and (3) a consumer demand module. However, the models differ substantially in their level of aggregation, substitution possibilities, and their representation of the dynamics of energy supply and demand. The energy-economy models included in the study are BESOM/H-J, ETA-MACRO, and Parikh WEM. All of these models are nationally aggregated. Table 3-2 provides a summary comparison of the energy-economy models.

BESOM/HUDSON-JORGENSON [1]

The full implementation of the linked BESOM/H-J system is achieved through information transfers between the Hudson-Jorgenson model and the Brookhaven National Laboratory static Energy System Optimization Model (BESOM, Figure 3-3). The econometric representation of producer and consumer behavior from the Hudson-Jorgenson model is combined with the detailed engineering representation of the energy sector from BESOM. The Hudson-Jorgenson model provides information on energy demand and economic effects, but not energy supply. BESOM supplies information on energy supply, conversion, and end-use technologies, but not energy demand. Therefore, the models interface at the point of energy demand. The complete integrated system was not used in this experiment. Rather, summary representations of the BESOM/H-J interactions, based on previous runs of the linked system, were used to modify BESOM's exogenously specified energy demands.

Supply

Hydroelectric, geothermal, solar, and nuclear energy are specified exogenously. Oil, gas, and coal are endogenously modeled.

Table 3-2

SUMMARY COMPARISON OF ENERGY-ECONOMY MODELS

Model	Supply		Conversion			Demand				
	Product Aggregation	Spatial Aggregation	Electric Utilities	Oil Refinery	Synfuels Conversion	Spatial Aggregation	Product Aggregation	Point of Measurement	Substitution Possibilities	Dynamics
BESOM/H-J	oil, gas, coal, uranium	U.S. total	explicit process model	no explicit model; refinery losses considered	represented in terms of price	U.S. total	end-use	end-use	own-price elasticities GNP/energy elasticities	static
ETA-MACRO	oil and gas, coal, uranium	U.S. total	explicit process model	not considered	represented in terms of price	U.S. total	electric energy nonelectric energy	busbar refinery gate	own- and cross-price elasticities; energy-economy interactions through aggregate production function	constant retirement rate for utilizing capital stock
Parikh WEM	oil, gas, coal, uranium	U.S. total	explicit process model	variable coefficient I/O sector	explicit process model	U.S. total	consumers and industry by fuel	input to end-use device	hierarchical homothetic demand structure including pairwise substitution elasticities	short- and long-run substitution elasticities

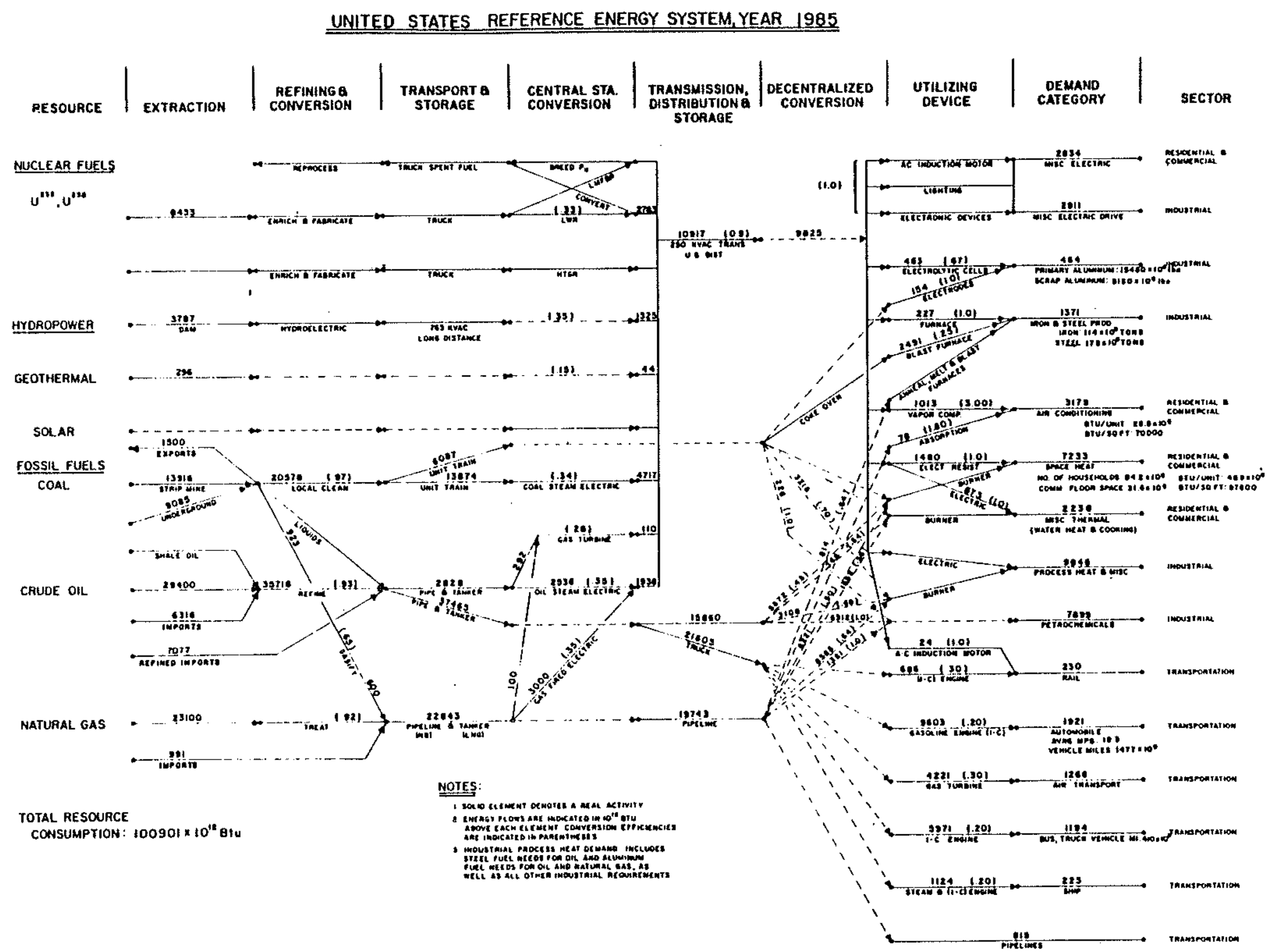


Figure 3-3 BESOM Flow Diagram

Conversion

Although refinery losses are considered, the model does not contain an explicit refinery model. Synthetic fuels are given a process representation. The model does, however, include a detailed nationally aggregated model of the electricity generation sector. The load profile of electricity demands is an endogenous function of the load characteristics of end-use demands that are satisfied by electricity.

Demand

In BESOM, end-use demands for 15 to 20 energy end-use products (e.g., space heat) in 3 consuming sectors, residential/commercial, industrial, and transportation, are considered. In the original BESOM, demands for these end-use products are specified exogenously. In the combined BESOM/H-J system, they are determined endogenously. For the current study, price elasticities and energy-economy elasticities estimated from previous runs of the combined model are used to modify the end-use demands from their base values. This was accomplished in the following manner: (1) a price index for end-use energy is calculated using prices from BESOM and Reference case quantities as weights, (2) a price elasticity for aggregate energy estimated from previous combined model runs is used to calculate the change in aggregate energy demand resulting from the price change, (3) aggregate energy-economy elasticities, also estimated from previous runs of the combined model, are used to calculate the changes in the level and structure of economic activity (output and input patterns) due to the change in aggregate energy use, (4) these changes are used to adjust the end-use demands in BESOM, and finally, BESOM is rerun with the new end-use demands. The procedure is truncated at this point; no attempt was made to revise the demands again in response to the new prices calculated by BESOM. Since BESOM is a linear program in which alternative fuels compete for the end-use demands on a least-cost basis, the cross-price elasticities between fuels are characteristically discontinuous. In the version of BESOM used here, the demands, which are measured at the point of end-use, are responsive to both price changes and energy-GNP feedback effects.

ETA-MACRO [2]

As shown in Figure 3-4, ETA-MACRO is an expanded version of the Energy Technology Assessment (ETA) model developed by Alan Manne. It adds a very simple representation of the interactions between the energy system and the rest of the economy to ETA's process representation of the energy supply system.

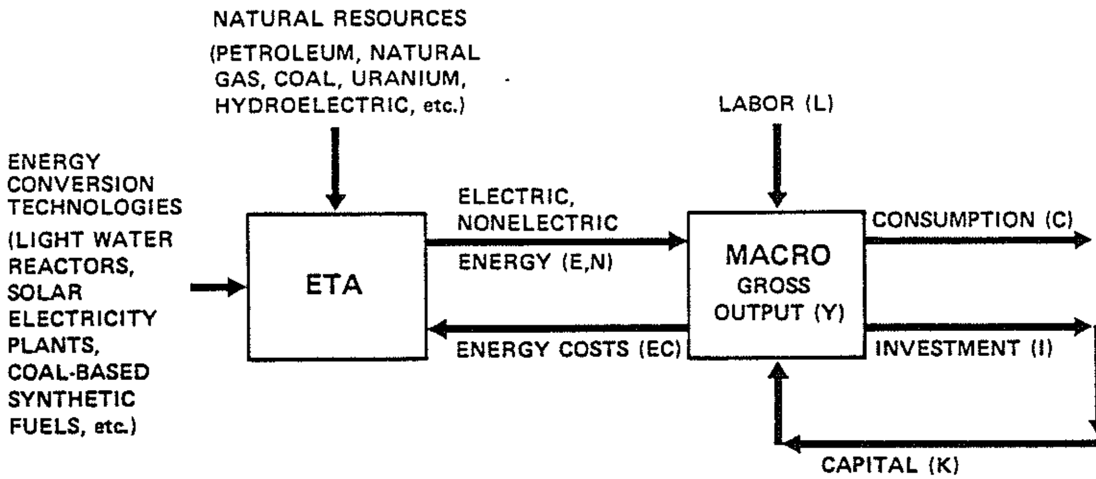


Figure 3-4 An Overview of ETA-MACRO

Supply

Oil and gas (aggregated together), coal, and uranium are explicitly represented in the model. Other fuels, hydroelectric and geothermal energy, etc. are specified exogenously.

Conversion

Oil refining is not considered in the model, electricity conversion is given an explicit process representation, and coal synthetics are represented in terms of their market price. Coal and nuclear fuel are the two possible choices for baseload electricity generation; the load structure of electricity demands is not considered. The model is dynamic and assumes perfect foresight on the part of energy producers. A 30-year plant life is assumed.

Demand

The economy is assumed to produce a single aggregate form of output from capital, labor, and energy inputs. A macroeconomic growth model provides for substitution between capital, labor, and energy inputs. The energy inputs are disaggregated into electricity measured at the busbar and nonelectric energy measured at the refinery gate. A Cobb-Douglas production function provides for substitution between the two energy forms. The demand side of the model is dynamic and assumes perfect foresight on the part of energy demanders. For example, savings and investment decisions are modeled so that consumers will receive equal benefits from an additional dollar's worth of current consumption and a dollar's worth of investment. Society's capital stock is assumed to be retired at a constant rate.

PARIKH WEM [3,4]

In the Parikh WEM, Figure 3-5, nonenergy sector production and consumer's utility are modeled using hierarchical homothetic functions. This model is a part of the Stanford PILOT Energy-Economic Model [5], which integrates a dynamic fixed coefficients input-output model with a detailed dynamic model of the energy sector.

Supply

Long-run marginal cost curves are used for oil, gas, and uranium. The dynamics of resource pricing provide for inclusion of scarcity rents. Other fuels and hydroelectric and geothermal energy are specified exogenously.

Conversion

The model's supply side consists of an input-output model with seven nonenergy and five energy sectors and a detailed process model of the energy sector. Oil refining is one of the input-output sectors. The electricity generation sector (including the nuclear fuel cycle) and synthetic fuels from coal and oil shale constitute a large portion of the process description of the energy sector. Substitution across energy inputs in the energy sector is modeled using engineering relationships. However, the nonenergy inputs into energy production are modeled with a variable coefficient I/O structure. The model is dynamic and assumes perfect foresight. The intertemporal link is provided through nonhomogeneous capital stocks and resource depletion.

Demand

Comprehensive modeling of demand-side substitutions between the 12 products represented in the Parikh WEM is implemented through a hierarchy of substitution functions for the consumers and each of the seven nonenergy industrial sectors. For consumers, time varying substitution elasticities characterize the ease of making adjustments in the input proportions of the following pairs of aggregates: electric/nonelectric, energy/nonenergy, and energy intensive goods and services/energy nonintensive goods and services. For each of the seven nonenergy industrial sectors, these elasticities characterize the ease of making adjustments in the input proportions of substitutable industrial energy, labor, and capital. The substitutable industrial energy is provided through a mix of coal, oil, gas, and electricity. It is modeled

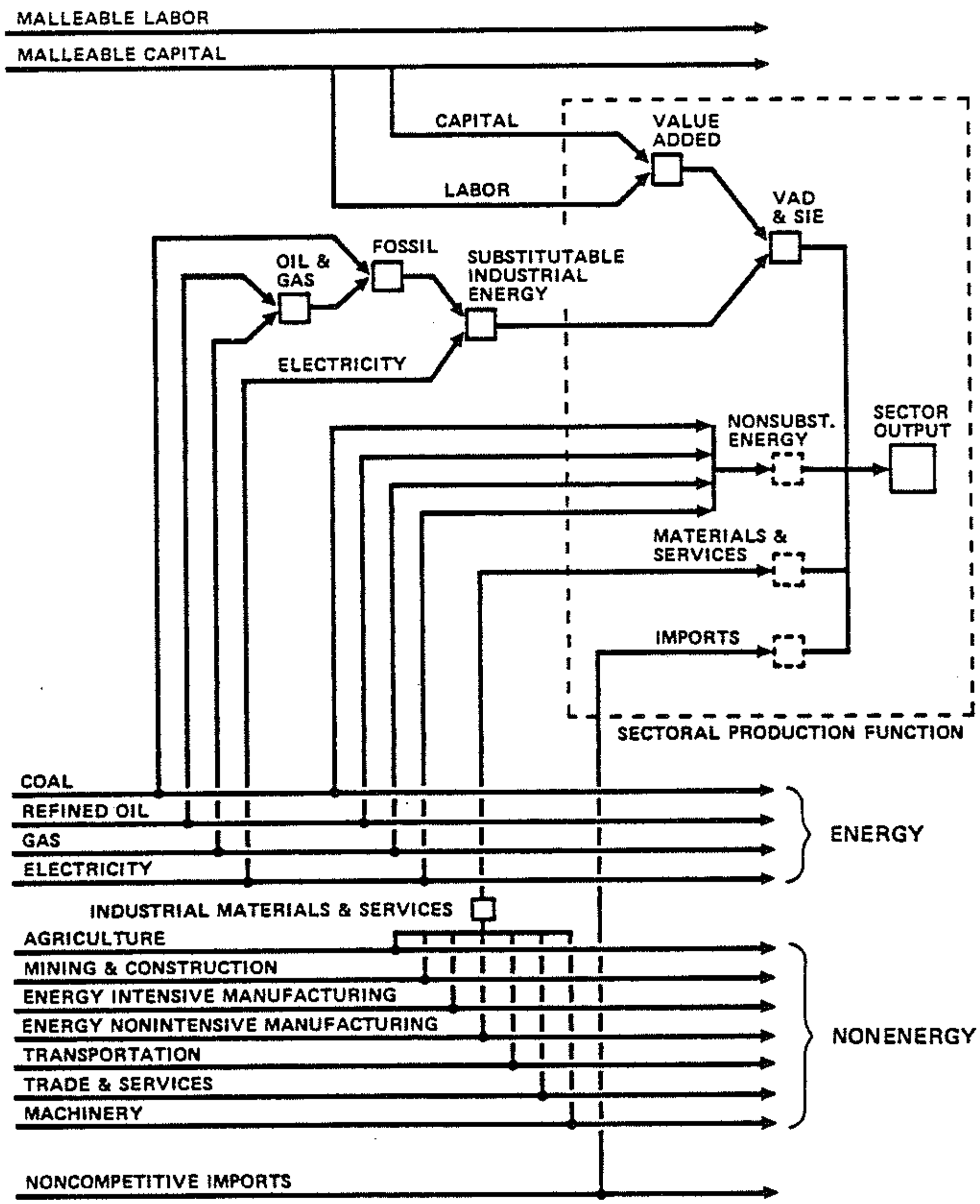


Figure 3-5 Hierarchical Structure of Sectoral Production Function in the WEM

through substitution functions that characterize the ease of making adjustments in the input proportions of the following pairs of fuels: electric/nonelectric, coal/oil-gas, and oil/gas. Industrial energy used as raw materials, such as metallurgical coal, feedstocks into chemicals and plastics, and liquid fuel needed by the transportation sector, etc., is assumed to depend on the level of output of the industrial sector. The demand side of the model is dynamic and assumes perfect foresight on the part of the energy consumers.

For each pair of variables in a substitution function, a constant elasticity of substitution (CES) function is developed using the assumed substitution elasticity and anchoring the reference isoquant at reference year quantities. This assumes that the reference year quantities are the profit (utility) maximizing choices at reference year prices.

Section 3

ENERGY SYSTEM MODELS

The energy system models include consideration of all energy demands, but exclude explicit representation of interactions between the energy sector and the remainder of the economy. These models typically include demand curves for energy fuels or products at various points of transaction within the energy system. However, the models differ substantially in their level of aggregation, substitution possibilities, and representations of the dynamics of energy supply and demand. The energy system models included in the study are Baughman-Joskow, EPM, FOSSIL1, Griffin OECD, MEFS, and Pindyck. Table 3-3 provides a summary comparison of the energy system models.

BAUGHMAN-JOSKOW [6]

The Baughman-Joskow model, Figure 3-6, consists of three major components or submodels: demand, supply, and financial/regulatory. The model is regionally disaggregated according to the nine census regions.

Supply

The supply of hydroelectric energy is specified exogenously. Coal, oil, gas, and nuclear fuels are endogenously represented in the model.

Conversion

Oil refining and synthetic fuels conversion are not included in the model. The supply submodel is a detailed model of the decision-making processes involved in operating and expanding an electricity supply system.

A detailed representation of the load-duration profile of electricity demand is included for each of the nine census regions. Hydroelectric capacity is specified exogenously. A cost minimizing "rule of thumb" is used to mimic the supply decisions made by electric utility companies. Firm expectations regarding fuel costs, plant construction costs, and plant operating characteristics are exogenous inputs into

Table 3-3

SUMMARY COMPARISON OF ENERGY SYSTEM MODELS

Model	Supply		Conversion			Demand					
	Product Aggregation	Spatial Aggregation	Electric Utilities	Oil Refinery	Synfuels Conversion	Spatial Aggregation	Product Aggregation	Point of Measurement	Factor Substitution	Interfuel Substitution	Dynamics
Baughman-Joskow	coal, oil, gas, nuclear fuel	9 regions	process model with exogenous load curve	not considered	not considered	9 regions	residential, commercial, industrial	input to end-use device	own-price elasticities	cross-price elasticities	explicit behavioral lag structure
EPM	oil natural gas coal uranium	12 regions 12 regions 4 regions U.S. total	hydro exogenous; explicit process model for oil, gas, coal, and nuclear	explicit process model	explicit process model	9 regions	end-use service demands	point of end-use	own-price elasticities	market share curves	behavioral lag curves
FOSSIL1	oil, natural gas, coal	U.S. total	explicit process model	not considered	explicit process model	U.S. total	fuel	input to end-use device	own-price elasticities	cross-price elasticities	explicit behavioral lag structure
Griffin OECD	oil, natural gas, coal	U.S. total	hydro and nuclear exogenous; econometric model for oil, gas, and coal	no explicit model; refinery losses accounted for	not considered	U.S. total	residential, industrial, transportation	input to end-use device	own-price elasticities	cross-price elasticities	explicit behavioral lag structure
MEFS	oil natural gas coal uranium	13 regions 14 regions 12 regions U.S. total	process model with exogenous load curve	process model aggregated from detailed model	not considered	9 regions	sector	input to end-use device	own-price elasticities	cross-price elasticities	explicit behavioral lag structure
Pindyck	not considered	not considered	not considered	crude oil markup	not considered	U.S. total	residential, industrial, "other fuels"	input to end-use device	own-price elasticities	cross-price elasticities	none

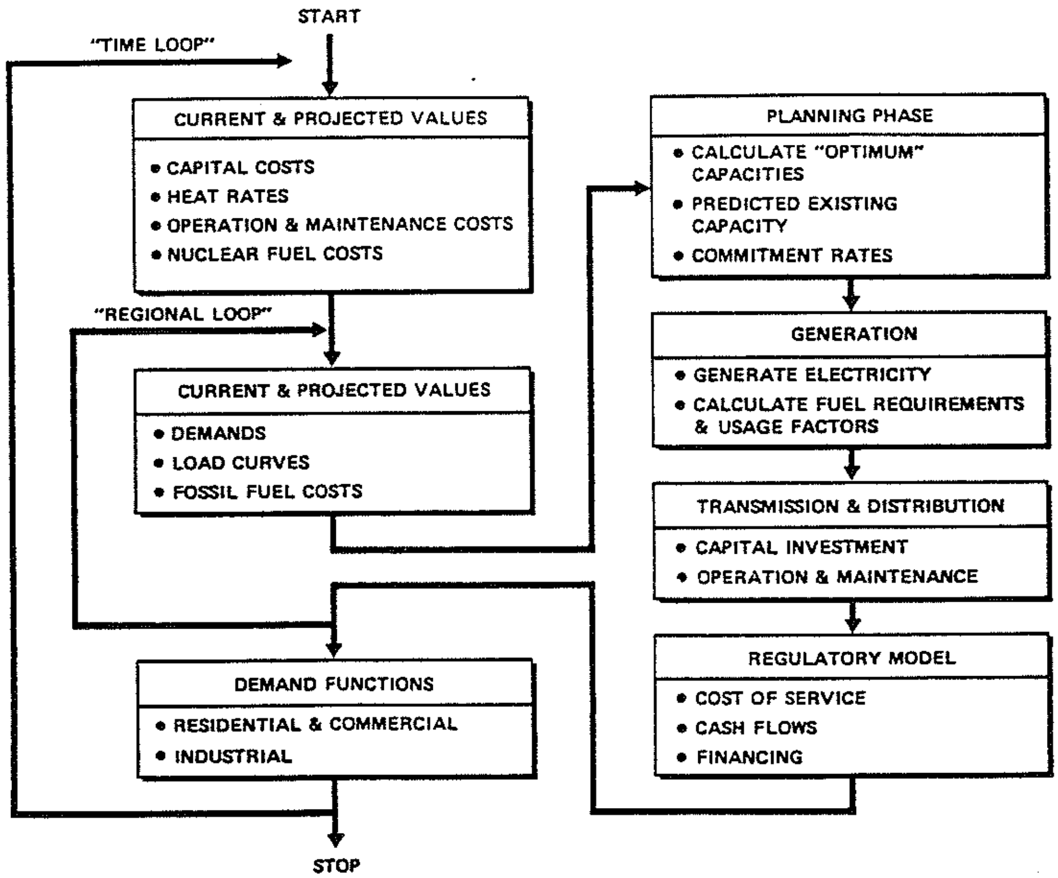


Figure 3-6 Flow Diagram of Electricity Model Interconnection with Regulatory Model and Demand Submodels for the Baughman-Joskow Model

the model, but electricity demand expectations by utilities are exponentially-weighted, moving averages with a trend adjustment.

Demand

The demand model consists of a set of demand equations for electricity, oil, natural gas, and coal for the residential/commercial and industrial sectors.* Industrial feedstock demands and transportation demands are not considered.

*Coal is not included in the residential/commercial sector.

For the residential/commercial sector, the demand model consists of an equation which estimates total energy consumption per capita as a function of a weighted energy price index (weighted by both consumption and the end-use efficiency of the various fuels) and incomes. A Koyck lag structure is employed. In addition, a set of fuel-split equations is utilized to divide total energy consumption into oil, natural gas, and electricity consumption.

For the industrial sector, a similar formulation is utilized. Total industrial energy consumption is estimated as a function of an energy price index and valued added in manufacturing. Nationally aggregated time-series data are utilized here. Next, a locational equation is used to determine total energy consumption in each of the states. Finally, a set of fuel-split equations is utilized to allocate the total energy consumption in each state among the four basic fuels--electricity, oil, natural gas, and coal.

EPM [7]

The Energy Policy Model is a descendant of the SRI/Gulf model [8]. It consists of a large number of mass balance relationships; these balance equations can be nonlinear. An iterative solution procedure is employed to compute an approximate solution. Thus, it does not use an optimization technique to compute a market equilibrium.

The EPM starts with a detailed engineering representation of the U.S. energy system. Substitution among alternative conversion processes is considered. In addition, the allocation of the various fuels that can be input to a conversion process is accomplished with a set of market-share equations that allocate fuels to demands according to their relative prices. This procedure avoids the discontinuities in the response of demand to relative fuel prices characteristic of mathematical programming models. Furthermore, market penetration (behavioral lag) curves retard responses in the allocations from instantaneous adjustment to a perfect foresight allocation based on the relative prices of the inputs. Thus, the stock adjustments are neither myopic nor clairvoyant.

Supply

Hydroelectric energy production is specified exogenously. Coal, oil, gas, and nuclear fuels are endogenously represented in the model. The amount of regional disaggregation of the fuel supplies varies from none for nuclear fuel to 12 oil and gas supply regions.

Conversion

Oil refinery, electricity generation, and synthetic fuels conversion are all explicitly represented in the model.

Demand

The alternative processes that can be used to satisfy a particular end-use demand are explicitly modeled. And the allocation of the end-use demands among the alternative sources is accomplished through market-share curves, as in the case of the energy producers. The model includes price-sensitive demands so that substitution of nonenergy goods for end-use energy products is explicitly considered. The EPM is formulated as a generalized network flow problem. It is solved through an iterative procedure. The model's objective is to find market clearing prices (prices at which supplies just equal demands) for the inputs and outputs of each energy conversion process.

FOSSIL1 [9]

The FOSSIL1 model, Figure 3-7, is a systems dynamics simulation model of the U.S. energy system developed by the Dartmouth College System Dynamics Group. A previous version of this model was called COAL2. The model considers nationally aggregated fuel demands and prices.

Supply

Gas, oil, and coal supply are nationally aggregated, although coal is differentiated according to high- versus low-sulfur content and surface versus deep mine. Nuclear supply is modeled at both the reactor and reactor fuel levels.

Conversion

Oil refining is not considered. Synthetic fuels and electricity conversion are explicitly modeled. The production capacity of a conversion technology is limited by the availability of physical capital. The use of fuels within each sector is determined by an allocator mechanism which divides the available capital between the various production alternatives. The allocator computes the marginal return on investment (ROI) for each alternative, shifting investment as the changing operating cost, capital cost, and output price of each production process alter that alternative's marginal ROI. The production process with the highest ROI gets the greatest proportion of capital investment funds. The allocation mechanism operates so as to maximize future industry profits. Capital stocks are

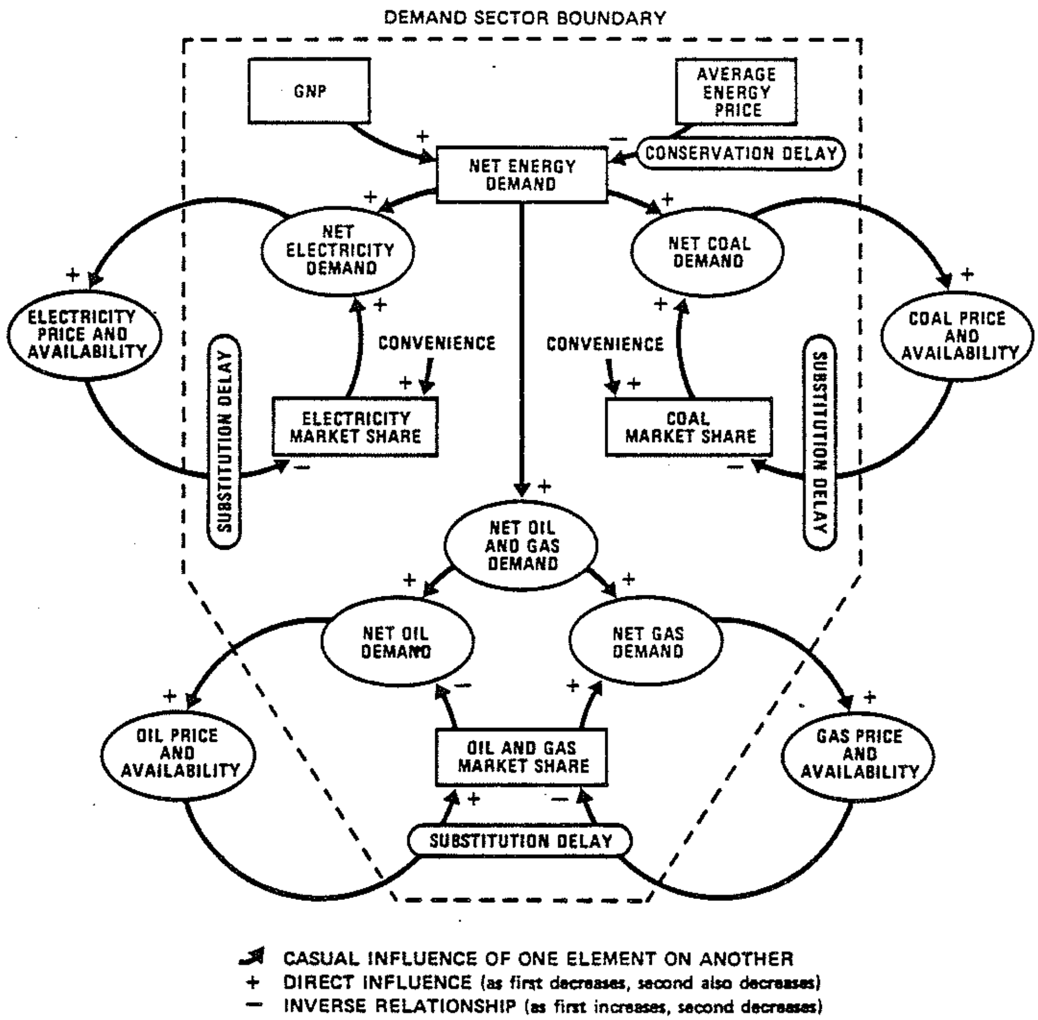


Figure 3-7 FOSSIL1 Demand Sectoral Causal Diagram

adjusted continuously, but myopically, with delays for construction lead times being taken into account explicitly.

Demand

Aggregate demands for electricity, gas, oil, and coal delivered to users are considered. Total demand is a function of income and average energy price. The market shares of the four fuels are determined by their relative prices, convenience of use factors, and saturation effects. The demand side of the model adjusts continuously but myopically. The demand sector is assumed to require a full 15 years to turn over its capital in response to an energy price change.

THE GRIFFIN OECD MODEL [10]

The Griffin OECD model, Figure 3-8, is an econometric energy demand model estimated with pooled cross-sectional data from 18 OECD (Organization for Economic Cooperation and Development) countries. All prices and quantities in the model are nationally aggregated.

Supply

Hydroelectric and nuclear energy are specified exogenously in the model. Coal, gas, and oil are endogenously represented.

Conversion

Synthetic fuels and oil refining are not explicitly considered in the model. However, aggregate refinery losses are estimated. Nuclear and hydroelectric inputs to electricity generation are specified exogenously, but the cost shares of the fossil fuels in the rest of the electricity load depend on relative fuel prices according to a 13-year distributed lag pattern.

Demand

Demands are disaggregated according to three consuming sectors: residential, industrial (including commercial activities), and transportation. The nonfeedstock industrial and residential demand modules have similar structures. Aggregate, thermally adjusted (i.e., end use) energy input to the sector is estimated as a function of income related variables and a five-year distributed lag* on the thermally adjusted aggregate energy input price of fuels to the sector. Cost shares for coal, gas, electricity, and oil are functions of 10-year distributed lags on relative fuel prices. Demands for industrial feedstocks are specified exogenously.

In the transportation sector, gasoline, automotive diesel, and aviation fuel demand are determined separately from other uses, such as rail and water transport. Gasoline

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*A five-year distributed lag is assumed in the industrial sector and a seven-year lag in the residential sector.

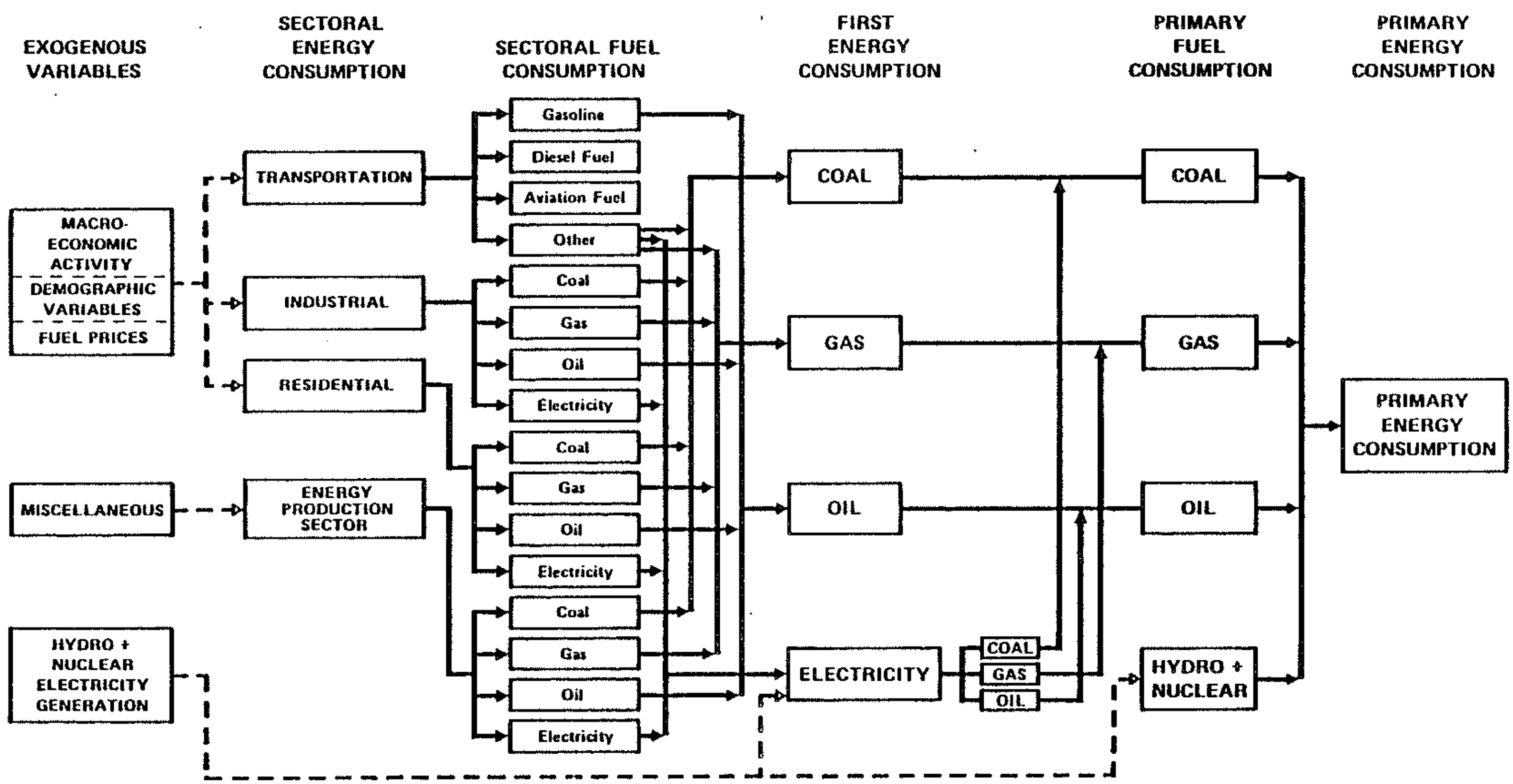

Figure 3-8 Flow Diagram of Griffin OECD Energy Demand Model

consumption is based on a simultaneous determination of car stock and car utilization. The stock of cars is estimated as a function of per capita income and a four-year distributed lag on gasoline price; gasoline consumption per car depends on per capita income, the stock of cars per capita, and a 10-year distribution lag on the real price of gasoline. Diesel consumption in trucks, buses, and automobiles is a function of real income and cars per capita; aviation fuel consumption is a function of economic activity and population; other transportation fuel uses are a function of aggregate economic activity and the real price of energy.

MEFS DEMAND MODEL

The Project Independence Evaluation System (PIES) was initially developed at the Federal Energy Administration during the work of the Project Independence Task Force [11]. Since then it has been substantially revised and augmented [12], critically reviewed [13], documented [14], and renamed. It is now called the Mid-term Energy Forecasting System (MEFS), and resides at the Energy Information Administration in the Department of Energy.

The MEFS system is a collection of independent models designed to be used either singly or in groups. The major components are the Mid-term Energy Market Model (formerly the PIES Integrating Model), Oil and Gas Model, National Coal Model, Refinery Model, and Demand Model. The Mid-term Energy Market Model is a static linear programming model designed to compute a market equilibrium solution given demand curves, supply curves, conversion characteristics, and pricing regulations. The Demand Model is a dynamic econometric model of energy demand disaggregated by consuming sector. Only the Demand Model (Figure 3-9) was employed in the current study.

Demand

The Mid-term Energy Demand Model is an econometric model that projects fuel demands according to the 10 federal regions. Twenty fuels are considered. Except for feedstocks, which are treated separately, the residential, commercial, and industrial sectors have a similar specification. However, different parameter values and exogenous variables are used in the implementation for the several sectors. Each of the three sectoral models has two parts. First, an index of total energy demand depends on the absolute level of a deflated fuel price index and exogenously

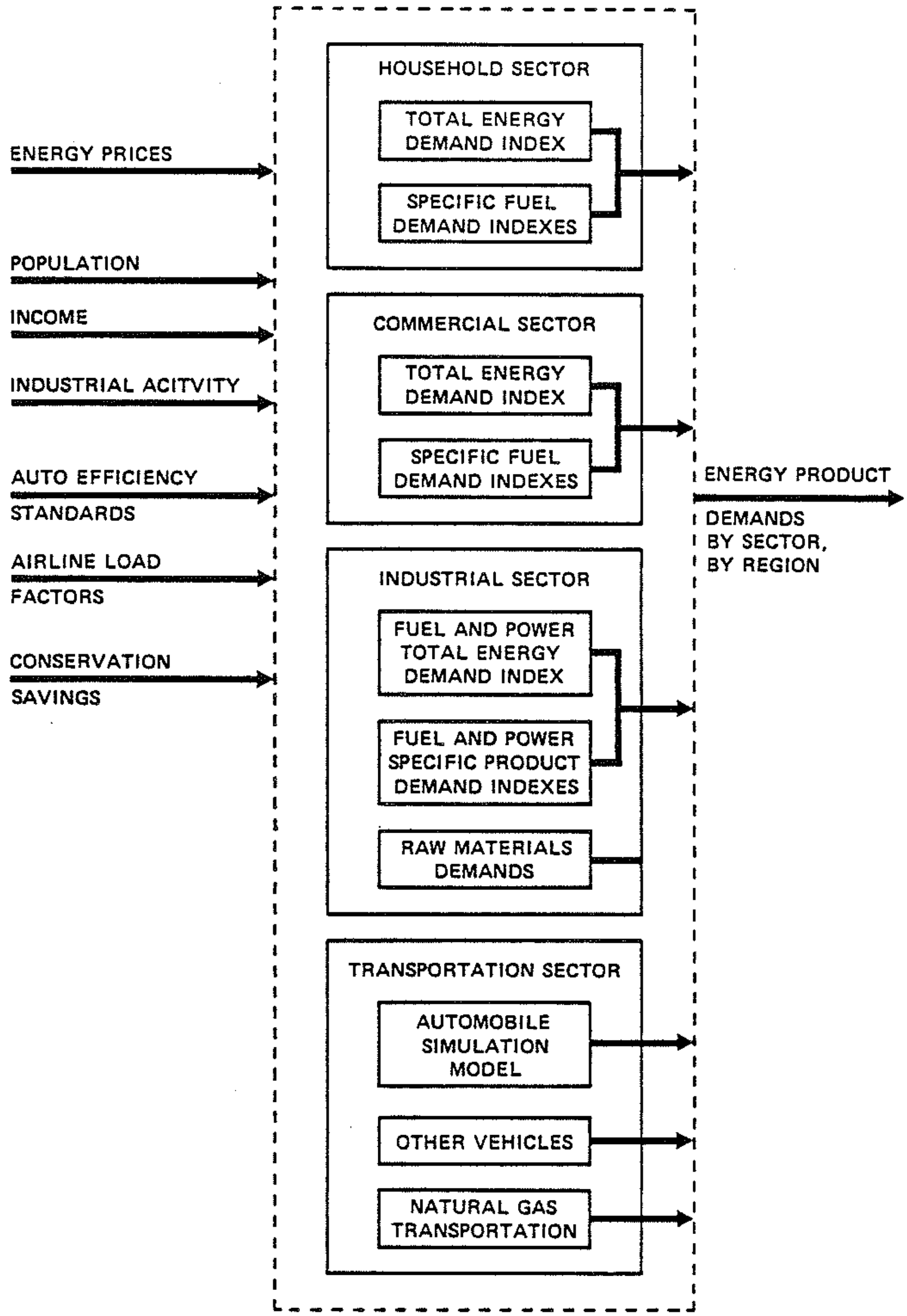

Figure 3-9 MEFS Demand Model: Basic Configuration

specified measures of the sector's activity level. Second, the ratio of each specific fuel demand to the total energy index is specified to depend on the relative price of the fuel.

The regional structure of the data is exploited by allowing the intercepts in each regional equation to vary but constraining the coefficients of all other variables to be the same across all regions. Consumers' behavioral lags to price changes are explicitly considered in the formulation. Fuels used as raw materials are related to projections of industrial indexes for the industry where the fuel has its primary application and are allocated to census regions using projections of regional income originating from those industries.

The specifications of transportation demand in the Demand Model are markedly different from those employed in the other sectors. In that sector, lack of inter-fuel competition and interregion climatological effects obviate the need for regional estimation. The major elements of transportation demand include automobile highway gasoline use, nonautomobile highway gasoline and diesel fuel use, rail diesel fuel use, and commercial jet fuel use. These fuel demand equations were estimated using time-series single equation techniques for national consumption and then disaggregated using regional income and population projections. The automotive gasoline demand module is a revised version of the Sweeney Auto model described later in this chapter.

PINDYCK MODEL [15]

The Pindyck model is an econometric energy demand model. All prices and quantities in the model are nationally aggregated.

Supply

Energy supply is not considered in the model.

Conversion

Energy conversion processes are not considered in the model, except that transportation, refining, marketing, and tax markups are added to the crude price to obtain refined oil product prices.

Demand

Demands are disaggregated according to three consuming sectors: residential, industrial, and "other fuels." The industrial sector includes commercial services. The other fuels sector includes transportation fuels (diesel and motor gas), as well as "remainder fuel" not included in any of the other categories.

In the residential and industrial sectors, a two-stage estimation procedure similar to that utilized in the MEFS Demand Model is employed. First, an aggregate energy price is computed as a translog function of the prices of oil, gas, coal, and electricity. Then the share of energy in aggregate consumption expenditures is computed as a translog function of the aggregate energy price and the aggregate price of nonenergy goods. Finally, the share of oil, gas, coal, and electricity, in total energy expenditures, is computed as a translog function of relative fuel prices.

Section 4

SECTORAL MODELS

INTRODUCTION

In this section, the seven dynamic sectoral models included in the study are described; Table 3-4 summarizes the comparison. The FEA-Faucett, Sweeney Auto, and Wharton MOVE models describe domestic gasoline demand for automobiles. The Jackson Commercial model considers commercial energy use; BECOM, consumption in residential and commercial buildings; ISTUM, industrial energy use; and Hirst Residential, residential energy demand.

Unlike energy system and energy-economy models, the scope of the sectoral models is confined to the explicit, endogenous representation of demand for energy fuels in particular sectors of the economy. Sectoral models typically contain three basic components:

- one that projects variables exogenous to the fuel demand forecasting section, such as demographic and supply forecasts, and feeds them into the rest of the model;
- one that computes fuel demands from those forecasts and from the characteristics and amount of fuel consuming equipment available (e.g., the automobile in the transportation sector); and
- a dynamic component that revises the amount and characteristics of the consuming equipment over time.

<u>Component 1: Demographic and Energy Supply Variable Projections</u>

Projections of the demographic and energy supply variables employed in the sectoral models may be assumed or formally or informally linked to other demographic assumptions. For example, housing stock, commercial floor space, industrial output, and vehicle registration projections can depend on population and/or economic growth rate assumptions. In the EMF experiment, standardization of fuel prices minimizes differences in assumptions about energy supply variables. However, economic growth rates are the only demographic variables that are standardized. This leaves open the possibility of differences in results due to differences in the modelers translation of economic growth rate assumptions into demographic projections. These projections are used to derive the sectoral energy fuel demand forecasts.

Table 3-4

SUMMARY COMPARISON OF SECTORAL MODELS

Model	Type	Sector	Spatial Aggregation	Fuels	End-Use Disaggregation	Additional Disaggregation	End-Use Conversion	Point of Measurement	Interfuel Substitution	End-Use Price Responsiveness	Dynamics
BECOM	optimization	residential, commercial	4 regions	oil, gas, electricity, coal, solar	4 end uses	4 shell types	explicit process model	end-use services	explicit process model	no	intertemporal optimization
FEA-Faucett	simulation	transportation	U.S. total	gasoline	vehicle miles only	auto classes	interclass shifts only	transportation services	none	yes	behavioral lag on usage
Hirst Residential	simulation	residential	U.S. total 10 federal regions	oil, gas, electricity, other	8 end uses	3 housing types	capital/fuel trade-off curves	end-use services	cross-price elasticities	yes	50% behavioral lag on usage response in first year
ISTUM	optimization	industrial	9 regions	oil, gas, electricity, other	26 industries, 23 service sectors	100 technologies	explicit process model	end-use services	explicit process model	no	behavioral lag on usage and new investments
Jackson Commercial	simulation	commercial	U.S. total 10 federal regions	oil, gas, electricity, other	5 end uses	10 building types	capital/fuel trade-off curves	end-use services	cross-price elasticities	yes	behavioral lag on usage.
Sweeney Auto	simulation	transportation	U.S. total	gasoline	vehicle miles only	none	fuel efficiency depends on gasoline price	transportation services	none	yes	behavioral lag on usage
Wharton MOVE	simulation	transportation	U.S. total	gasoline	vehicle miles only	auto classes	interclass shifts only	transportation services	none	yes	behavioral lag on usage

Component 2: Demand Forecasting

However, it is within the second component, fuel demand forecasting, that the greatest variance among models occurs. Much attention is given to the data collection, model formulation, and parameter estimation required to project fuel demands given values for demographic variables and characteristics of consuming equipment. Simulation and cost minimization are the two methods of fuel demand forecasting used in the EMF 4 models.

The most commonly used method, the simulation approach, derives total fuel demands in any year via a simulation that continually combines and updates three types of information:

(1) the total stock of all energy consuming equipment (e.g., automobiles);

(2) the percentage or market share of that stock of equipment which have the same technical characteristics (e.g., automobiles grouped by mileage ratings or space heaters grouped by fuel utilized and year installed); and

(3) the intensity with which each market share of the stock of equipment is used (e.g., an automobile with a high mileage rating may be driven farther over a given period than one with a low mileage rating).

Using this information, the models simulate the evolution of sectoral energy demands. Each year existing consuming equipment is utilized at a rate that depends on the cost of providing energy services (e.g., space heat, automobile vehicle miles, etc.). The higher the cost of utilizing existing consuming equipment, the less it is used. Since 90 to 95% of consuming equipment used during a particular year is in place at the beginning of that year, reductions in usage rates are the only way most consumers can respond to sudden increases in energy prices in the short run. However, those consumers who buy new equipment or replace worn out equipment can take new price information into account in making their choice. Reductions in fuel demands by those purchasing new, more fuel efficient equipment is greater than by those who own existing equipment. In the simulation models, the type, amount, and rate of use of newly installed equipment depends on the projected lifetime usage cost. The more it costs to provide end-use services from the new equipment, the less is installed.

The FEA-Faucett, Hirst Residential, Jackson Commercial, Sweeney Auto, and Wharton MOVE models employ the simulation approach to project fuel demands. Figure 3-10 provides a schematic representation of the simulation approach.

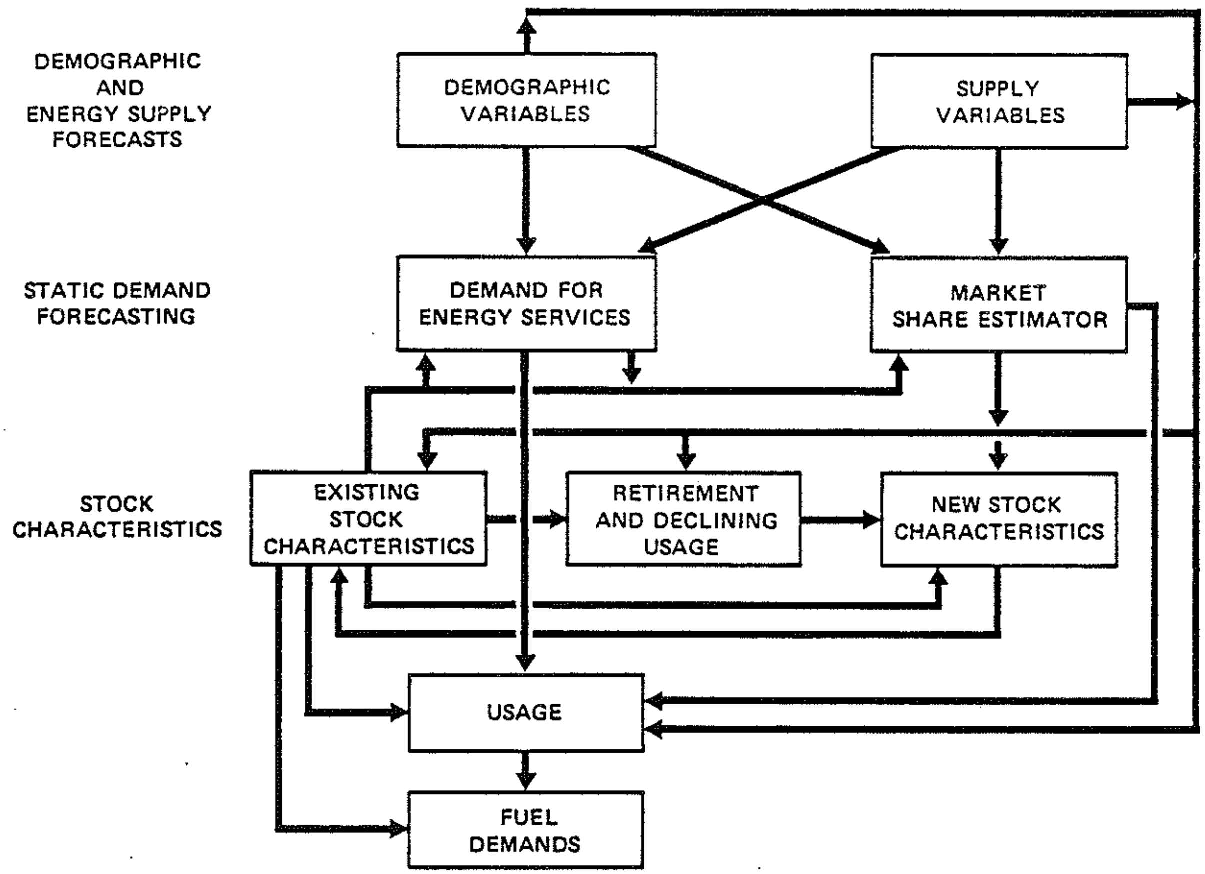

Figure 3-10 "Simulation" Approach to Sectoral Fuel Demand Projections

The second method of fuel demand forecasting, cost minimization, is utilized in the ISTUM and BECOM models. These two models estimate intensity and market shares simultaneously.

In ISTUM, the total demand for useful energy (the amount of energy demanded net of end-use conversion losses) within each industry is linked to the economic growth rate projections; a unitary income elasticity is assumed. Market shares for the technologies capable of meeting those demands are determined by comparing exogenously specified fuel and capital cost curves and choosing the set of energy and capital resources that minimize each industry's energy costs. Total demand for a particular type of fuel in a given year is determined by aggregating the fuel demands of all industrial users.

BECOM uses an analogous cost minimizing methodology. In that model, buildings are assumed to require some acceptable level of comfort or service. The comfort level and the number of different types of buildings are forecast outside the system. However, decisionmakers are permitted to choose among different types of energy service systems and levels of insulation. The amounts chosen are presumed to depend upon which combination of service system(s) and insulation will minimize the cost of achieving the acceptable level of comfort or service. Figure 3-11 is a schematic of the cost minimization method of demand forecasting.

Component 3: Consuming Equipment Projections

Population, fuel prices, and income vary over time as existing equipment is retired. Thus, one would expect the amount and technical characteristics of new equipment to fluctuate. The stock of consuming equipment of a given vintage can be expected to decline over time, due to retirement and/or less intensive use. Accordingly, sectoral models represent both the retirement of existing vintages and the changing characteristics of newly installed consuming equipment. However, the models represent these effects in different ways.

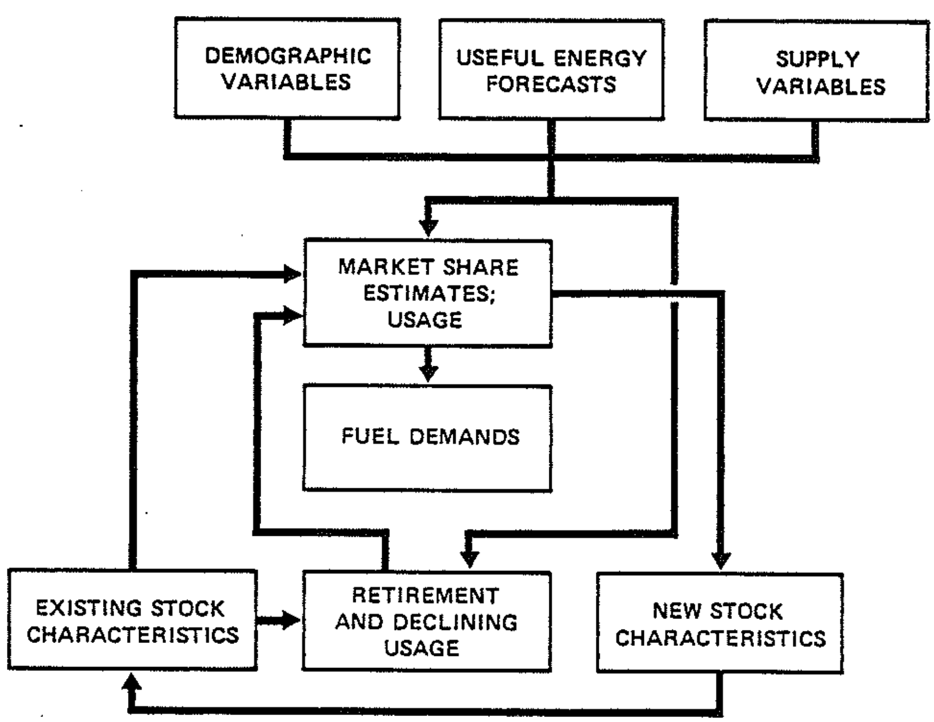

Figure 3-11 "Cost Minimization" Approach to Sectoral Fuel Demand Projections

The BECOM, Hirst Residential, Jackson Commercial, ISTUM, and Sweeney Auto models assume retirement rates that are not responsive to other system variables, such as fuel prices and national income. For example, the Jackson Commercial model postulates that the demand for energy services is proportional to the total area of commercial floor space and that a given vintage of floor space is retired at an exponential rate over time. In the Sweeney Auto model, constant proportional automobile scrappage and usage rates are specified. The FEA-Faucett and Wharton MOVE models, on the other hand, calculate retirement rates endogenously. Both models allow the scrappage of automobiles of a given vintage to depend upon economic variables, such as the unemployment rate and the price of automobiles. Neither model, however, accounts for the dependence of usage on age.

Sectoral models also represent changes in total demand and the market shares and technical efficiencies of newly sold consuming equipment. Most models, specifically, FEA-Faucett, Hirst Residential, Jackson Commercial, and Wharton MOVE, use standard econometric methods to describe the dependence of market shares and total demands upon relevant economic variables.

A number of methods are used to estimate changes in technical efficiency. FEA-Faucett and Wharton MOVE assume that the technical efficiencies of consuming equipment within each class remain constant over time. Sweeney Auto, Hirst Residential, and Jackson Commercial relate technical efficiencies to capital costs and fuel prices. In these models, increasing fuel prices are presumed to induce substitution of capital for energy, generally resulting in higher technical efficiencies among newly introduced consuming equipment. Furthermore, the extent of efficiency changes is determined through the use of elasticity estimates. Similarly, ISTUM relates changes in technical efficiencies to fuel prices and capital costs. However, it uses cost minimization to determine which types of technologies will be used in a given industrial subsector. It also assumes that the cost curves for a particular technology do not change over time. Finally, ISTUM includes behavioral lags upon the date at which new systems are introduced.

BECOM [16]

The Buildings Energy Conservation Optimization Model was developed at Brookhaven National Laboratory. It is a cost minimizing model of energy use in buildings. The emphasis in BECOM is upon market share rather than demand growth projections. Stocks of different building types and basic energy demands for those types are exogenously specified. Various end-use energy conversion systems are chosen in a way to minimize the cost of meeting those basic demands. This process is dynamic and includes consideration of retirement of old technologies and new technology introduction to the year 2000.

BECOM is a highly detailed model. Energy use in buildings is disaggregated by fuel, conversion device, such as gas heating converted by burners or heat pumps, shell, such as single or multifamily dwelling or mobile homes, region, and end use. Data relating to building stocks, technology costs, and shell and conversion efficiencies are taken from the Arthur D. Little data base for buildings. Other exogenous data were supplied by BESOM (Brookhaven Energy System Optimization Model).

Model outputs can be aggregated in three different ways: (1) building types, (2) conversion device, or (3) fuel by end use for each year. BECOM utilizes exogneous forecasts of oil, gas, electricity, coal, and solar power. Demand forecasts are derived through a multistep process. Stocks of consuming equipment, different building types, and basic energy demands are exogenously specified. The specific determination of how those demands are met is solved by formulating a building network problem.

The structure of BECOM is similar to that of a transshipment problem, with the BESOM* aggregate demand points for residential/commercial space heat, air conditioning, water heating, and appliance services as sources for the transshipment. The destinations for this problem are the different buildings markets, each of which requires a certain amount of energy to provide an acceptable level of comfort or service. Shipments from sources to destinations are made through intermediate transfer points or transshipment points.

Each conversion device is characterized by an efficiency coefficient and an annualized capital cost. Devices in existing buildings have a cost coefficient of zero since they are already in place. The thermal shell is similarly characterized by an efficiency coefficient which compares its heat loss characteristics with those of a reference shell, and by a cost coefficient which is the annualized capital cost of upgrading a structure relative to the reference shell. Nonupgraded buildings and reference new buildings have an efficiency of 1. Upgraded shells have higher efficiencies. The energy demand in each market can be met by any

*See the description of BESOM/H-J in Section 2.

combination of firing device and thermal shell. The model selects those combinations of conversion devices and thermal integreties that minimize the cost of satisfying the demands in the various markets subject to the technical constraints.

FEA-FAUCETT [17]

The FEA-Faucett automobile gasoline model falls neatly into the market share sectoral model framework described previously. No supply detail is specified and there are no conversion activities. Additionally, no energy-economy feedbacks are considered. The modeling approach incorporates a vintage capital stock structure. Parameters are estimated econometrically from historical data.

Gasoline is the only fuel considered. The model produces aggregate national projections of gasoline use. The estimates, depending upon both consumers' and manufacturers' behavior, are derived separately; a simulation model is used to combine the estimates. A number of automobile size classes are considered. Dynamics are provided by a constant turnover in the capital stock of automobiles. A scrappage function is computed as a function of the unemployment rate and new car prices which are exogenous to the consumer model. Vehicle miles traveled are then computed, leading to the desired estimate of gasoline usage.

HIRST RESIDENTIAL [18]

The Hirst Residential model, Figure 3-12, was developed at Oak Ridge National Laboratory (ORNL). It simulates energy use in the residential sector from 1970 through 2000. It is a typical sectoral model in the sense that there is little detail on energy supply, no consideration of energy-economy feedbacks, and no modeling of conversion activities. The model consists of four basic submodels: (1) a demographics submodel that calculates the stocks of occupied housing units for each year; (2) an economics submodel that determines the responsiveness of households to changes in income, fuel prices, and equipment prices by estimating the relevant price elasticities; (3) a technology submodel that describes the characteristics of energy consuming equipment from an engineering perspective; and (4) a simulation model which combines the outputs of the other submodels. The model was validated for the 1960-1975 period and performed well in energy use by fuel predictions.

The Hirst Residential model considers four fuel categories: electricity, gas, oil, and other. It should be noted that the electricity figures include losses in generation, transmission, and distribution. The oil and gas figures do not include losses associated with refining and transportation.

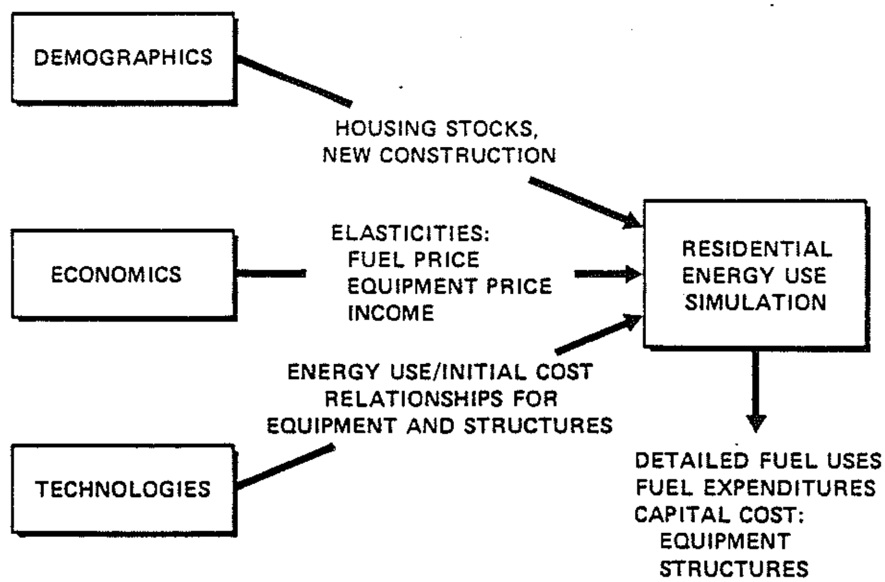

Figure 3-12 Schematic of Hirst Residential Energy-Use Model

The Hirst Residential model provides projections of residential energy use for four fuels (electricity, gas, oil, other), eight end uses (space heating, water heating, refrigeration, food freezing, cooking, air conditioning, lighting, other), and three types of housing units (single-family dwellings, apartments, mobile homes). The model operates at either the national level or 10 disaggregated federal regions. Only the national version was used in this study. The modeling approach is to express household energy use in terms of the stock of occupied housing units, equipment ownership, thermal properties of housing units, efficiency of equipment, and usage factors (e.g., thermostat setting, lighting intensity).

Substitution possibilities are reflected in cross-price elasticities provided to the simulator by the economics submodel. Hirst, however, notes that the statistical basis for many of these elasticities is weak. The dynamic nature of the model is evident as it incorporates changing equipment characteristics over time and a constant turnover of the housing stock. Other aspects of the dynamics are the behavioral lag structure of the model and usage rate changes. The Hirst Residential model assumes 50% of the usage response occurs in the first year and that changes in equipment market shares are constrained only by equipment lifetimes.

ISTUM [19]

The Energy and Environmental Analysis (EEA) Industrial Sector Technology Use Model (ISTUM) is a detailed representation of the industrial sector designed to predict market penetration for various energy technologies to the year 2000. To make forecasts, a cost minimization algorithm is used. Moreover, the focus of the model is on projecting the relative use of the various energy fuels and technologies, as opposed to the growth rates of energy demands. "Useful" energy demands (the amount of energy utilized after all conversion and generation losses have been subtracted) are exogenously specified. The demand for and market shares of specific types of energy consuming equipment (e.g., industrial boilers) are endogenously forecasted. Additionally, ISTUM includes the retirement process and the changing characteristics of newly introduced equipment.

Like other sectoral models, ISTUM is disaggregated by energy source and end use (industry submarket). ISTUM is also disaggregated by year to include dynamic effects, combustor size, and combustor load factor. However, the demand for energy is not disaggregated spatially.

Furthermore, because modeling the costs of using alternative energy consuming devices is central in the ISTUM approach, all energy-related costs are carefully specified. Fixed and variable costs are included and time preferences for capital are accounted for. Furthermore, in constrast to the other models, costs in each ISTUM submarket are presented in the form of a distribution, showing what fraction of the demand in a cell will experience each possible cost. Finally, as they become feasible, new technologies are introduced and allowed to compete with conventional technologies.

ISTUM includes a detailed, although exogenous, representation of energy fuels supplied to the industrial sector. Capital cost and fuel price distributions for a variety of technologies and energy sources are introduced in the model. The more than 100 end-use conversion technologies considered are categorized into six groups: conventional, fossil energy, conservation, cogeneration, solar, and geothermal technologies. Cost distributions for new technologies are introduced as those technologies become feasible.

JACKSON COMMERCIAL [20]

The Jackson Commercial model, Figure 3-13, is a disaggregated economic-engineering model of commercial energy use also developed at ORNL. Structurally similar to the Hirst Residential model, the Jackson Commercial model provides detailed annual forecasts of commercial energy use to the year 2000. In order to restrict consideration to what is typically referred to as commercial energy use, the ORNL group defines the commercial sector to include the BEA Standard Industrial Classifications E, F, G, H, I, and J* but excludes transportation, electricity power generation, and feedstock energy involved in these divisions. Thus, there were problems in compiling a consistent historical set of data upon which to base the model. Additionally, the level of aggregation of the demand side of the model necessitated development of energy-use data for 10 building types, four fuels, and five end uses. These problems and their solutions are discussed in detail in [21].

The modeling approach taken by Jackson explicitly recognizes the fact that energy is consumed in capital equipment to provide desired services. Thus, the problem can be viewed as one of estimating the equipment stock and its utilization rate. Furthermore, since the stock of equipment is dependent upon the amount of floor space served, the estimation can be approached from a floor space, utilization rate perspective. The model was validated by simulating commercial energy use from 1969 to 1975; it produced small forecast errors but demonstrated an ability to predict turning points in market shares.

The Jackson Commercial model is a demand model and thus has a very simple supply side. Four fuel categories, oil, natural gas, electricity, and other, are considered. The other category includes coal and liquid natural gas demands, which are specified exogenously in accordance with historical trends.

The model provides national aggregate projections of commercial energy use by 10 building types, five end uses (space heating, cooling, water heating, lighting and other i.e., cooking and electromechanical uses), and four fuel types. All

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*E = transportation, communication, electricity, gas, and sanitary services.
F = wholesale trade.
G = retail trade.
H = finance, insurance, and real estate.
I = services.
J = public administration.

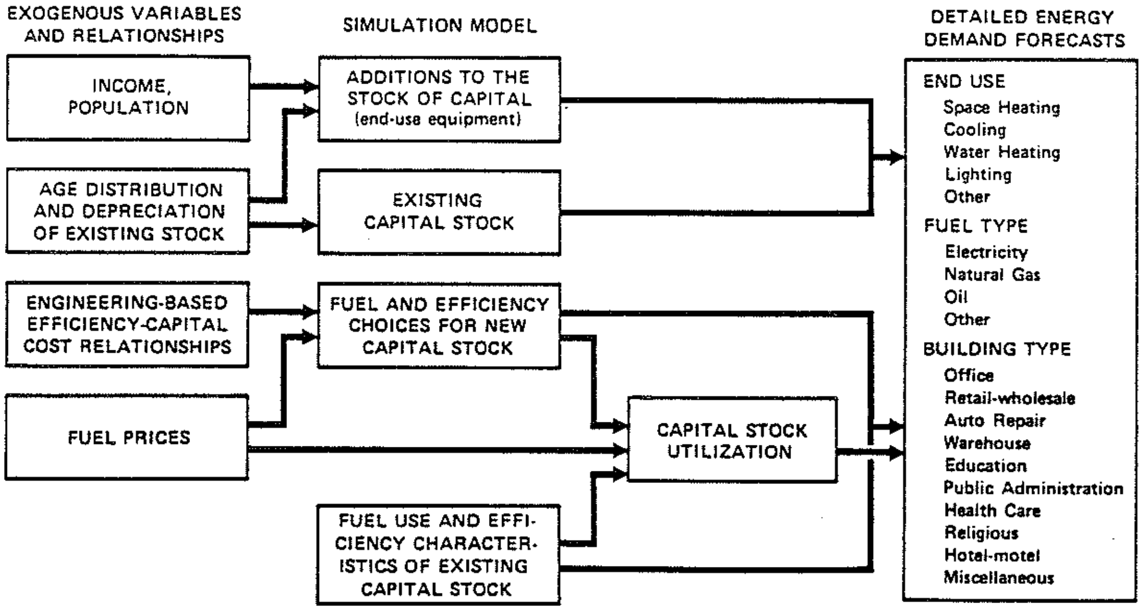

Figure 3-13 Jackson Commercial Model

measurements are made at the point of end use, making this model much better suited for this study's secondary, rather than primary, elasticity experiment.

As previously mentioned, the Jackson Commercial model takes a vintage capital-stock approach. There is a precise division between long-run and short-run effects. The main elements of the model are a series of equations: $Q = U(P) \times S$ where Q = energy use, $U(P)$ = utilization rate, which is dependent on price of equipment services, and S = stock of equipment. In the short run (one year), only utilization rate changes representing behavioral changes are allowed, with the stock of equipment held constant. In the long run, providing the dynamics of the model, the stock of equipment changes with additions made in some areas and retirements made in others. It is interesting to note that in determining fuel use for the future, the Jackson Commercial model makes use of short-run own-price elasticity estimates from the Baughman-Joskow model.

SWEENEY AUTO [22]

The Sweeney Automobile model, Figure 3-14, is similar to the Hirst Residential and Jackson Commercial models in the sense that all three employ the vintage capital stock approach. That is, emphasis is placed upon the processes of capital stock adjustment and stock utilization. Such an approach makes it possible to examine the influence of policy-directed technological changes or usage changes in addition to policies acting on economic variables. In the Sweeney Auto model, solutions are derived using a simulation model whose parameters have been estimated econometrically.

The supply side of the model is very simple; only gasoline is considered. Thus, no interfuel substitution is possible. The demand side of the model produces aggregate national projections of gasoline use through 1990. The basic approach is to view gasoline demand as derived from the consumer's desire for mobility. Thus, estimated gasoline demand is simply the desired number of miles traveled divided by the overall automobile fleet efficiency. Dynamics are introduced by detailed modeling of the evolution of the fleet of automobiles, including endogenous projections of the size and technical efficiency of each vintage. There is an annual changeover in the structure of the auto fleet. Age is a critical factor in the model, both because more cars are retired and are generally used less as they get older. Given the efficiency of autos of each vintage, the relative miles driven by autos of different vintages, and the number of autos in existence from each vintage, the average fleet efficiency can be estimated. Then vehicle mile demand is estimated as a function of income, driving costs, and other economic variables. An estimate for gasoline demand can be obtained by dividing the demand for vehicle miles by the fleet efficiency.

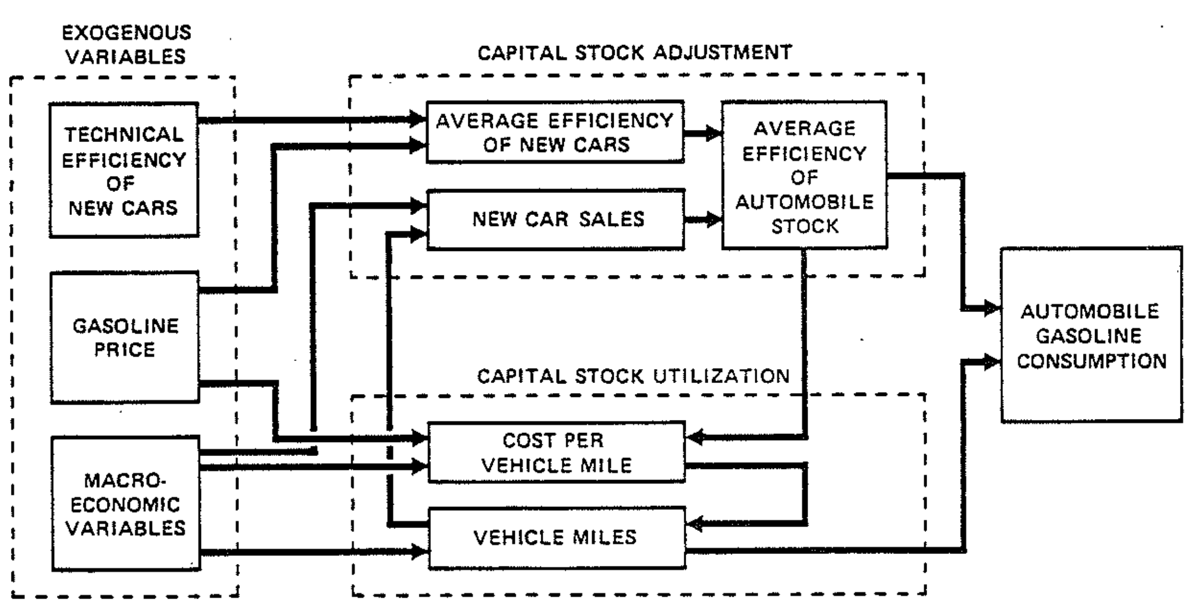

Figure 3-14 Sweeney Automobile Model Overview

WHARTON MOVE [23]

The Wharton MOVE analysis of the automobile market directly estimates the total stock, market shares, and usage (vehicle miles) of automobiles to derive automobile gasoline demand estimates. Exogenous inputs to the system include demographic indicators, variables, such as fuel prices and automotive efficiencies, and macroeconomic variables, such as the unemployment rate, real income, and inflation. The model can be used to project the size of the total automobile stock, automobile gasoline consumption, and total automobile vehicle miles traveled.

Like the FEA-Faucett model, the Wharton MOVE model uses a classical structural approach. Total stock, market shares, and vehicle miles of automobiles are directly estimated; a market share approach is employed; and an updating process is used. Both models also estimate the long-term demand elasticity for gasoline to be approximately 0.1 - 0.2 in the automobile market.

The Wharton MOVE model does contain some unique characteristics. The most important of these characteristics is its attempt to fully account for the costs of driving an automobile and to use an equilibrium approach in the estimation process. Unlike other models, Wharton MOVE accounts for the effects of changes in all relevant costs on the automobile market including both purchase price and variable costs, such as financing, gasoline consumption, and insurance. Through this process, an average cost per mile is found and then combined with other variables to produce a steady-state "desired stock per family." Desired stock values are then compared with "actual stock" levels to give estimates of newly introduced and scrapped automobiles (the consuming equipment represented in the model).

As in the Sweeney and FEA-Faucett models, gasoline demand is considered in the Wharton MOVE model. The representation of gasoline demand is very similar to FEA-Faucett's. The number of automobiles in a given vintage is computed first, and then market shares are derived to give the number of automobiles in each class for a specific vintage. Vehicle mile demand is simultaneously calculated, and dynamic effects are taken into account. Gasoline consumption can then be predicted from the class, stock, and vehicle mile projections.

CONCLUSION

A comparison of the key characteristics of the models provides essential background for interpreting the results from the comparative model experiments. A priori, many differences in model characteristics seem capable of producing differences in their aggregate elasticity estimates. But careful scrutiny of the results of the study within the framework established here is required to ascertain which characteristics are indeed most telling. In Chapter 8, Griffin and Wood make this very interesting ex post comparison.

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Chapter 4

THE THEORY AND PRACTICE OF ENERGY AGGREGATION*

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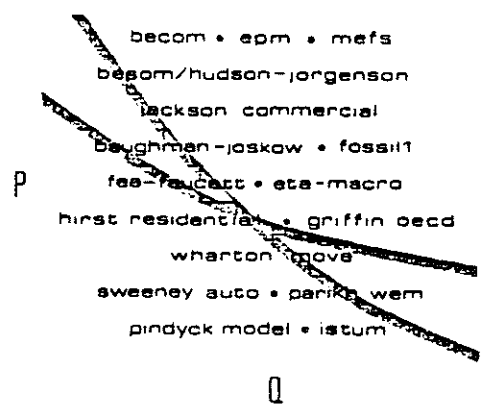


*This chapter formerly was Working Paper EMF 4.3.

Chapter 4

THE THEORY AND PRACTICE OF ENERGY AGGREGATION

INTRODUCTION

The fundamental purpose of the "Aggregate Elasticity of Energy Demand" study was to determine the implicit price elasticity of aggregate energy demand in 16 models. If energy were a single good, this would have simply entailed the fitting of the best constant elasticity curve to the energy price and quantity data from the models. However, since energy is an aggregation of numerous individual energy goods, this determination was substantially more difficult. In particular, it entailed three distinct tasks--aggregating the price and quantity data for individual energy goods using price and quantity indexes, calculating the price elasticity of energy demand using these indexes in a practical and meaningful manner, and interpreting the results of these calculations. This chapter discusses the first two of these tasks:

- the theory of energy price and quantity indexes, and
- the advantages and disadvantages of several methods for cälculating the price elasticity of energy demand using these indexes.

THE THEORY OF ENERGY PRICE AND QUANTITY INDEXES

Definitions

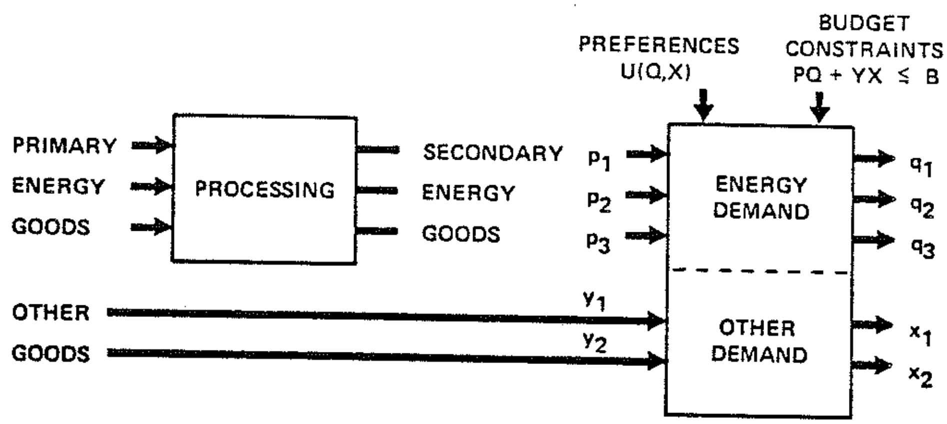
Figure 4-1 contains a simple model of the demand for energy. In this model, the behavior of society can be characterized by a single aggregate preference or production function over all goods:

$$U(Q, X) = f(q_1, q_2, q_3, \ldots x_1, x_2, \ldots) \quad , \tag{4-1}$$

where

$Q = (q_1, q_2, \ldots)$ represents the quantities of energy goods or inputs, and

$X = (x_1, x_2, \ldots)$ represents the quantities of nonenergy goods or inputs.



p_k = PRICE OF ENERGY GOOD K
P = VECTOR OF ENERGY PRICES
q_k = QUANTITY OF ENERGY GOOD K
Q = VECTOR OF ENERGY QUANTITIES
y_k = PRICE OF OTHER GOOD K
Y = VECTOR OF OTHER PRICES
x_k = QUANTITY OF OTHER GOOD K
X = VECTOR OF OTHER QUANTITIES

Figure 4-1 The Demand For Energy

The preference (or production) function $U(Q, X)$ is defined so that the society prefers combinations of Q and X that have higher values of $U(Q, X)$ over combinations of Q and X that have lower values. The society is indifferent among combinations of Q and X which give identical values of $U(Q, X)$. The society is assumed to act so as to maximize U subject to constraints. Furthermore, for X (nonenergy goods) held constant, the behavior of society can be characterized by a preference function over Q (energy goods). This energy preference function, $E_X(Q)$, is defined by normalizing $U(0, X)$ to zero:

$$E_X(Q) = U(Q, X) - U(0, X) \quad . \qquad (4\text{-}2)$$

For X held fixed, the society is indifferent among combinations of energy commodities that give equal values to $E_X(Q)$. Society can be envisioned as making choices which maximize $E_X(Q)$ subject to an "energy budget constraint," determined by the total expenditure on all commodities minus the expenditures on nonenergy goods. If $E_X(Q)$ is scaled so that $E_X(\alpha Q) = \alpha \cdot E_X(Q)$, it is convenient to regard $E_X(Q)$ as the "quantity" of aggregate energy consumed.

Modeling energy demand as a constrained societal maximization allows one to adapt the cost of living interpretation of price and quantity indexes to energy. Following Samuelson and Swamy [1], we can define the price and quantity indexes for energy as follows:

- Given two price situations, P^0 and P^A, the true price index, π, is the ratio of the minimum expenditures for a reference quantity of energy.
- Given two quantity situations, Q^0 and Q^A, the true quantity index, θ, is the ratio of the minimum expenditures for their respective quantities of energy in the face of a reference price situation.

Notationally, the minimum expenditure for a given quantity of aggregate energy $E_X(Q^j)$ in a given price situation (P^i) is*

$$e(P^i, Q^j) = \min_Q (P^i \cdot Q) \quad , \qquad (4\text{-}3)$$

$$\text{such that} \quad E_X(Q) = E_X(Q^j) \quad .$$

Then, the price and quantity indexes are as follows:

$$\pi(P^0, P^A, Q^r) = \frac{e(P^A, Q^r)}{e(P^0, Q^r)} \quad , \qquad (4\text{-}4)$$

$$\theta(Q^0, Q^A, P^r) = \frac{e(P^r, Q^A)}{e(P^r, Q^0)} \quad , \qquad (4\text{-}5)$$

where Q^r represents the Reference case energy quantity vector, and P^r represents the reference price situation.

In Eqs. 4-4 and 4-5, it should be noted that $\pi(P^0, P^0, Q^r) = \theta(Q^0, Q^0, P^r) = 1$.

These definitions have an appealing intuitive interpretation, based on a comparison of the choices of optimizing consumers. The price index, π, indicates how the minimum cost for the reference quantity of energy differs in price situations A

*The notation $P^i \cdot Q$ denotes the inner product of the two vectors: $\sum_k p_k^i q_k$.

and O. The difference in cost reflects the difference in the prices of the energy commodities. The quantity index, θ , indicates how the minimum cost under reference prices differs for the quantity of energy associated with situation A and the quantity of energy associated with situation O. The difference in cost reflects the difference in the quantity of energy consumed. Note that both indexes are based upon some reference situation.

If preferences were known, it would be a relatively straightforward matter to calculate the true price and quantity indexes.

For example, assume that society's preferences can be characterized by the function:

$$U(q_1, q_2, X) = q_1^{0.5} q_2^{0.2} X^{0.2} , \tag{4-6}$$

where q_1 is the quantity of oil consumed, q_2 is the quantity of coal consumed, and X is the quantity of the single nonenergy good consumed. In this case, since $U(0, X) = 0$, the energy preference function is identical to $U(Q, X)$, with X fixed:

$$E_X(Q) = U(Q, X) . \tag{4-7}$$

Under Eqs. 4-6 and 4-7 the minimum expenditure for a given quantity of energy can be shown to equal:

$$e(P^i, Q^j) = K(p_1^i q_1^j)^{5/7} (p_2^i q_2^j)^{2/7} , \tag{4-8}$$

where K is a positive constant. Note that doubling both energy quantities or energy prices doubles expenditures.

The true price index can be calculated using Eqs. 4-4 and 4-8:

$$\pi(P^0, P^A, Q^r) = \left(\frac{p_1^A}{p_1^0}\right)^{5/7} \left(\frac{p_2^A}{p_2^0}\right)^{2/7} . \tag{4-9}$$

In this situation, the price index is independent of the reference quantity levels. Doubling both prices doubles the price index.

The true quantity index can be calculated using Eqs. 4-5 and 4-8:

$$\theta(Q^0, Q^A, P^r) = \left(\frac{q_1^A}{q_1^0}\right)^{5/7} \left(\frac{q_2^A}{q_2^0}\right)^{2/7} \quad . \tag{4-10}$$

In this situation, the quantity index is independent of reference price levels. Doubling both quantities doubles the quantity index.

With Eqs. 4-9 and 4-10, nonproportional price and quantity changes can be considered. Letting $p_1^0 = 1$ and $p_2^0 = 2$ and $p_1^A = 2$ and $p_2^A = 1$, then the price index is $(2)^{5/7}(1/2)^{2/7}$ or 1.35. Alternatively if $p_1^A = 1$ and $p_2^A = 1$, then the price index is $(1/2)^{2/7}$, or 0.82. Similar calculations can be made for quantity changes.

Figure 4-2 illustrates one of the calculations graphically. Based on the energy quantity curve represented by $E_X(q_1, q_2) = 1$, cost-minimizing quantities are illustrated for each price combination, simplifying calculation of cost-minimizing expenditures. To calculate the price index, we must compare the expenditures at two points on this energy quantity curve, one for each price situation.

Properties of the True Indexes*

Because these indexes are defined in terms of optimizing behavior, they possess some important properties. First, for nonenergy quantities, reference prices, and reference quantities all fixed, the values of the price and quantity indexes (π, θ) map out a single valued demand curve for aggregate energy, E_X. For the true indexes if there is no change in the price index, there will be no change in the quantity index, even though changes in the price vector may change the quantity vector.

As is clear from the definitions, the precise form of both the true price and the true quantity indexes depends upon the function, E_X, which is not observable. Therefore, one normally cannot construct true indexes, but must rely upon approximations. Fortunately, several properties of the true indexes make the approximation process feasible.

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*Much of this material is based on Samuelson and Swamy [1] and Blackorby [2].

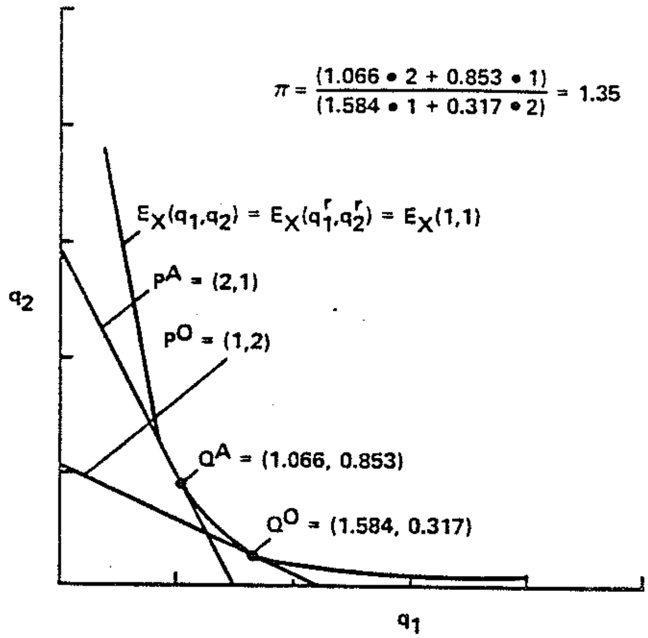

Figure 4-2 Calculation of the Price Index

Small Change Properties

For infinitesimally small price and quantity changes, an index with the same gradients as the true index approximates the true index.* If the reference situation is chosen as (P^O, Q^O), the gradient of the true price index is proportional to the quantity vector and the gradient of the true quantity index is proportional to the price vector. Thus, any index with these gradients, such as the Paasche, Laspeyres, or Ideal index,† can approximate the true index. Futhermore, since these gradients are independent of the preference structure, the approximations are similarly independent for small changes.

*The gradient of a function $g(x)$ is $[\partial g/\partial x_1, \partial g/\partial x_2, \ldots, \partial g/\partial x_n]$, a vector perpendicular to the level sets of $g(x)$.

†The Paasche and Laspeyres indexes are defined mathematically by Eqs. 4-15 through 4-18. The Ideal index is the geometric mean of the Paasche and Laspeyres indexes. Specifically, $\pi_I = \sqrt{\pi_P \pi_\ell}$ and $\theta_I = \sqrt{\theta_P \theta_\ell}$.

Specifically, for $Q^r = Q^0$, we have

$$\pi = \frac{e(P^A, Q^0)}{e(P^0, Q^0)} .$$

Letting $\nabla_P e$ denote the gradient of e with respect to price, we can approximate π as follows for small price changes:

$$\pi \simeq 1 + \frac{\nabla_P e \cdot (P^A - P^0)}{e(P^0, Q^0)} .$$

Furthermore, since $e(P^A, Q^0) = \min \left\{ P^A \cdot Q \text{ , such that } E_X(Q) = E_X(Q^0) \right\}$, it follows that:

$$\nabla_P e = Q + P^A \cdot \nabla_P Q .$$

Because $e(P^A, Q^0)$ represents a minimization, small changes in Q which keep $E_X(Q)$ constant will not change the expenditure, i.e.,

$$P^A \cdot \nabla_P Q = 0 ,$$

and therefore:

$$\nabla_P e = Q . \tag{4-11}$$

For small price changes, with $Q \approx Q^0$, the true price index can be approximated as

$$\pi \approx 1 + \frac{(P^A - P^0) \cdot Q^0}{P^0 \cdot Q^0} . \tag{4-12}$$

In other words, changes in the true price index can be approximated by the change in energy expenditures for quantities held constant at their initial values, divided by the initial expenditure.

Similarly we can approximate the quantity index:

$$\theta \approx 1 + \frac{\nabla_Q e \cdot (Q^A - Q^0)}{e(P^0, Q^0)} .$$

The gradient of e is simply evaluated

$$\nabla_Q e = P . \tag{4-13}$$

For small price changes, with $P \approx P^0$, the quantity index can be approximated as

$$\theta \approx 1 + \frac{P^0 \cdot (Q^A - Q^0)}{P^0 \cdot Q^0} . \tag{4-14}$$

In other words, changes in the true quantity index can be approximated by the changes in energy expenditure for prices held constant, divided by the initial expenditure.

Because Eqs. 4-12 and 4-13 rely upon observable data only, they can be readily evaluated. However, because they are only local approximations, they deteriorate in quality for large changes in prices and quantitites.

Bounds

For large changes, establishing upper and lower bounds on the values of the indexes that are not dependent upon the precise form of the preference function would be very helpful. Unfortunately, this goal cannot be achieved without specifying further properties of the preference function.

One-sided bounds can be found when indexes are evaluated at the initial and the alternative price and quantity situations:

$$\pi(P^0, P^A, Q^0) \leq \frac{P^A \cdot Q^0}{P^0 \cdot Q^0} \triangleq \pi_\ell , \tag{4-15}$$

$$\pi(P^0, P^A, Q^A) \geq \frac{P^A \cdot Q^A}{P^0 \cdot Q^0} \triangleq \pi_\rho , \tag{4-16}$$

$$\theta(Q^0, Q^A, P^0) \leq \frac{P^0 \cdot Q^A}{P^0 \cdot Q^0} \triangleq \theta_\ell \quad , \tag{4-17}$$

$$\theta(Q^0, Q^A, P^A) \geq \frac{P^A \cdot Q^A}{P^A \cdot Q^0} \triangleq \theta_\rho \quad , \tag{4-18}$$

where π_ℓ and π_ρ are the Laspeyres and Paasche price indexes, respectively and θ_ℓ and θ_ρ are Laspeyres and Paasche quantity indexes, respectively.

The logic behind these bounds is easily seen. For example, consider the Laspeyres price index. If consumers are optimizing, then the minimum expenditure at P^0 is simply P^0Q^0 ; hence, the denominator of Eq. 4-15. If the price changes to P^A , one must evaluate the minimum expenditure needed to provide a level of aggregate energy equivalent to that provided by Q^0. Clearly Q^0 provides sufficient energy and can be purchased at a cost of P^AQ^0. Thus the minimum expenditure is no greater than P^AQ^0 ; hence, the numerator of Eq. 4-15. A similar logic allows derivation of each bound above.

Inequalities 4-15 through 4-18 provide one-sided bounds on the various indexes. Two-sided bounds are based on additional properties--separability and homotheticity.

Separability Property

Separability can be defined as follows:

$$E_X \text{ is separable if } E_{X^i}(Q^A) > E_{X^i}(Q^B) \text{ implies}$$

$$E_{X^j}(Q^A) > E_{X^j}(Q^B) \text{ for all } X^i , X^j \ . \tag{4-19}$$

Separability implies that preferences among energy goods are independent of the quantities of nonenergy goods consumed. If $E_X(Q)$ is scaled to be linear in Q , E_X can be interpreted as the quantity of energy consumed, where this quantity is independent of X . E_X may be effectively separable within some small range of X , the nonenergy good consumed, even though it is not separable over a wide range. Thus, in some cases, separability can be induced by restricting the possible range of X's considered.

If E_X is not separable, the price and quantity indexes are functions of X, the quantity of other goods consumed. For instance, if the earlier example were based on a nonseparable utility function such as the following:

$$U(q_1, q_2, x_1, x_2) = q_1^{0.5}x_1^{0.5} + q_2^{0.2}x_2^{0.2} ,$$

the calculated indexes would have been different for different values of x_1 and x_2. If $x_1 = 0$, the price of energy would equal the price of energy type 1 (coal), and if $x_2 = 0$, the price of energy would equal the price of energy type 2 (oil). (If $x_1 = 0$, then the expenditure minimizing level of q_1 would be zero, since q_1 does not enter the preference function. Only one energy type would be purchased (type 2) and therefore the ratio of expenditures would be simply the ratio of p_2's in two situations. The reasoning is analogous when $x_2 = 0$.)

Such nonseparable preference functions are certainly plausible. In this example, where oil is consumed as gasoline and coal as electricity, x_1 might represent the quantity of automobiles and x_2 the quantity of electric heaters. If no autos were owned, no gasoline would be purchased; no electric heaters implies no use of coal.

As noted above, if E_X is separable, there is an invariant E that reflects the preferences of society among energy goods. The price and quantity indexes are determined solely by E and by the prices and quantities of energy goods. This is the case in the example of Eq. 4-6. In this example, for any X, $E_X(Q)$ can be scaled to equal $(q_1)^{5/7}(q_2)^{2/7}$. This function therefore reflects the preferences among energy goods irrespective of X. Thus, if two situations are being compared, it is unimportant so far as the indexes are concerned that X may be different in the two cases.

Separability is an important property for the application of price and quantity indexes to energy, since without separability, the concept of aggregate energy has little meaning. Without separability, a change in the quantities of nonenergy goods may change the energy price and quantity indexes even though all energy prices and quantities remain unchanged. If separability does hold, the price and quantity indexes do not vary with the quantity of nonenergy goods consumed; aggregate energy can be treated analytically like any individual good. In this case, the concept of an energy aggregate could be useful.

Homotheticity Property

The second important possible property of E_X is homotheticity. Homotheticity can be defined as follows:

$$E_X \text{ is homothetic if } E_X(Q^i) > E_X(Q^j) \text{ implies}$$

$$E_X(aQ^i) > E_X(aQ^j) \text{ for all } a > 0 \ . \qquad (4\text{-}20)$$

An equivalent definition is that E_X must be expressible as a monotonically increasing transformation of a linear homogeneous function. Given homotheticity, preferences are proportional, i.e., if the expenditure on energy increases or decreases by a factor β , the purchase quantity of each energy commodity increases or decreases by the same factor β. Essentially, the society when rich has the same spending pattern for energy as the society when poor. In a fashion analogous to separability, E_X may be effectively homothetic within a small range of Q , even if it is not homothetic over a wide range. Thus, homotheticity can be induced by restricting the possible range of Q's considered.

In the general case, we cannot assume E_X is homothetic. Without homotheticity, the price and quantity indexes based on E_X lack several useful properties. First, these indexes will not be unique; that is, given (P^0, Q^0) , the price index will depend on the reference quantity situation and the quantity index will depend on the reference price situation. Second, there are nontrivial bounds on the indexes that are independent of the preference structure. Third, the indexes do not satisfy the factor reversal property: the product of the price and quantity indexes does not equal the expenditure index.* These conclusions are elaborated below.

Without homotheticity, the price index will be dependent upon the reference quantity situation used to compare two price situations; the quantity index will be dependent upon the reference price situation used to compare two quantity situations. If the price and quantity indexes of some (P^A, Q^A) situation are being

*The factor reversal property is simply that the product of the price and quantity indexes equals the expenditure index: $\pi\theta = (P^AQ^A)/(P^0Q^0)$. This property, also called value equivalence, has an intuitive appeal and is useful for calculating one index from the other when total expenditures are known.

calculated, either Q^0, Q^A, or any other Q conceivably can be used as the reference quantity. Likewise, either P^0, P^A, or any other P conceivably can be used as the reference price situation. Thus, we can identify two basic price indexes $\pi(P^0,P^A,Q^0)$ and $\pi(P^0,P^A,Q^A)$, two basic quantity indexes $\theta(Q^0,Q^A,P^0)$ and $\theta(Q^0,Q^A,P^A)$, and an infinite number of other indexes, each using a different reference price or quantity. The basic price indexes compare the cost of purchasing quantitites that were actually chosen; the basic quantity indexes compare the cost of purchasing quantitites at prices actually faced.

To illustrate that nonhomotheticity leads to nonunique price and quantity indexes, let us assume that preferences are described by the function

$$U = 2q_1^{1/2} + q_2 \quad ,$$

which is not homothetic in q_1 and q_2. Under this utility function, the minimum expenditure to obtain utility equal to U^0 is

$$e(P,Q) = \begin{cases} P_2U^0 - P_2^2/P_1 \ , & \text{for } U^0 \geq 2P_2/P_1 \\ P_1U_0^2/4 \ , & \text{for } U^0 \leq 2P_2/P_1 \end{cases}$$

The quantity index is therefore

$$\theta(Q^0,Q^A,P^r) = \frac{U^A - P_{2r}/P_{1r}}{U^0 - P_{2r}/P_{1r}} \quad ,$$

$$\text{for } U^A \text{ and } U^0 \geq 2P_2/P_1$$

and the price index is

$$\pi(P^0,P^A,Q^r) = \frac{(U^r - P_2^A/P_1^A)P_2^A}{(U^r - P_2^0/P_1^0)P_2^0} \quad ,$$

$$\text{for} \quad U^r \geq 2P_2^A/P_1^A$$

$$\text{and} \quad U^r \geq 2P_2^0/P_1^0 \; .$$

The quantity index depends upon the reference price ratio. As the ratio P_{2r}/P_{1r} decreases, the quantity index increases, approaching U^A/U^o as P_{2r}/P_{1r} approaches zero. Similarily, the price index depends upon the reference quantities, or more precisely the reference utilities. For U_r very large, the price index approximately equals P_2^A/P_2^0. The index may either increase or decrease as U^r declines.

With homotheticity, the price index will not depend upon the reference quantity situation and the quantity index will not depend upon the reference price situation. Examples of this property can be seen in Eq. 4-8 which shows that the price index does not include the reference quantities and by Eq. 4-9 which shows that the quantity index does not include the reference prices.

The general result depends upon the realization that, with homothetic preferences, the cost minimizing proportions of the various energy commoditites depends upon the prices but not upon the initial quantities. Thus, various reference quantities scale both the numerator and the denominator of Eq. 4-4 up or down proportionately, leaving the ratio unchanged. Similarily, in Eq. 4-5, the cost-minimizing quantities given Q^A are fixed multiples of the cost-minimizing quantities given Q^0. Thus the ratio of expenditures in the two quantity situations is independent of reference prices.

At an earlier point, one-sided bounds to the true indexes were presented. If the true price and quantity indexes depend upon the reference situation, either upper <u>or</u> lower bounds can be derived, but it is not possible to derive <u>both</u> nontrivial upper <u>and</u> lower bounds. In the case of nonhomothetic preferences, only the following trivial bounds can be established as valid for all preference structures:

$$\min_i \left\{ \frac{p_i^A}{p_i^0} \right\} \leq \pi \leq \max_i \left\{ \frac{p_i^A}{p_i^0} \right\} ,$$

$$\min_i \left\{ \frac{q_i^A}{q_i^0} \right\} \leq \theta \leq \max_i \left\{ \frac{q_i^A}{q_i^0} \right\} ,$$

where p_i is the price of the i^{th} energy good, and q_i is the quantity of i^{th} energy good. These bounds are not particularly helpful.

However, in the homothetic case, the indexes are independent of the reference situation. Therefore, the one-sided bounds (inequalities 4-15 through 4-18) can be combined to produce two-sided bounds based on the Paasche and Laspeyres indexes,

$$\pi_\rho \leq \pi \leq \pi_\ell \quad \text{and} \tag{4-21}$$

$$\theta_\rho \leq \theta \leq \theta_\ell \ . \tag{4-22}$$

These inequalities provide fairly tight bounds.

A final property which depends upon homotheticity is the factor reversal property of indexes. Factor reversal is the property that the ratio of expenditures in two situations $(P^A \cdot Q^A / P^O \cdot Q^O)$ will equal the ratios of the products of price and quantity indexes in the two situations $(\pi^A \cdot \theta^A / \pi^O \cdot \theta^O)$. Because the price and quantity indexes depend upon reference quantities and prices, $\pi \cdot \theta$ will not, in general, be equal to $P^A \cdot Q^A / P^O \cdot Q^O$: the factor reversal property will not hold if preferences are not homothetic.

In general, however, using Eqs. 4-4 and 4-5 it can be easily verified that

$$\pi(P^O,P^A,Q^O) \cdot \theta(Q^O,Q^A,P^A) = \pi(P^O,P^A,Q^A) \cdot \theta(Q^O,Q^A,P^O) = \frac{P^A Q^A}{P^O Q^O} \ . \tag{4-23}$$

Thus, <u>some</u> combinations of price and quantity indexes do satisfy the factor reversal property. By Eq. 4-23, if total expenditures on energy are known, two basic indexes can be used to calculate the other two.

With homotheticity, the indexes are unique. Thus, Eq. 4-23 establishes that the factor reversal property must hold for all true indexes which reflect homothetic preferences.

Like separability, homotheticity is important for the application of price and quantity indexes to energy. However, this importance is primarily a practical, rather than a theoretical, matter. In particular, even if $E_X(Q)$ is unknown, homotheticity allows one to bound the true price and quantity indexes and to calculate approximations of these indexes independent of the preference structure and the reference situation. Furthermore, if the expenditures on energy are known, it allows one to use the factor reversal property to calculate one index from the other. These capabilities can be very useful.

Conclusion

Price and quantity indexes for energy are intimately connected to the society's preference structure. Only when this preference structure is well characterized can the true indexes be precisely defined. Even then, the definitions may be unsatisfactory from a single-good perspective for large changes in prices and quantities. In particular, if the preference structure is not separable, the indexes vary with respect to the quantity of nonenergy goods consumed. If the preference structure is separable, but not homothetic, the indexes vary with respect to the energy price and quantity reference situations. Only if the preference structure is separable and homothetic will the indexes exhibit the most desirable behavior. For infinitestimal price and quantity changes from a given situation (P^0,Q^0) used as the reference, we can obtain approximations of the true index for any separable preference structure. In this case, homotheticity provides the added advantage of independence from the reference situation.

To the extent that we are examining very small changes from a given situation (P^0,Q^0) , using this situation as our reference point, knowledge of the preference structure is not required to define meaningful price and quantity indexes. For larger changes, significant assumptions regarding the preference structure are required in order to define meaningful indexes. Meaningful price and quantity indexes are, however, only the first step. The equally important second step is developing an operational technique for using such indexes to interpret data.

CALCULATING ENERGY ELASTICITIES USING THE INDEXES

The Approaches

The theory of price and quantity indexes indicates how the preference structure of society regarding energy (E_X) can be used to define price and quantity indexes. However, the actual values of these indexes, and thus the calculated elasticities, come from the data on the prices and quantities of individual energy goods. There are three approaches to using these data to calculate the price elasticity of energy demand, implicit in the models:

- functional,
- direct, and
- decomposition.

Each of these approaches is described below together with its advantages and disadvantages. In all cases, separability is assumed. Thus, E(Q) represents the preference ordering among the energy goods irrespective of the nonenergy goods consumed.

Methodology and Implementation

In the functional approach, the (P, Q) data points are first used to estimate the parameters of a preference function, E(Q) , of some particular form, e.g., homogeneous quadratic $(Q^TAQ)^{1/2}$. Each functional form has corresponding index number formulae; e.g., for the homogeneous quadratic, the true price and quantity indexes are the Ideal indexes [3,4]. These index number formulae are then applied to the new, smoothed points on the E(Q) curve (P, Q) to generate the (π, θ) data points. Because these points are derived from the application of the true indexes to data points that precisely reflect the appropriate preferences, they lie on a well defined demand curve. To calculate the elasticity, the (π, θ) data points are fit to a constant elasticity function. Note that a particular functional form for E(Q) may be selected for each individual model, or a single functional form may be selected for all models.

In the direct approach, index number formulae are derived either by assuming a particular functional form for E(Q) and using the corresponding indexes, or by assuming homotheticity and using the Ideal (or Paasche or Laspeyres) index. These formulae are then applied directly to the (P, Q) data, without smoothing, to generate the (π, θ) data points. Because these points are derived from the application of indexes that do not precisely reflect the preferences, the points do not lie precisely on a well defined demand curve. In particular, the curve may appear to be more of a point-to-set rather than a point-to-point mapping with the same price index associated with many quantity indexes. Nevertheless, to calculate the elasticity, a constant elasticity function is fitted to the data points. Again, assumptions regarding E(Q) may be made for each model individually or for all models collectively.

Implementation of the functional and direct approaches involves choosing the form of E(Q) one wishes to assume for each model. Once this decision is made, the indexes, whether true or approximate, are determined and calculation of the elasticities is direct.

The decomposition approach is fundamentally different from the other two in the assumptions necessary regarding preferences and the manner in which the (π, θ) data points are utilized to calculate elasticities. Changes from the Reference case (P^0, Q^0) are normalized, making all (P, Q) data points representative of unit price changes. One of the approximate indexes (Paasche, Laspeyres, or Ideal) is applied to these normalized (P, Q) data points using (P^0, Q^0) as the reference situation to generate (π, θ) data. The (π, θ) data will lie in a demand region rather than on a demand curve. Points are fitted to describe a region (rather than a curve) that reflects the possible changes in the quantity of energy consumed for a unit price change. This region is characterized by two parameters: an average elasticity reflecting changes in the quantity of energy consumed as a result of changes in the price level, and an adjustment factor reflecting the sensitivity of the elasticity to the composition of the price change. This approach is discussed more fully in Chapter 9, "Price and Quantity Change Decomposition for Aggregated Commodities."

As in implementation of the functional and direct approaches, implementation of the decomposition approach also involves choosing the form of $E(Q)$ one wishes to assume for each model. However, once the indexes are determined, the (π, θ) points are used to estimate an average elasticity and a directional sensitivity, instead of simply calculating the fitted elasticity. The three approaches are contrasted graphically in Figure 4-3.

Advantages and Disadvantages

As noted above, the purpose of the "Aggregate Elasticity of Energy Demand" study was to determine the implicit price elasticity of demand for energy in a number of models. However, the elasticities calculated on the basis of the (P, Q) data from each model may be highly dependent on the approach taken and the indexes chosen for the given approach. In choosing an approach and indexes, the fundamental considerations are:

- Is the elasticity calculated for each model an accurate and useful reflection of its behavior?
- Do the elasticities provide a meaningful way of comparing models?

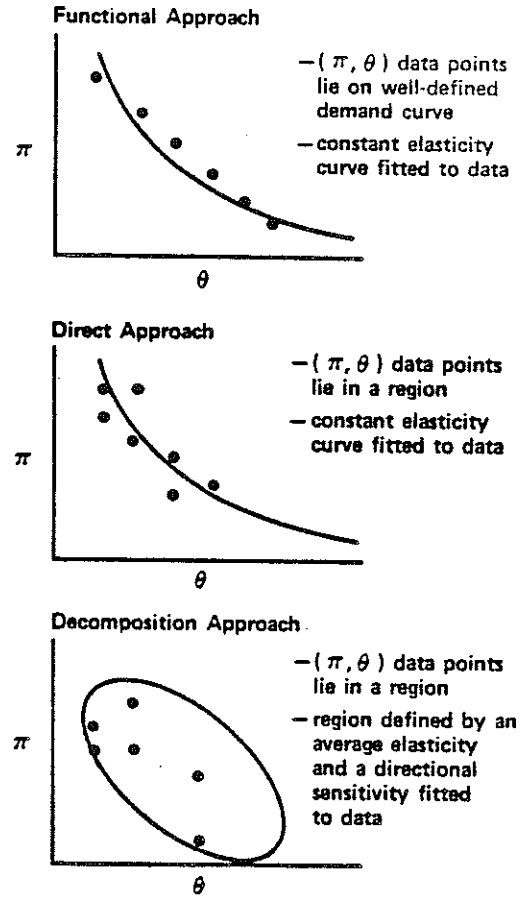

Figure 4-3 Three Approaches to Estimating Price Elasticity

It should be noted that these considerations are somewhat contradictory. To accurately reflect the behavior of a given model, the approach best suited to that model should be used. However, this would lead to comparisons among models based on entirely different behavioral assumptions. To compare models, exactly the same approach for each model should be used. However, the approach may be inconsistent with the assumptions of some models. Some compromise is needed.

The functional approach is based on the assumption that each model's behavior can be characterized by some functional form and that one has sufficient information to determine that form. This assumption normally is invalid when only nine data points are available. However, the advantages obtained with this approach are significant. In particular, the (π, θ) data points obtained through this method (like data points for a single good) precisely reflect the demand curve inherent in the $E(Q)$ preference function. Thus, the elasticity calculated will be as good a measure of the elasticity inherent in the $E(Q)$ function as in any constant elasticity estimate for a single good and, to the extent that this function is appropriate for the model, a good measure of the model's behavior. It is unlikely that any one functional form would be best for all models.

The direct approach is also based on the assumption that each model's behavior can be characterized. However, it is not necessary to assume a given functional form, only the property of homotheticity. Because the (P, Q) data are bound to deviate from the behavioral assumption and because, in the case of a homotheticity assumption, the indexes are approximate, the (π, θ) data points that are obtained will not precisely reflect the demand curve inherent in the behavioral assumption. The end result is a point-to-set mapping. One problem with this mapping is that the corresponding best fit elasticity is highly sensitive to the magnitude of the price changes considered in the experiment. Because the points do not lie along a single curve, this can lead to counterintuitive results, such as an increasing price resulting in an increasing quantity consumed. The advantage of this approach for comparing models is that it may be more reasonable to impose the assumption of homotheticity than to require some particular functional form for $E(Q)$.

The decomposition approach limits the calculation of elasticity to small changes from a given reference situation, which ensures the applicability of the approximate indexes. Furthermore, this approach acknowledges that the (π, θ) data resulting from an application of the indexes will not, in general, precisely determine a demand curve. Rather than attempting to characterize this locus of points with a single elasticity, one obtains an average elasticity for a random set of normalized price directions and an adjustment factor indicating the sensitivity of this elasticity to the direction of the price change. Through the directional sensitivity, it provides an indication of the degree to which the model can be characterized by a single elasticity or conversely the degree to which the elasticity depends upon the composition of the price changes. This directional sensitivity will vary from zero as a result both of nonlinearity of the price and quantity changes and of nonexistence of a single-valued demand curve.

The functional approach requires the most rigid assumptions of the three and is the most unsuited for general application to all models. The direct approach requires only a weaker assumption, homotheticity, and is generally applicable but produces an elasticity that is sample dependent and fails to indicate the degree to which it fits the model. The decomposition approach is limited to the small changes for which the indexes are more accurately approximated and has the additional advantage of less sample dependence and the availability of an indicator of the fit of the elasticity. The disadvantage is that it relies on linearity in normalizing price and quantity changes.

In our study, the functional approach was rejected for two reasons. First, because the experiment involved only nine data points, sufficient data did not exist to determine the functional form underlying the preferences in each model. Second, because comparison of the models was fundamental to the study, selecting a single functional form for all models would have been necessary. This would defeat the best-fit advantage of this approach. For similar reasons, the indexes used in the other two approaches were selected, not on the basis of specific functional forms, but for their general applicability. The Btu-weighted index was selected since it is so commonly used, even if it lacks theoretical justification. The Paasche and Laspeyres indexes were selected for their simplicity and because they are applicable to a wide range of roughly homothetic preference structures.

Specific results and their interpretations appear in Chapters 1 and 9 and hence are not reported here. Only a few summary comments will be offered.

Generally, for a given approach, the elasticity reported for a model is roughly the same for the Paasche and Laspeyres indexes. For example, out of the 10 models reporting the elasticity for total energy, the Paasche and Laspeyres numbers differ by 0.1 or less in 9 cases for the decomposition approach. This similarity is understandable, since they are both first order approximations of the true index. The same cannot be said, however, for the Btu-weighted index. It differs from the Paasche index by more than 0.1 in 4 out of 10 cases for both approaches.

The elasticity results differ more between approaches than among indexes. The Paasche index results differ by more than 0.1 in 6 out of 10 cases. A similar variation occurs for the Btu-weighted index. Interestingly, the same four cases have the least disparity in both the Paasche and Btu-weighted indexes.

The data from the two approaches can be used to determine what kind of qualifications must be placed on the elasticities reported for the various models. For some models, the results from the two approaches differ minimally and the sensitivity to composition of price changes is small. This indicates that the (π, θ) data lie in a narrow, demand-curve-like region, well characterized by a single elasticity. For other models, the results from the two approaches vary greatly and the sensitivity to price change composition is large. This indicates that the data do not lie in a demand-curve-like region and are not well characterized by a single elasticity.

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Chapter 5

ENERGY SYSTEM ELASTICITIES: DEFINITION AND COMPUTATION

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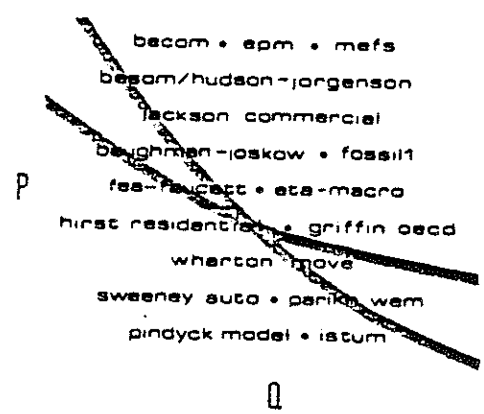

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Chapter 5

ENERGY SYSTEM ELASTICITIES: DEFINITION AND COMPUTATION

There are strikingly different opinions about the magnitude of the aggregate elasticity of energy demand. Much of that diversity of opinion stems from differences in the way the aggregate elasticity is defined, such as differences in where it is measured, which fuels it includes, and how those fuels are aggregated. The effects of these definitional differences have not generally been taken into account by those producing aggregate elasticity estimates. Hence, they often remain implicit and undocumented, complicating the comparison of the aggregate elasticity estimates.

Even when these purely definitional differences are resolved (as in the EMF experiment), significant differences in elasticity estimates can result from restrictions on the capability of certain sectors of the economy to adjust to changing energy prices that are included in some models, but not others. For example, ISTUM includes a detailed and disaggregated description of alternate means to produce industrial process heat. However, it omits consideration of the effects of energy price changes on the amount of process heat used in the production of industrial products and on the level of demand for those products. When isolated, restrictions of this sort can explain sizable differences in aggregate elasticity estimates. This chapter develops a generalized framework to rationalize the development and examination of aggregate elasticity estimates.

INTRODUCTION

Energy models can be typically organized into sectors and markets. Within a sector, quantity and price decisions are determined by the goal-seeking behavior of representative agents given technologies, expectations, endowments of stocks of resources and capital goods, and market conditions. A sector's behavior can be characterized by the elasticities (logarithmic derivatives) of its quantity choices, or in other words, its demands and supplies with respect to the prices it faces in markets.

Market behavior is determined by the competitive or regulated equilibration of agents' supplies and demands in the markets with intermediate transactions between agents. When some condition in an energy system varies, say an external price, policy, technology, or endowment, all quantity flows and prices between sectors may vary. Logarithmic ratios of quantity changes and price changes may be defined as system elasticities providing important characterizations of system behavior. These system elasticities depend both on sector elasticities and sector relationships--phenomena which should be distinguished when comparing and analyzing models of system behavior.

Further, since prices and quantities are jointly determined, part of the system must be conceptually isolated to define the system elasticities. Specifically, a quantity variation at a specific point depends on what quantity variations are induced and accounted for elsewhere in the system. Similarly, relative price variations vary at different places in the system. To define a system elasticity then, two things must be specified:

- The behavioral coverage of the elasticity--in what sector are quantity variations included?
- The location of the price measurement of the elasticity--where is the "independent" price variation indexed and measured?

Typically, energy systems have a hierarchical organization with a natural direction of energy flow from primary to secondary to final (or bottom to top) energy demand sectors. Language like "primary energy elasticity" and "secondary energy elasticity" can distinguish a different price measurement location with or without a change in the behavioral coverage of the elasticity in question, e.g., are induced changes in conversion losses included? Of course, different specifications will yield different characterizations of the same system, which can lead to confusion in analysis of sector or system differences.

This chapter develops explicit methods for specifying and calculating alternative energy system elasticities. The methodology is developed and illustrated as follows: (1) a simple energy demand system, (2) some applications of the simple system, (3) extensions of the constant-returns-to-scale energy system, (4) the EMF experiment in perspective, (5) extensions and further applications, and (6) conclusions.

A SIMPLE ENERGY DEMAND SYSTEM

This section derives the primary energy elasticity for a simple energy demand system. When the intermediate sector exhibits constant returns to scale, this elasticity is a simple function of sector elasticities and sector heat rates (energy input-output coefficients or energy input value shares). This function includes, as a special case, the standard wholesale-retail elasticity relationship.

Figure 5-1 depicts a simple energy demand system which presents essentially all the problems discussed in the introduction. There are three sectors in the complete system: primary supply, conversion, and final demand. The system is divided between primary supply and conversion; the primary supply function is replaced by an exogenously specified supply price, P_0. We may now consider primary energy demand, Q_0, as a function of P_0 and evaluate the system primary demand elasticity,

$$\frac{d \ln Q_0}{d \ln P_0} ,$$

as determined by sector behavior and intersector equilibration throughout the system. Total differentials describe system behavior and partial differentials describe sector behavior.

Final demand, Q_1, depends on its delivered price, P_1, and on real income, u. The final demand price elasticity is δ^F, and the final demand real income elasticity is δ_u^F. The representative consumer's real income depends on his attainable consumption set which in turn may depend on the primary supply price, P_0. For purposes of this analysis, this energy-economy interaction is summarized in the primary price elasticity of real income, η^F, representing the behavior of some energy-economy model. In the conversion sector (sector 1), the heat rate h^1 is the gross energy input-output ratio:

$$h^1 \triangleq Q_0/Q_1 \quad . \tag{5-1}$$

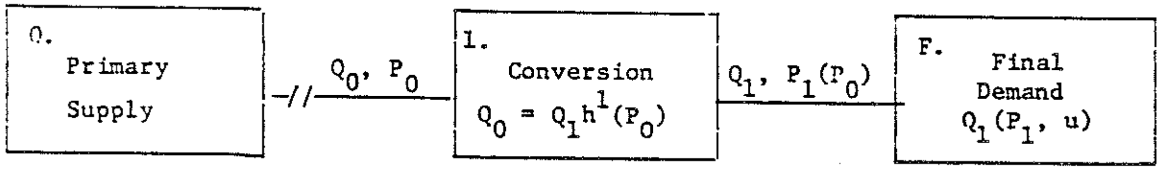
In this introductory section, the heat rate is assumed to be determined only by the input price and not by the output level,

$$h^1 = h^1(P_0) \quad . \tag{5-2}$$

(This amounts to assuming constant returns to scale in this sector as we shall see.) The price elasticity of h^1 is denoted δ^1. Now use the heat rate definition to state the conversion sector factor demand function,

$$Q_0 = h^1(P_0)Q_1 \quad . \tag{5-3}$$

Network Specification



Behavior Characterization

- Primary supply:

  P_0 - the primary supply price is exogenous.

- Conversion:

  $h^1 = Q_0/Q_1, h^1$ is the heat rate of the conversion sector,

  $h^1 = h^1(P_0)$, $\delta^1 = \dfrac{\partial \ln h^1}{\partial \ln P_0}$,

  $Q_0 = h^1(P_0)Q_1$, the conversion sector demand function,

  $P_1 = P_1(P_0)$, the conversion sector inverse supply function.

- Final demand:

  $Q_1 = Q_1(P_1, u)$, the final demand function depends on price, P_1, and real income, u,

$$\delta^F = \frac{\partial \ln Q_1}{\partial \ln P_1}, \quad \delta^F_u = \frac{\partial \ln Q_1}{\partial \ln u} .$$

  $u = u(P_0)$, real income, a function of the attainable demand set, may depend on the primary supply price, characterizing energy-economy interactions,

$$\frac{\partial \ln u}{\partial \ln P_0} = \eta^F .$$

Figure 5-1 A Simple Energy Demand System

The conversion sector supply price, P_1, is determined by its input price, P_0, in a way to be specified,

$$P_1 = P_1(P_0) \quad .$$

This relation will serve as the inverse supply function for the conversion sector.

The determinants of the system primary demand elasticity may now be derived. Since

$$Q_0 = h^1(P_0)\, Q_1[u,\, P_1(P_0)] \quad , \tag{5-4}$$

$$\frac{d \ln Q_0}{d \ln P_0} = \frac{\partial \ln h^1}{\partial \ln P_0} + \frac{d \ln Q_1}{d \ln P_0} = \delta^1 + \frac{d \ln Q_1}{d \ln P_0} \quad , \tag{5-5}$$

where

$$\frac{d \ln Q_1}{d \ln P_0} = \frac{\partial \ln Q_1}{\partial \ln u} \frac{\partial \ln u}{\partial \ln P_0} + \frac{\partial \ln Q_1}{\partial \ln P_1} \frac{\partial \ln P_1}{\partial \ln P_0} \quad , \tag{5-6}$$

$$= \delta_u^F \eta^F + \delta^F \frac{\partial \ln P_1}{\partial \ln P_0} \quad .$$

Note that there are at least five distinct demand elasticity notions in this derivation:

1. the final demand price elasticity, δ^F;
2. the final demand "displaced price" elasticity, $\delta^F \frac{\partial \ln P_1}{\partial \ln P_0}$, which depends in part on the supply price pass-through;
3. the final demand primary price elasticity, $\frac{d \ln Q_1}{d \ln P_0}$, which involves the income effect as well;
4. the intermediate sector heat rate demand elasticity, δ^1; and
5. the system primary demand elasticity, $\frac{d \ln Q_0}{d \ln P_0}$, which depends on all of the above.

These may be combined to yield

$$\frac{d \ln Q_0}{d \ln P_0} = \delta^1 + \delta^F \frac{\partial \ln P_1}{\partial \ln P_0} + \delta_u^F \eta^F \quad . \tag{5-7}$$

It remains to characterize the intermediate sector cost pass-through behavior simply. When the intermediate sector minimizes cost subject to a constant-returns-to-scale production constraint, the marginal cost pass-through equals the heat rate,

$$\frac{\partial P_1}{\partial P_0} = h^1 \quad ,$$

and its elasticity equals the gross energy input value share,

$$\frac{\partial \ln P_1}{\partial \ln P_0} = \frac{P_0}{P_1} \frac{\partial P_1}{\partial P_0} = \frac{P_0 h^1}{P_1} = \frac{P_0 Q_0}{P_1 Q_1} \triangleq v^1 \quad .$$

This amounts to proving that the displaced final demand elasticity equals the input-output value share of energy in the intervening sector(s) times the final demand elasticity,

$$\left(\frac{\partial \ln Q_1}{\partial \ln P_0}\right)_u = \frac{\partial \ln Q_1}{\partial \ln P_1} \frac{\partial \ln P_1}{\partial \ln P_0} = \delta^F v^1 \quad . \tag{5-8}$$

When the intervening heat rate is identically unity so that there is no conversion loss (and, a fortiori, no variation in intermediate demand),

$$v^1 = \frac{P_0 h^1}{P_1} = \frac{P_0}{P_1} \quad .$$

This reduces to the familiar relationship between wholesale and retail price elasticities, [1]

$$\text{wholesale elasticity} = \left(\frac{\text{wholesale price}}{\text{retail price}}\right) \times \text{retail elasticity.}$$

The force of the Shephard's lemma contribution is that Eq. 5-8 is true independent of the price variability of the intermediate sector heat rate.

The results of this section may be summarized by the following expression for the system primary energy demand elasticity,

$$\begin{bmatrix}\text{System Primary}\\\text{Price Elasticity}\end{bmatrix} = \begin{bmatrix}\text{Conversion}\\\text{Heat Rate}\\\text{Elasticity}\end{bmatrix} + \begin{bmatrix}\text{Final Demand}\\\text{Price Elasticity}\end{bmatrix} \times \begin{bmatrix}\text{Gross Energy}\\\text{Input Value}\\\text{Share}\end{bmatrix}$$

$$+ \begin{bmatrix}\text{Final Demand}\\\text{Income Elasticity}\end{bmatrix} \times \begin{bmatrix}\text{Elasticity of Income}\\\text{with respect to Primary}\\\text{Energy Price}\end{bmatrix}$$

$$\frac{d \ln Q_0}{d \ln P_0} = \delta^1 + \delta^F v^1 + \delta_u^F \eta^F \tag{5-9}$$

which holds when the intervening sector is cost-minimizing with constant returns to scale.

EXAMPLE APPLICATIONS OF THE SIMPLE SYSTEM

Three examples may give some feel for the magnitude of the ingredients of the primary demand elasticity.

The Wholesale Gasoline Elasticity

Suppose gasoline costs 75¢/gallon at the terminal and $1.25/gallon at the pump (these prices may go rapidly out of date), and there is 2% wastage between the terminal and the pump which is price invariant. Then

$$P_0 = 75¢/\text{gallon} \ ,$$

$$P_1 = \$1.25/\text{gallon} \ ,$$

$$h^1 = 1.02 \ ,$$

$$\delta^1 = 0 \ ,$$

$$v^1 = (0.75/1.25)\ 1.02 = 0.612 \ .$$

Suppose that the retail price elasticity is 0.7 and the retail income elasticity is 1.0,

$$\delta^F = 0.7 \ ,$$

$$\delta^F_u = 1.0 \ .$$

Finally suppose in the analysis in question, the event which will drive the wholesale price up 10% will lower a consumer's real income by 0.5%, so

$$\eta^F = -0.05 \ .$$

(The importance of explicit scenario specification and analysis in determining δ^F is clear.)

Then,

$$\frac{d \ln Q_0}{d \ln P_0} = \delta^1 + \delta^F v^1 + \delta^F_u \eta^F \ ,$$

$$= 0 + (0.7)\ (0.612) + (1.0)\ (-0.05) \ ,$$

$$= -0.43 - 0.05 = -0.48 \ .$$

The intermediate demand effect is zero. The retail demand elasticity is attenuated by the intermediate value share of 61% and the real income effect is small but not trivial.

<u>The Coal Price Elasticity of Electricity Demand</u>

Suppose the price of coal is $\$1.00/10^6$Btu ($25 for a $25\text{x}10^6$Btu ton) and the price of delivered electricity is $0.03/kWh delivered. Let the heat rate for electricity generated be 10,000 Btu/kWh generated ($0.01\text{x}10^6$Btu/kWh generated) and let transmission losses be 9% (0.91 kWh delivered/kWh generated),

$$P_0 = \$1.00/10^6\text{Btu} \ ,$$

$$P_1 = \$0.03/\text{kWh delivered} \ ,$$

$$h^1 = (0.01\ 10^6\text{Btu/kWh generated}) \div (0.91\ \text{kWh generated/kWh delivered}) \ , \text{ and}$$

$$v^1 = (1.00/0.03)\ (0.01/0.91) = 0.366 \ .$$

Suppose doubling this coal price would cause the generation heat rate to drop to 9500 Btu/kWh generated but leave the transmission loss constant so

$$\delta^1 \simeq -0.05 \quad .$$

Let the retail price elasticity for electricity be -0.5, the income elasticity be 1.0, and suppose the events which raise the coal price have a real income elasticity of -0.05:

$$\delta^F = -0.5 \quad ,$$

$$\delta_u^F = 1.0 \quad ,$$

$$\eta^F = -0.05 \quad .$$

Then

$$\frac{d \ln Q_0}{d \ln P_0} = \delta^1 + \delta^F v^1 + \delta_u^F \eta^F \quad ,$$

$$= -0.05 + (-0.5)\ (0.366) + (1.0)\ (-0.05) \quad ,$$

$$= -0.05 - 0.183 - 0.05 = -0.283 \quad .$$

An Environmental Policy Impact Analysis

To analyze the impact of a complex environmental policy which affects the coal price directly and also affects coal and other factor inputs to generation as well, retain the specification above. Parameterize the severity of implementation of this policy by α so that

$$\frac{d \ln P_0}{d \alpha} = 1 \quad .$$

Suppose the impact of the policy change on the level of fuel and other generation input requirements, at constant factor prices, are 20% and 40% of the impact on the coal price respectively,

$$\frac{\partial \ln h^1}{\partial \alpha} = 0.2 \quad ,$$

$$\frac{\partial \ln x^1}{\partial \alpha} = 0.4 \quad .$$

Given the cost function

$$P_1 = C_1(P_0, w; \alpha) = P_0(\alpha)\, h^1(P_0, w; \alpha) + w\, x^1(P_0, w; \alpha),$$

then

$$\frac{d \ln P_1}{d \alpha} = v^1 \frac{d \ln P_0}{d \alpha} + v^1 \frac{\partial \ln h^1}{\partial \alpha} + (1 - v^1) \frac{\partial \ln x^1}{\partial \alpha}$$

$$= (0.366)\,(1) + (0.366)\,(0.2) + (0.634)\,(0.4) \quad ,$$

$$= 0.686 \quad .$$

Then

$$Q_0 = h^1(P_0, \alpha)\, Q_1[u(P_0), P_1] \quad ,$$

$$\frac{d \ln Q_0}{d \alpha} = \frac{\partial \ln h^1}{\partial \ln P_0} \frac{d \ln P_0}{d \alpha} + \frac{\partial \ln h^1}{\partial \alpha}$$

$$+ \frac{\partial \ln Q_1}{\partial \ln u} \frac{\partial \ln u}{\partial \ln P_0} \frac{d \ln P_0}{d \alpha} + \frac{\partial \ln Q_1}{\partial \ln P_1} \frac{d \ln P_1}{d \alpha} \quad ,$$

$$= (-0.05)\,(1.0) + (0.2)$$

$$+ (1.0)\,(-0.05)\,(1.0) + (-0.5)\,(0.686) \quad ,$$

$$= -0.243 \quad .$$

This example illustrates the type of complex and offsetting effects which can be analyzed in this framework.

EXTENSIONS OF THE CONSTANT-RETURNS-TO-SCALE ENERGY SYSTEM

This section treats some extensions of the derivation of the primary energy demand elasticity in the simple energy demand system. For each case, the assumption of constant returns to scale (CRS) in intermediate sectors simplifies both the analysis and the implementation. Essentially the CRS assumptions mean that marginal costs equal average costs and marginal input-output ratios equal average input-output ratios.

The extensions analyzed here are:

- multiple intermediate sectors (in series),
- multiple final demand sectors (in parallel), and
- multiple inputs (including the supply aggregation problem).

Multiple Intermediate Sectors

Let there be N intermediate sectors arranged hierarchically as in Figure 5-2. Then the primary energy demand elasticity is a weighted sum of intermediate and final demand elasticities. These elasticities are attenuated by the product of the energy input value shares through the system.

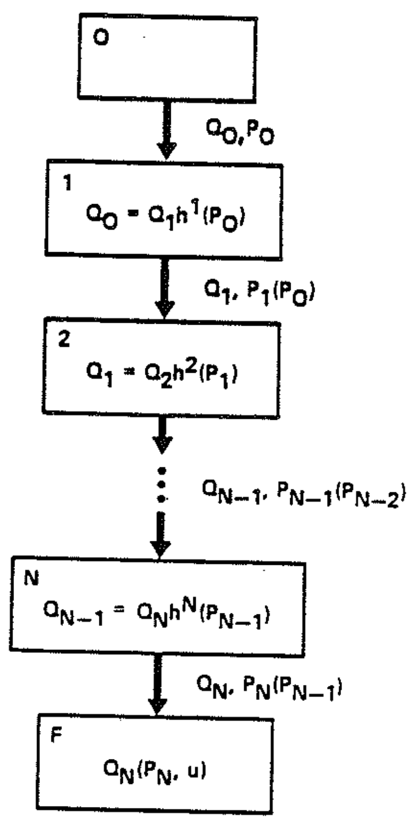


Figure 5-2 An Energy Demand System with Multiple Intermediate Sectors

From the notation presented in Figure 5-2, we have

$$Q_0 = h^1 \dots h^N Q_N \quad , \tag{5-10}$$

$$h^i = h^i(P_{i-1}), \; \frac{\partial \ln h^i}{\partial \ln P_{i-1}} = \delta^i \quad , \tag{5-11}$$

$$Q_N = Q_N(P_N, u), \; \frac{\partial \ln Q_N}{\partial \ln P_N} = \delta^F \quad , \tag{5-12}$$

$$\frac{\partial \ln Q_N}{\partial \ln u} = \delta_u^F \quad ,$$

$$u = u(P_0), \quad \frac{\partial \ln u}{\partial \ln P_0} = \eta^F \quad , \tag{5-13}$$

$$v^{i,1} \triangleq \prod_{j=1}^{i} v^j \quad . \tag{5-14}$$

Then since

$$\frac{\partial \ln P_i}{\partial \ln P_{i-1}} = v^i,$$

$$\frac{\partial \ln P_i}{\partial \ln P_0} = v^{i,1} \quad . \tag{5-15}$$

Differentiating Eq. 5-10 gives

$$\frac{d \ln Q_0}{d \ln P_0} = \sum_{i=1}^{N} \frac{d \ln h^i}{d \ln P_0} + \frac{d \ln Q_N}{d \ln P_0} \quad , \tag{5-16}$$

but

$$\frac{d \ln h^i}{d \ln P_0} = \delta^i \, v^{i-1,1} \quad ,$$

$$\frac{d \ln Q_N}{d \ln P_0} = \delta^F v^{N,1} + \delta_u^F \eta^F .$$

So

$$\frac{d \ln Q_0}{d \ln P_0} = \sum_{i=1}^{N} \delta^i v^{i-1,1} + \delta^F v^{N,1} + \delta_u^F \eta^F . \qquad (5\text{-}17)$$

In fact, the system can be entered at any point if the real income elasticity is appropriately corrected,

$$\frac{d \ln Q_j}{d \ln P_j} = \sum_{i=j+1}^{N} \delta^i v^{i-1,j+1} + \delta^F v^{N,j+1} + \delta_u^F \eta^F / v^{j,1} . \qquad (5\text{-}18)$$

Multiple Final Demand Sectors

Multiple "parallel" final demand sectors, as shown in Figure 5-3, can be represented, at least locally, by an aggregate final demand sector whose elasticity is the quantity-weighted average of the individual sector elasticities. We will ignore the income effect in this section.

Let

$$Q_N = \sum_{i=1}^{m} Q_N^i (P_N)$$

be the aggregate final demand function. Then

$$\delta^N = \frac{\partial \ln Q_N}{\partial \ln P_N} = \sum_{i=1}^{m} \frac{Q_N^i}{Q_N} \frac{\partial \ln Q_N^i}{\partial \ln P_N}$$

$$= \sum_{i=1}^{m} q_N^i \delta^{F,i} , \qquad (5\text{-}19)$$

where

$$q_N^i \triangleq \frac{Q_N^i}{Q_N} , \quad \delta^{F,i} \triangleq \frac{\partial \ln Q_N^i}{\partial \ln P_N} . \qquad (5\text{-}20)$$

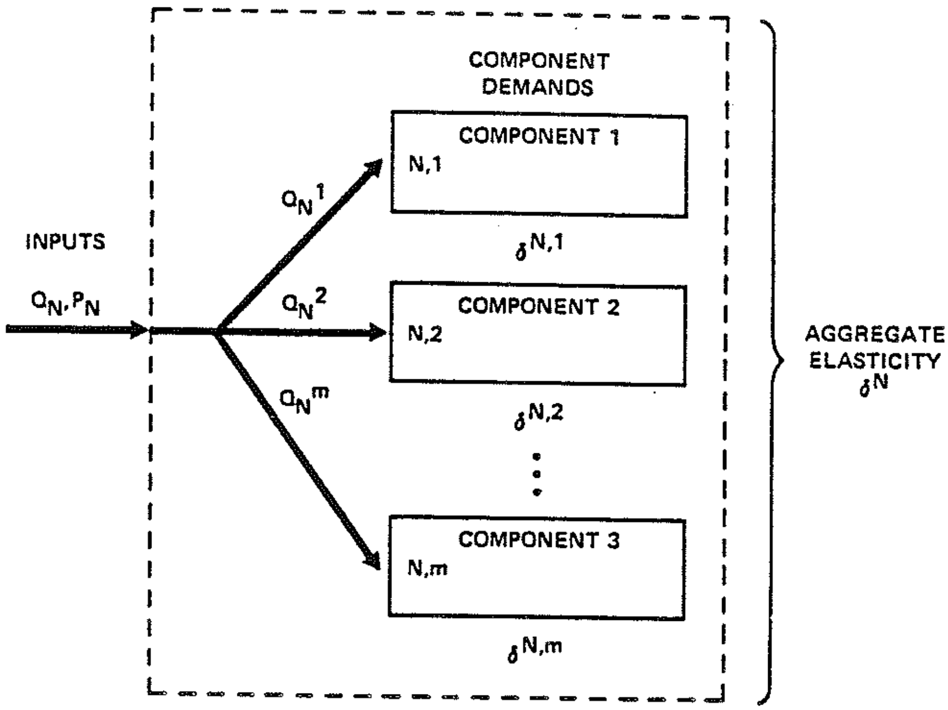


Figure 5-3 An Energy System with Multiple Final Demand Sectors

Multiple Inputs

The demand analysis of a system with multiple primary supplies or multiple inputs to the intermediate sector shown in Figure 5-3 is straightforward but doesn't go very far without introducing aggregation rules whose meaning in practice is still obscure.

Assuming constant returns to scale in sector 1, its factor demand can be written

$$Q_{0j} = Q_1 \, h_j^1(P_{01} \ldots P_{0m}, w), j = 1, \ldots, m \quad . \tag{5-21}$$

Then

$$\frac{d \ln Q_{0j}}{d \ln P_{0j'}} = \frac{\partial \ln h_j^1}{\partial \ln P_{0j'}} + \frac{\partial \ln Q_1}{\partial \ln P_1} \frac{\partial \ln P_1}{\partial \ln P_{0j'}} \quad . \tag{5-22}$$

Denote

$$\delta^1_{jj'} = \frac{\partial \ln h^1_j}{\partial \ln P_{0j'}} ,$$

$$\delta^F = \frac{\partial \ln Q_1}{\partial \ln P_1} .$$

Let the unit cost function for this sector be

$$P_1 = \sum_{j=1}^{m} P_{0j} h^1_j + wx^1 .$$

By Shephard's lemma

$$\frac{\partial P_1}{\partial P_{0j}} = h^1_j \triangleq \frac{Q_{0j}}{Q_1} ,$$

so

$$\frac{\partial \ln P_1}{\partial \ln P_{0j}} \triangleq \frac{P_{0j}}{P_1} \frac{Q_{0j}}{Q_1} \triangleq v^1_j .$$

Thus, Eq. 5-22 can be written

$$\frac{d \ln Q_{0j}}{d \ln P_{0j'}} = \delta^1_{jj'} + \delta^F v^1_{j'} , \tag{5-23}$$

which reduces to the elasticity expression derived for a single input when the subscripts are suppressed.

There are m^2 of these expressions, one for each input quantity price pair. Much of the design work for the EMF experiment was devoted to devising quantity and price aggregators for reducing such arrays to a scalar "elasticity" in a consistent and, hopefully, meaningful way.

THE EMF EXPERIMENT IN PERSPECTIVE

The methodology developed here provides a natural perspective for understanding many important differences in the results from the models included in the EMF "Agregate Elasticity of Energy Demand" experiment. Analytical transformations of all the detailed elasticities included in each model to a single consistently defined aggregate measure were not attempted nor perhaps even feasible. However, a simple stylized model of the U.S. energy system can be used to illustrate the effects of differences in point of measurement and restrictions on the ability of the economy to respond to changes in energy prices on the aggregate elasticity estimates obtained.

A Stylized Representation of the U.S. Energy System

A stylized representation of the U.S. energy system provides a common framework for interpreting the results of the EMF experiment. Many of the definitions employed in the simple system parallel those used in the design of the experiment (Chapter 2). In this simple system, oil, gas, and coal are considered as components of an aggregate called primary energy. Primary energy is combined with material inputs to produce electricity or combined directly with electric energy in an aggregate called secondary energy. Secondary energy, in turn, is transported and distributed becoming aggregate delivered energy. Figure 5-4 shows the linkages between the aggregate sectors.

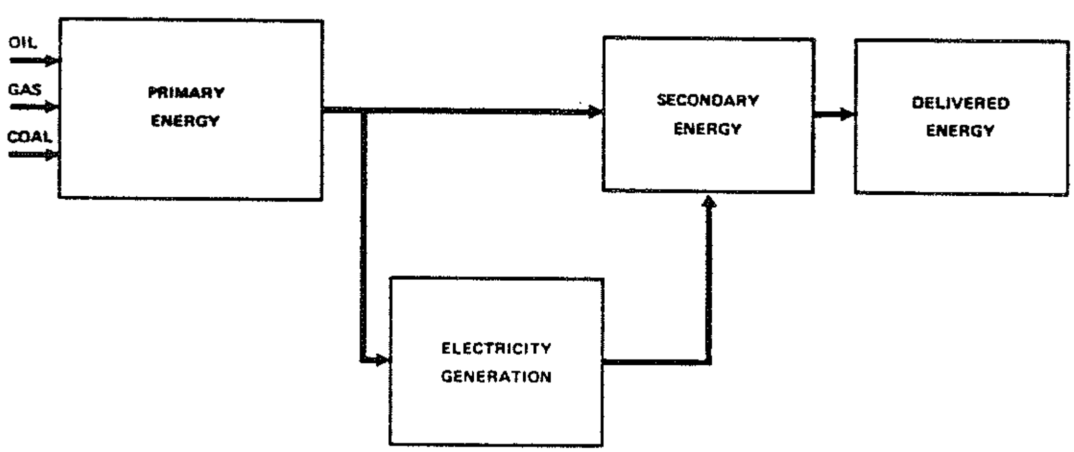


Figure 5-4 Sectoral Linkages in the Simple U.S. Energy System Model

In general, the models include detailed representations of each aggregate sector. However, simple stylized representations can be used to illustrate several key conclusions regarding the energy system elasticity estimates produced by the models.

It is assumed that the Cobb-Douglas functional form can be used to summarize the aggregation of the individual primary energy fuels. The unit cost function for primary energy can be written as

$$P_E = C_E P_O^{\alpha_O} P_G^{\alpha_G} P_C^{\alpha_C} , \tag{5-24}$$

where

P_E = the price of aggregate energy;

C_E = a constant (calibrated to 1972 data);

P_O, P_G, P_C = the prices of oil, gas, and coal, respectively; and

α_O, α_G, α_C = the value shares of oil, gas, and coal, respectively, in aggregate primary energy.

A similar Cobb-Douglas rule is assumed to summarize the aggregation of material and primary energy inputs into electricity. The unit cost function for electricity can be written as

$$P_{EL} = C_{EL} P_E^{\alpha_E} P_M^{\alpha_M} , \tag{5-25}$$

where

P_{EL} = the price of electricity (at the busbar);

C_{EL} = a constant (calibrated to 1972 data);

P_M = the price of materials; and

α_E, α_M = the value shares of primary energy and materials, respectively, in the production of electricity.

Another Cobb-Douglas formulation is used to capture the aggregate substitution between electric and nonelectric energy (here assumed to be equivalent to primary energy) in the composition of aggregate secondary energy. Thus, the price of secondary energy can be expressed as

$$P_S = C_S P_E^{\alpha_{NE}} P_{EL}^{\alpha_{EL}} , \quad (5\text{-}26)$$

where

P_S = the price of secondary energy,

C_S = a constant (calibrated to 1972 data), and

α_{NE}, α_{EL} = the value shares of nonelectric and electric energy, respectively.

Finally, the price of aggregate delivered energy, P_D, is assumed to be equal to the price of aggregate secondary energy, P_S, plus a constant, K,

$$P_D = P_S + K . \quad (5\text{-}27)$$

Equations 5-24 through 5-27 provide a straightforward way to calculate the price of delivered energy from the price of primary energy. In the simple system, it is assumed that the demand for delivered energy, Q_D, can be calculated from the following isoelastic demand function:

$$Q_D = C_D G(P_D)^{-\delta_D} , \quad (5\text{-}28)$$

where

C_D = a constant (calibrated to 1972 data),

G = Gross National Product, and

δ_D = the price elasticity of aggregate delivered energy.

It is assumed that there are no physical losses in the final transportation and distribution of energy. Thus, the demand for secondary energy, Q_S, is equal to the demand for delivered energy:

$$Q_S = Q_D . \quad (5\text{-}29)$$

The simple Cobb-Douglas structure of the primary, secondary, and electric energy sectors provides a set of simple rules that can be used to derive the demand for electricity, Q_{EL}, and primary energy, Q_E, from the demand for secondary energy

$$Q_{EL} = \frac{\alpha_{EL} \; P_S Q_S}{P_{EL}} \tag{5-30}$$

and

$$Q_E = \frac{\alpha_{NE} \; P_S Q_S}{P_E} + \frac{\alpha_E P_{EL} Q_{EL}}{P_E}$$

$$= \frac{\alpha_{NE} P_S Q_S}{P_E} + \frac{\alpha_E P_{EL} \; \alpha_{EL} \; P_S Q_S}{P_{EL}}$$

$$Q_E = (\alpha_{NE} + \alpha_E \alpha_{EL}) \frac{P_S Q_S}{P_E} \; . \tag{5-31}$$

The complete specification of the simple U.S. energy system model is shown in Figure 5-5.

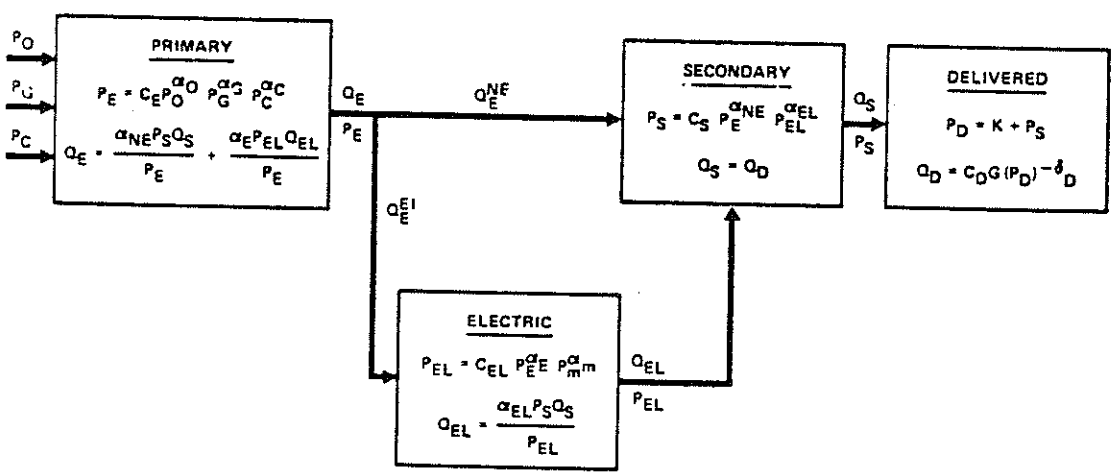


Figure 5-5 Sectoral Specification of the Simple U.S. Energy System Model

The EMF experiment could be implemented with the simple energy system model defined by Eqs. 5-24 through 5-31. Primary energy prices could be varied and used to derive variations in secondary and delivered energy prices which could in turn be used to derive changes in the demand for secondary and, finally, primary energy. Then iso-elastic aggregate primary and secondary elasticity estimates could be calculated. This, however, ignores the valuable insights that can be drawn from the methodology developed in the previous sections of the present paper. The simple rules that were developed to compute aggregate elasticities have a straightforward application in the simple energy system model. For example, since no conversion losses in secondary energy production are assumed, the aggregate secondary elasticity (δ^S) can be related to the aggregate delivered elasticity as follows:

$$\delta^S = \frac{P_S}{P_D} \delta^D \quad . \tag{5-32}$$

Now from Eq. 5-18, the elasticity for nonelectric inputs to secondary energy is

$$\delta^{NE} = \alpha_{NE} \frac{P_S}{P_D} \delta^D \quad . \tag{5-33}$$

The electricity demand elasticity is

$$\delta^{EL} = \alpha_{EL} \frac{P_S}{P_D} \delta^D \quad . \tag{5-34}$$

Additionally, the heat rate for the electricity conversion sector can be represented as

$$h_{EL} = \frac{\alpha_E P_{EL} Q_{EL}}{P_E} \Big/ Q_{EL} = \frac{\alpha_E P_{EL}}{P_E} \quad , \tag{5-35}$$

and the heat rate elasticity with respect to the energy inputs price as

$$\delta^{h_{EL}} = \frac{\partial \ln h_{EL}}{\partial \ln P_E} = -(1 - \alpha_E) \quad . \tag{5-36}$$

Thus, from Eq. 5-18, the elasticity for primary energy inputs to electricity generation is

$$\delta^{EI} = \alpha_E \alpha_{EL} \frac{P_S}{P_D} \delta^D - (1 - \alpha_E) \tag{5-37}$$

Finally, following Eq. 5-19 we can express the demand elasticity for aggregate primary energy as a quantity weighted average of the elasticities of demand for inputs to electricity and direct contributions to secondary energy production:

$$\delta^E = \frac{Q_E^{NE}}{Q_E} \delta^{NE} + \frac{Q_E^{EI}}{Q_E} \delta^{EI} \quad , \tag{5-38}$$

or, using Eq. 5-31, substituting Eqs. 5-33 and 5-37, and simplifying:

$$\delta^E = \left[\frac{\alpha_E \alpha_{EL}}{\alpha_{NE} + \alpha_E \alpha_{EL}} \alpha_E \alpha_{EL} + \frac{\alpha_{NE}}{\alpha_{NE} + \alpha_E \alpha_{EL}} \alpha_{NE} \right] \frac{P_S}{P_D} \delta^D - \frac{\alpha_E \alpha_{EL}}{\alpha_{NE} + \alpha_E \alpha_{EL}} (1 - \alpha_E) \ . \tag{5-39}$$

Additionally, Eqs. 5-25 through 5-27 can be substituted into Eq. 5-39, yielding an expression for the primary energy elasticity as a function of the delivered elasticity and the primary energy price.

Properties of the Simple Energy System

Several observations can be drawn from the key equations relating the system elasticities characterizing the simple energy system. From Eq. 5-39, we observe that even if the delivered elasticity is zero, the calculated primary energy elasticity will still be positive, although not as large as if the responsivness of end-use demands to higher prices was included.

A second observation concerns the relationship between relative prices and relative elasticities. From Eq. 5-32 we get

$$\frac{\delta^s}{\delta^d} = \frac{P_s}{P_d} \ .$$

So the simple price proportionality rule of thumb for elasticity translation holds. This rule holds for the simple system and is a good approximation to the aggregate behavior of many of the detailed models included in the study. For primary energy, on the other hand, the situation is more complicated. Shifts in the relative prices of capital and energy fuels can be expected to cause a shift in the mix of electric and nonelectric energy fuels and in the mix of capital and energy inputs to electricity generation. In general, increases in primary energy prices lead to a decrease in the demand for delivered and secondary energy, an increase in the proportion of electricity in secondary energy inputs, an increase in the efficiency of electricity generation, with an indeterminant effect on total primary energy inputs to electricity generation, and hence total primary energy demand. Rearrangement of Eq. 5-39 yields the following expression for the resulting primary elasticity to delivered elasticity ratio:

$$\frac{\delta_E}{\delta_D} = \left[\frac{\alpha_E\alpha_{EL}}{\alpha_{NE} + \alpha_E\alpha_{EL}}\,\alpha_E\alpha_{EL} + \frac{\alpha_{NE}}{\alpha_{NE} + \alpha_E\alpha_{EL}}\,\alpha_{NE}\right]\frac{P_S}{P_D} - \frac{\alpha_E\alpha_{EL}\,(1 - \alpha_E)}{(\alpha_{NE} + \alpha_E\alpha_{EL})\,\delta^D} \;.$$

Note that in this case the elasticity ratio is not proportional to the price ratio nor is it independent of the value of the delivered energy elasticity. In fact, in this case it is conceivable that the elasticity ratio exceeds unity; i.e., the primary elasticity exceeds the delivered elasticity.

An explanation of the effects of changes in primary energy prices on the system elasticities brings this set of rather stark equations to life. The left-hand side of Table 5-1 shows the electricity, secondary energy, and delivered energy prices calculated from the primary energy prices shown in the first column. The right-hand side of the table shows the key price and elasticity ratios characterizing the system at each primary energy price. These ratios are plotted and compared in Figure 5-6. The secondary to delivered elasticity ratio is identically equal to the corresponding price ratio. The relationship between the two primary to delivered ratios is far more complicated. At very low primary energy prices the primary elasticity exceeds the delivered elasticity as the heat rate elasticity dominates. As the primary energy price is increased, however, the primary elasticity becomes progressively smaller than the delivered elasticity because of the decreasing role of primary energy inputs in electricity and secondary energy production.

EXTENSIONS AND FURTHER APPLICATIONS

Extensions of the methodology developed here to more complicated systems is clear, albeit notationally complex. Nissen [2] develops system elasticity relationships for a constant-returns-to-scale technology system with feedback (i.e., a system where one of a sector's outputs is also one of its inputs), and for a system with nonhomothetic (nonconstant-returns-to-scale) technologies. Finally, he develops the machinery required to derive system elasticities in a network with a general hierarchial structure.

Table 5-1

PRICES AND ELASTICITIES FOR THE SIMPLE U.S. ENERGY SYSTEM MODEL

P_E	P_{EL}	P_S	P_D	P_S/P_D	δ^S/δ^D	P_E/P_D	δ^E/δ^D
$1	$ 7.93	$1.70	$3.44	0.49	0.49	0.29	0.37
2	10.68	2.77	4.51	0.61	0.61	0.44	0.42
3	12.72	3.73	5.47	0.68	0.68	0.54	0.43
4	14.39	4.61	6.35	0.73	0.73	0.63	0.48
5	15.84	5.44	7.17	0.76	0.76	0.70	0.49
6	17.14	6.22	7.96	0.78	0.78	0.75	0.53

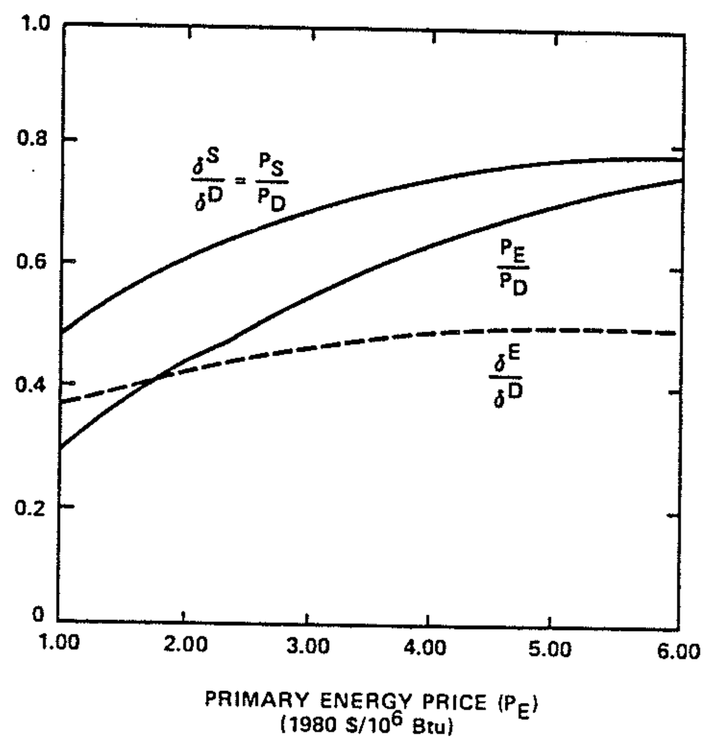


Figure 5-6 Comparison of Energy System Price and Elasticity Ratios for the Simple U.S. Model

There are a wealth of additional applications for the methods developed here (and further in [2]). For a given system with an explicit statement of sector elasticities, system (network) specification, system state (the quantity of flows and prices on network links) sector elasticities, and system elasticities can be defined, reported, and analyzed. For example, in exercises like the EMF experiment, differences in system behavior could be attributed to differences in system structure, sector behavior specification, and data.

Cost functions like those used in the stylized representation of the EMF experiment are natural minimodels of sector behavior. With standardized definitional structure they serve as natural vehicles for assessing the sensitivity of system behavior with respect to alternative sectoral specifications.

Sector analyses are partial equilibrium analyses. Minimodel representations of the remainder of the appropriate energy system components can be developed using the methods developed here. And these minimodels can be used to develop bounds for the general equilibrium consequences of the partial equilibrium analysis based on a detailed sector model.

The methods developed here offer obvious opportunities for algorithmic compression in both network visitation and programming algorithms. For example, in intermediate iterations in a network visitation algorithm, an analytic evaluation may replace a complicated sequence of process model visitations. In fact, the details of a complicated sector that is only of indirect importance in a particular application could be completely suppressed for that application. Thus, the energy system elasticity methodology could provide a useful discipline for the art, theory, and practice of combining energy models; see [3]. Both system and sector model research and development can be more successfully developed and integrated when their separate roles and interactions are explicitly understood.

CONCLUSIONS

The energy system elasticity methodology developed here helps illustrate the dependence of any aggregate elasticity estimate on where it is measured, which fuels it includes, and how those fuels are aggregated. When these factors are not standardized, large differences in elasticity estimates can result, even when they represent two different measurements from the same underlying system.

In addition to definitional differences, which were resolved during the EMF experiment, the methodology can be used to illustrate the effect of restrictions on some sectors of the economy to respond to changing energy prices on the measured energy system elasticity. Such restrictions were included in some of the models included in the EMF experiment, but not in others.

Finally, there are a wealth of applications for the methodology developed here outside of its pedagogical role in the present EMF study. For example, it can be used to structure future model comparisons, to speed the convergence of existing energy system modeling algorithms, and to guide the flexible and efficient composition of the energy models of the future.

REFERENCES

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Chapter 6

THE DATA BASE FOR ELASTICITY ESTIMATES

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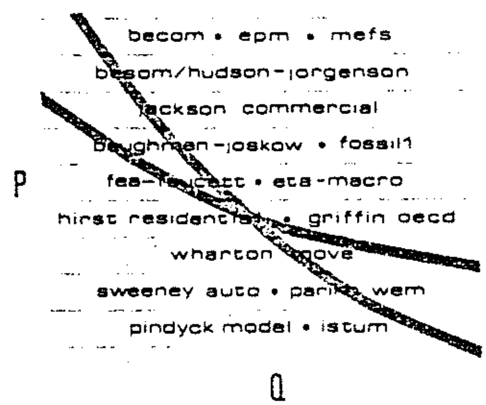

Chapter 6

THE DATA BASE FOR ELASTICITY ESTIMATES

The elasticity estimates presented in this chapter were based on the following points of measurement.

Primary Energy

- Oil and Gas: Includes shale oil and biomass. Quantity of oil measured at the refinery input; quantity of natural gas measured at the city gate. Composite price measured at the refinery input.
- Coal: Quantity of coal measured after cleaning; price measured delivered to electric utilities.
- Nuclear: Includes nuclear, solar, geothermal, and hydroelectric generation, as well as geothermal and solar heat. Primary energy equivalents (quantities) for nonfossil energy sources that are converted to electricity are computed as the amount of electric energy generated using the resource divided by the approximate thermal efficiency for electricity generation from fossil fuels equal to 0.36. Primary energy equivalents for nonfossil energy sources that are used directly as thermal energy are computed as the amount of fossil fuel energy replaced. This is assumed to be at the rate of $(1/0.36)(1.09) = 3.03$ input Btu/end-use Btu. This assumes 100% efficiency for replacement of delivered electricity, accounts for a 9% transmission loss, and converts to fossil input equivalent. In both cases, the price for the primary energy input is computed as the price of enough nuclear fuel to provide 10^6 Btu (293 kWh) of electricity at the busbar times a nuclear fuel-to-electricity conversion efficiency of 0.36.

Secondary Energy

- Oil and Gas: Quantity of oil measured at the refinery output; quantity of natural gas measured at the city gate; composite price measured at the refinery output.
- Coal: Net of coal used for electricity generation and synthetic fuels production; price measured delivered to industry.
- Electricity: Includes geothermal and solar heat. Quantity and price of electricity measured at the busbar. Primary energy equivalents for nonfossil energy sources that are used directly as thermal energy computed at 1.09 input Btu/end-use Btu. This assumes 100% efficiency for replacement of delivered electricity and accounts for a 9% transmission loss.

Baughman-Joskow

The Baughman-Joskow model reported 15- and 25-year primary and secondary energy demand estimates for the total demand, residential/commercial, and industrial sectors. All nine price cases were considered.

Primary Energy:

Price Case	Fuel	1990 (15-year)				2000 (25-year)			
		Price	Total Demand	Residential/ Commercial	Industrial	Price	Total Demand	Residential/ Commercial	Industrial
1. NNN	Oil & Gas	2.69	18.65	9.51	9.14	3.28	17.69	10.42	7.27
	Coal	0.87	26.52	10.20	16.32	0.96	28.47	10.97	17.50
	Nuclear	0.46	15.00	6.37	8.63	0.51	36.79	15.86	20.93
2. HNN	Oil & Gas	3.36	15.35	8.16	7.19	4.10	14.05	8.72	5.33
	Coal	0.87	28.82	10.49	18.33	0.96	30.89	11.52	19.37
	Nuclear	0.46	15.00	6.07	8.93	0.51	37.37	15.62	21.74
3. NHN	Oil & Gas	2.69	19.52	9.63	9.89	3.28	18.30	10.48	7.87
	Coal	1.09	22.84	8.83	14.01	1.20	18.29	6.79	11.50
	Nuclear	0.46	16.87	7.11	9.76	0.51	45.58	19.28	26.30
4. NNH	Oil & Gas	2.69	18.85	9.56	9.29	3.28	17.62	10.32	7.30
	Coal	0.87	27.73	13.71	17.02	0.96	34.97	13.72	21.25
	Nuclear	0.51	13.16	5.58	7.58	0.64	28.87	12.45	16.42
5. HHH	Oil & Gas	3.36	16.10	8.23	7.87	4.10	15.04	8.89	6.15
	Coal	1.09	26.57	9.74	16.83	1.20	25.74	9.47	16.27
	Nuclear	0.51	15.12	6.05	9.07	0.64	38.82	15.91	22.91
6. LNN	Oil & Gas	2.02	24.40	11.75	12.65	2.46	23.46	12.83	10.63
	Coal	0.87	23.70	9.32	13.72	0.96	26.86	11.08	15.78
	Nuclear	0.46	13.50	6.20	7.30	0.51	33.15	16.18	16.97
7. NLN	Oil & Gas	2.69	17.76	9.45	8.31	3.28	16.27	10.12	6.15
	Coal	0.65	30.65	11.67	18.98	0.72	39.88	15.63	24.25
	Nuclear	0.46	12.94	5.55	7.39	0.51	28.60	12.54	16.06
8. NNL	Oil & Gas	2.02	18.48	9.48	9.00	3.28	17.07	10.28	6.79
	Coal	0.87	24.92	9.50	15.42	0.96	20.51	7.55	12.96
	Nuclear	0.35	17.42	7.36	10.06	0.38	47.64	20.35	27.29
9. LLL	Oil & Gas	2.02	22.69	11.42	11.27	2.46	21.57	12.53	9.04
	Coal	0.65	26.29	10.82	15.47	0.72	30.94	12.54	18.40
	Nuclear	0.35	14.34	6.58	7.76	0.38	35.07	16.06	19.01

Baughman-Joskow Secondary Energy:

Price Case	Fuel	1990 (15-year)				2000 (25-year)			
		Price	Total Demand	Residential/ Commercial	Industrial	Price	Total Demand	Residential/ Commercial	Industrial
1. NNN	Oil & Gas	3.34	16.94	8.78	8.16	4.05	16.73	10.01	6.72
	Coal	1.03	2.48	--	2.48	1.15	3.02	--	3.02
	Electricity	3.91	12.23	5.19	7.04	3.81	19.14	8.25	10.89
2. HNN	Oil & Gas	3.94	13.64	7.47	6.17	4.83	13.05	8.30	4.75
	Coal	1.03	2.87	--	2.87	1.15	3.33	--	3.33
	Electricity	4.06	12.81	5.18	7.63	3.84	19.97	8.35	11.62
3. NHN	Oil & Gas	3.34	17.69	8.86	8.83	4.05	17.37	10.09	7.28
	Coal	1.24	1.89	--	1.89	1.39	2.23	--	2.23
	Electricity	4.15	11.91	5.02	6.89	3.80	18.96	8.02	10.94
4. NNH	Oil & Gas	3.34	17.09	8.81	8.28	4.05	17.09	10.09	7.00
	Coal	1.03	2.52	--	2.52	1.15	3.15	--	3.15
	Electricity	4.04	12.05	5.12	6.93	4.11	18.55	8.00	10.55
5. HHH	Oil & Gas	3.94	14.45	7.57	6.88	4.83	14.02	8.47	5.55
	Coal	1.24	2.25	--	2.25	1.39	2.65	--	2.65
	Electricity	4.48	12.34	4.94	7.40	4.32	19.06	7.83	11.23
6. LNN	Oil & Gas	2.74	21.97	10.63	11.34	3.27	22.72	12.49	10.23
	Coal	1.03	1.97	--	1.97	1.15	2.60	--	2.60
	Electricity	3.73	11.30	5.19	6.11	3.89	17.62	8.05	9.57
7. NLN	Oil & Gas	3.34	16.02	8.70	7.32	4.05	15.75	9.89	5.87
	Coal	0.82	3.44	--	3.44	0.91	4.23	--	4.23
	Electricity	3.59	12.57	5.39	7.18	3.50	19.62	8.60	11.02
8. NNL	Oil & Gas	3.34	16.75	8.75	8.00	4.05	16.21	9.91	6.30
	Coal	1.03	2.43	--	2.43	1.15	2.84	--	2.84
	Electricity	3.70	12.50	5.28	7.22	3.25	20.04	8.56	11.48
9. LLL	Oil & Gas	2.74	20.67	10.49	10.18	3.27	20.98	12.26	8.72
	Coal	0.82	2.73	--	2.73	0.91	3.57	--	3.57
	Electricity	3.29	11.99	5.50	6.49	3.20	18.97	8.69	10.28

BECOM

The BECOM model reported 15-year and 25-year secondary energy demand estimates for the residential, residential/commercial, and commercial sectors. All nine price cases were considered for oil and gas and electricity.

Secondary Energy:

Price Case	Fuel	1990 (15-year)				2000 (25-year)			
		Price	Residential	Residential/ Commercial	Commercial	Price	Residential	Residential/ Commercial	Commercial
1. NNN	Oil & Gas	2.92	5.88	7.40	1.52	3.57	5.64	7.56	1.92
	Electricity	11.16	4.20	6.80	2.60	10.29	6.25	10.12	3.87
2. HNN	Oil & Gas	3.65	5.78	7.28	1.50	4.46	5.32	7.13	1.81
	Electricity	12.02	4.20	6.79	2.59	10.36	6.21	10.07	3.86
3. NHN	Oil & Gas	2.92	5.90	7.42	1.52	3.57	5.64	7.56	1.92
	Electricity	11.16	4.18	6.78	2.60	10.29	6.25	10.12	3.87
4. NNH	Oil & Gas	2.92	5.98	7.41	1.43	3.57	5.66	7.61	1.95
	Electricity	11.16	4.20	6.80	2.60	10.29	6.32	10.31	3.99
5. HHH	Oil & Gas	3.65	4.94	6.43	1.49	4.46	5.32	7.13	1.81
	Electricity	12.02	4.19	6.77	2.58	10.36	6.21	10.07	3.86
6. LNN	Oil & Gas	2.20	6.19	7.73	1.54	2.67	5.98	8.00	2.02
	Electricity	10.81	4.16	6.76	2.60	10.26	6.35	10.24	3.89
7. NLN	Oil & Gas	2.92	5.88	7.40	1.52	3.57	5.64	7.56	1.92
	Electricity	11.16	4.21	6.81	2.60	10.29	6.25	10.12	3.87
8. NNL	Oil & Gas	2.92	5.88	7.40	1.52	3.57	5.64	7.56	1.92
	Electricity	11.16	4.20	6.80	2.60	10.29	6.25	10.12	3.87
9. LLL	Oil & Gas	2.20	6.15	7.70	1.55	2.67	7.96	9.99	2.03
	Electricity	10.81	4.20	6.81	2.61	10.26	6.35	10.24	3.89

BESOM/H-J

The BESOM/H-J model reported 15- and 25-year primary energy demand estimates for the total demand sector and 15- and 25-year secondary estimates for the total demand, residential/commercial, industrial, and transportation sectors. All nine price cases were used.

Primary Energy:

Price Case		Fuel	1990 (15-year) Price	1990 (15-year) Total Demand	2000 (25-year) Price	2000 (25-year) Total Demand
1.	NNN	Oil & Gas	3.00	64.72	3.55	67.40
		Coal	1.19	25.78	1.28	31.55
		Nuclear	0.46	19.65	0.51	38.14
2.	HNN	Oil & Gas	3.67	57.46	4.35	58.23
		Coal	1.19	25.05	1.28	24.23
		Nuclear	0.46	19.63	0.51	43.28
3.	NHN	Oil & Gas	3.00	63.36	3.53	63.98
		Coal	1.41	25.39	1.52	30.87
		Nuclear	0.46	20.16	0.51	39.00
4.	NNH	Oil & Gas	3.00	63.03	3.55	66.65
		Coal	1.19	24.92	1.28	29.50
		Nuclear	0.58	20.23	0.64	39.29
5.	HHH	Oil & Gas	3.67	56.96	4.34	57.57
		Coal	1.41	22.79	1.52	25.66
		Nuclear	0.58	20.86	0.64	40.70
6.	LNN	Oil & Gas	2.33	71.18	2.72	72.94
		Coal	1.19	27.89	1.28	35.27
		Nuclear	0.46	19.21	0.51	36.93
7.	NLN	Oil & Gas	3.00	64.51	3.54	67.80
		Coal	0.97	27.39	1.04	34.59
		Nuclear	0.46	19.31	0.51	37.33
8.	NNL	Oil & Gas	3.00	64.75	3.55	68.07
		Coal	1.19	26.71	1.28	32.22
		Nuclear	0.35	19.07	0.38	37.02
9.	LLL	Oil & Gas	2.33	72.36	2.72	76.63
		Coal	0.97	28.60	1.04	36.42
		Nuclear	0.35	18.77	0.38	36.58

BESOM/H-J Secondary Energy:

Price Case	Fuel	1990 (15-year)					2000 (25-year)				
		Price	Total Demand	Residential/ Commercial	Industrial	Transportation	Price	Total Demand	Residential/ Commercial	Industrial	Transportation
1. NNN	Oil & Gas	3.65	53.47	14.25	19.34	19.88	4.25	57.41	11.57	22.97	22.87
	Coal	1.33	7.27	--	7.27	--	1.42	11.63	--	11.63	--
	Electricity	8.83	13.89	6.97	6.74	0.18	8.03	20.88	10.03	10.55	0.30
2. HNN	Oil & Gas	4.38	47.44	12.43	16.54	18.47	5.12	50.17	9.77	19.29	21.11
	Coal	1.33	7.91	--	7.91	--	1.42	11.54	--	11.54	--
	Electricity	8.88	13.48	6.79	6.52	0.17	7.29	20.30	9.51	10.51	0.28
3. NHN	Oil & Gas	3.65	52.24	13.98	18.57	19.69	4.25	54.52	10.92	21.20	22.40
	Coal	1.56	7.64	--	7.64	--	1.67	12.30	--	12.30	--
	Electricity	9.63	13.86	6.93	6.75	0.18	8.09	20.83	9.92	10.61	0.30
4. NNH	Oil & Gas	3.65	53.31	14.24	19.25	19.82	4.20	56.84	11.47	22.70	22.67
	Coal	1.33	7.27	--	7.27	--	1.42	11.59	--	11.59	--
	Electricity	9.03	13.84	6.93	6.73	0.18	8.09	20.66	9.93	10.43	0.30
5. HHH	Oil & Gas	4.38	46.86	12.28	16.28	18.30	5.12	49.42	9.62	18.98	20.82
	Coal	1.56	7.77	--	7.77	--	1.67	11.40	--	11.40	--
	Electricity	9.06	13.29	6.69	6.69	0.18	7.60	20.38	9.51	10.59	0.28
6. LNN	Oil & Gas	2.92	58.88	15.69	21.76	21.43	2.98	62.08	13.25	24.17	24.66
	Coal	1.33	7.24	--	7.24	--	1.42	11.67	--	11.67	--
	Electricity	8.84	14.50	7.27	7.04	0.19	8.36	21.67	10.34	11.01	0.32
7. NLN	Oil & Gas	3.65	53.44	14.41	18.95	20.08	4.25	58.21	11.82	23.05	23.34
	Coal	1.11	8.02	--	8.02	--	1.18	12.47	--	12.47	--
	Electricity	8.66	13.95	7.02	6.75	0.18	7.91	21.11	10.15	10.66	0.30
8. NNL	Oil & Gas	3.65	53.51	14.21	19.37	19.91	4.25	57.94	12.08	22.96	22.90
	Coal	1.33	7.27	--	7.27	--	1.42	11.63	--	11.63	--
	Electricity	8.80	13.97	7.02	6.77	0.18	7.98	20.69	9.78	10.61	0.30
9. LLL	Oil & Gas	2.92	60.10	16.13	22.32	21.65	2.98	65.47	13.68	26.69	25.10
	Coal	1.11	7.04	--	7.04	--	1.18	11.81	--	11.81	--
	Electricity	8.50	14.58	7.31	7.07	0.20	8.00	21.85	10.41	11.12	0.32

EPM

The EPM model reported 15-, 25-, and 35-year primary energy demand estimates for the total demand sector and 15-, 25-, and 35-year secondary estimates for the total demand, residential/commercial, industrial, and transportation sectors. Three price cases were used: 1(NNN), 5(HHH), and 9(LLL).

Primary Energy:

Price Case	Fuel	1990 (15-year)		2000 (25-year)		2010 (35-year)	
		Price	Total Demand	Price	Total Demand	Price	Total Demand
1. NNN	Oil & Gas	2.70	60.71	3.28	69.64	4.00	75.87
	Coal	0.87	29.57	0.96	36.75	1.06	44.81
	Nuclear	0.48	9.17	0.51	14.11	0.57	20.69
5. HHH	Oil & Gas	3.36	51.84	4.10	56.45	5.00	37.29
	Coal	1.09	28.26	1.21	32.36	1.33	61.59
	Nuclear	0.58	10.35	0.63	17.44	0.70	24.36
9. LLL	Oil & Gas	2.02	73.98	2.46	92.10	3.00	101.22
	Coal	0.65	28.95	0.72	33.93	0.79	47.08
	Nuclear	0.40	4.03	0.41	4.86	0.44	8.44

Secondary Energy:

Price Case	Fuel	1990 (15-year)				
		Price	Total Demand	Residential/ Commercial	Industrial	Transportation
1. NNN	Oil & Gas	3.17	50.63	13.05	19.84	17.74
	Coal	1.40	6.66	--	6.66	--
	Electricity	9.40	10.73	5.38	5.35	--
5. HHH	Oil & Gas	3.88	45.58	11.85	17.06	16.67
	Coal	1.64	6.79	--	6.79	--
	Electricity	10.29	10.24	5.18	5.06	--
9. LLL	Oil & Gas	2.44	58.22	14.61	24.41	19.20
	Coal	1.14	5.88	--	5.88	--
	Electricity	8.35	11.38	5.64	5.74	--

EPM Secondary Energy (continued):

Price Case	Fuel	2000 (25-year) Price	Total Demand	Residential/ Commercial	Industrial	Transportation
1. NNN	Oil & Gas	3.79	57.78	14.28	21.84	21.65
	Coal	1.50	10.46	--	10.46	--
	Electricity	9.55	14.31	6.86	7.42	0.04
5. HHH	Oil & Gas	4.66	51.07	12.60	18.49	19.98
	Coal	1.76	10.33	--	10.33	--
	Electricity	10.44	13.66	6.62	6.98	0.05
9. LLL	Oil & Gas	2.91	68.46	16.55	27.89	24.02
	Coal	1.22	9.54	--	9.54	--
	Electricity	8.44	15.29	7.21	8.04	0.03
		2010 (35-year)				
1. NNN	Oil & Gas	4.53	63.38	15.09	22.35	25.94
	Coal	1.60	14.55	--	14.55	--
	Electricity	9.90	18.85	9.39	9.35	0.10
5. HHH	Oil & Gas	5.34	55.93	13.05	19.04	23.84
	Coal	1.89	14.04	--	14.04	--
	Electricity	10.62	18.16	9.18	8.85	0.13
9. LLL	Oil & Gas	3.48	75.68	17.94	28.61	29.12
	Coal	1.30	14.06	--	14.06	--
	Electricity	8.84	20.05	9.83	10.15	0.07

ETA-MACRO

The ETA-MACRO model reported 15-, 25-, and 35-year energy demand estimates at the primary and secondary energy level for the total demand sector. All price cases were used except 7(NLN).

Primary Energy:

Price Case	Fuel	1990 (15-year)		2000 (25-year)		2010 (35-year)	
		Price	Total Demand	Price	Total Demand	Price	Total Demand
1. NNN	Oil & Gas	2.70	61.40	3.30	69.40	4.00	76.50
	Coal	0.90	41.60	1.00	54.80	1.10	48.80
	Nuclear	0.74	6.00	0.79	12.50	0.84	43.10
2. HNN	Oil & Gas	3.40	55.80	4.10	61.70	5.00	66.90
	Coal	0.90	44.80	1.00	59.30	1.10	52.70
	Nuclear	0.74	6.00	0.79	13.00	0.84	46.50
3. NHN	Oil & Gas	2.70	62.00	3.30	70.20	4.00	77.10
	Coal	1.10	30.20	1.20	38.60	1.30	35.60
	Nuclear	0.74	16.10	0.79	26.70	0.84	54.30
4. NNH	Oil & Gas	2.70	61.40	3.30	69.40	4.00	76.70
	Coal	0.90	41.60	1.00	60.20	1.10	82.60
	Nuclear	0.85	5.90	0.91	7.20	0.97	8.80
5. HHH	Oil & Gas	3.40	56.70	4.10	62.70	5.00	68.00
	Coal	1.10	32.60	1.20	41.70	1.30	36.50
	Nuclear	0.85	16.00	0.91	27.60	0.97	58.50
6. LNN	Oil & Gas	2.00	74.70	2.50	85.90	3.00	90.60
	Coal	0.90	32.80	1.00	44.40	1.10	44.10
	Nuclear	0.74	6.00	0.79	11.80	0.84	39.20
8. NNL	Oil & Gas	2.70	61.20	3.30	69.00	4.00	75.70
	Coal	0.90	31.90	1.00	40.80	1.10	36.20
	Nuclear	0.64	16.00	0.67	27.40	0.71	57.80
9. LLL	Oil & Gas	2.00	68.40	2.50	78.90	3.00	88.30
	Coal	0.70	40.00	0.70	57.90	0.80	79.90
	Nuclear	0.64	5.90	0.67	7.20	0.71	8.80

ETA-MACRO Secondary Energy:

Price Case	Fuel	1990 (15-year)		2000 (25-year)		2010 (35-year)	
		Price	Total Demand	Price	Total Demand	Price	Total Demand
1. NNN	Oil & Gas	2.70	61.40	3.30	69.40	4.00	76.50
	Coal	0.90	6.10	1.00	7.40	1.10	9.00
	Electricity	6.92	14.33	7.21	20.48	7.38	28.33
2. HNN	Oil & Gas	3.40	55.80	4.10	61.70	5.00	66.90
	Coal	0.90	6.10	1.00	7.40	1.10	9.00
	Electricity	6.92	15.36	7.21	22.18	7.38	30.72
3. NHN	Oil & Gas	2.70	62.00	3.30	70.20	4.00	77.10
	Coal	1.10	6.10	1.20	7.40	1.30	9.00
	Electricity	7.27	13.65	7.59	19.80	7.65	27.65
4. NNH	Oil & Gas	2.70	61.40	3.30	69.40	4.00	76.70
	Coal	0.90	6.10	1.00	7.40	1.10	9.00
	Electricity	6.92	13.99	7.18	20.48	7.47	27.99
5. HHH	Oil & Gas	3.40	56.70	4.10	62.70	5.00	68.00
	Coal	1.10	6.10	1.20	7.40	1.30	9.00
	Electricity	7.47	14.68	7.74	21.16	7.91	29.35
6. LNN	Oil & Gas	2.00	74.70	2.50	85.90	3.00	90.60
	Coal	0.90	6.10	1.00	7.40	1.10	9.00
	Electricity	6.92	12.97	7.21	18.43	7.38	25.26
8. NNL	Oil & Gas	2.70	61.20	3.30	69.00	4.00	75.70
	Coal	0.90	6.10	1.00	7.40	1.10	9.00
	Electricity	6.83	14.33	7.03	20.82	7.15	29.01
9. LLL	Oil & Gas	2.00	68.40	2.50	78.90	3.00	88.30
	Coal	0.70	6.10	0.70	7.40	0.80	9.00
	Electricity	6.27	13.65	6.48	19.80	6.68	27.30

FEA-Faucett

The FEA-Faucett model reported 15-year primary and secondary energy demand estimates for the transportation sector. Oil and gas prices from three price cases were examined: 1(NNN), 5(HHH), and 9(LLL). Additionally the FEA-Faucett estimates are divided into two categories: Those with CAFE (corporate average fuel efficiency) standards and those without.

Primary Energy:

Price Case	Fuel	1990 (CAFE)		1990 (No CAFE)	
		Price ($)	Transportation	Price ($)	Transportation
1. NNN	Oil & Gas	2.66	26.63	2.66	29.60
5. HHH	Oil & Gas	3.32	25.98	3.32	28.69
9. LLL	Oil & Gas	1.99	27.33	1.99	30.65

Secondary Energy:

1. NNN	Oil & Gas	3.31	24.23	3.31	26.94
5. HHH	Oil & Gas	4.04	23.64	4.04	26.11
9. LLL	Oil & Gas	2.58	24.87	2.58	27.89

FOSSIL1

The FOSSIL1 model reported 15-, 25-, and 35-year primary and secondary energy demand estimates for the total demand sector. All nine price cases were used.

Primary Energy:

Price Case	Fuel	1990 (15-year) Price ($)	1990 (15-year) Total Demand	2000 (25-year) Price ($)	2000 (25-year) Total Demand	2010 (35-year) Price ($)	2010 (35-year) Total Demand
1. NNN	Oil & Gas	2.69	74.70	3.29	71.00	4.02	54.50
	Coal	0.87	30.20	0.96	56.70	1.06	99.70
	Nuclear	0.46	17.20	0.51	35.60	0.56	59.70
2. HNN	Oil & Gas	3.13	73.60	4.23	68.00	5.71	48.20
	Coal	0.87	30.60	0.96	58.00	1.06	104.00
	Nuclear	0.46	17.30	0.51	36.00	0.56	61.80
3. NHN	Oil & Gas	2.69	75.00	2.93	71.90	4.02	56.40
	Coal	1.01	29.30	1.23	53.10	1.51	91.60
	Nuclear	0.46	17.40	0.51	37.00	0.56	63.70
4. NNH	Oil & Gas	2.69	74.60	3.29	70.90	4.02	54.50
	Coal	0.87	30.40	0.96	57.70	1.06	101.50
	Nuclear	0.53	17.00	0.65	34.60	0.80	57.40
5. HHH	Oil & Gas	3.13	73.90	4.23	68.90	5.71	50.00
	Coal	1.01	29.90	1.23	55.50	1.51	97.70
	Nuclear	0.53	17.40	0.65	36.50	0.80	63.60
6. LNN	Oil & Gas	2.32	75.80	2.56	74.20	2.83	60.90
	Coal	0.87	29.70	0.96	54.90	1.06	95.10
	Nuclear	0.46	17.00	0.51	34.90	0.56	57.60
7. NLN	Oil & Gas	2.69	74.30	3.29	70.00	4.02	52.70
	Coal	0.75	31.00	0.75	60.20	0.75	107.20
	Nuclear	0.46	16.90	0.51	34.20	0.56	56.20
8. NNL	Oil & Gas	2.69	74.70	3.29	71.10	4.02	54.50
	Coal	0.87	29.90	0.96	55.70	1.06	98.10
	Nuclear	0.40	17.30	0.40	36.40	0.40	61.80
9. LLL	Oil & Gas	2.32	75.40	2.56	73.30	2.83	59.10
	Coal	0.75	30.30	0.75	57.30	0.75	100.80
	Nuclear	0.40	16.90	0.40	34.50	0.40	56.20

FOSSIL1 Secondary Energy:

Price Case	Fuel	1990 (15-year) Price ($)	1990 (15-year) Total Demand	2000 (25-year) Price ($)	2000 (25-year) Total Demand	2010 (35-year) Price ($)	2010 (35-year) Total Demand
1. NNN	Oil & Gas	2.69	65.70	3.29	71.70	4.02	74.80
	Coal	0.87	5.26	0.96	6.80	1.06	8.60
	Electricity	7.46	16.50	8.29	26.20	9.48	38.40
2. HNN	Oil & Gas	3.13	64.60	4.23	68.60	5.71	68.30
	Coal	0.87	5.47	0.96	7.50	1.06	9.90
	Electricity	7.58	16.60	8.36	26.70	9.39	40.10
3. NHN	Oil & Gas	2.69	66.00	3.29	72.50	4.02	76.50
	Coal	1.01	5.00	1.23	6.00	1.51	6.90
	Electricity	7.63	16.40	8.62	26.00	9.90	37.90
4. NNH	Oil & Gas	2.69	65.70	3.29	71.70	4.02	74.90
	Coal	0.87	5.30	0.96	6.80	1.06	8.60
	Electricity	7.58	16.50	8.44	26.20	9.76	38.20
5. HHH	Oil & Gas	3.13	64.80	4.23	69.30	5.71	69.90
	Coal	1.01	5.20	1.23	6.60	1.51	8.30
	Electricity	7.81	16.60	8.87	26.50	10.13	39.40
6. LNN	Oil & Gas	2.32	66.90	2.56	75.10	2.83	81.40
	Coal	0.87	5.00	0.96	6.10	1.06	7.20
	Electricity	7.36	16.30	8.26	25.60	9.61	36.70
7. NLN	Oil & Gas	2.69	65.50	3.29	70.90	4.02	73.10
	Coal	0.75	5.50	0.75	7.70	0.75	10.30
	Electricity	7.31	16.50	8.00	26.50	9.14	38.00
8. NNL	Oil & Gas	2.69	65.70	3.29	71.70	4.02	74.70
	Coal	0.87	5.30	0.96	6.80	1.06	8.60
	Electricity	7.52	16.50	8.16	26.30	9.26	38.60
9. LLL	Oil & Gas	2.32	66.70	2.56	74.20	2.83	79.70
	Coal	0.75	5.32	0.75	6.90	0.75	8.90
	Electricity	7.17	16.40	7.85	25.80	9.06	37.40

FOSSIL1 Conservation

The FOSSIL1 Conservation model, a variant of the FOSSIL1 model, employs disaggregated elasticities twice as large as the baseline FOSSIL1 system. It produced energy demand estimates in the same categories as the FOSSIL1 model.

Primary Energy:

Price Case		Fuel	1990 (15-year)		2000 (25-year)		2010 (35-year)	
			Price ($)	Total Demand	Price ($)	Total Demand	Price ($)	Total Demand
1.	NNN	Oil & Gas	2.69	68.66	3.29	59.95	4.02	38.96
		Coal	0.87	27.84	0.96	50.04	1.06	85.02
		Nuclear	0.46	16.35	0.51	32.39	0.56	51.81
2.	HNN	Oil & Gas	3.13	67.33	4.23	56.59	5.71	32.66
		Coal	0.87	28.06	0.96	50.82	1.06	87.56
		Nuclear	0.46	16.40	0.51	32.55	0.56	52.69
3.	NHN	Oil & Gas	2.69	68.95	3.29	60.66	4.02	40.10
		Coal	1.01	26.97	1.23	47.05	1.51	79.04
		Nuclear	0.46	16.57	0.51	33.67	0.56	54.57
4.	NNH	Oil & Gas	2.69	68.91	3.29	59.85	4.02	38.85
		Coal	0.87	28.05	0.96	50.87	1.06	86.24
		Nuclear	0.53	16.20	0.65	31.50	0.80	50.13
5.	HHH	Oil & Gas	3.13	67.56	4.23	57.22	5.71	33.76
		Coal	1.01	27.44	1.23	48.68	1.51	82.74
		Nuclear	0.53	16.48	0.65	32.99	0.80	53.80
6.	LNN	Oil & Gas	2.32	70.04	2.56	63.52	2.83	45.51
		Coal	0.87	27.51	0.96	48.84	1.06	82.12
		Nuclear	0.46	16.28	0.51	32.00	0.56	50.86
7.	NLN	Oil & Gas	2.69	68.39	3.29	59.21	4.02	37.76
		Coal	0.75	28.66	0.75	52.94	0.75	90.52
		Nuclear	0.46	16.13	0.51	31.15	0.56	49.41
8.	NNL	Oil & Gas	2.69	68.70	3.29	50.03	4.02	38.98
		Coal	0.87	27.63	0.96	49.29	1.06	83.89
		Nuclear	0.40	16.49	0.40	33.19	0.40	53.32
9.	LLL	Oil & Gas	2.32	69.80	2.56	62.89	2.83	44.30
		Coal	0.75	28.15	0.75	51.00	0.75	86.63
		Nuclear	0.40	16.21	0.40	31.63	0.40	50.07

FOSSIL1 Conservation Secondary Energy:

Price Case		Fuel	1990 (15-year)		2000 (25-year)		2010 (35-year)	
			Price ($)	Total Demand	Price ($)	Total Demand	Price ($)	Total Demand
1.	NNN	Oil & Gas	2.69	60.44	3.29	61.41	4.02	59.77
		Coal	0.87	4.84	0.96	5.81	1.06	6.84
		Electricity	7.61	15.23	8.64	22.86	10.27	31.53
2.	HNN	Oil & Gas	3.13	59.08	4.23	57.99	5.71	53.45
		Coal	0.87	5.01	0.96	6.33	1.06	7.71
		Electricity	7.74	15.29	8.74	23.01	10.25	32.37
3.	NHN	Oil & Gas	2.69	60.66	3.29	61.97	4.02	60.86
		Coal	1.01	4.57	1.23	5.10	1.51	5.48
		Electricity	7.78	15.17	8.98	22.64	10.71	31.11
4.	NNH	Oil & Gas	2.69	60.44	3.29	61.38	4.02	59.72
		Coal	0.87	4.84	0.96	5.81	1.06	6.82
		Electricity	7.65	15.22	8.81	22.80	10.54	31.35
5.	HHH	Oil & Gas	3.13	59.26	4.23	58.52	5.71	54.47
		Coal	1.01	4.76	1.23	5.60	1.51	6.43
		Electricity	7.95	15.23	9.25	22.77	10.99	31.75
6.	LNN	Oil & Gas	2.32	61.89	2.56	65.09	2.83	66.41
		Coal	0.87	4.64	0.96	5.29	1.06	5.80
		Electricity	7.50	15.13	8.62	22.49	10.33	30.60
7.	NLN	Oil & Gas	2.69	60.26	3.29	60.82	4.02	58.68
		Coal	0.75	5.09	0.75	6.56	0.75	8.17
		Electricity	7.45	15.28	8.37	23.06	9.89	31.93
8.	NNL	Oil & Gas	2.69	60.45	3.29	61.44	4.02	59.82
		Coal	0.87	4.84	0.96	5.82	1.06	6.86
		Electricity	7.56	15.24	8.51	22.91	10.06	31.68
9.	LLL	Oil & Gas	2.32	61.68	2.56	64.53	2.83	65.23
		Coal	0.75	4.92	0.75	6.03	0.75	7.25
		Electricity	7.30	15.20	8.20	22.77	9.74	31.21

Griffin OECD

The Griffin OECD model reported 15- and 25- year primary and secondary energy demand estimates for seven price cases (4(NNH) and 8(NNL) excepted). Total demand, residential, commercial/industrial, and transportation were the sectors examined.

Primary Energy:

Price Case	Fuel	1990 (15-year) Price ($)	Total Demand	Residential	Commercial/ Industrial	Transportation	2000 (25-year) Price ($)	Total Demand	Residential/ Commercial	Commercial/ Industrial	Transportation
1. NNN	Oil & Gas	2.68	49.22	13.29	15.14	20.79	3.25	56.26	13.42	18.46	24.38
	Coal	0.87	22.47	9.63	12.79	0.05	0.95	27.65	11.89	15.71	0.05
	Nuclear	0.46	15.92	7.95	7.93	0.04	0.51	26.43	13.24	13.14	0.05
2. HNN	Oil & Gas	3.35	42.90	9.81	14.24	15.85	4.06	48.59	8.70	17.49	22.40
	Coal	0.87	24.90	10.88	13.97	0.05	0.95	31.62	14.74	16.83	0.05
	Nuclear	0.46	15.92	8.06	7.83	0.03	0.51	26.43	13.28	13.11	0.04
3. NHN	Oil & Gas	2.68	50.92	14.23	15.91	20.78	3.25	58.14	14.54	19.24	24.36
	Coal	1.08	19.48	8.33	11.10	0.05	1.19	23.97	10.28	13.64	0.05
	Nuclear	0.46	15.92	7.77	8.11	0.04	0.51	26.42	12.93	13.44	0.05
5. HHH	Oil & Gas	3.35	44.19	10.60	14.75	18.84	4.06	50.76	10.27	18.11	22.38
	Coal	1.08	22.13	9.64	12.44	0.05	1.19	27.25	11.93	15.27	0.05
	Nuclear	0.58	15.92	7.87	8.02	0.03	0.64	26.43	13.10	13.28	0.05
6. LNN	Oil & Gas.	2.01	58.92	18.49	16.90	23.53	2.44	67.15	19.57	20.31	27.27
	Coal	0.87	19.06	7.96	11.04	0.06	0.95	23.35	9.75	13.54	0.06
	Nuclear	0.46	15.93	7.83	8.05	0.05	0.51	26.44	13.03	13.34	0.07
7. NLN	Oil & Gas	2.68	47.51	12.37	14.34	20.80	3.25	54.54	12.42	17.78	24.39
	Coal	0.65	25.78	11.00	14.72	0.06	0.71	31.36	13.38	17.93	0.05
	Nuclear	0.46	15.93	8.19	7.70	0.04	0.51	26.42	13.65	12.72	0.05
9. LLL	Oil & Gas	2.01	56.63	17.34	15.75	23.54	2.44	64.58	18.21	19.09	27.28
	Coal	0.65	22.61	9.39	13.16	0.06	0.71	27.70	11.49	16.14	0.07
	Nuclear	0.35	15.92	8.04	7.83	0.05	0.38	26.43	13.41	12.96	0.06

Griffin OECD Secondary Energy:

Price Case		Fuel	1990 (15-year)					2000 (25-year)				
			Price ($)	Total Demand	Residential	Commercial/ Industrial	Transportation	Price ($)	Total Demand	Residential/ Commercial	Commercial/ Industrial	Transportation
1.	NNN	Oil & Gas	3.34	37.70	9.19	9.15	19.36	3.96	43.00	9.38	10.99	22.63
		Coal	0.98	3.18	--	3.17	0.01	1.07	3.74	--	3.73	0.01
		Electricity	6.46	12.65	6.32	6.31	0.03	6.57	17.32	8.68	8.61	0.03
2.	HNN	Oil & Gas	4.07	32.17	6.67	8.10	17.40	4.85	36.70	6.39	9.69	20.62
		Coal	0.98	3.27	--	3.26	0.01	1.07	5.81	1.80	4.00	0.01
		Electricity	6.54	13.14	6.65	6.46	0.03	6.66	17.76	8.93	8.81	0.03
3.	NHN	Oil & Gas	3.34	38.30	9.55	9.40	19.36	3.96	43.76	9.84	11.29	22.63
		Coal	1.20	2.29	--	2.28	0.01	1.30	2.79	--	2.78	0.01
		Electricity	6.84	12.27	5.99	6.25	0.03	6.97	16.79	8.22	8.54	0.03
5.	HHH	Oil & Gas	4.07	32.76	7.02	8.34	17.39	4.85	37.55	6.95	9.99	20.61
		Coal	1.20	2.49	--	2.48	0.01	1.30	3.01	--	3.00	0.01
		Electricity	6.97	12.70	6.28	6.39	0.03	7.06	17.42	8.64	8.76	0.03
6.	LNN	Oil & Gas	2.60	45.82	13.08	10.61	22.13	3.07	52.30	13.91	12.82	25.57
		Coal	0.98	2.73	--	2.72	0.01	1.07	3.39	--	3.38	0.01
		Electricity	6.29	12.14	5.97	6.14	0.04	6.45	16.53	8.14	8.34	0.04
7.	NLN	Oil & Gas	3.34	36.93	8.78	8.79	19.36	3.96	42.06	8.86	10.56	22.64
		Coal	0.76	4.25	--	4.24	0.01	0.83	5.30	--	5.29	0.01
		Electricity	6.03	13.06	6.71	6.31	0.03	6.16	17.82	9.20	8.59	0.03
9.	LLL	Oil & Gas	2.60	45.03	12.65	10.25	22.13	3.07	51.31	13.35	12.38	25.57
		Coal	0.76	3.89	--	3.88	0.01	0.83	4.87	--	4.86	0.01
		Electricity	5.94	12.45	6.29	6.12	0.04	6.08	16.96	8.60	8.32	0.04

Hirst Residential

The Hirst Residential model reported 15- and 25-year primary and secondary energy demand estimates for the residential sector. All price cases except 4(NNH) and 8(NNL) were reported.

Primary Energy:

Price Case		Fuel	1990 (15-year)		2000 (25-year)	
			Price ($)	Residential	Price ($)	Residential
1.	NNN	Oil & Gas	2.66	9.75	3.21	9.97
		Coal	0.87	13.35	0.96	17.79
		Nuclear	--	--	--	--
2.	HNN	Oil & Gas	3.54	8.65	4.28	8.72
		Coal	0.87	14.71	0.96	19.92
		Nuclear	--	--	--	--
3.	NHN	Oil & Gas	2.66	9.86	3.19	10.04
		Coal	1.67	11.17	1.79	14.80
		Nuclear	--	--	--	--
5.	HHH	Oil & Gas	3.54	8.70	4.28	8.70
		Coal	1.67	12.40	1.79	16.79
		Nuclear	--	--	--	--
6.	LNN	Oil & Gas	1.82	11.38	2.30	11.85
		Coal	0.87	12.07	0.96	15.63
		Nuclear	--	--	--	--
7.	NLN	Oil & Gas	2.66	9.83	3.20	10.21
		Coal	0.07	16.50	0.14	22.11
		Nuclear	--	--	--	--
9.	LLL	Oil & Gas	1.82	11.38	2.30	11.96
		Coal	0.07	14.96	0.14	19.56
		Nuclear	--	--	--	--

Hirst Residential Secondary Energy:

Price Case	Fuel	1990 (15-year)		2000 (25-year)	
		Price ($)	Residential	Price ($)	Residential
1. NNN	Oil & Gas	3.31	7.84	3.92	7.53
	Coal	--	--	--	--
	Electricity	7.44	4.88	7.70	6.50
2. HNN	Oil & Gas	4.04	6.60	4.80	6.04
	Coal	--	--	--	--
	Electricity	7.44	5.37	7.70	7.28
3. NHN	Oil & Gas	3.31	8.21	3.92	7.95
	Coal	--	--	--	--
	Electricity	9.68	4.08	9.99	5.41
5. HHH	Oil & Gas	4.04	6.92	4.80	6.38
	Coal	--	--	--	--
	Electricity	9.68	4.53	9.99	6.14
6. LNN	Oil & Gas	2.58	9.60	3.02	9.65
	Coal	--	--	--	--
	Electricity	7.44	4.41	7.70	5.71
7. NLN	Oil & Gas	3.31	6.03	3.92	7.25
	Coal	--	--	--	--
	Electricity	5.22	7.55	5.41	8.08
9. LLL	Oil & Gas	2.58	9.26	3.02	9.29
	Coal	--	--	--	--
	Electricity	5.22	5.47	5.41	7.15

ISTUM

The ISTUM model, reported 15- and 25-year secondary energy demand estimates for the industrial sector. All price cases were examined except 4(NNH) and 7(NLN).

Secondary Energy:

Price Case		Fuel	1990 (15-year)		2000 (25-year)	
			Price ($)	Industrial	Price ($)	Industrial
1.	NNN	Oil & Gas	2.83	20.65	3.43	25.10
		Coal	0.98	10.35	1.08	18.63
		Electricity	5.30	5.32	5.53	7.44
2.	HNN	Oil & Gas	3.52	18.22	4.25	20.54
		Coal	0.98	12.73	1.08	23.30
		Electricity	5.30	5.40	5.53	7.53
3.	NHN	Oil & Gas	2.83	21.31	3.43	26.68
		Coal	1.21	9.84	1.31	17.29
		Electricity	5.85	5.20	6.14	7.23
5.	HHH	Oil & Gas	3.52	19.07	4.25	21.96
		Coal	1.21	11.93	1.31	21.91
		Electricity	5.85	5.35	6.14	7.46
6.	LNN	Oil & Gas	2.16	22.44	2.59	29.02
		Coal	0.98	9.14	1.08	15.47
		Electricity	5.30	5.02	5.53	6.97
7.	NLN	Oil & Gas	2.83	19.77	3.43	23.34
		Coal	0.76	11.17	0.84	20.38
		Electricity	4.75	5.40	4.92	7.56
9.	LLL	Oil & Gas	2.16	21.67	2.59	27.68
		Coal	0.76	9.69	0.84	16.39
		Electricity	4.75	5.24	4.92	7.32

Jackson Commercial

The Jackson Commercial model reported 15- and 25-year primary and secondary energy demand estimates for the commercial sector. All price cases except 4(NNH) and 8(NNL) were examined.

Primary Energy:

Price Case	Fuel	1990 (15-year) Price ($)	1990 (15-year) Commercial	2000 (25-year) Price ($)	2000 (25-year) Commercial
1. NNN	Oil & Gas	2.66	7.83	3.21	11.30
	Coal	0.87	11.00	0.96	17.34
	Nuclear	--	--	--	--
2. HNN	Oil & Gas	3.54	7.42	4.28	10.66
	Coal	0.87	11.16	0.96	17.64
	Nuclear	--	--	--	--
3. NHN	Oil & Gas	2.66	7.76	3.19	11.18
	Coal	1.67	9.84	1.79	15.27
	Nuclear	--	--	--	--
5. HHH	Oil & Gas	3.54	7.37	4.28	10.55
	Coal	1.67	9.97	1.79	15.51
	Nuclear	--	--	--	--
6. LNN	Oil & Gas	1.82	8.43	2.30	12.21
	Coal	0.07	10.85	0.96	17.04
	Nuclear	--	--	--	--
7. NLN	Oil & Gas	2.66	7.93	3.20	11.48
	Coal	0.07	12.63	0.14	20.30
	Nuclear	--	--	--	--
9. LLL	Oil & Gas	1.82	8.53	2.30	12.40
	Coal	0.07	12.43	0.14	19.89
	Nuclear	--	--	--	--

Jackson Commercial Secondary Energy:

Price Case		Fuel	1990 (15-year)		2000 (25-year)	
			Price ($)	Commercial	Price ($)	Commercial
1.	NNN	Oil & Gas	3.31	6.19	3.92	8.73
		Coal	--	--	--	--
		Electricity	7.44	4.02	7.70	6.34
2.	HNN	Oil & Gas	4.04	5.78	4.80	8.09
		Coal	--	--	--	--
		Electricity	7.44	4.08	7.70	6.46
3.	NHN	Oil & Gas	3.31	6.26	3.92	8.85
		Coal	--	--	--	--
		Electricity	9.68	3.60	9.99	5.58
5.	HHH	Oil & Gas	4.04	5.87	4.80	8.23
		Coal	--	--	--	--
		Electricity	9.68	3.65	9.99	5.67
6.	LNN	Oil & Gas	2.58	6.78	3.02	9.62
		Coal	--	--	--	--
		Electricity	7.44	3.96	7.70	6.23
7.	NLN	Oil & Gas	3.31	6.10	3.92	8.56
		Coal	--	--	--	--
		Electricity	5.22	4.62	5.41	7.43
9.	LLL	Oil & Gas	2.58	6.69	3.02	9.47
		Coal	--	--	--	--
		Electricity	5.21	4.55	5.41	7.28

MEFS

The MEFS model reported 15-year primary energy demand estimates and 15- and 25-year secondary estimates. Estimates were reported for all nine price cases and the following sectors: total demand, residential, residential/commercial, commercial, industrial, and transportation.

Primary Energy:

Price Case	Fuel	1990 (15-year)						
		Price ($)	Total Demand	Residential	Residential/ Commercial	Commercial	Industrial	Transportation
1. NNN	Oil & Gas	1.97	78.32	13.07	22.49	9.42	28.32	27.51
	Coal	1.08	24.14	5.25	9.56	4.31	14.58	--
	Nuclear	0.67	15.11	4.23	7.72	3.49	7.39	--
2. HNN	Oil & Gas	2.72	76.27	13.09	22.37	9.28	28.55	25.35
	Coal	1.08	25.70	5.24	9.55	4.31	16.15	--
	Nuclear	0.67	15.98	5.02	8.51	3.49	7.47	--
3. NHN	Oil & Gas	1.97	78.32	13.07	22.49	9.42	28.32	27.51
	Coal	1.39	24.14	5.25	9.56	4.31	14.58	--
	Nuclear	0.67	15.11	4.23	7.72	3.49	7.39	--
4. NNH	Oil & Gas	1.97	76.10	13.04	22.43	9.39	28.16	27.51
	Coal	1.08	24.00	5.25	9.55	4.30	14.45	--
	Nuclear	0.79	15.01	4.23	7.72	3.49	7.30	--
5. HHH	Oil & Gas	2.72	76.01	13.07	22.34	9.27	28.32	25.35
	Coal	1.39	24.14	5.25	9.56	4.31	14.58	--
	Nuclear	0.79	15.11	4.23	7.72	3.49	7.39	--
6. LNN	Oil & Gas	1.15	79.42	13.04	22.50	9.46	28.01	28.91
	Coal	1.08	23.96	5.25	9.55	4.30	14.41	--
	Nuclear	0.67	14.99	4.23	7.71	3.48	7.28	--
7. NLN	Oil & Gas	1.97	78.32	13.07	22.49	9.42	28.32	27.51
	Coal	0.78	24.14	5.25	9.56	4.31	14.58	--
	Nuclear	0.67	15.11	4.23	7.72	3.49	7.49	--
8. NNL	Oil & Gas	1.97	78.64	13.10	22.52	9.42	28.61	27.51
	Coal	1.08	24.28	5.24	9.55	4.31	14.73	--
	Nuclear	0.56	16.00	5.02	8.51	3.49	7.49	--
9. LLL	Oil & Gas	1.15	79.79	13.07	22.56	9.49	28.32	28.91
	Coal	0.78	24.14	5.25	9.56	4.31	14.58	--
	Nuclear	0.56	15.00	4.23	7.72	3.49	7.28	--

MEFS Secondary Energy:

Price Case	Fuel	1990 (15-year) Price ($)	Total Demand	Residential	Residential/ Commercial	Commercial	Industrial	Transportation
1. NNN	Oil & Gas	2.44	59.95	9.98	16.87	6.89	22.10	20.98
	Coal	1.22	5.12	0.02	0.02	--	5.10	--
	Electricity	7.27	10.84	2.92	5.28	2.35	5.55	0.01
2. HNN	Oil & Gas	3.35	53.49	8.39	14.07	5.68	20.41	19.02
	Coal	1.21	5.16	0.02	0.02	--	5.14	--
	Electricity	7.23	11.31	3.04	5.45	2.42	5.84	0.01
3. NHN	Oil & Gas	2.44	60.00	9.98	16.87	6.89	22.15	20.98
	Coal	1.52	4.87	0.02	0.02	--	4.85	--
	Electricity	7.27	10.87	2.92	5.28	2.35	5.58	0.01
4. NNH	Oil & Gas	2.40	61.56	10.39	17.63	7.24	22.95	20.98
	Coal	1.22	5.21	0.02	0.02	--	5.19	--
	Electricity	9.39	9.35	2.43	4.37	1.94	4.97	0.01
5. HHH	Oil & Gas	3.30	54.85	8.71	14.66	5.95	21.17	19.02
	Coal	1.52	4.99	0.02	0.02	--	4.97	--
	Electricity	9.33	9.77	2.53	4.52	1.99	5.24	0.01
6. LNN	Oil & Gas	1.54	69.92	12.51	21.38	8.87	24.65	23.89
	Coal	1.22	5.07	0.02	0.02	--	5.05	--
	Electricity	7.33	10.30	2.79	5.06	2.27	5.22	0.01
7. NLN	Oil & Gas	2.44	59.90	9.98	16.87	6.89	22.05	20.98
	Coal	0.92	5.48	0.02	0.02	--	5.46	--
	Electricity	7.28	10.81	2.92	5.28	2.35	5.52	0.01
8. NNL	Oil & Gas	2.48	58.11	9.51	16.00	6.49	21.13	20.98
	Coal	1.21	5.02	0.02	0.02	--	5.00	--
	Electricity	5.13	13.15	3.69	6.70	3.01	6.43	0.01
9. LLL	Oil & Gas	1.58	67.48	11.87	20.19	8.32	23.39	23.89
	Coal	0.92	5.31	0.02	0.02	--	5.29	--
	Electricity	5.19	12.42	3.52	6.42	2.91	5.98	0.01

MEFS Secondary Energy (continued):

Price Case	Fuel	2000 (25-year)						
		Price ($)	Total Demand	Residential	Residential/ Commercial	Commercial	Industrial	Transportation
1. NNN	Oil & Gas	2.78	69.19	10.71	18.31	7.59	27.49	23.40
	Coal	1.28	5.83	0.02	0.02	--	5.81	--
	Electricity	7.84	13.75	3.79	6.79	2.99	6.95	0.01
2. HNN	Oil & Gas	3.75	61.79	9.07	15.35	6.28	25.38	21.05
	Coal	1.28	5.88	0.02	0.02	--	5.86	--
	Electricity	7.78	14.34	3.93	7.00	3.07	7.33	0.01
3. NHN	Oil & Gas	2.77	69.25	10.71	18.31	7.59	27.54	23.40
	Coal	1.60	5.53	0.02	0.02	--	5.51	--
	Electricity	7.83	13.79	3.79	6.79	2.99	6.99	0.01
4. NNH	Oil & Gas	2.73	71.05	11.14	19.11	7.97	28.54	23.40
	Coal	1.28	5.94	0.02	0.02	--	5.92	--
	Electricity	10.10	11.97	3.21	5.73	2.52	6.22	0.01
5. HHH	Oil & Gas	3.70	63.37	9.40	15.98	6.58	26.33	21.05
	Coal	1.60	5.67	0.02	0.02	--	5.65	--
	Electricity	10.05	12.49	3.33	5.91	2.59	6.57	0.01
6. LNN	Oil & Gas	1.80	80.61	13.34	23.07	9.74	30.63	26.91
	Coal	1.28	5.78	0.02	0.02	--	5.76	--
	Electricity	7.90	13.06	3.62	6.52	2.90	6.53	0.01
7. NLN	Oil & Gas	2.78	69.14	10.71	18.31	7.59	27.43	23.40
	Coal	0.96	6.28	0.02	0.02	--	6.26	--
	Electricity	7.84	13.72	3.79	6.79	2.99	6.92	0.01
8. NNL	Oil & Gas	2.82	67.05	10.24	17.39	7.15	26.26	23.40
	Coal	1.28	5.70	0.02	0.02	--	5.68	--
	Electricity	5.54	16.52	4.70	8.43	3.74	8.07	0.01
9. LLL	Oil & Gas	1.84	77.80	12.68	21.82	9.14	29.07	26.91
	Coal	0.96	6.06	0.02	0.02	--	6.04	--
	Electricity	5.61	15.60	4.49	8.11	3.62	7.49	0.01

Parikh WEM

The Parikh WEM model reported 15-, 25-, and 35-year primary and secondary energy demand estimates for the total demand sector. Five price cases were used: 1(NNN), 2(HNN), 5(HHH), 5(LNN), and 9(LLL).

Primary Energy:

Price Case	Fuel	1990 (15-year)		2000 (25-year)		2005 (30-year)	
		Price ($)	Total Demand	Price ($)	Total Demand	Price ($)	Total Demand
1. NNN	Oil & Gas	4.00	59.96	5.09	56.68	5.71	47.09
	Coal	1.21	25.72	1.34	35.14	1.36	42.07
	Nuclear	0.55	15.57	0.62	27.31	0.68	35.54
2. HNN	Oil & Gas	4.93	56.28	6.37	55.62	7.15	43.00
	Coal	1.21	26.62	1.36	37.61	1.37	44.92
	Nuclear	0.55	15.57	0.61	27.31	0.67	35.54
5. HHH	Oil & Gas	4.89	57.44	6.36	52.53	7.15	45.27
	Coal	1.48	25.10	1.65	34.04	1.68	40.64
	Nuclear	0.58	15.57	0.69	27.31	0.81	35.54
6. LNN	Oil & Gas	3.23	60.66	3.86	60.93	4.32	55.67
	Coal	1.21	23.91	1.33	32.14	1.35	38.61
	Nuclear	0.56	15.57	0.62	26.81	0.68	32.85
9. LLL	Oil & Gas	3.24	58.08	3.87	58.69	4.32	52.35
	Coal	0.94	25.73	1.04	34.78	1.05	42.83
	Nuclear	0.53	15.57	0.54	26.81	0.54	32.35

Parikh WEM Secondary Energy:

Price Case	Fuel	1990 (15-year) Price ($)	1990 (15-year) Total Demand	2000 (25-year) Price ($)	2000 (25-year) Total Demand	2005 (30-year) Price ($)	2005 (30-year) Total Demand
1. NNN	Oil & Gas	4.50	49.81	5.70	53.71	6.43	49.83
	Coal	1.35	9.52	1.48	11.14	1.50	12.07
	Electricity	6.25	12.94	7.66	17.78	5.46	18.84
2. HNN	Oil & Gas	5.48	49.17	7.06	52.71	[illegible]	45.99
	Coal	1.35	9.52	1.50	11.41	1.51	11.42
	Electricity	4.72	13.24	5.72	18.60	6.30	20.07
5. HHH	Oil & Gas	5.49	50.26	7.04	49.82	7.93	48.12
	Coal	1.62	9.60	1.79	11.14	1.82	11.94
	Electricity	8.18	12.66	7.09	17.41	6.67	18.33
6. LNN	Oil & Gas	3.65	53.30	4.35	57.75	4.89	58.08
	Coal	1.35	9.81	1.47	11.54	1.49	12.41
	Electricity	5.07	12.18	6.49	16.55	6.43	17.44
9. LLL	Oil & Gas	3.66	52.76	4.36	55.64	4.90	54.97
	Coal	1.08	9.83	1.18	11.28	1.19	11.93
	Electricity	4.78	12.80	5.24	17.61	6.37	18.94

Pindyck Model

The Pindyck model reported both 15- and 25-year primary and secondary energy demand estimates. All price cases and the following sectors were used: total demand, residential, commercial/industrial, and transportation.

Primary Energy:

Price Case	Fuel	1990 (15-year)					2000 (25-year)				
		Price ($)	Total Demand	Residential	Commercial/ Industrial	Transportation	Price ($)	Total Demand	Residential	Commercial/ Industrial	Transportation
1. NNN	Oil & Gas	2.66	50.43	11.69	2.26	36.48	3.20	62.42	12.21	2.37	47.84
	Coal	0.87	7.03	--	7.03	--	0.96	8.13	--	8.13	--
	Nuclear	0.87	18.40	14.38	4.02	--	0.96	25.58	21.12	4.46	--
2. HNN	Oil & Gas	3.54	44.14	9.40	1.84	32.90	4.28	55.45	10.33	1.94	43.18
	Coal	0.87	8.09	0.01	8.08	--	0.96	9.25	--	9.25	--
	Nuclear	0.87	18.36	14.51	3.85	--	0.96	26.05	21.28	4.78	--
3. NHN	Oil & Gas	2.66	50.90	11.91	2.52	36.47	3.20	63.44	12.97	2.63	47.84
	Coal	1.12	5.23	--	5.23	--	1.24	6.11	--	6.11	--
	Nuclear	0.87	18.06	14.15	3.91	--	0.96	25.18	20.81	4.37	--
4. NNH	Oil & Gas	2.66	49.82	11.24	2.10	36.48	3.20	62.18	12.12	2.22	47.84
	Coal	0.87	7.84	1.01	--	6.83	0.96	7.97	--	7.97	--
	Nuclear	1.67	14.97	11.35	3.62	--	1.79	20.72	16.67	4.06	--
5. HHH	Oil & Gas	3.54	43.96	9.15	1.91	32.90	4.28	55.16	9.96	2.02	43.19
	Coal	1.12	5.95	--	5.95	--	1.24	6.29	--	6.29	--
	Nuclear	1.67	14.64	11.26	3.38	--	1.79	20.34	16.55	3.79	--
6. LNN	Oil & Gas	1.82	60.48	15.63	3.08	41.77	2.30	74.69	16.85	3.15	54.69
	Coal	0.87	5.62	--	5.62	--	0.96	6.62	--	6.62	--
	Nuclear	0.87	18.43	14.31	4.23	--	0.96	25.62	20.90	4.72	--
7. NLN	Oil & Gas	2.66	49.86	11.44	1.94	36.48	3.20	62.29	12.41	2.04	47.84
	Coal	0.62	10.15	--	10.15	--	0.69	11.60	--	11.60	--
	Nuclear	0.87	17.28	15.82	4.61	--	0.96	26.67	21.58	5.09	--
8. NNL	Oil & Gas	2.66	51.32	12.39	2.45	36.48	3.20	64.03	13.63	2.56	47.84
	Coal	0.87	7.26	--	7.26	--	0.96	8.30	--	8.30	--
	Nuclear	0.07	24.07	19.50	4.57	--	0.14	33.42	28.38	5.04	--
9. LLL	Oil & Gas	1.82	58.18	13.62	2.79	41.77	2.30	74.99	17.40	2.90	54.69
	Coal	0.62	8.68	--	8.68	--	0.69	9.93	--	9.93	--
	Nuclear	0.07	24.27	18.73	5.55	--	0.14	34.36	28.91	5.45	--

Pindyck Model Secondary Energy:

Price Case	Fuel	1990 (15-year) Price ($)	Total Demand	Residential	Commercial/ Industrial	Transportation	2000 (25-year) Price ($)	Total Demand	Residential	Commercial/ Industrial	Transportation
1. NNN	Oil & Gas	3.31	44.53	9.56	1.77	33.20	3.92	54.70	9.34	1.83	43.53
	Coal	1.01	7.03	--	7.03	--	1.10	8.13	--	8.13	--
	Electricity	7.44	6.93	5.42	1.52	--	7.70	9.64	7.96	1.68	--
2. HNN	Oil & Gas	4.04	38.63	7.32	1.37	29.94	4.80	48.19	7.48	1.42	39.29
	Coal	1.01	8.09	0.01	8.08	--	1.10	9.25	--	9.25	--
	Electricity	7.44	6.91	5.46	1.45	--	7.70	9.61	8.01	1.60	--
3. NHN	Oil & Gas	3.31	45.04	9.81	2.04	33.19	3.92	55.77	10.14	2.10	43.53
	Coal	1.26	5.23	--	5.23	--	1.39	6.11	--	6.11	--
	Electricity	7.44	7.84	5.33	1.47	--	7.70	9.48	7.84	1.65	--
4. NNH	Oil & Gas	3.31	44.38	9.52	1.66	33.20	3.92	55.10	9.84	1.73	43.53
	Coal	1.01	6.83	--	6.83	--	1.10	7.97	--	7.97	--
	Electricity	9.68	5.64	4.27	1.36	--	9.99	7.80	6.28	1.53	--
5. HHH	Oil & Gas	4.04	38.93	7.49	1.50	29.94	4.80	48.56	7.71	1.56	39.29
	Coal	1.26	5.95	--	5.95	--	1.39	6.29	--	6.29	--
	Electricity	9.68	5.52	4.24	1.28	--	9.99	7.66	6.24	1.43	--
6. LNN	Oil & Gas	2.58	54.01	13.44	2.56	38.01	3.02	66.29	13.95	2.57	49.77
	Coal	1.01	5.62	--	5.62	--	1.10	6.62	--	6.62	--
	Electricity	7.44	6.94	5.35	1.59	--	7.70	9.65	7.87	1.78	--
7. NLN	Oil & Gas	3.31	44.01	9.37	1.44	33.20	3.92	54.56	9.54	1.49	43.53
	Coal	0.76	10.15	--	10.15	--	0.83	11.60	--	11.60	--
	Electricity	5.22	6.51	5.09	1.42	--	7.70	9.84	8.13	1.71	--
8. NNL	Oil & Gas	3.31	44.69	9.60	1.89	33.20	3.92	55.30	9.83	1.94	43.53
	Coal	1.01	7.26	--	7.26	--	1.10	8.30	--	8.30	--
	Electricity	5.22	8.32	6.74	1.58	--	5.41	12.59	10.69	1.90	--
9. LLL	Oil & Gas	2.58	53.30	13.11	2.18	38.01	3.02	65.49	13.48	2.24	49.77
	Coal	0.76	8.68	--	8.68	--	0.83	9.93	--	9.93	--
	Electricity	5.22	8.52	6.81	1.71	--	5.41	12.94	10.89	2.05	--

Sweeney Auto

The Sweeney Auto model reported 15-year primary and secondary energy demand estimates for the transprotation sector. Additionally the estimates are divided into two categories: those with CAFE (corporate average fuel efficiency) standards and those without. Oil and gas estimates from three price cases were used: 1(NNN), 5(HHH), and 9(LLL).

Primary Energy:

Price Case	Fuel	1990 (CAFE)		1990 (No CAFE)	
		Price ($)	Transportation	Price ($)	Transportation
1. NNN	Oil & Gas	2.66	12.95	2.66	15.20
5. HHH	Oil & Gas	3.32	12.47	3.32	13.80
9. LLL	Oil & Gas	1.99	13.57	1.99	16.97

Secondary Energy:

Price Case	Fuel	1990 (CAFE) Price ($)	1990 (CAFE) Transportation	1990 (No CAFE) Price ($)	1990 (No CAFE) Transportation
1. NNN	Oil & Gas	3.31	11.78	3.31	13.83
5. HHH	Oil & Gas	4.04	11.35	4.04	12.56
9. LLL	Oil & Gas	2.58	12.25	2.58	15.44

Wharton MOVE

The Wharton MOVE model reported 15-year secondary energy demand estimates for the transportation sector. These elasticities are divided into two categories: those with CAFE (corporate average fuel efficiency) standards and those without. Oil and gas estimates from three price cases were reported: 1(NNN), 5(HHH), and 9(LLL).

Secondary Energy:

Price Case	Fuel	1990 (CAFE)		1990 (No CAFE)	
		Price ($)	Transportation	Price ($)	Transportation
1. NNN	Oil & Gas	2.25	8.87	2.52	10.70
5. HHH	Oil & Gas	3.19	8.44	3.19	10.16
9. LLL	Oil & Gas	1.87	9.38	1.87	11.35

Chapter 7

THE ELASTICITY ESTIMATES

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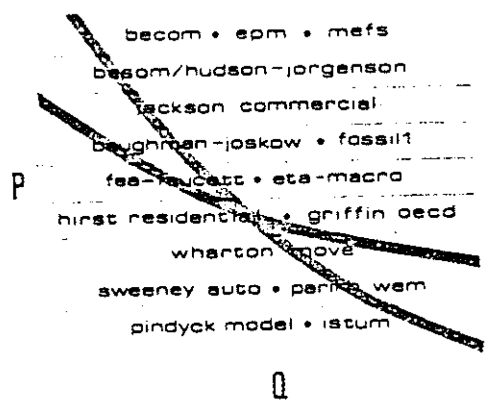


Chapter 7

THE ELASTICITY ESTIMATES

INTRODUCTION

This chapter presents elasticity estimates at the primary and secondary energy levels. These estimates are derived for each model from the raw fuel price and demand data presented in the previous chapter. The Paasche, Laspeyres, Tornquist, and Btu-weighted indexes are used to produce estimates of aggregate energy demand and its price in 1990, 2000, 2010. Paasche index scatter diagrams (aggregate price versus quantity plots) at the primary and secondary level are presented for each model by sector for the 25-year year period. Constant aggregate elasticities were estimated for each model, sector, index, year, and point of measurement. Fifteen- (1990), 25- (2000), and 35-year (2010) constant aggregate elasticity estimates are first presented in tabular form for each model, sector, index, and point of measurement. Then, the estimates based on the Paasche index are compared graphically over time for each model and sector at each point of measurement. A review of the sectors, points of measurement, and years reported by each model precedes the description of the index number and elasticity estimates.

SALIENT CHARACTERISTICS OF THE MODELS

Several key characteristics of the models provide the necessary background for the elasticity comparisons. Table 7-1 provides a list of the sixteen models. Nine models considered essentially all U.S. demands for energy, while the other seven considered only one or two energy consuming sectors. Table 7-2 lists the models and their sectoral coverage. All the models, except BECOM and ISTUM, reported both primary and secondary data. Some models reported only 15-year results, others 15- and 25-year, and still others 15-, 25-, and 35-year results.

INDEX NUMBERS

Four price-quantity index pair combinations were considered: Paasche, Laspeyres, Tornquist, and Btu-weighted. Table 7-3 shows the price index (P_E) and quantity index (E) formula for each index, where (p_i^o, q_i^o) are the price and quantities

Table 7-1

MODELS USED IN THE AGGREGATE ELASTICITY OF ENERGY DEMAND STUDY

Energy-Economy Models

- Brookhaven Energy System Optimization Model/Hudson-Jorgenson (BESOM/H-J), Brookhaven National Laboratory and Dale Jorgenson Associates
- Energy Technology Assessment-MACRO (ETA-MACRO), Alan Manne, Stanford University
- Parikh Welfare Equilibrium Model (Parikh WEM), Shailendra Parikh, Stanford University

Energy System Models

- Baughman-Joskow (Baughman-Joskow), Martin Baughman and Paul Joskow, University of Texas
- Energy Policy Model (EPM), Lawrence Livermore Laboratory
- FOSSIL1 (FOSSIL1), Dartmouth System Dynamics Group, Dartmouth College
- Griffin Organization for Economic Cooperation and Development (Griffin OECD), James Griffin, University of Houston
- Mid-Range Energy Forecasting System (MEFS), U.S. Department of Energy
- Pindyck International Study (Pindyck), Robert Pindyck, Massachusetts Institute of Technology

Sectoral Models

- Buildings Energy Conservation Optimization Model (BECOM), Brookhaven National Laboratory
- Federal Energy Administration-Faucett (FEA-Faucett), Carmen Difiglio and Damian Kulash, Federal Energy Administration
- Industrial Sector Technology Use Model (ISTUM), Energy and Environmental Analysis, Inc.
- Jackson Commercial (Jackson Commercial), Jerry Jackson, Oak Ridge National Laboratory
- The ORNL Residential Energy-Use Model (Hirst Residential), Eric Hirst and Janet Carney, Oak Ridge National Laboratory
- Sweeney Automobile Model (Sweeney Auto), James Sweeney, Stanford University
- Wharton Motor Vehicle Model (Wharton MOVE), Wharton Econometric Forecasting Associates

of the input fuels for the Reference price case for some model/sector/point-of-measurement/year combination, whereas (p_i^1, q_i^1) are the price and quantities of the input fuels for another price case for the same model/sector/point-of-measurement/year combination.

In preparation for the estimation of the aggregate elasticities, it is instructive to plot the aggregate price and quantity indexes for a particular model/sector/point-of-measurement/year combination to get some idea of the shape of the implied

Table 7-2

SECTORAL COVERAGE OF EMF 4 MODELS

Model	TD	R	R/C	C	C/I	I	T
Baughman-Joskow	x		x			x	
BECOM		x	x	x			
BESOM/H-J[a]	x		x			x	x
EPM[a]	x		x			x	x
ETA-MACRO	x						
FEA-Faucett[b]							x
FOSSIL1	x						
FOSSIL1 Conservation	x						
Griffin OECD	x		x		x		x
Hirst Residential		x					
ISTUM						x	
Jackson Commercial				x			
MEFS	x	x	x	x		x	x
Parikh WEM	x						
Pindyck	x	x			x		
Sweeney Auto[b]							x
Wharton MOVE[b]							x

[a]Reports only total demand at primary level.

[b]Automobile only.

Note: TD = total demand.
R = residential.
R/C = residential/commercial.
C = commercial.
C/I = commercial/industrial.
I = industrial.
T = transportation.

curve for aggregate energy. Figures 7-1 through 7-19 show the 25-year primary energy scatter diagrams for each model by sector using the Paasche index. Figures 7-20 through 7-54 show the corresponding secondary level diagrams.

ELASTICITY ESTIMATES

The available aggregate price and quantity index pair values for a given model/sector/point-of-measurement/year combination can be used to compute a constant elasticity estimate. The following functional form was assumed for the estimations:

Table 7-3

PRICE AND QUANTITY INDEX FORMULATIONS

Name	Price Index	Quantity Index
Paasche	$P_E = \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^1}$	$E = \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^1 q_i^0}$
Laspeyres	$P_E = \frac{\sum_{i=1}^{n} p_i^1 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$	$E = \frac{\sum_{i=1}^{n} p_i^0 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}$
Tornquist	$P_E = \exp 1/2 \sum_{i=1}^{n} \left[\frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} + \frac{p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^1 q_i^1} \right] (\ln p_i^1 - \ln p_i^0)$	$E = \frac{\sum_{i=1}^{n} p_i^1 q_i^1}{P_E \cdot \sum_{i=1}^{n} p_i^0 q_i^0}$
Btu-weighted	$P_E = \frac{\sum_{i=1}^{n} p_0^1 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$	$E = \frac{\sum_{i=1}^{n} q_i^1}{\sum_{i=1}^{n} q_i^0}$

$$\ln \frac{E}{GNP} = C - \varepsilon \ln P_E \ ,$$

where E = aggregate energy demand

GNP = gross national product

C = a constant

ε = the aggregate price elasticity

P_E = aggregate energy price .

The 15-, 25-, and 35-year primary elasticity estimates are presented for the model/sector/point-of-measurement combination in Tables 7-4 through 7-6 while the corresponding secondary elasticity estimates are recorded in Tables 7-7 through 7-9. Finally, the aggregate elasticity estimates are compared by sector over time for primary and secondary energy in Figures 7-55 through 7-68, respectively.

Table 7-4

15-YEAR ESTIMATED PRIMARY ENERGY DEMAND ELASTICITIES

Sector/Model	Paasche	Tornquist	Laspeyres	Btu-weighted
Total Demand				
Baughman-Joskow	0.39	0.44	0.40	0.16
BESOM/H-J	0.40	0.41	0.40	0.28
EPM	0.57	0.56	0.57	0.33
ETA-MACRO	0.24	0.26	0.25	0.06
FOSSIL1	0.07	0.09	0.07	0.04
FOSSIL1 Conservation	0.11	0.13	0.11	0.08
Griffin OECD	0.45	0.47	0.46	0.28
MEFS	0.04	0.07	0.04	0.03
Parikh WEM	0.02	0.05	0.02	-0.02
Pindyck	0.39	0.36	0.40	0.40
Residential				
Hirst Residential	0.21	0.04	0.22	0.21
MEFS	-0.01	0.04	-0.01	-0.02
Pindyck	0.38	0.30	0.42	0.44
Griffin OECD	0.82	0.84	0.83	0.43
Residential/Commercial				
Baughman-Joskow	0.43	0.47	0.44	0.27
MEFS	--	0.05	--	-0.01
Commercial				
Jackson Commercial	0.14	-0.04	0.15	0.18
MEFS	0.02	0.06	0.02	0.02
Commercial/Industrial				
Griffin OECD	0.14	0.16	0.14	0.06
Pindyck	0.38	0.63	0.41	0.42
Industrial				
Baughman-Joskow	0.33	0.37	0.33	0.03
MEFS	-0.02	0.01	-0.02	-0.03
Transportation				
FEA-Faucett CAFE[a]	0.10	0.10	0.10	0.10
FEA-Faucett No CAFE	0.13	0.13	0.13	0.13
Griffin OECD	0.60	0.60	0.59	0.60
MEFS	0.14	0.14	0.14	0.14
Pindyck	0.36	0.36	0.36	0.35
Sweeney Auto CAFE	0.17	0.17	0.17	0.17
Sweeney Auto No CAFE	0.40	0.40	0.40	0.40

[a]Corporate average fuel efficiency standards.

Table 7-5

25-YEAR ESTIMATED PRIMARY ENERGY DEMAND ELASTICITIES

Sector/Model	Paasche	Tornquist	Laspeyres	Btu-weighted
Total Demand				
Baughman-Joskow	0.49	0.55	0.51	0.17
BESOM/H-J	0.48	0.48	0.48	0.29
EPM	0.79	0.78	0.79	0.40
ETA-MACRO	0.30	0.32	0.31	0.05
FOSSIL1	0.11	0.15	0.11	0.05
FOSSIL1 Conservation	0.17	0.21	0.17	0.09
Griffin OECD	0.45	0.46	0.45	0.24
Parikh WEM	0.11	0.15	0.12	0.01
Pindyck	0.44	0.41	0.45	0.45
Residential				
Hirst Residential	0.24	0.03	0.25	0.22
Pindyck	0.52	0.45	0.52	0.53
Griffin OECD	0.93	0.95	0.94	0.40
Residential/Commercial				
Baughman-Joskow	0.59	0.62	0.59	0.36
Commercial				
Jackson Commercial	0.17	--	0.19	0.22
Commercial/Industrial				
Griffin OECD	0.12	0.14	0.12	0.05
Pindyck	0.43	0.45	0.43	0.44
Industrial				
Baughman-Joskow	0.37	0.42	0.40	0.01
Transportation				
Griffin OECD	0.39	0.39	0.39	0.39
Pindyck	0.38	0.38	0.38	0.38

Table 7-6

35-YEAR ESTIMATED PRIMARY ENERGY DEMAND ELASTICITIES

Sector/Model	Paasche	Tornquist	Laspeyres	Btu-weighted
Total Demand				
EPM	1.34	1.37	1.34	0.47
ETA-MACRO	0.31	0.34	0.33	0.02
FOSSIL1	0.16	0.23	0.16	0.03
FOSSIL1 Conservation	0.25	0.32	0.25	0.09
Parikh WEM	0.26	0.30	0.27	0.04

Table 7-7

15-YEAR ESTIMATED SECONDARY ENERGY DEMAND ELASTICITIES

Sector/Model	Paasche	Tornquist	Laspeyres	Btu-weighted
Total Demand				
Baughman-Joskow	0.59	0.60	0.61	0.63
BESOM/H-J	0.47	0.47	0.47	0.49
EPM	0.49	0.49	0.50	0.51
ETA-MACRO	0.14	0.15	0.16	0.30
FOSSIL1	0.06	0.06	0.06	0.10
FOSSIL1 Conservation	0.11	0.11	0.11	0.15
Griffin OECD	0.57	0.58	0.59	0.76
MEFS	0.30	0.31	0.31	0.32
Parikh WEM	0.04	0.04	0.04	0.05
Pindyck	0.68	0.68	0.68	0.66
Residential				
BECOM	0.17	0.17	0.17	0.39
Griffin OECD	0.90	0.93	0.96	1.35
Hirst Residential	0.47	0.49	0.51	0.44
MEFS	0.46	0.47	0.48	0.50
Pindyck	0.82	0.81	0.80	0.81
Residential/Commercial				
Baughman-Joskow	0.73	0.73	0.74	0.77
BECOM	0.14	0.14	0.14	0.33
BESOM/H-J	0.54	0.54	0.54	0.71
EPM	0.43	0.44	0.44	0.54
MEFS	0.49	0.50	0.51	0.52
Commercial				
BECOM	0.07	0.07	0.07	0.10
Jackson Commercial	0.34	0.34	0.34	0.30
MEFS	0.52	0.54	0.55	0.55
Commercial/Industrial				
Griffin OECD	0.28	0.26	0.30	0.54
Pindyck	0.52	0.56	0.52	0.68
Industrial				
Baughman-Joskow	0.46	0.47	0.48	0.52
BESOM/H-J	0.52	0.52	0.52	0.49
EPM	0.64	0.65	0.65	0.63
ISTUM	0.14	0.14	0.14	0.02
MEFS	0.16	0.16	0.17	0.16

Table 7-7 (continued)

Sector/Model	Paasche	Tornquist	Laspeyres	Btu-weighted
Transportation				
BESOM/H-J	0.29	0.29	0.29	0.29
EPM	0.30	0.30	0.30	0.30
FEA-Faucett CAFE[a]	0.11	0.11	0.11	0.11
FEA-Faucett No CAFE	0.15	0.15	0.15	0.15
Griffin OECD	0.54	0.54	0.54	0.54
MEFS	0.30	0.30	0.30	0.30
Pindyck	0.53	0.53	0.53	0.53
Sweeney Auto CAFE	0.17	0.17	0.17	0.17
Sweeney Auto No CAFE	0.46	0.46	0.46	0.46
Wharton MOVE CAFE	0.19	0.19	0.19	0.19
Wharton MOVE No CAFE	0.21	0.21	0.21	0.21

[a]Corporate average fuel efficiency standards.

Table 7-8

25-YEAR ESTIMATED SECONDARY ENERGY DEMAND ELASTICITIES

Sector/Model	Paasche	Tornquist	Laspeyres	Btu-weighted
Total Demand				
Baughman-Joskow	0.61	0.61	0.61	0.59
BESOM/H-J	0.42	0.43	0.44	0.46
EPM	0.57	0.57	0.57	0.59
ETA-MACRO	0.18	0.19	0.20	0.35
FOSSIL1	0.08	0.08	0.08	0.14
FOSSIL1 Conservation	0.16	0.16	0.16	0.23
Griffin OECD	0.55	0.56	0.57	0.68
MEFS	0.31	0.31	0.32	0.33
Parikh WEM	0.10	0.09	0.09	0.10
Pindyck	0.70	0.71	0.71	0.66
Residential				
BECOM	0.60	0.63	0.66	1.12
Griffin OECD	0.95	0.99	1.01	1.22
Hirst Residential	0.44	0.45	0.45	0.47
MEFS	0.45	0.46	0.47	0.48
Pindyck	0.98	0.98	0.99	0.96

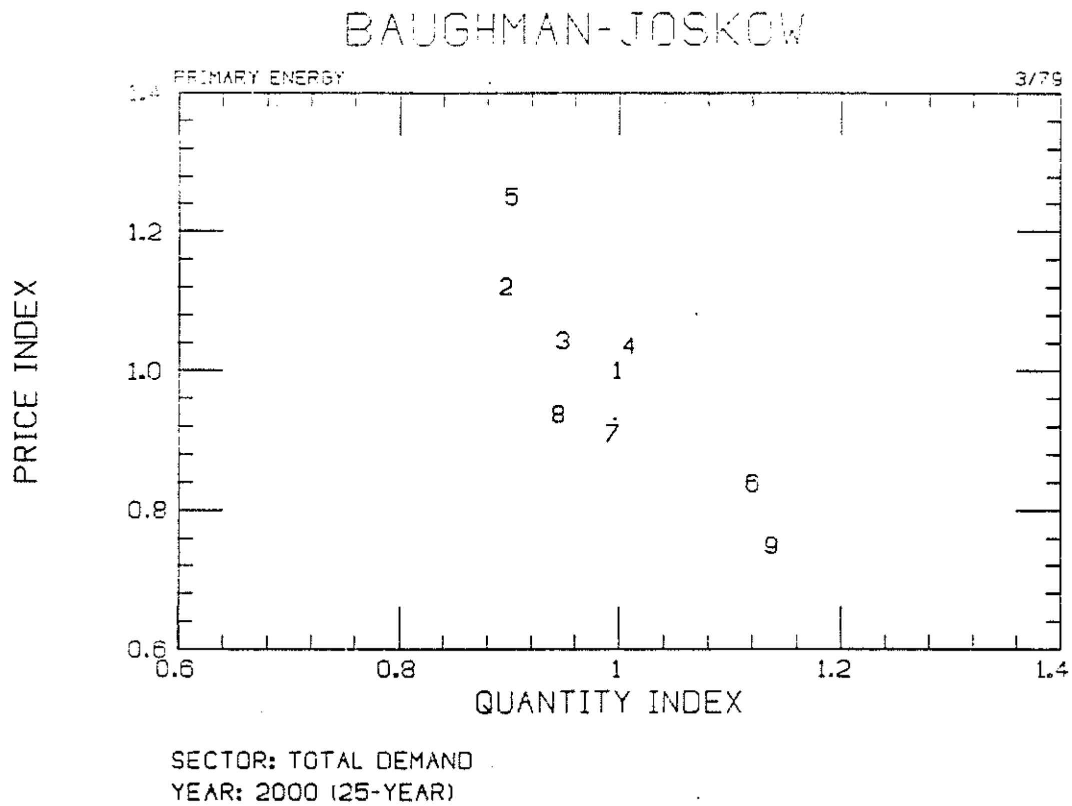
Table 7-8 (continued)

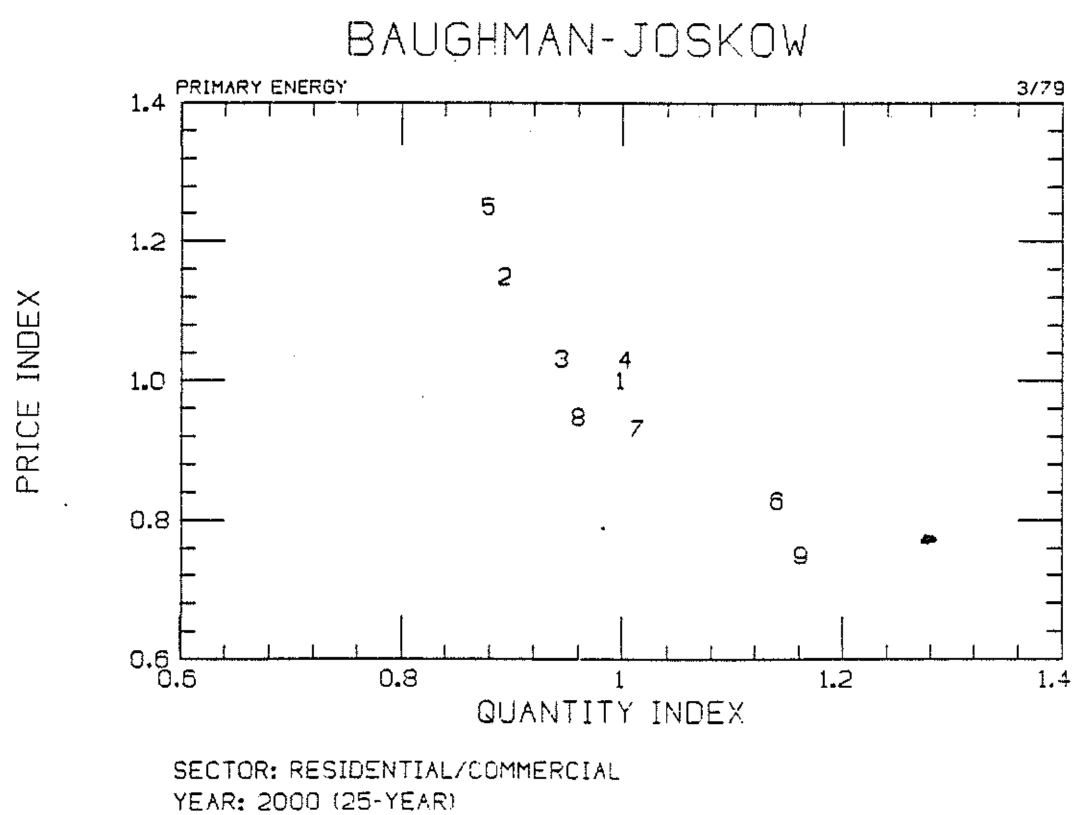
Sector/Model	Paasche	Tornquist	Laspeyres	Btu-weighted
Residential/Commercial				
Baughman-Joskow	0.76	0.76	0.76	0.75
BECOM	0.51	0.54	0.57	1.00
BESOM/H-J	0.71	0.77	0.82	1.05
EPM	0.51	0.52	0.52	0.64
MEFS	0.48	0.49	0.50	0.51
Commercial				
BECOM	0.28	0.28	0.28	0.52
Jackson Commercial	0.37	0.38	0.38	0.33
MEFS	0.51	0.52	0.53	0.53
Commercial/Industrial				
Griffin OECD	0.29	0.25	0.31	0.54
Pindyck	0.68	0.70	0.69	0.74
Industrial				
Baughman-Joskow	0.43	0.43	0.44	0.44
BESOM/H-J	0.48	0.48	0.47	0.46
EPM	0.71	0.72	0.73	0.68
ISTUM	0.24	0.24	0.24	0.01
MEFS	0.16	0.17	0.17	0.17
Transportation				
BESOM/H-J	0.23	0.23	0.23	0.23
EPM	0.38	0.38	0.38	0.39
Griffin OECD	0.47	0.47	0.47	0.47
MEFS	0.34	0.34	0.34	0.34
Pindyck	0.51	0.51	0.51	0.51

Table 7-9

35-YEAR ESTIMATED SECONDARY ENERGY DEMAND ELASTICITIES

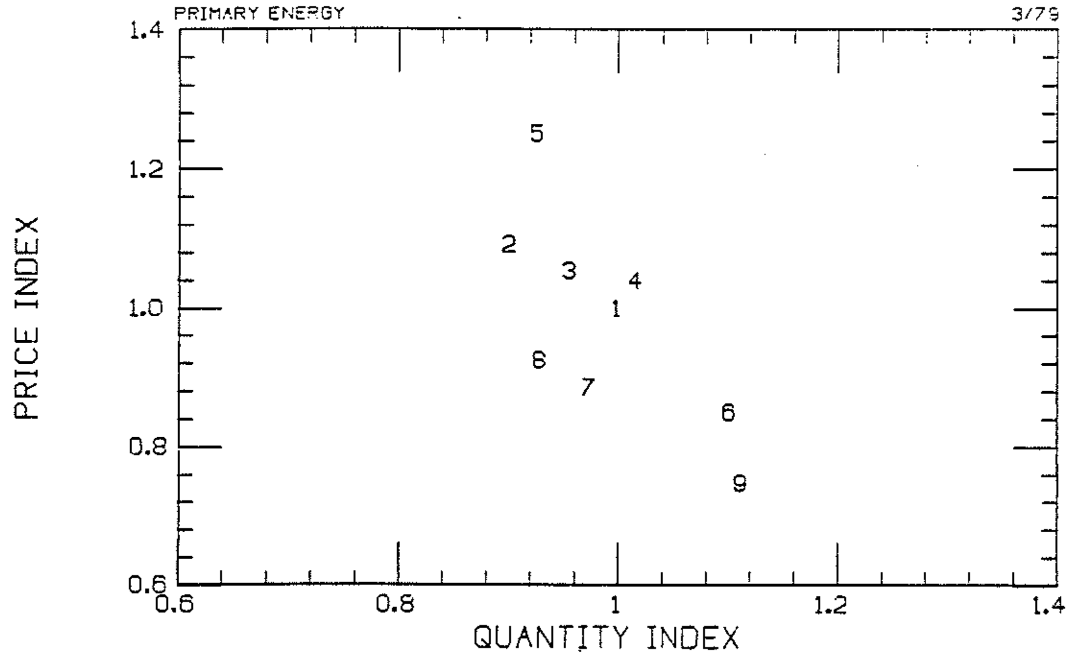
Sector/Model	Paasche	Tornquist	Laspeyres	Btu-weighted
Total Demand				
EPM	0.63	0.63	0.64	0.66
ETA-MACRO	0.15	0.16	0.17	0.29
FOSSIL1	0.08	0.08	0.08	0.18
FOSSIL1 Conservation	0.19	0.19	0.18	0.31
Parikh WEM	0.25	0.27	0.29	0.25
Residential/Commercial				
EPM	0.59	0.60	0.61	0.78
Industrial				
EPM	0.77	0.78	0.80	0.73
Transportation				
EPM	0.45	0.45	0.45	0.46





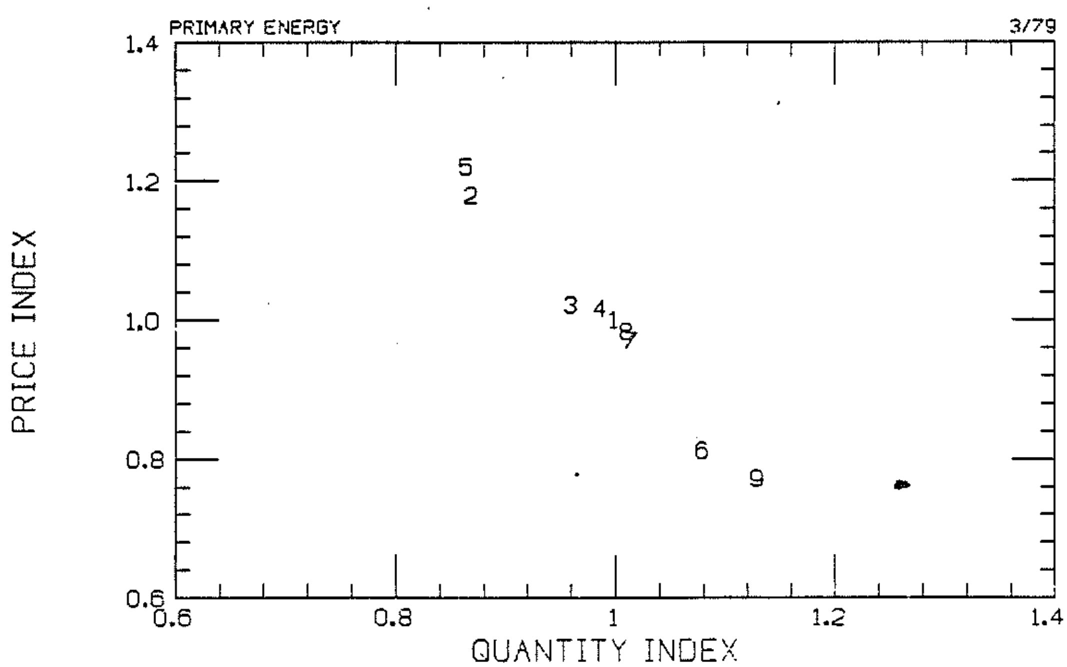
Figures 7-1 and 7-2

BAUGHMAN-JOSKOW



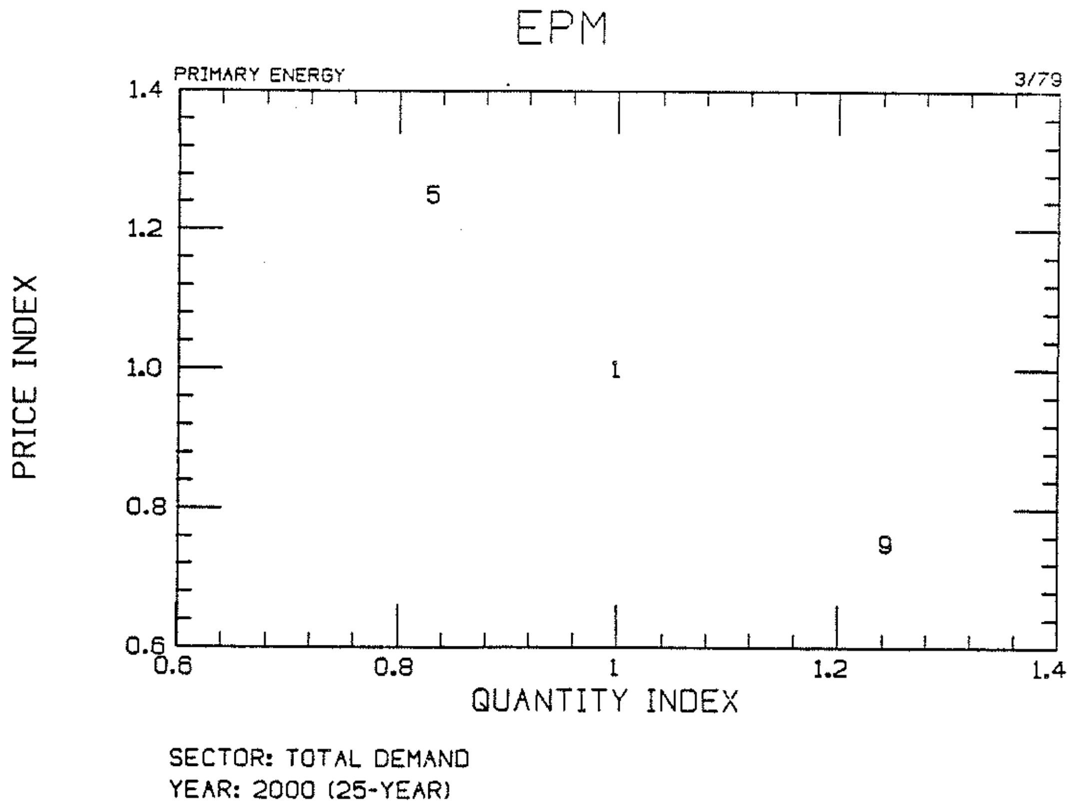
SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)

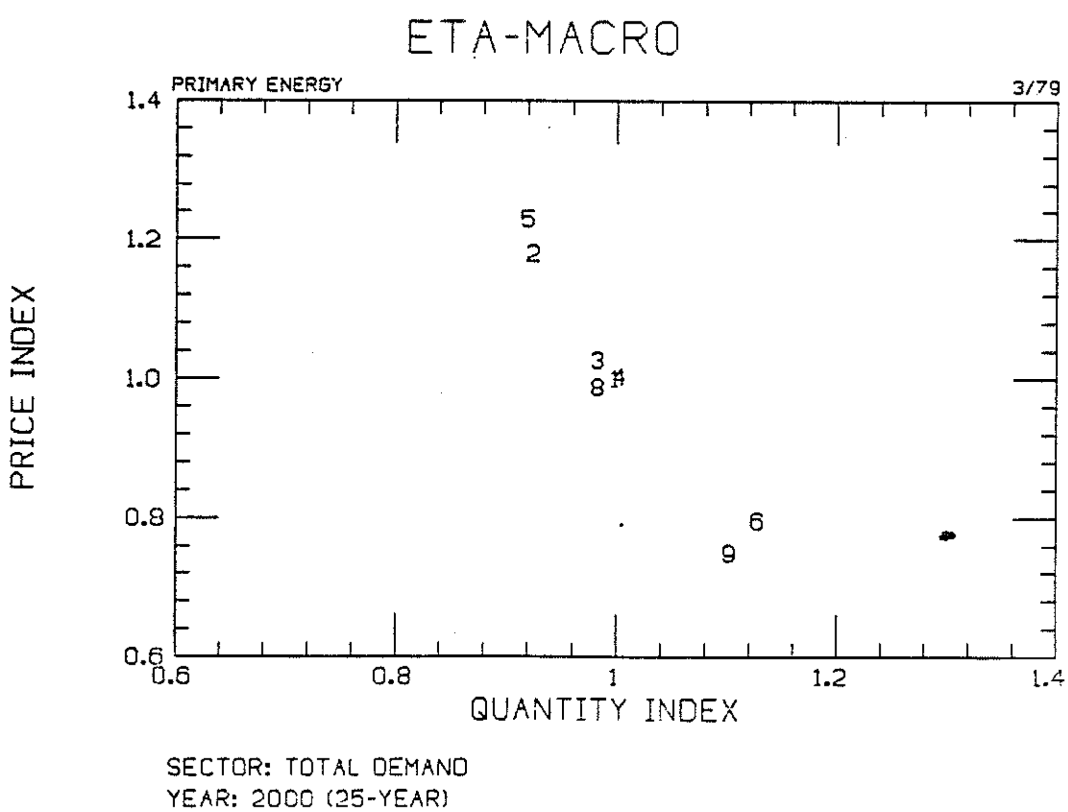
BESOM/H-J



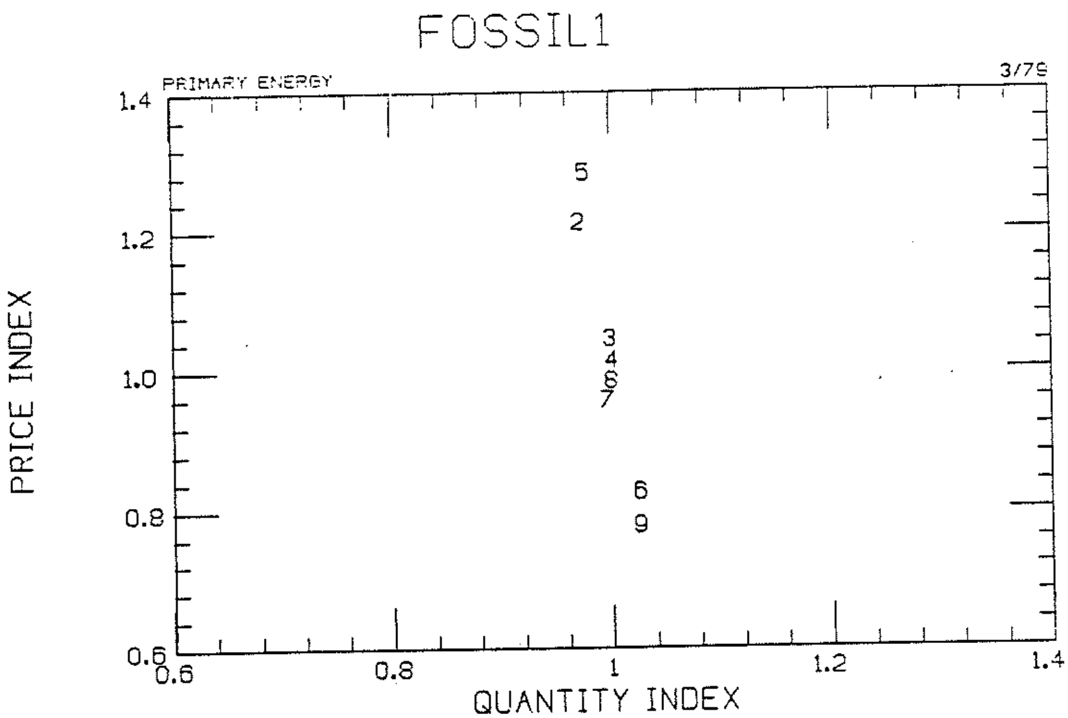
SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)

Figures 7-3 and 7-4

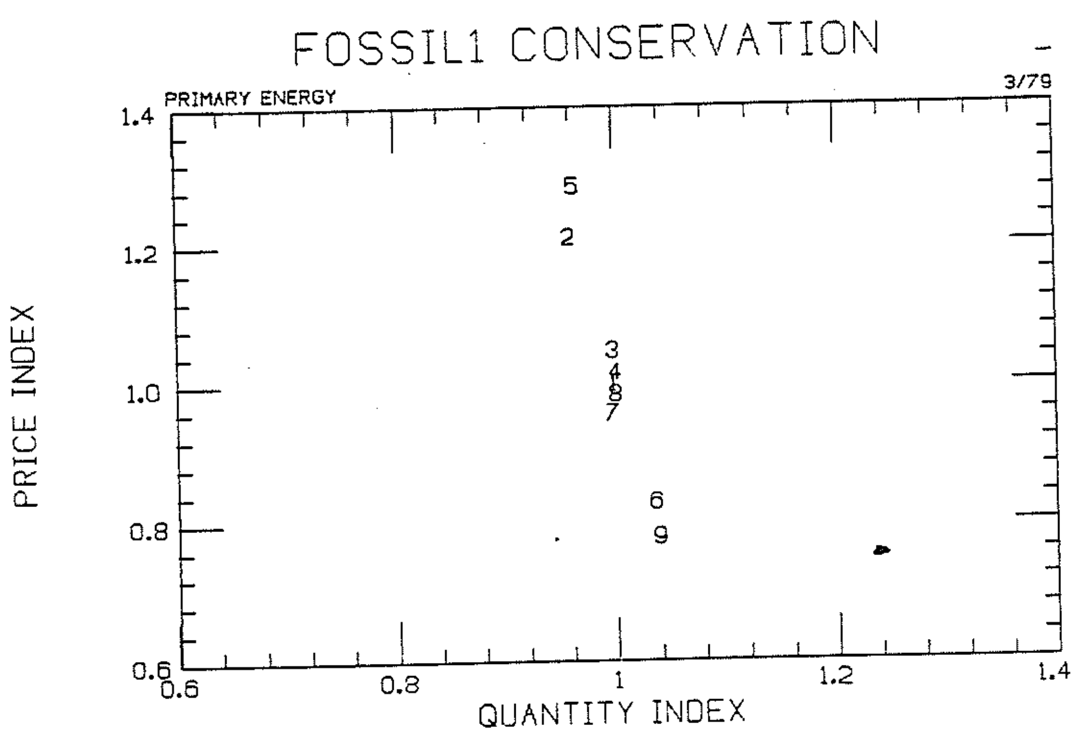




Figures 7-5 and 7-6

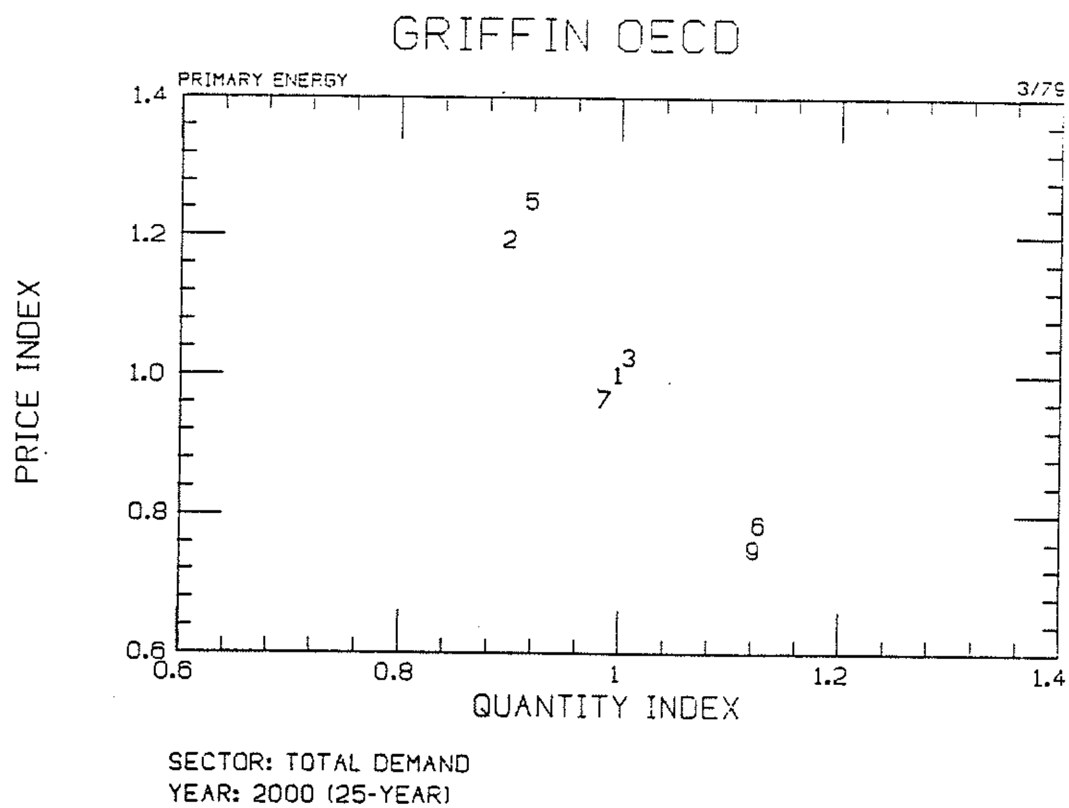


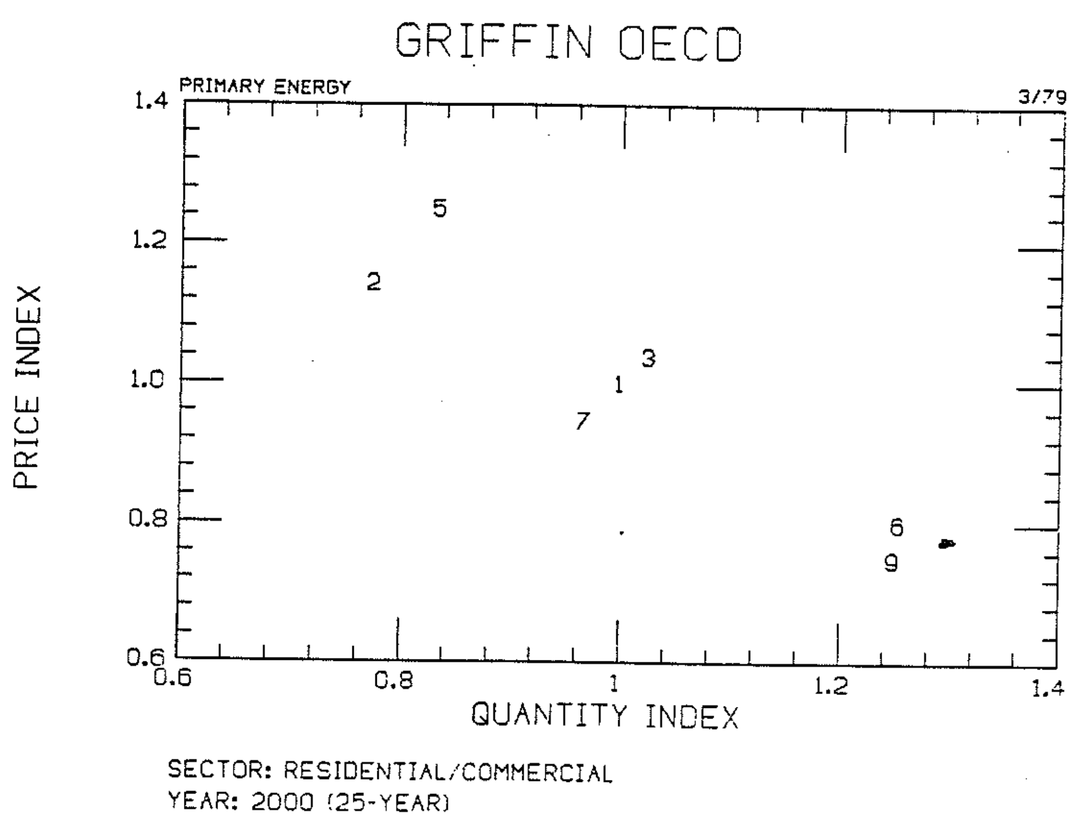
SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)



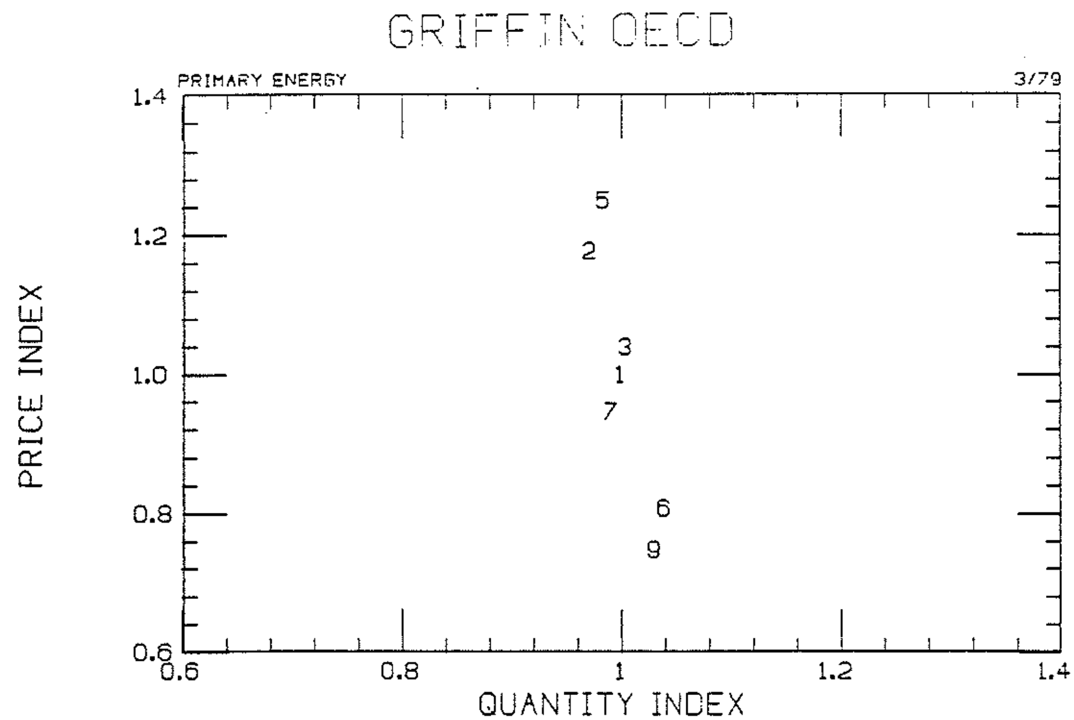
SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)

Figures 7-7 and 7-8

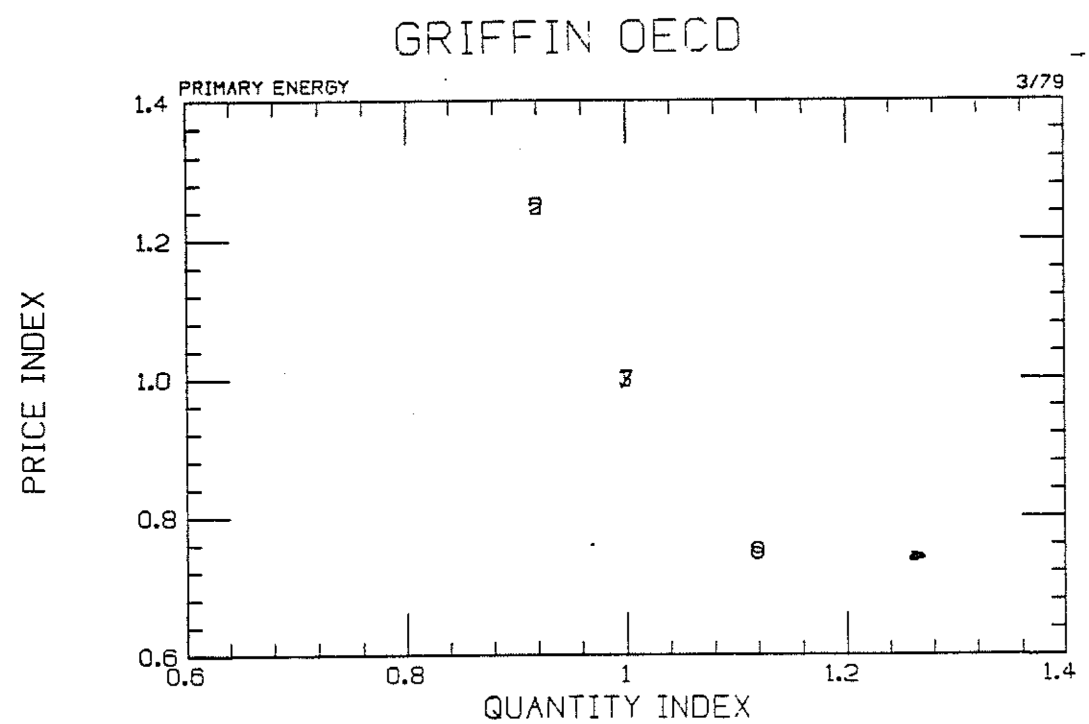




Figures 7-9 and 7-10

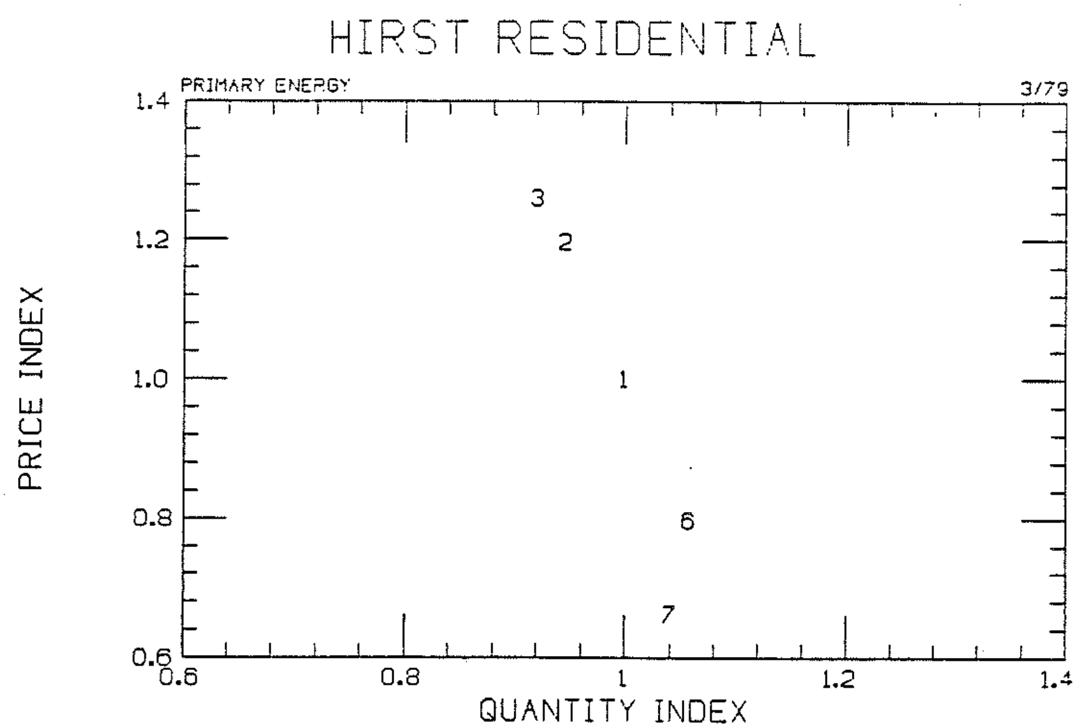


SECTOR: COMMERCIAL/INDUSTRIAL
YEAR: 2000 (25-YEAR)

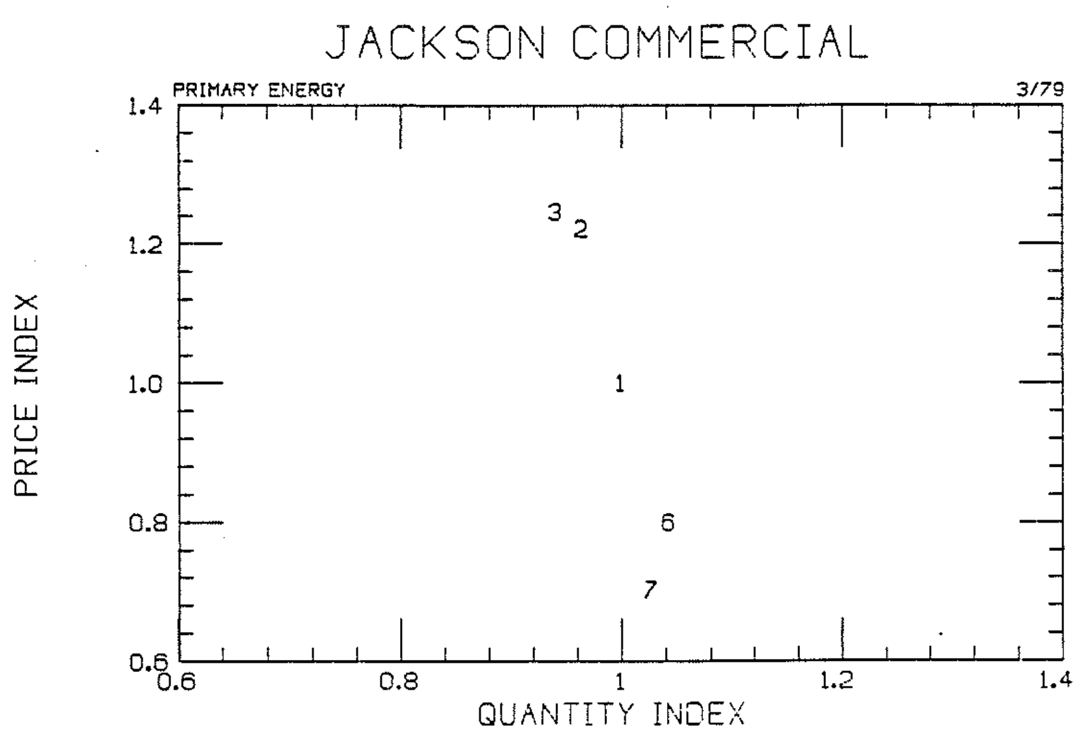


SECTOR: TRANSPORTATION
YEAR: 2000 (25-YEAR)

Figures 7-11 and 7-12

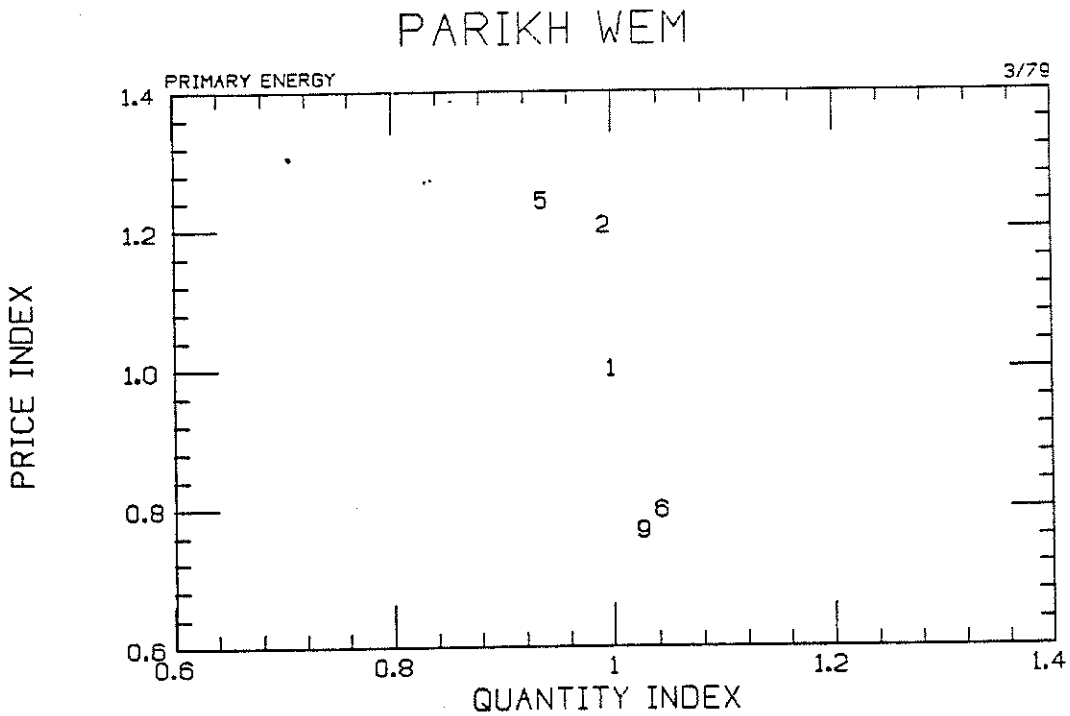


SECTOR: RESIDENTIAL
YEAR: 2000 (25-YEAR)

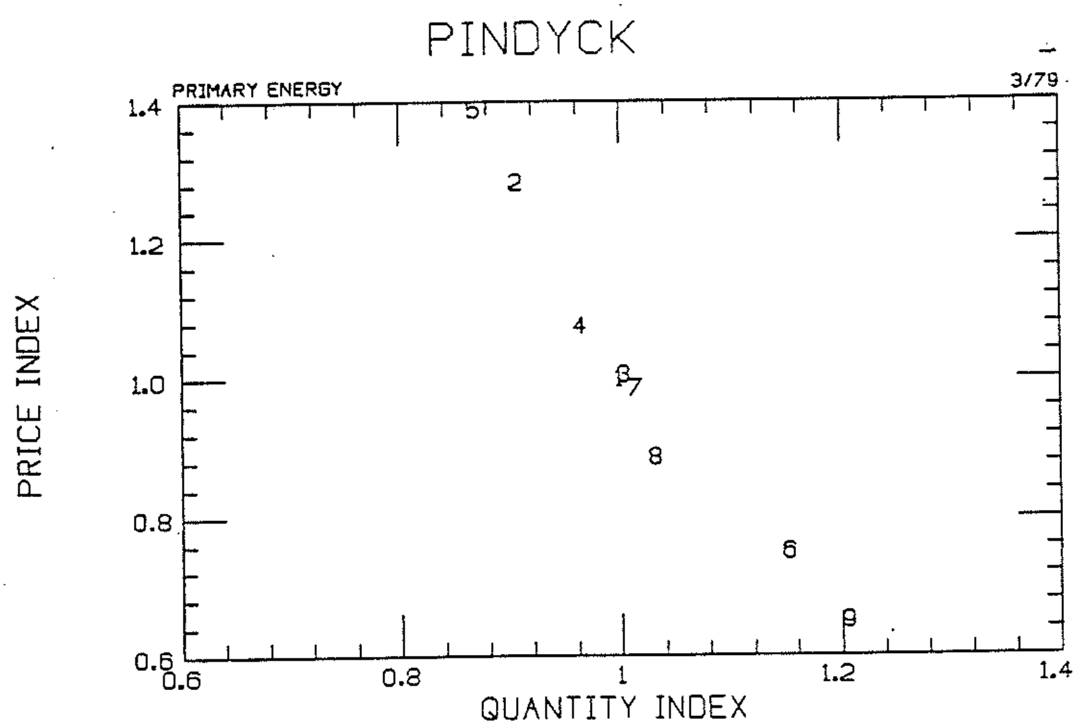


SECTOR: COMMERCIAL
YEAR: 2000 (25-YEAR)

Figures 7-13 and 7-14

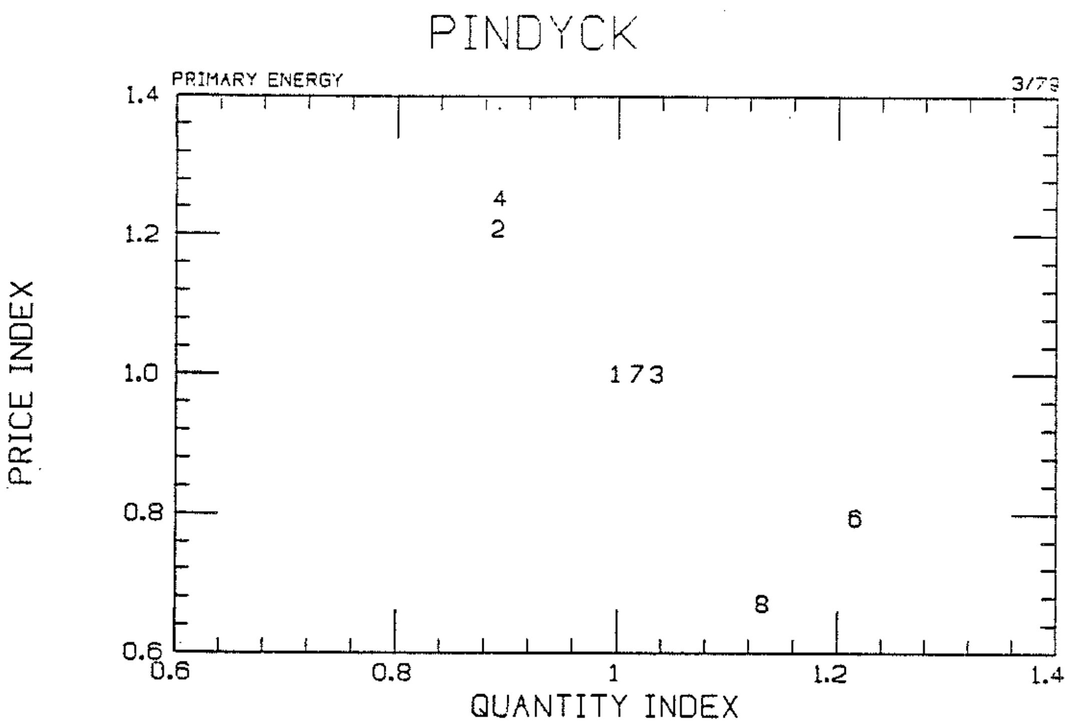


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)

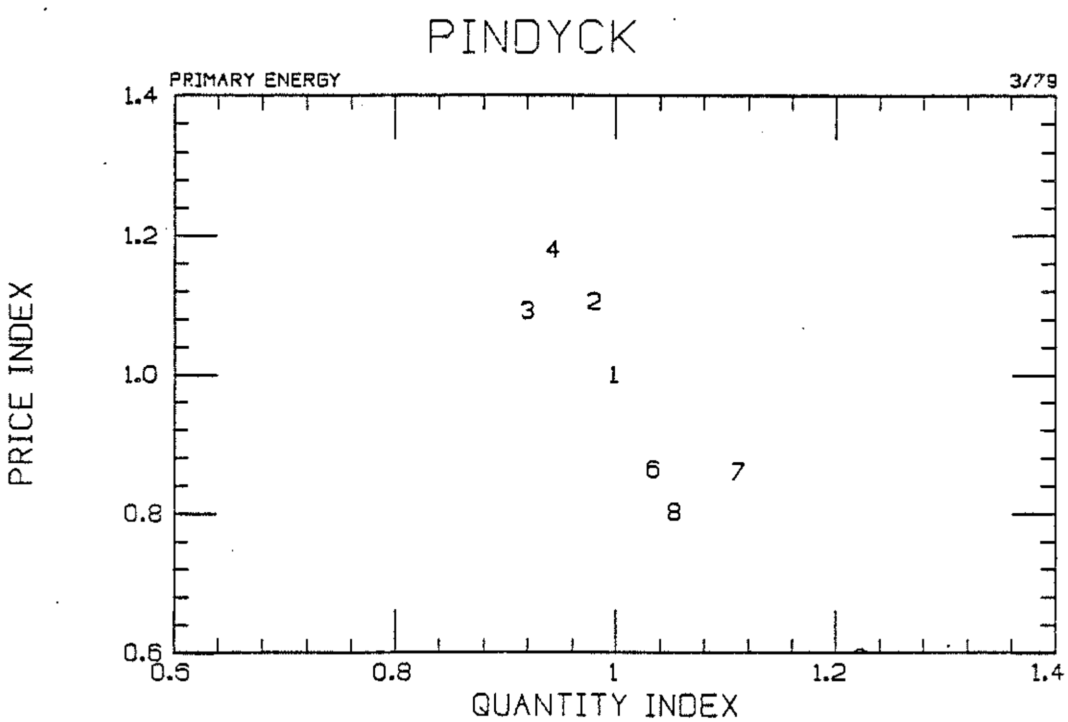


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)

Figures 7-15 and 7-16

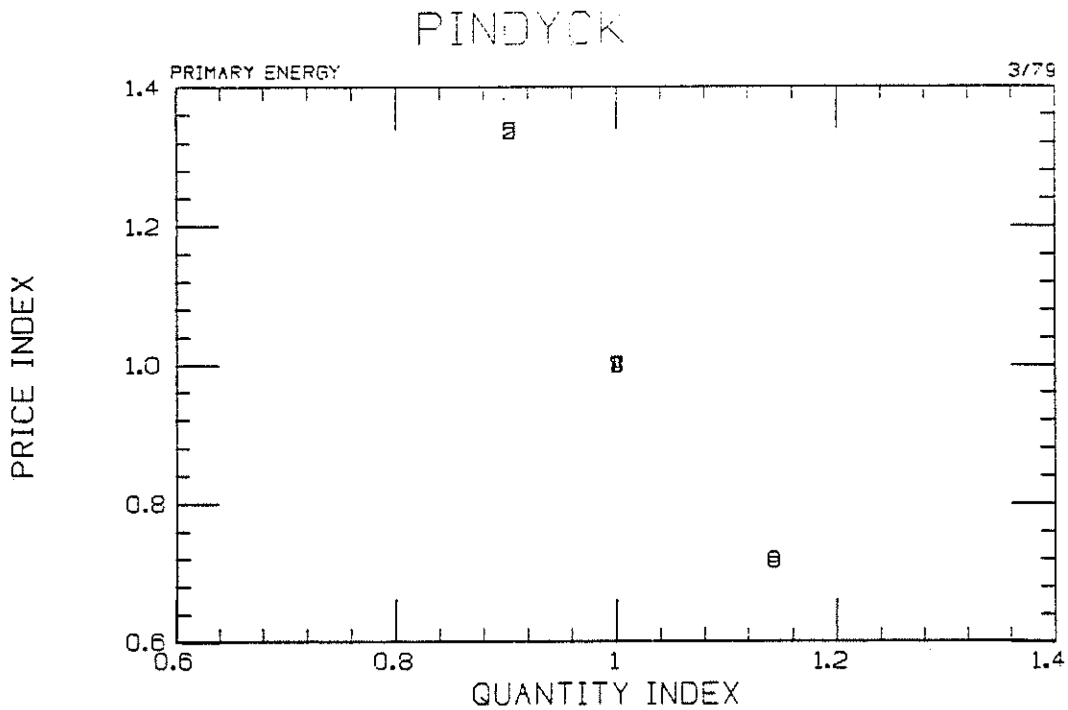


SECTOR: RESIDENTIAL
YEAR: 2000 (25-YEAR)



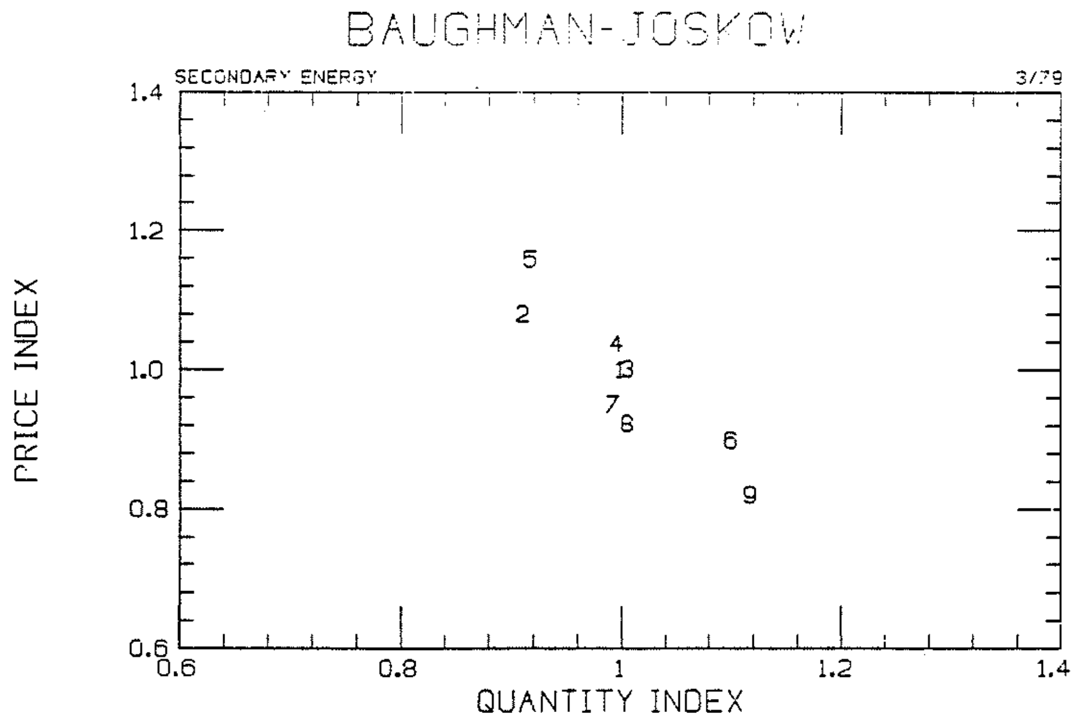
SECTOR: COMMERCIAL/INDUSTRIAL
YEAR: 2000 (25-YEAR)

Figures 7-17 and 7-18

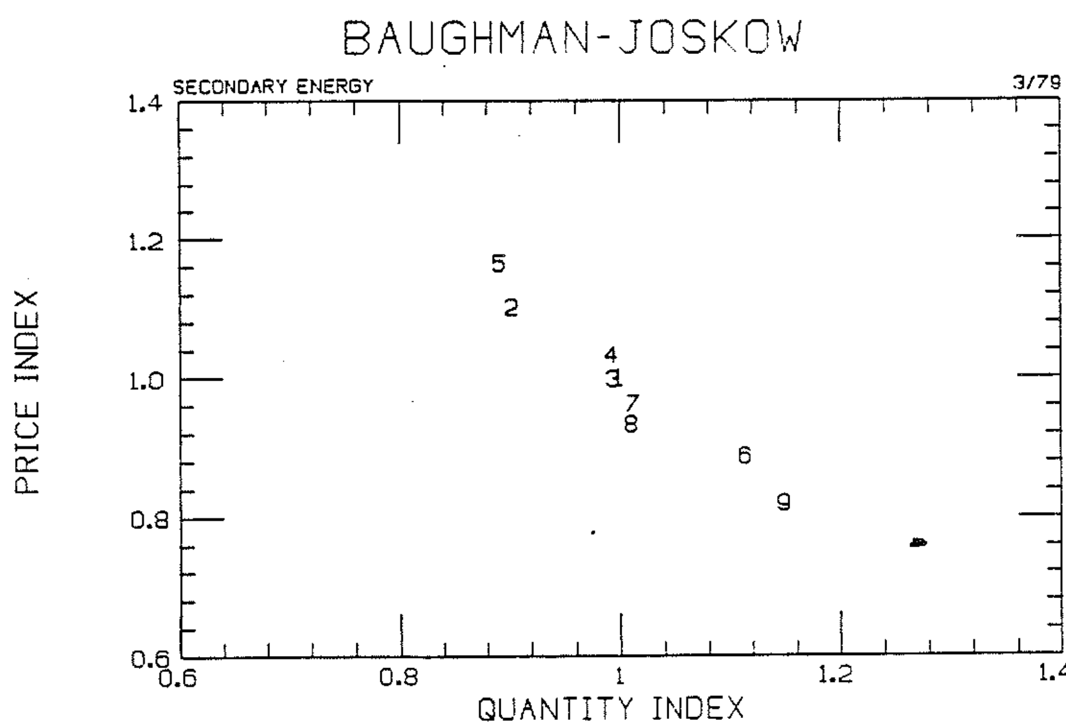


SECTOR: TRANSPORTATION
YEAR: 2000 (25-YEAR)

Figure 7-19

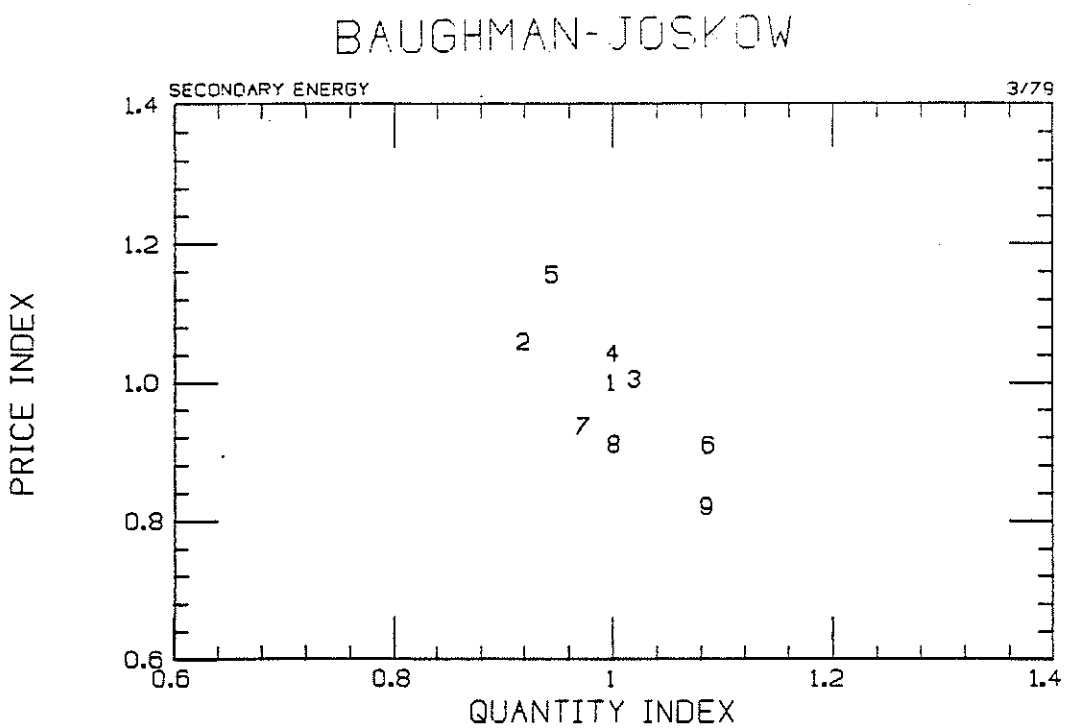


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)

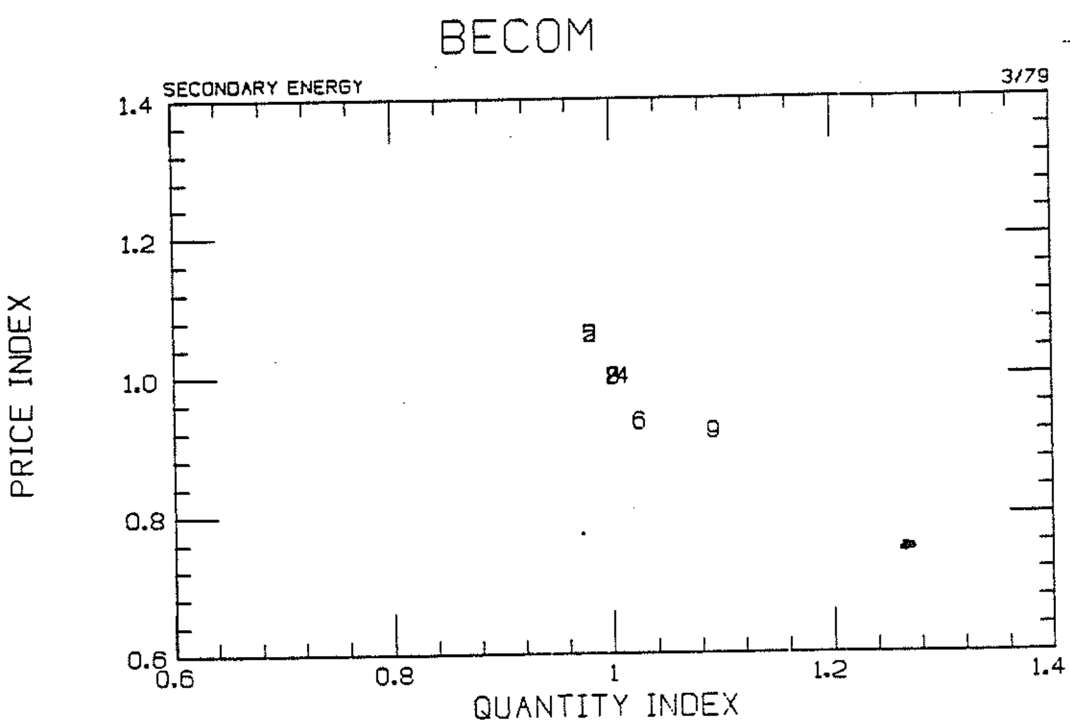


SECTOR: RESIDENTIAL/COMMERCIAL
YEAR: 2000 (25-YEAR)

Figures 7-20 and 7-21

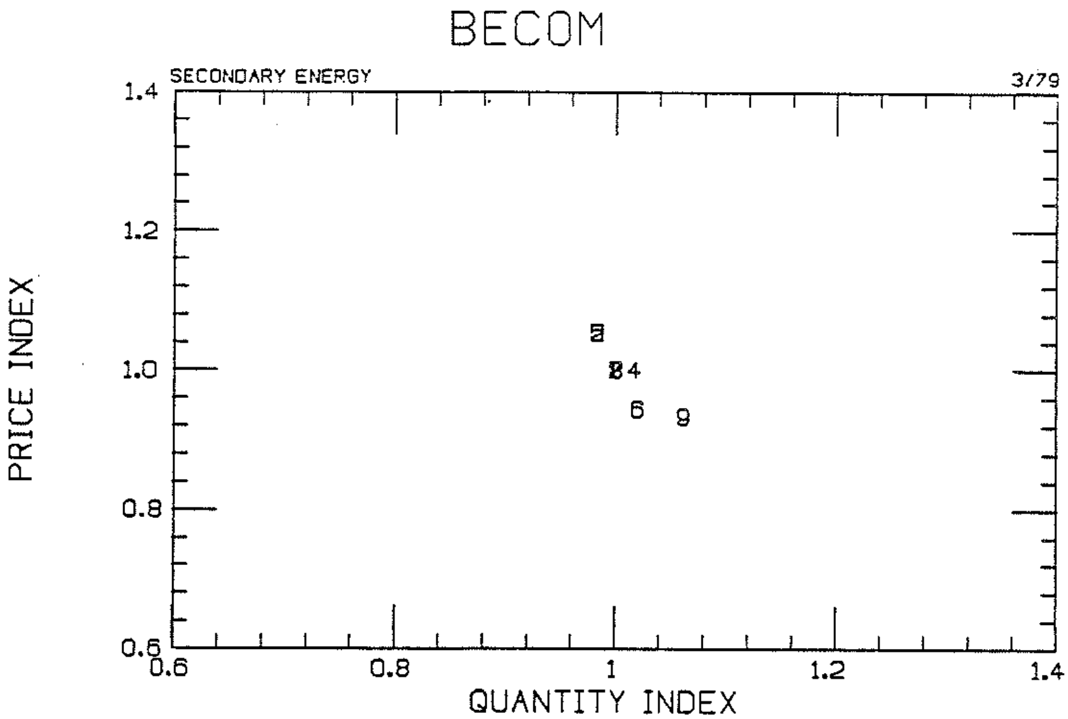


SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)

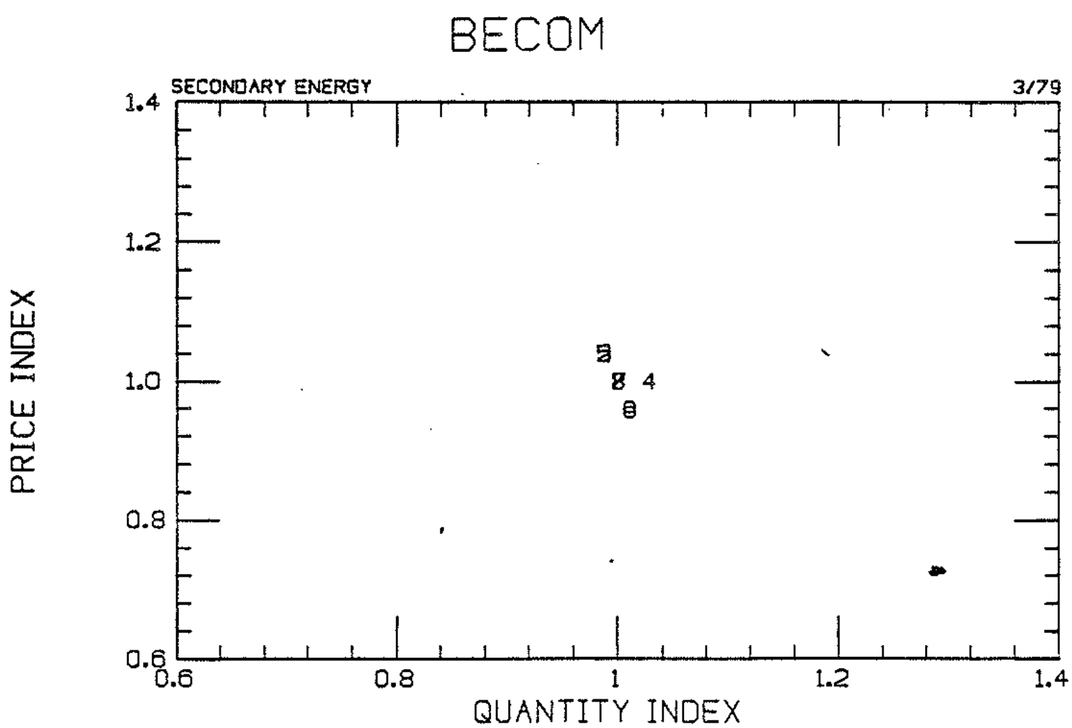


SECTOR: RESIDENTIAL
YEAR: 2000 (25-YEAR)

Figures 7-22 and 7-23

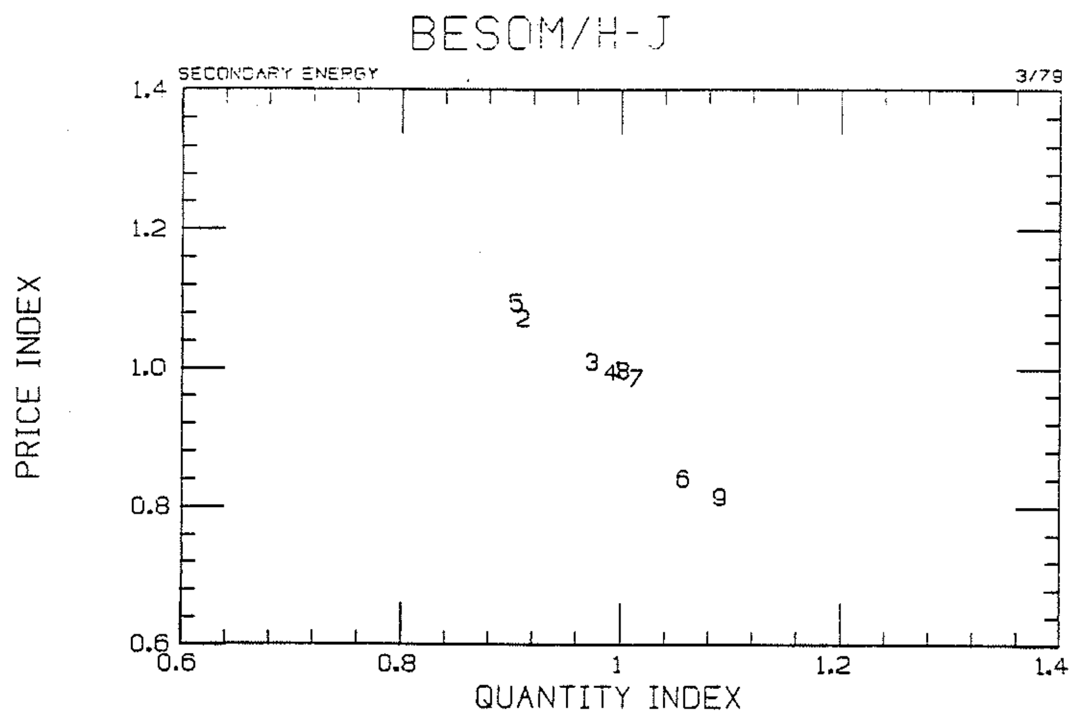


SECTOR: RESIDENTIAL/COMMERCIAL
YEAR: 2000 (25-YEAR)

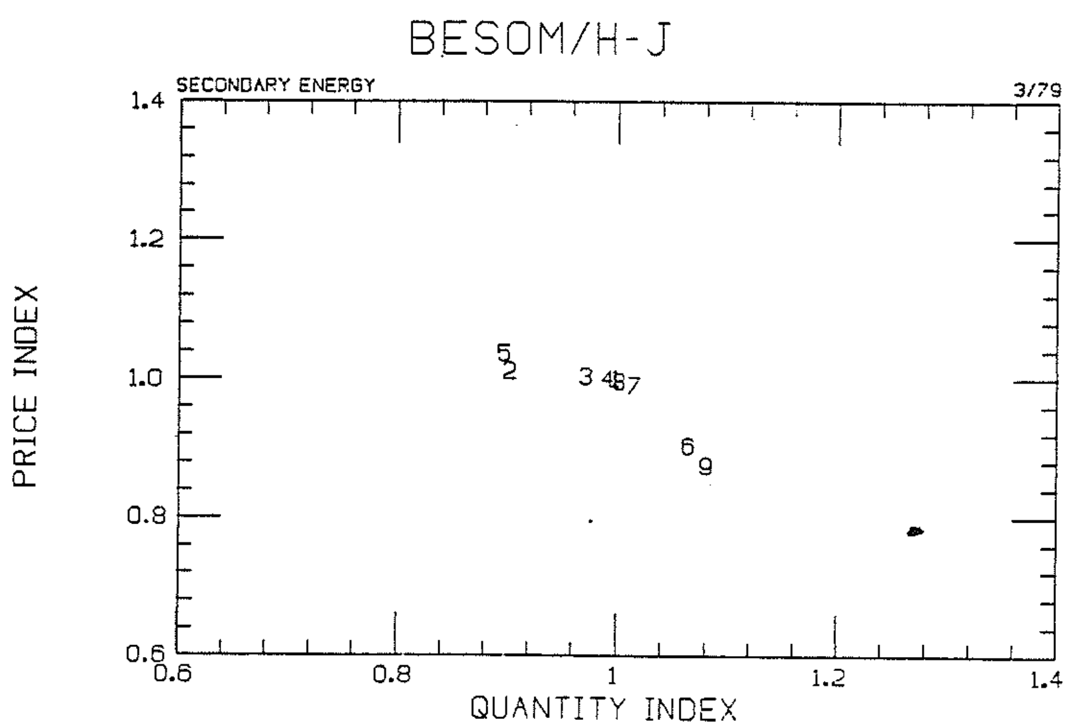


SECTOR: COMMERCIAL
YEAR: 2000 (25-YEAR)

Figures 7-24 and 7-25

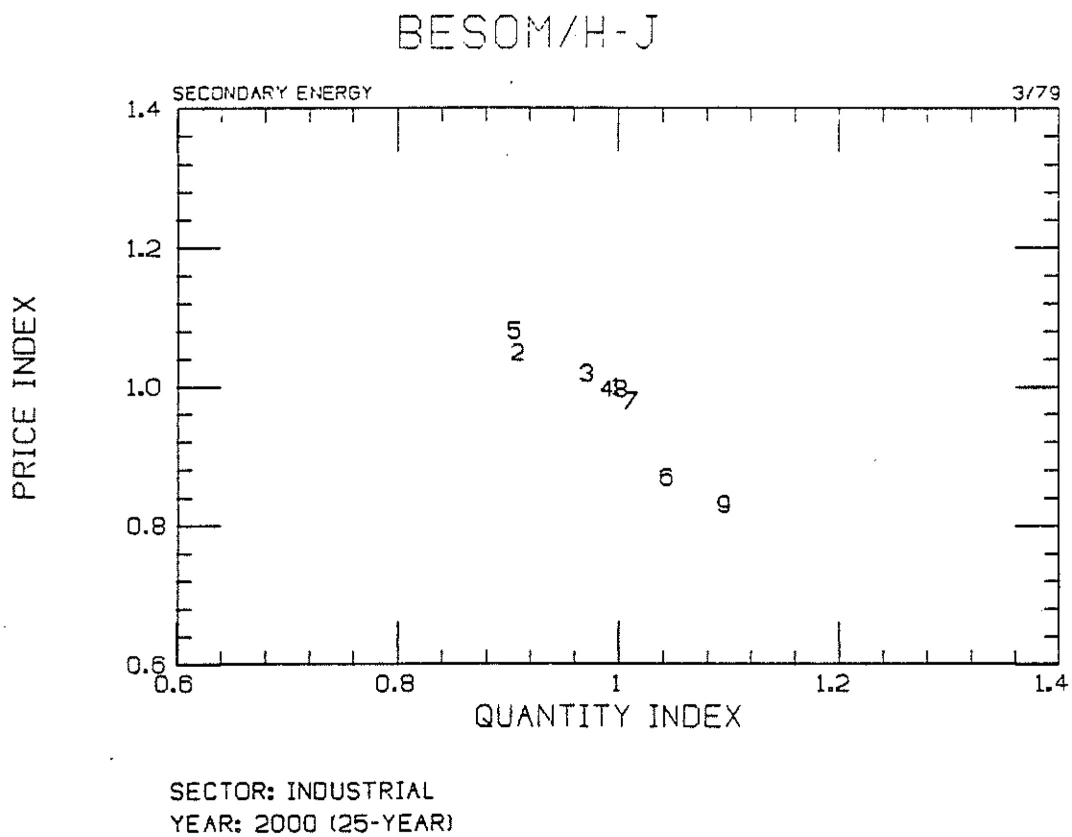


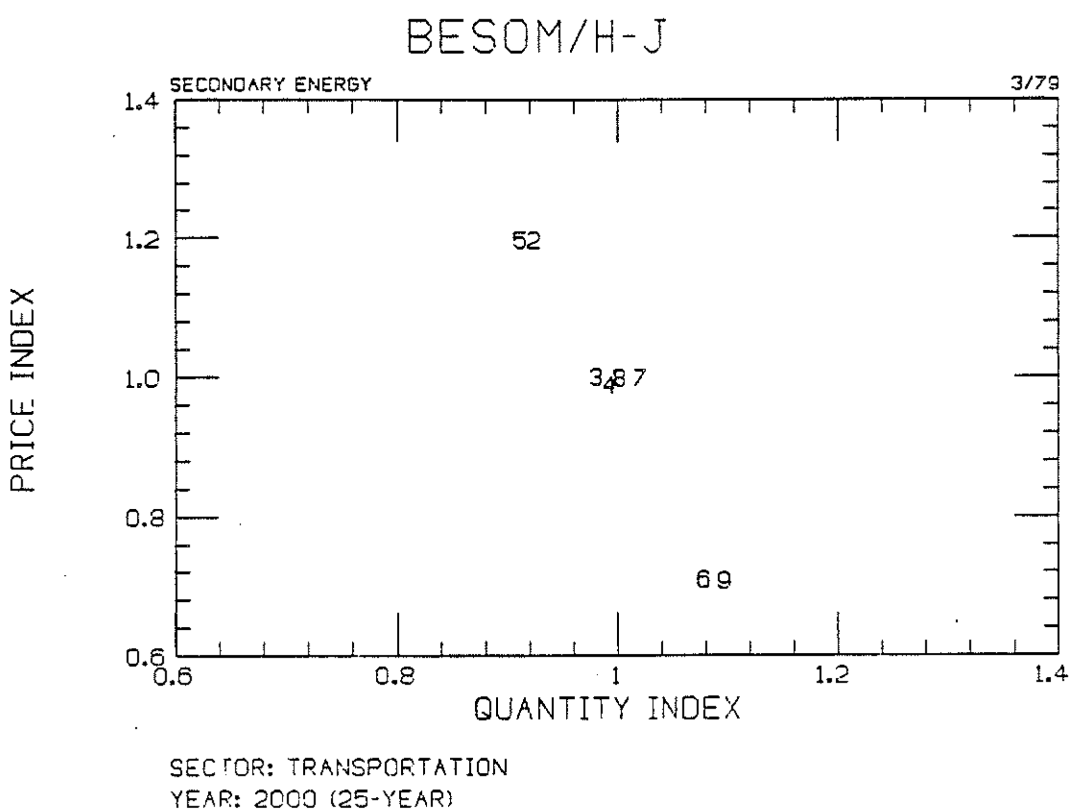
SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)



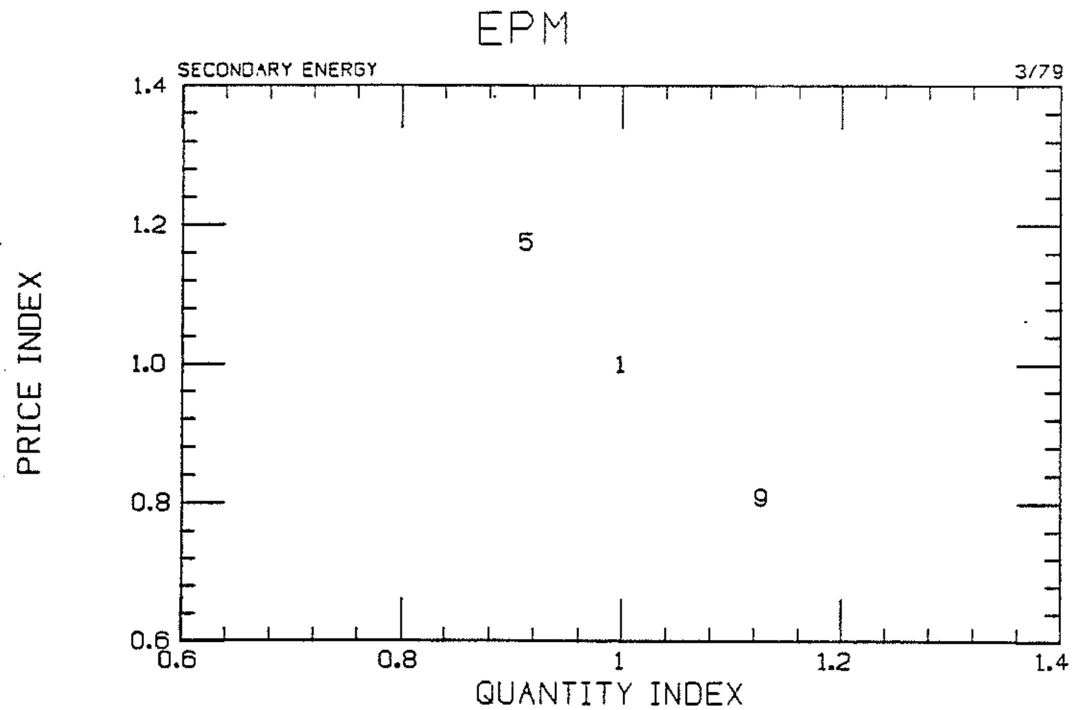
SECTOR: RESIDENTIAL/COMMERCIAL
YEAR: 2000 (25-YEAR)

Figures 7-26 and 7-27

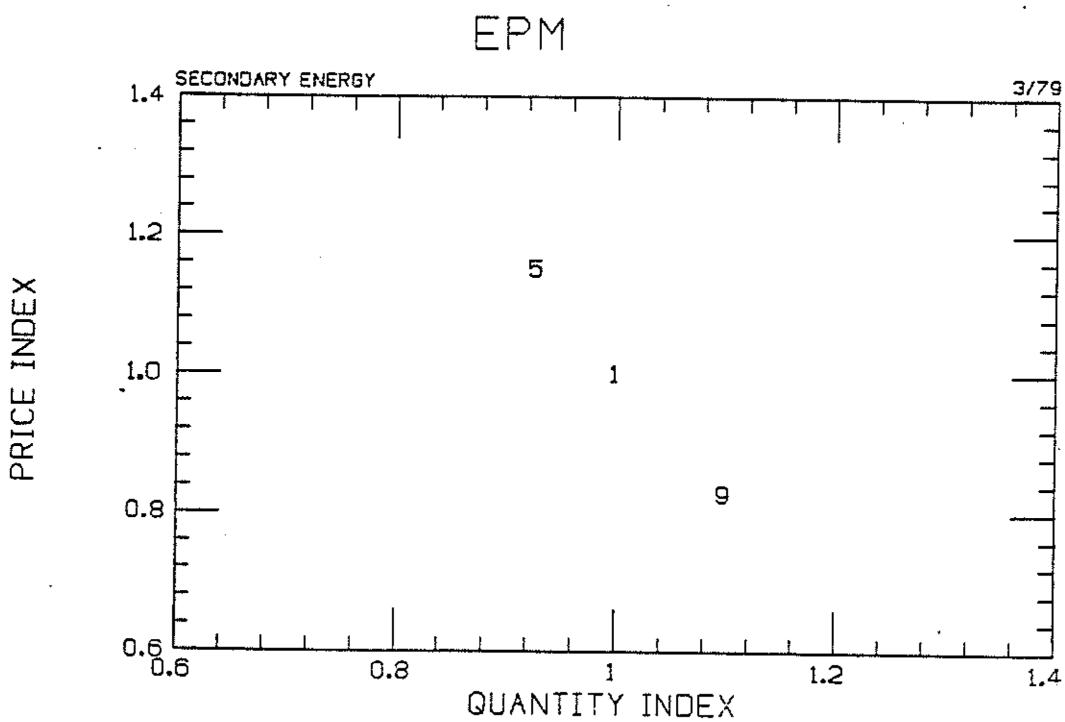




Figures 7-28 and 7-29

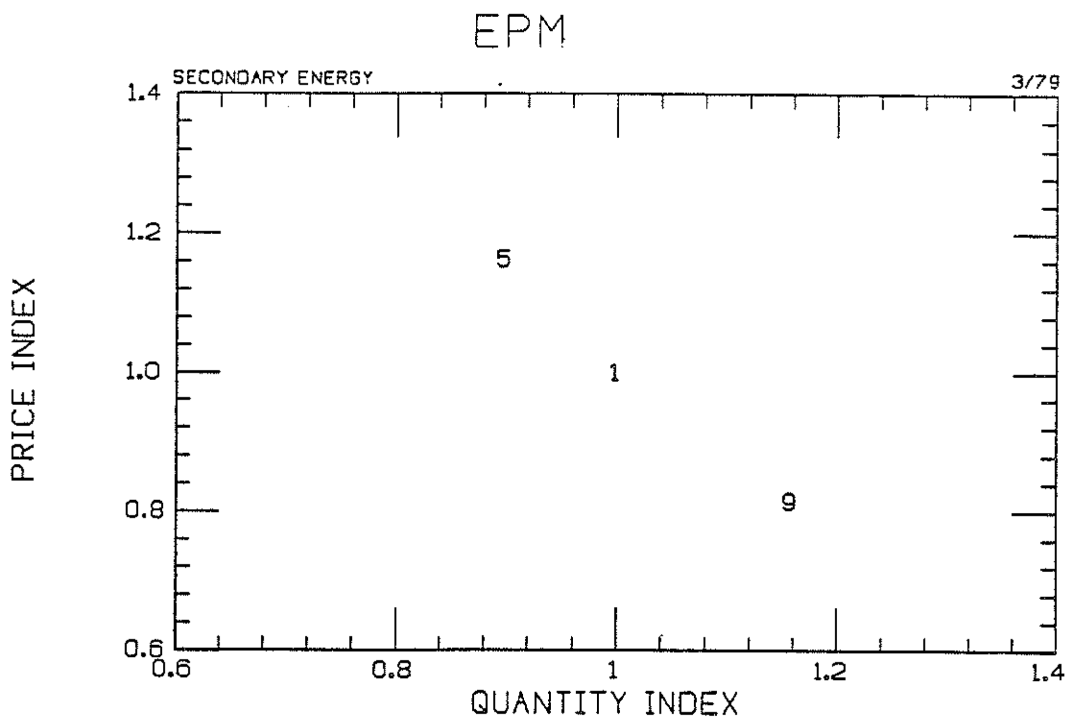


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)

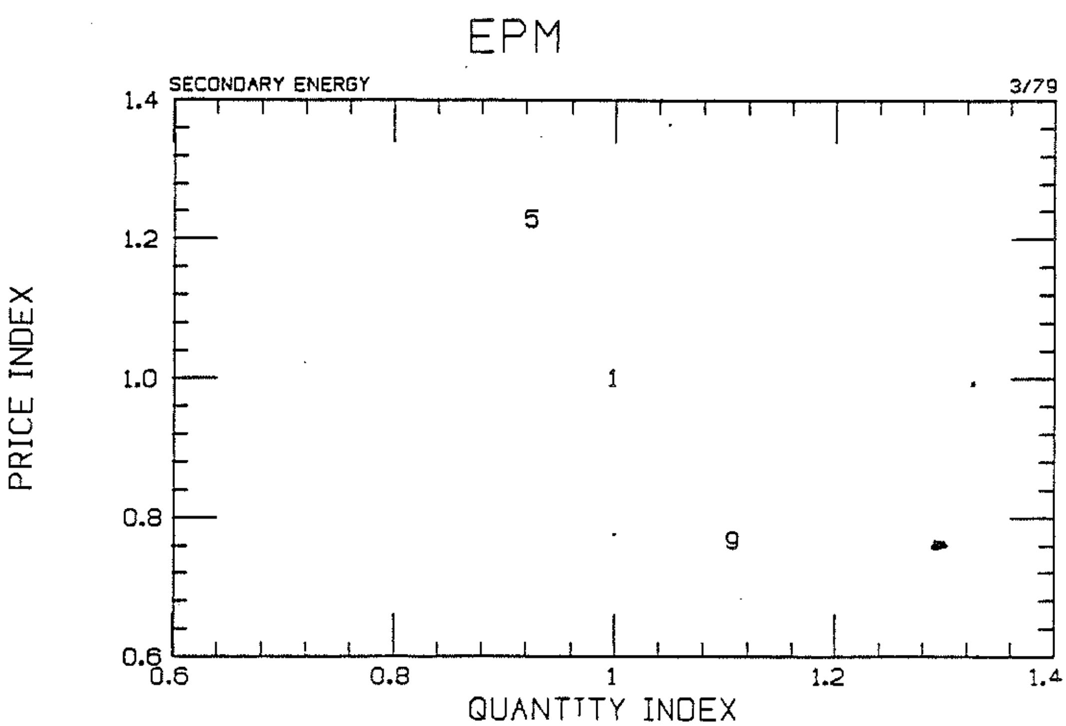


SECTOR: RESIDENTIAL/COMMERCIAL
YEAR: 2000 (25-YEAR)

Figures 7-30 and 7-31

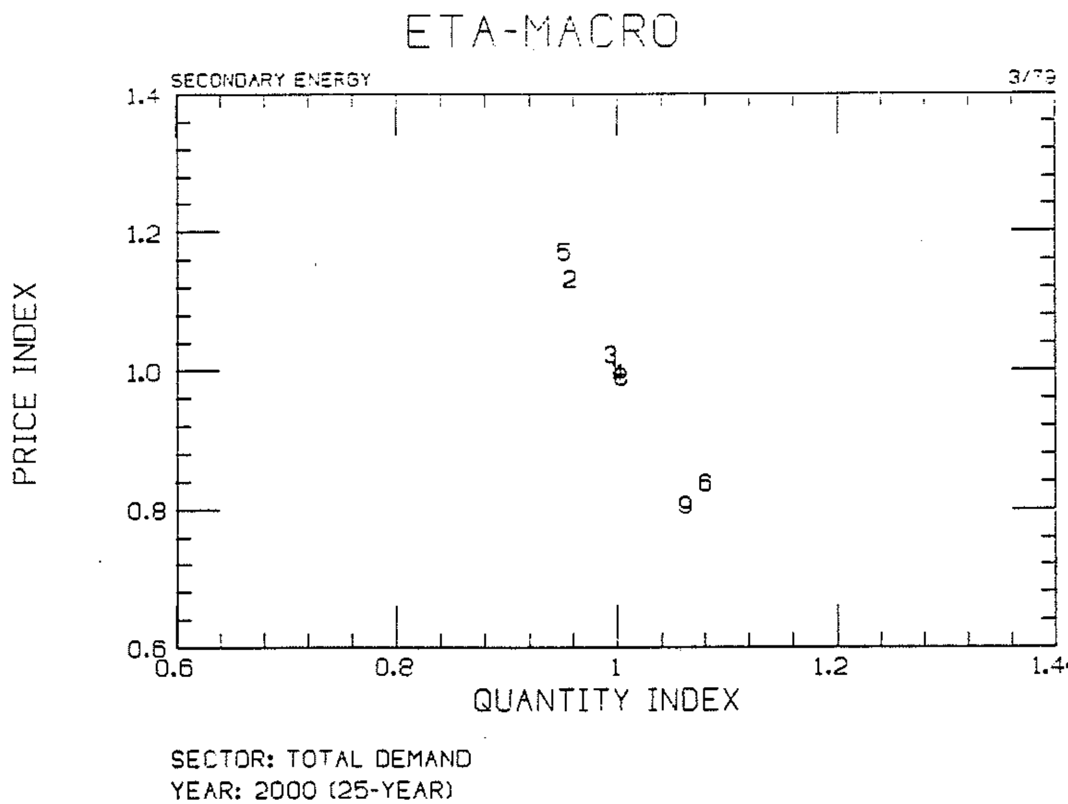


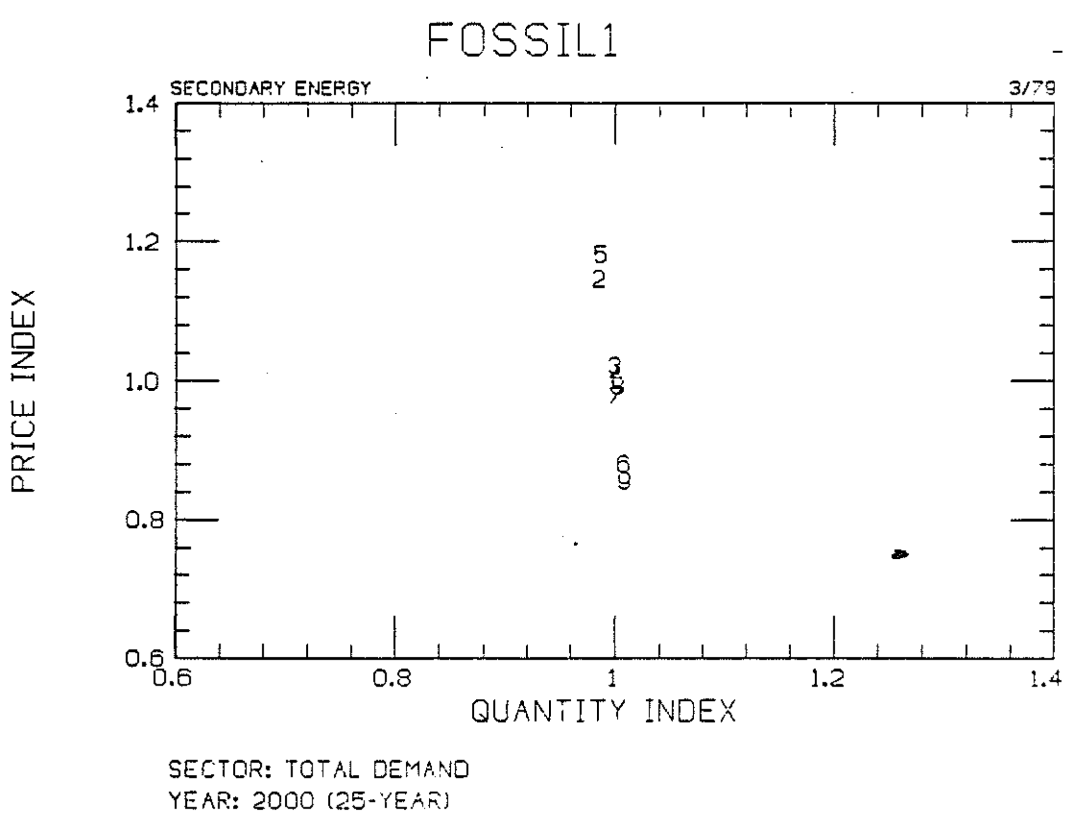
SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)



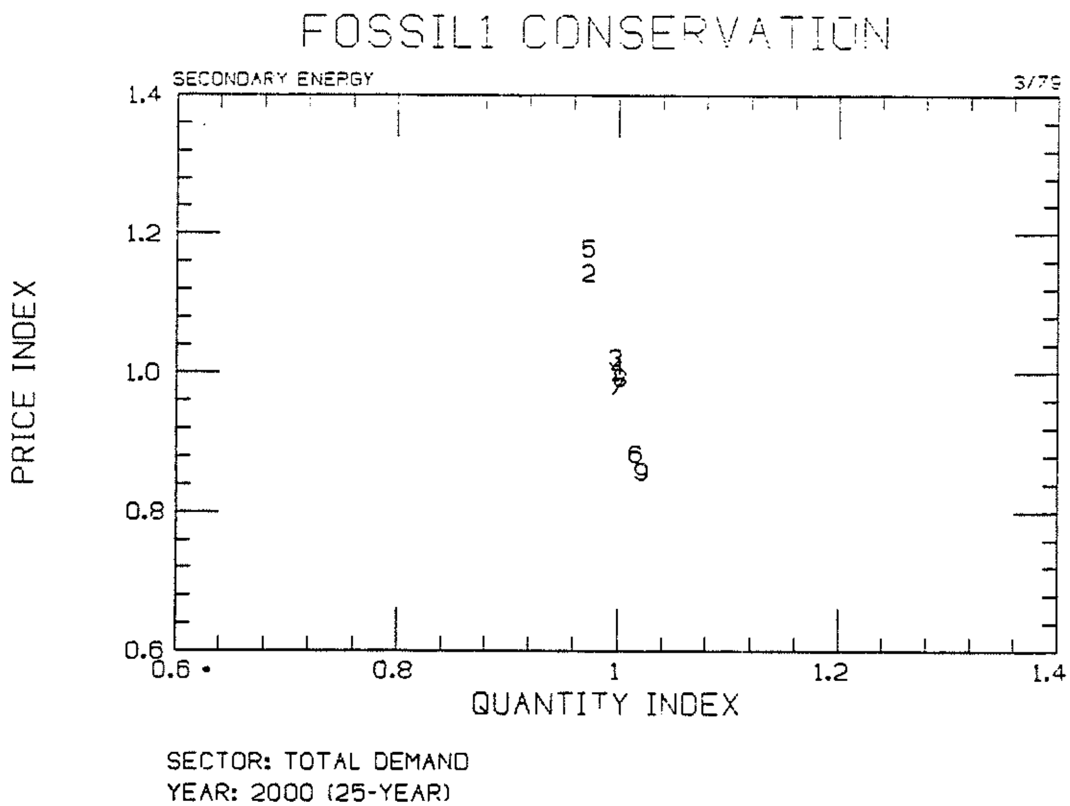
SECTOR: TRANSPORTATION
YEAR: 2000 (25-YEAR)

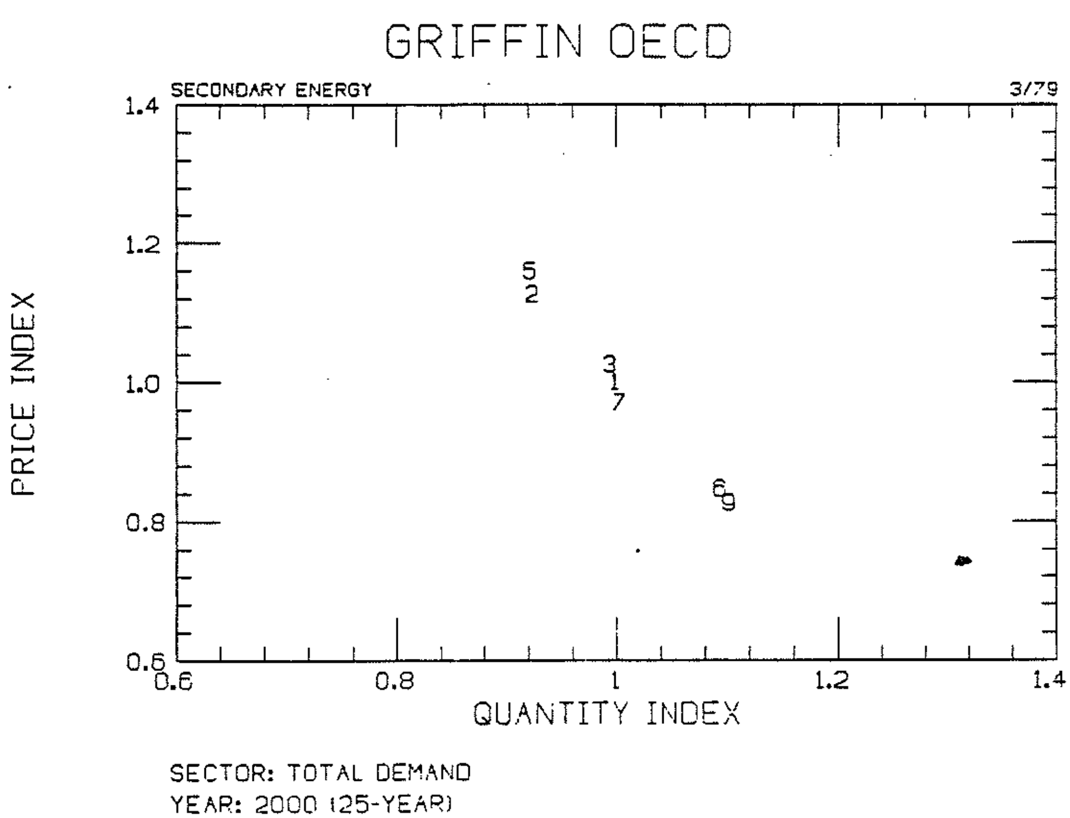
Figures 7-32 and 7-33



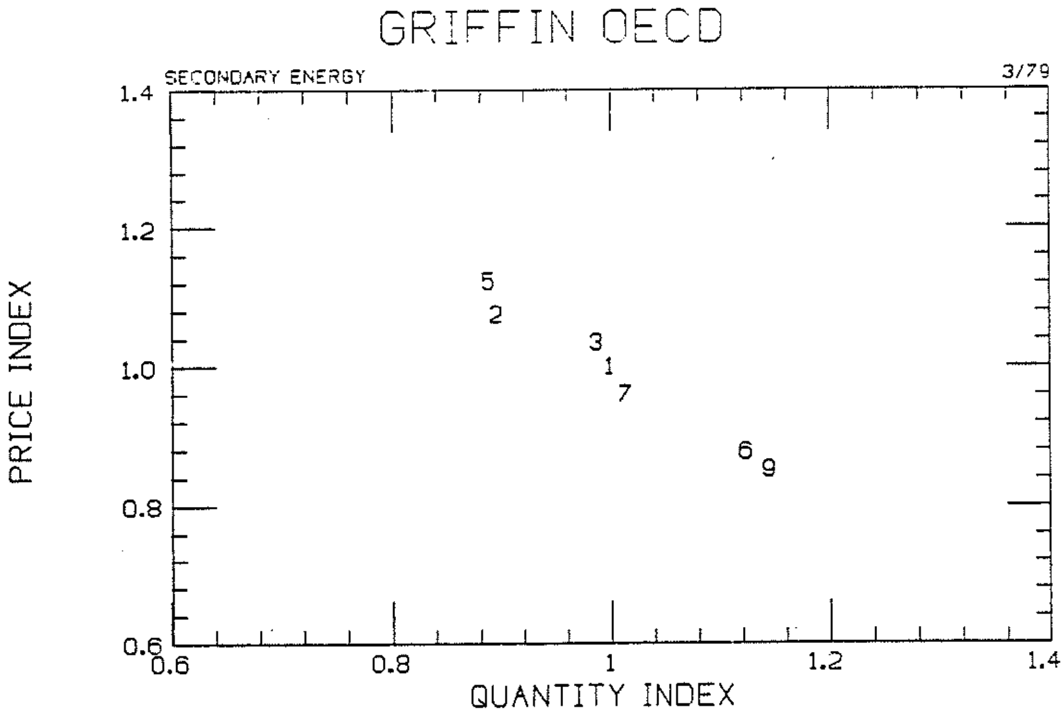


Figures 7-34 and 7-35

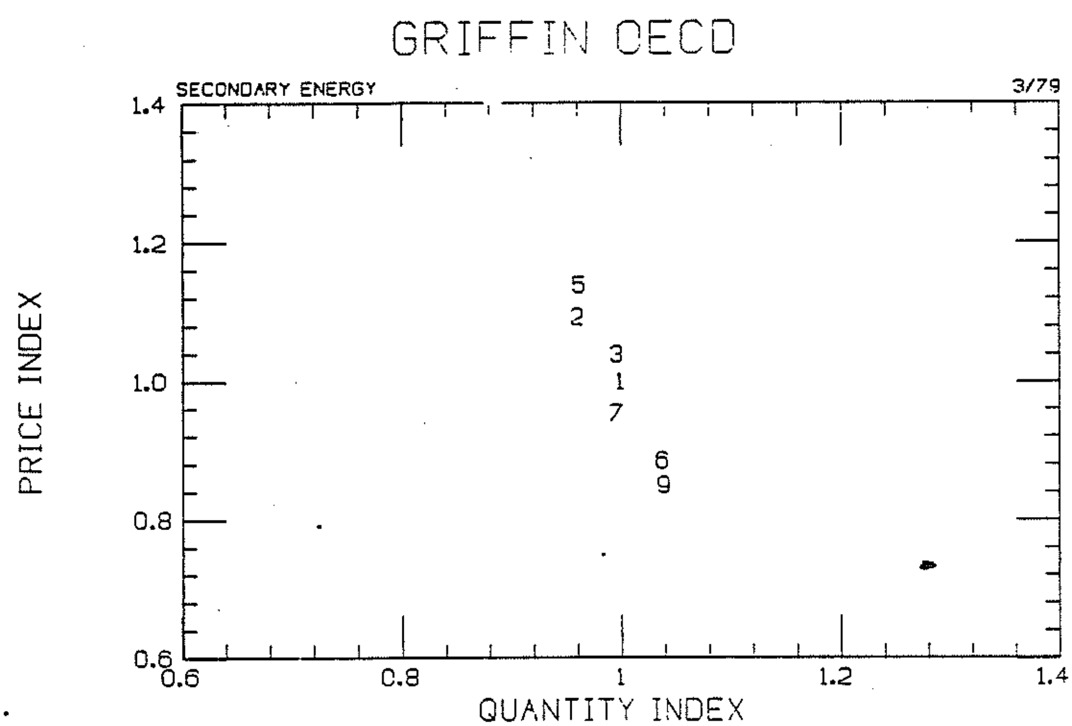




Figures 7-36 and 7-37

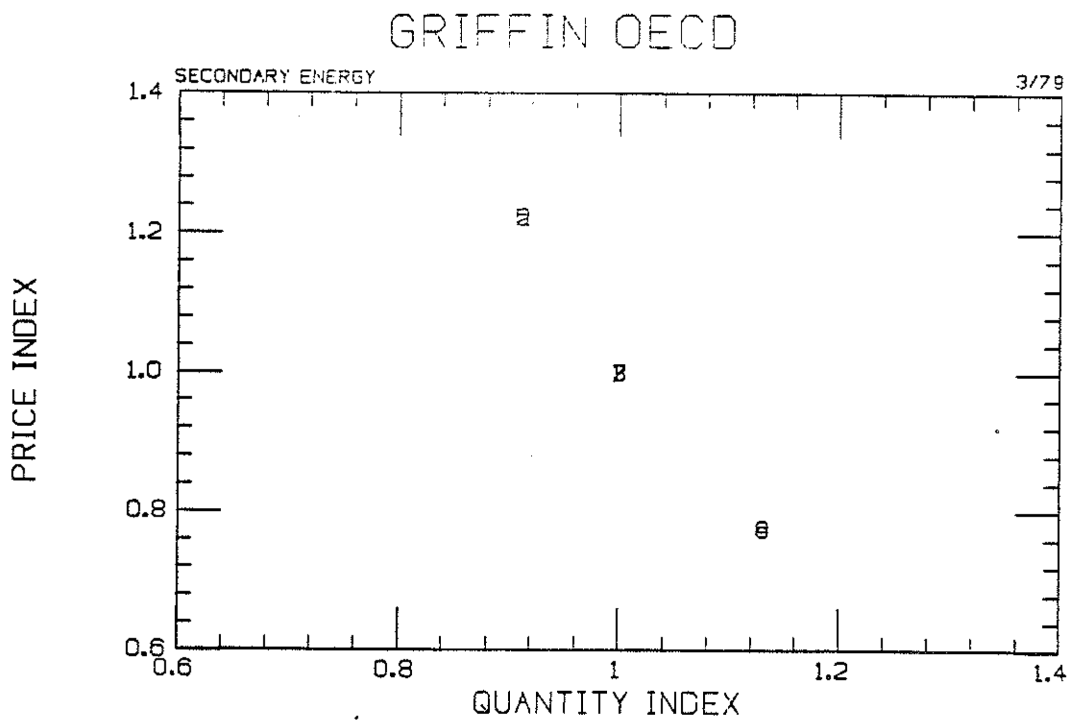


SECTOR: RESIDENTIAL
YEAR: 2000 (25-YEAR)

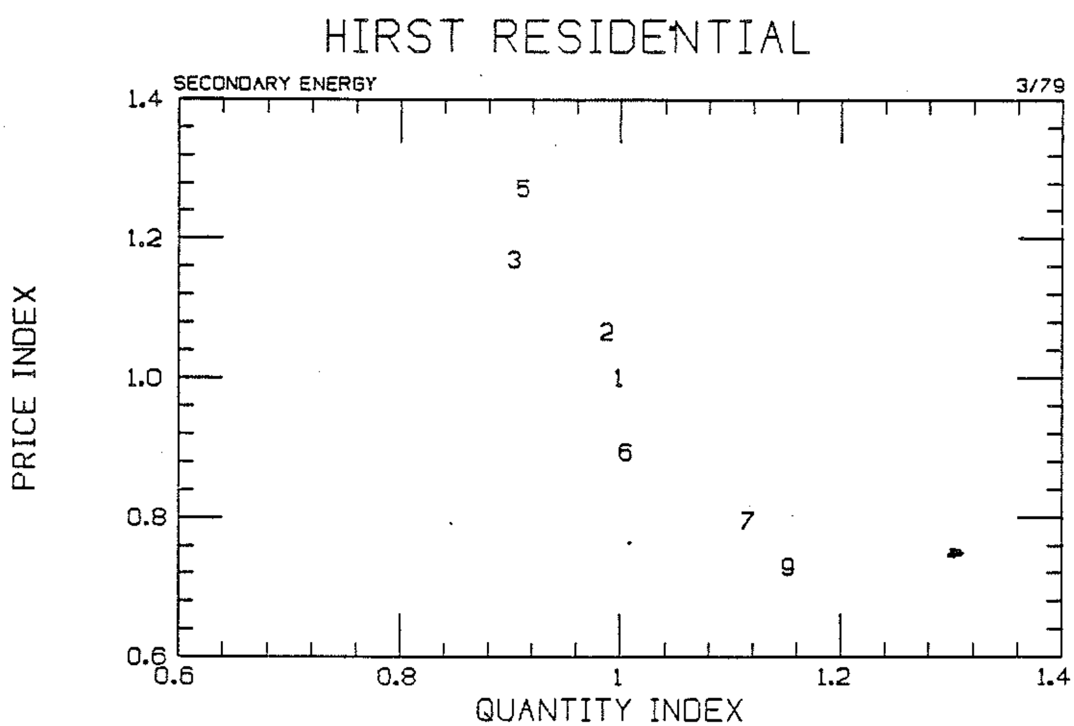


SECTOR: COMMERCIAL/INDUSTRIAL
YEAR: 2000 (25-YEAR)

Figures 7-38 and 7-39

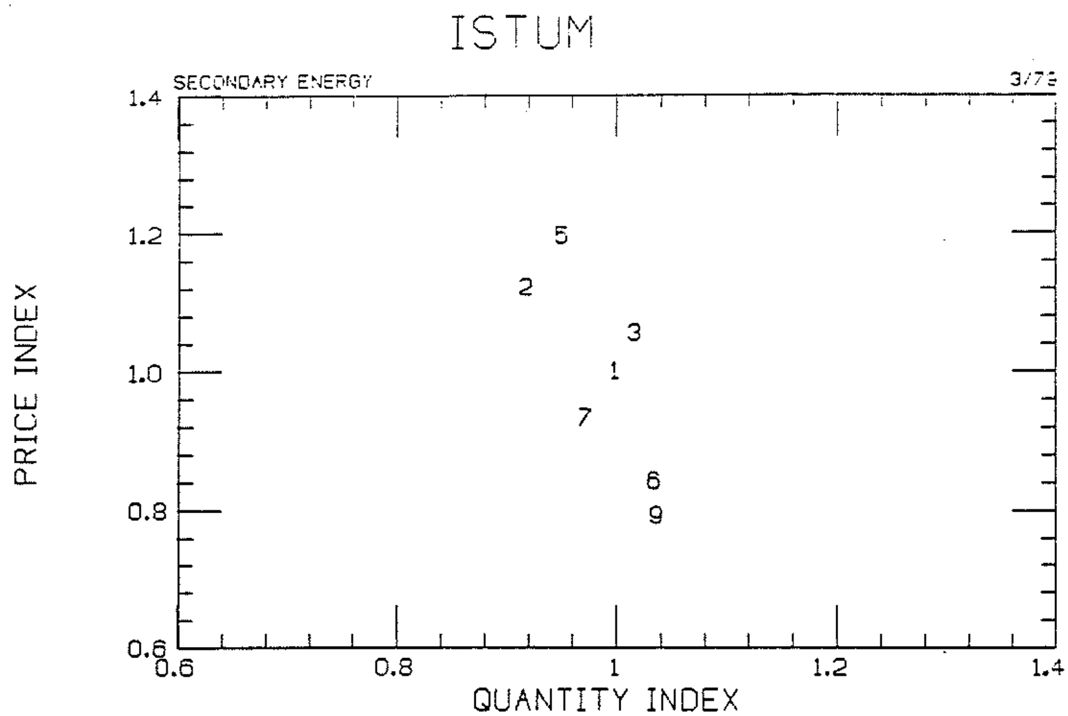


SECTOR: TRANSPORTATION
YEAR: 2000 (25-YEAR)

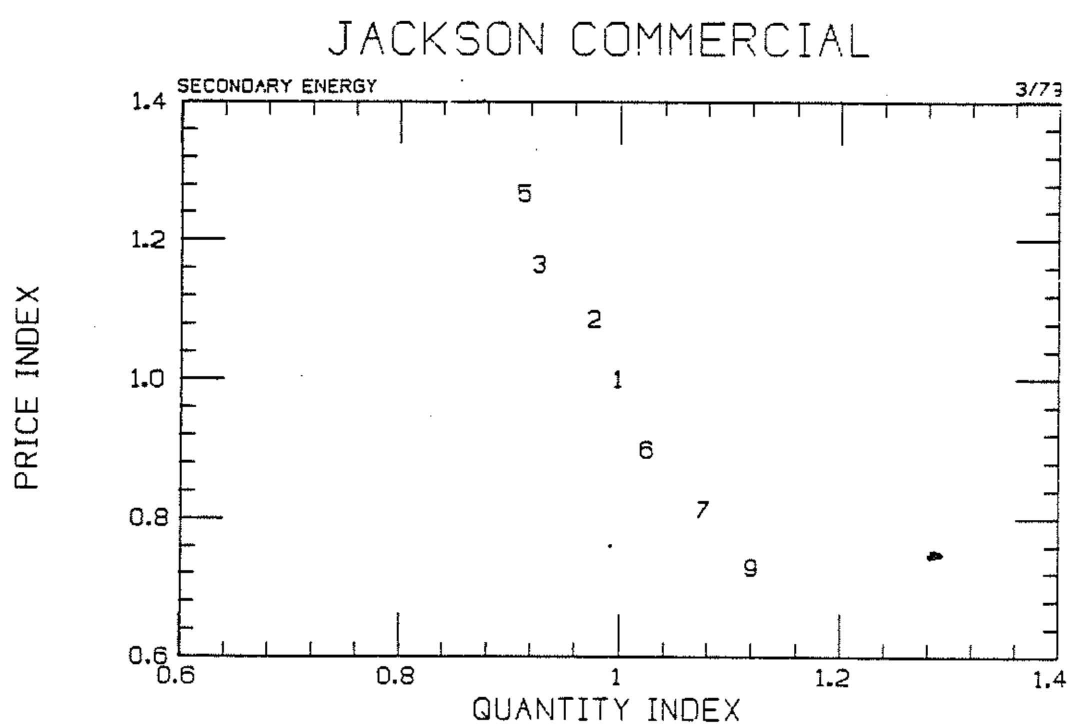


SECTOR: RESIDENTIAL
YEAR: 2000 (25-YEAR)

Figures 7-40 and 7-41

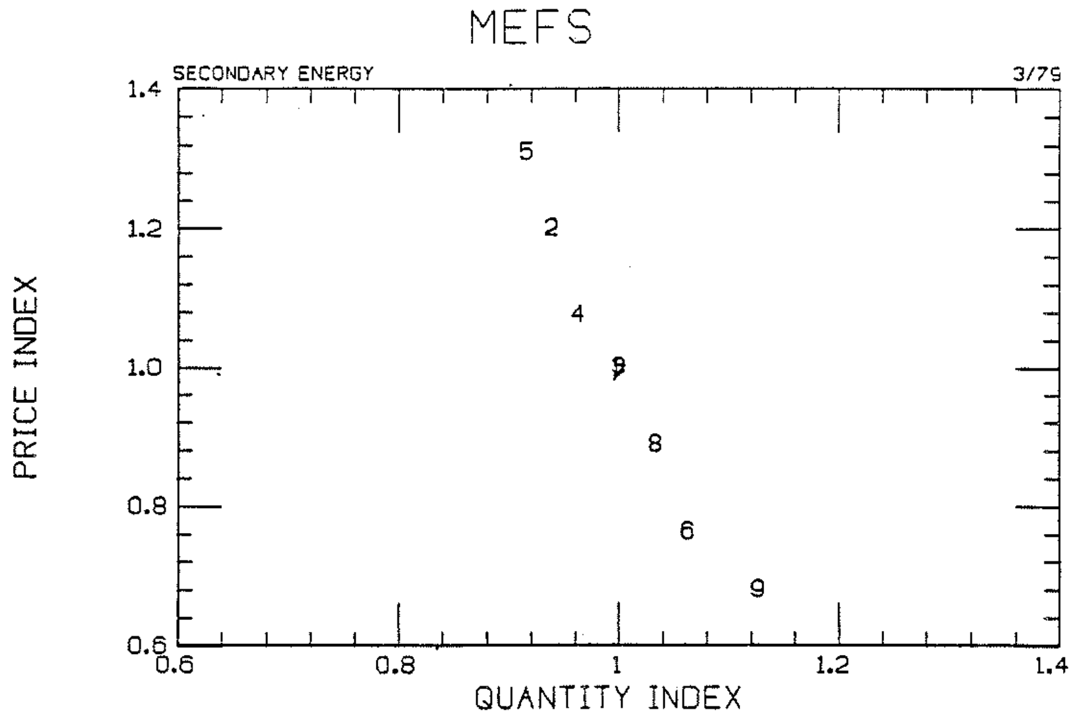


SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)

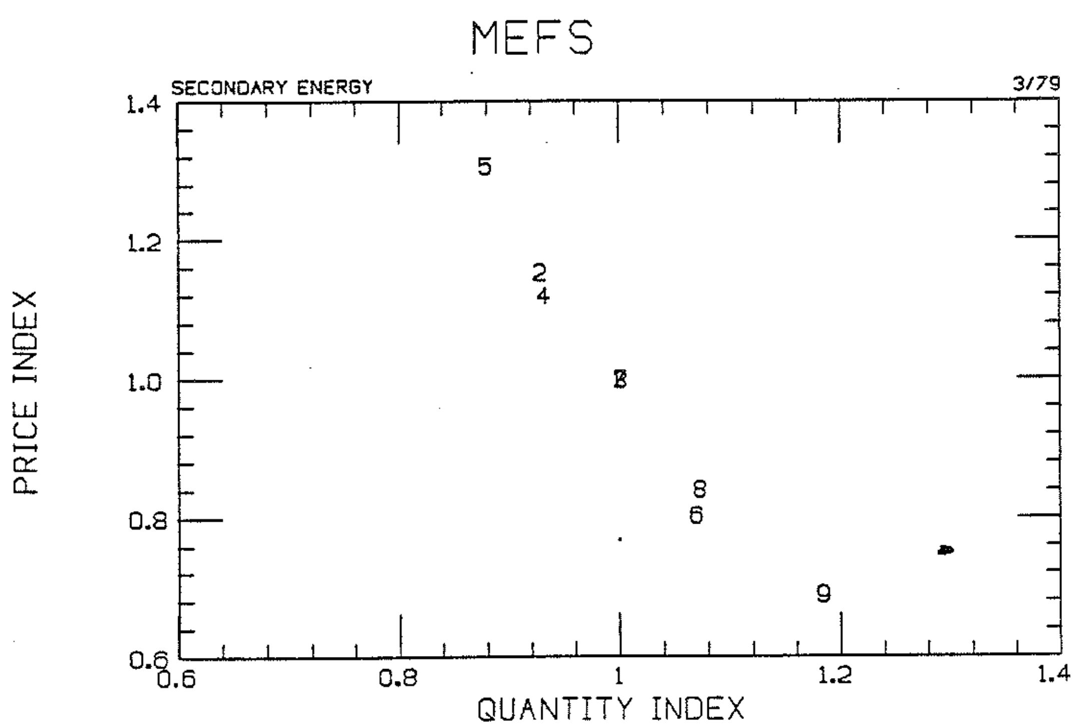


SECTOR: COMMERCIAL
YEAR: 2000 (25-YEAR)

Figures 7-42 and 7-43

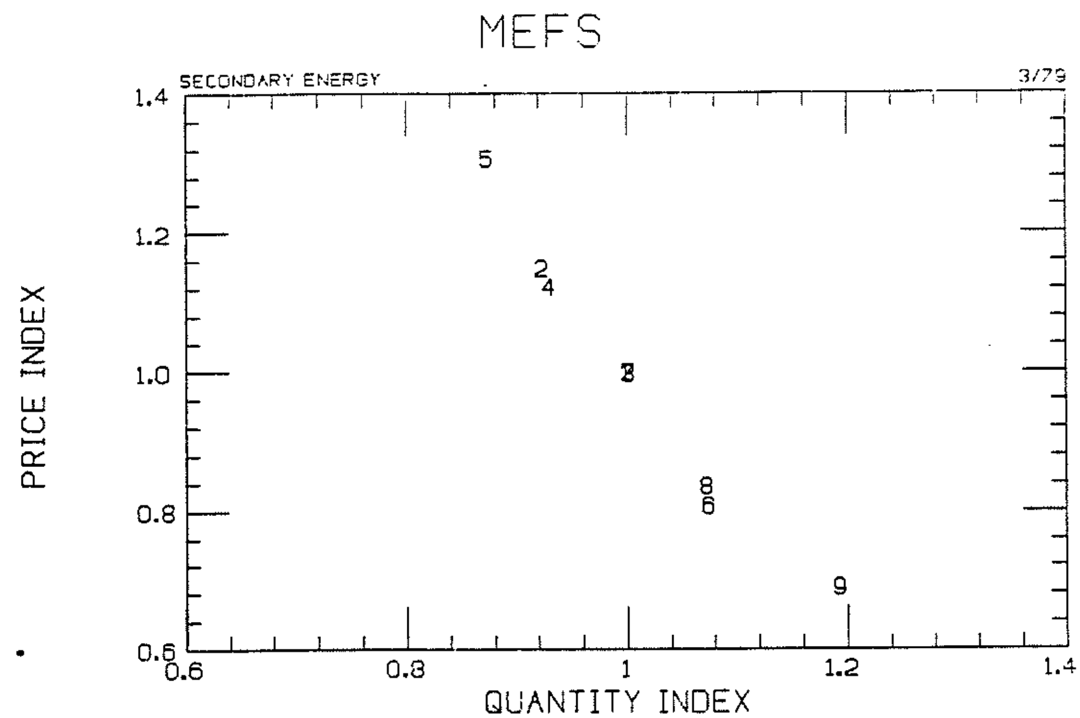


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)

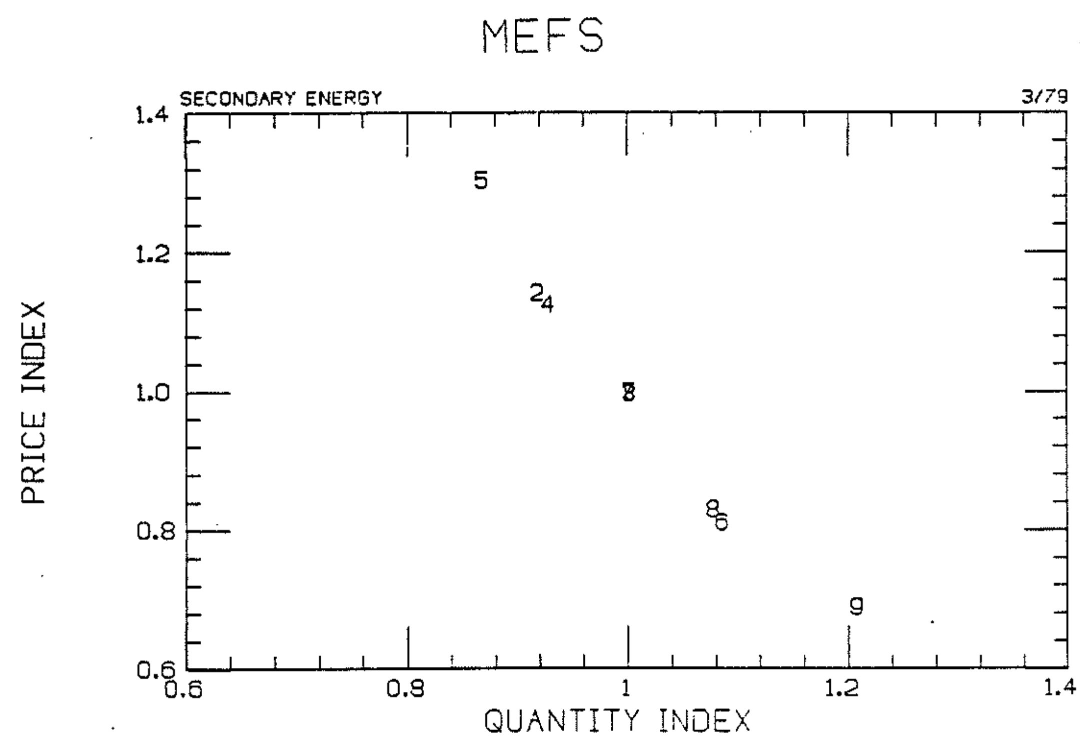


SECTOR: RESIDENTIAL
YEAR: 2000 (25-YEAR)

Figures 7-44 and 7-45

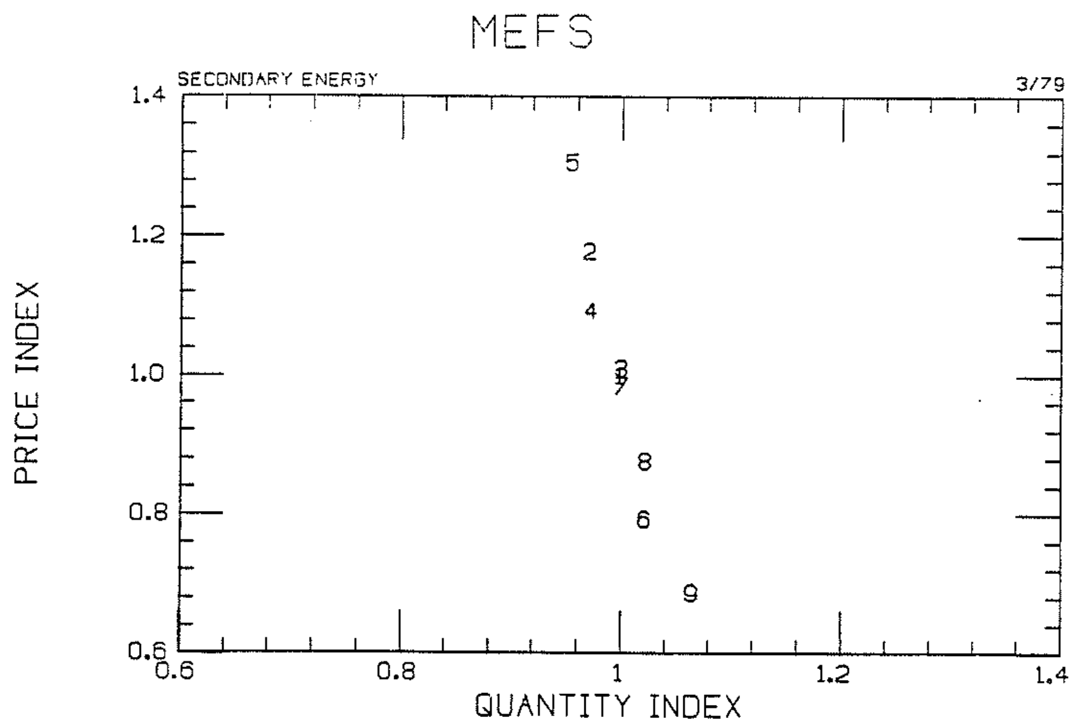


SECTOR: RESIDENTIAL/COMMERCIAL
YEAR: 2000 (25-YEAR)

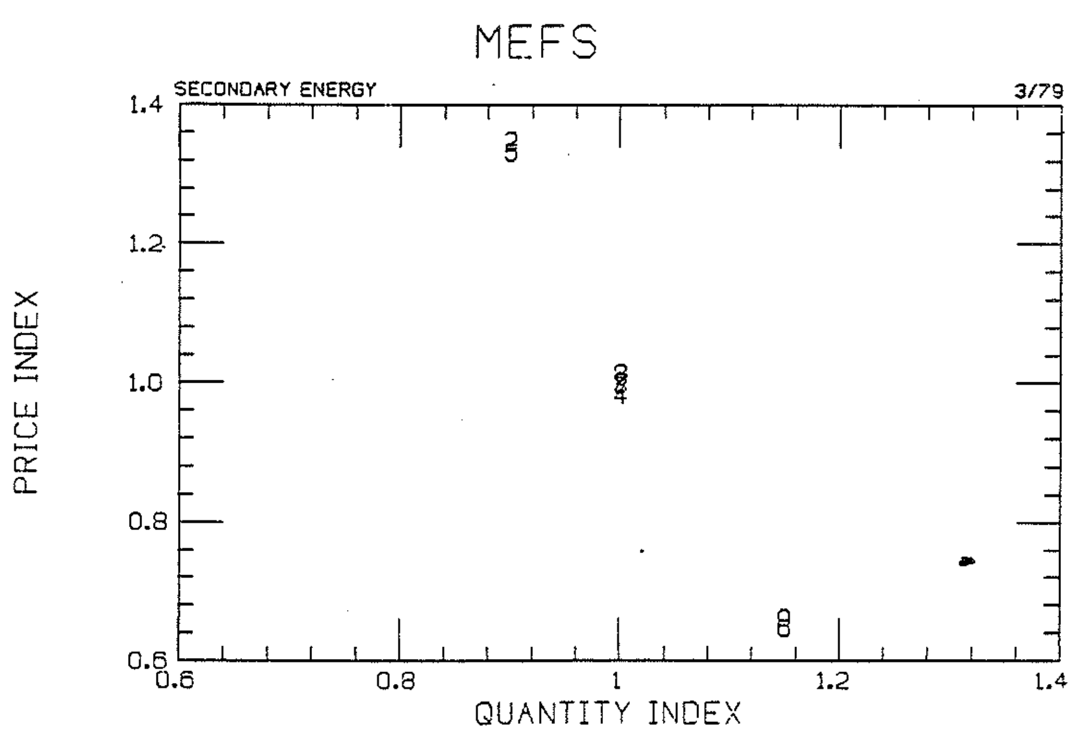


SECTOR: COMMERCIAL
YEAR: 2000 (25-YEAR)

Figures 7-46 and 7-47

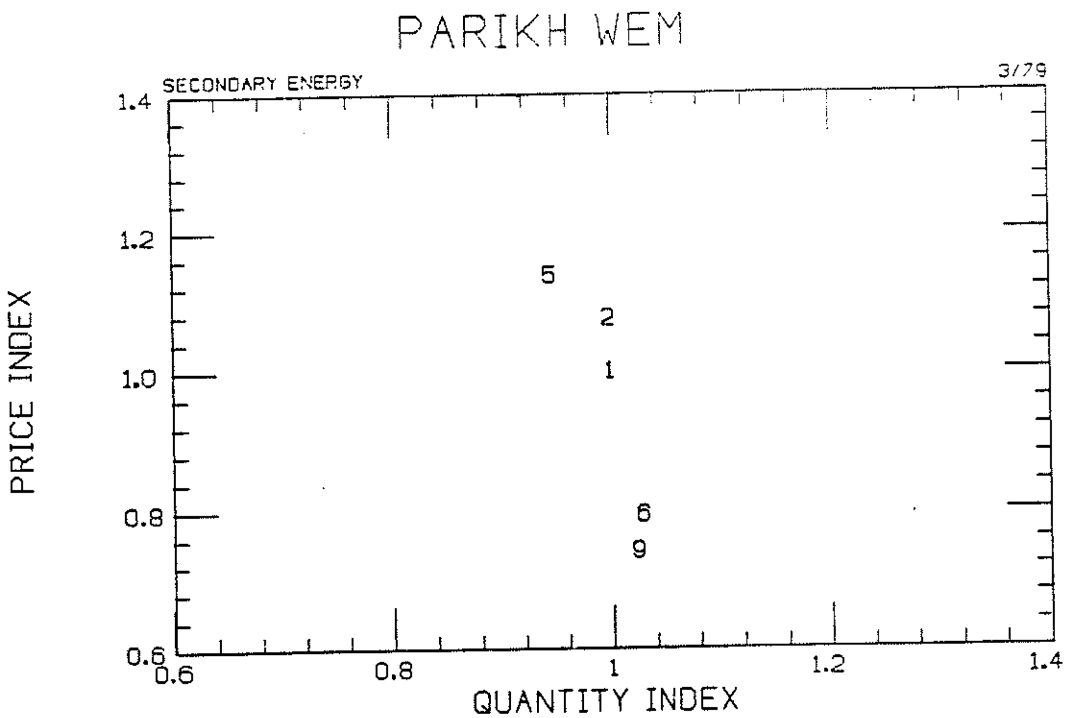


SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)

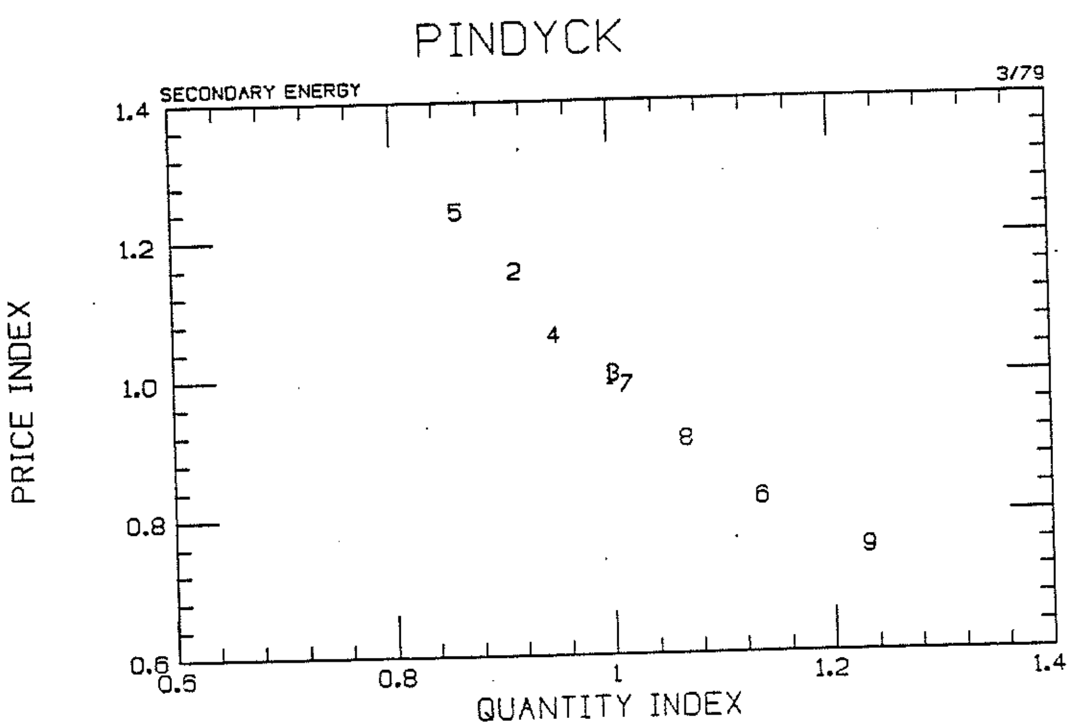


SECTOR: TRANSPORTATION
YEAR: 2000 (25-YEAR)

Figures 7-48 and 7-49

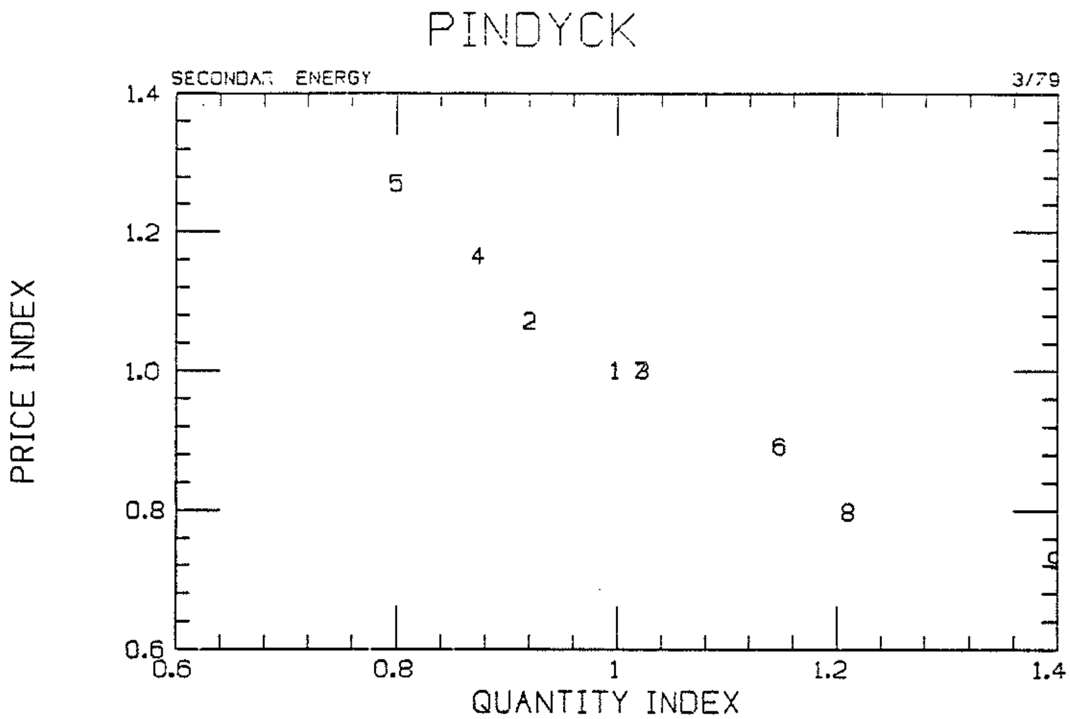


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)

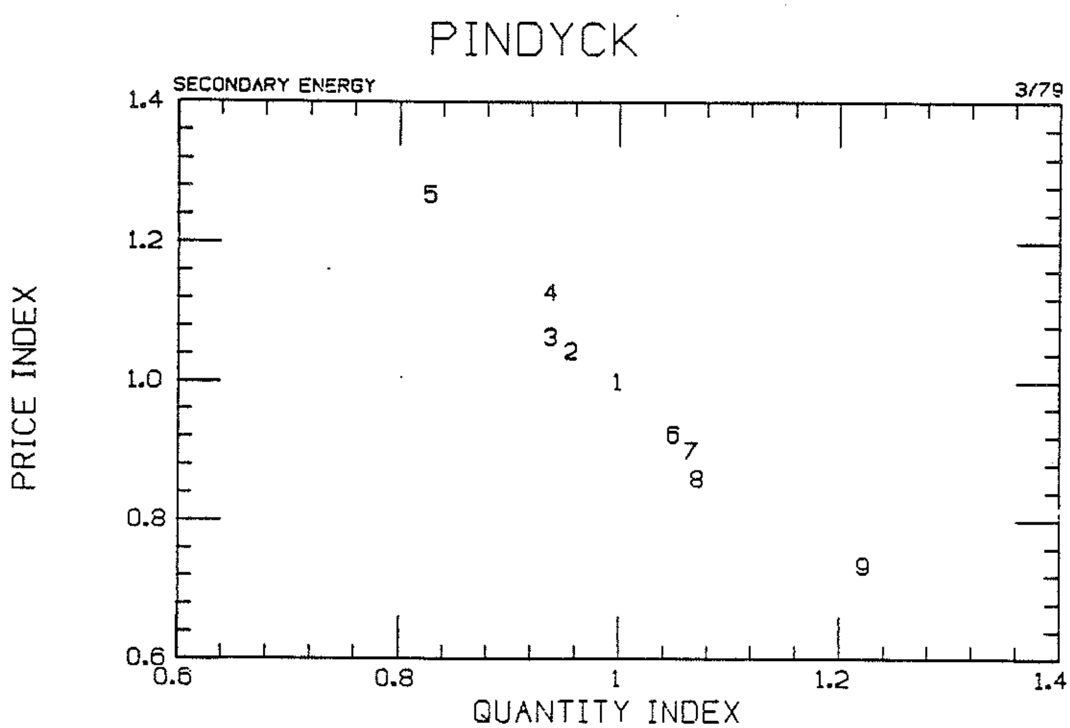


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)

Figures 7-50 and 7-51

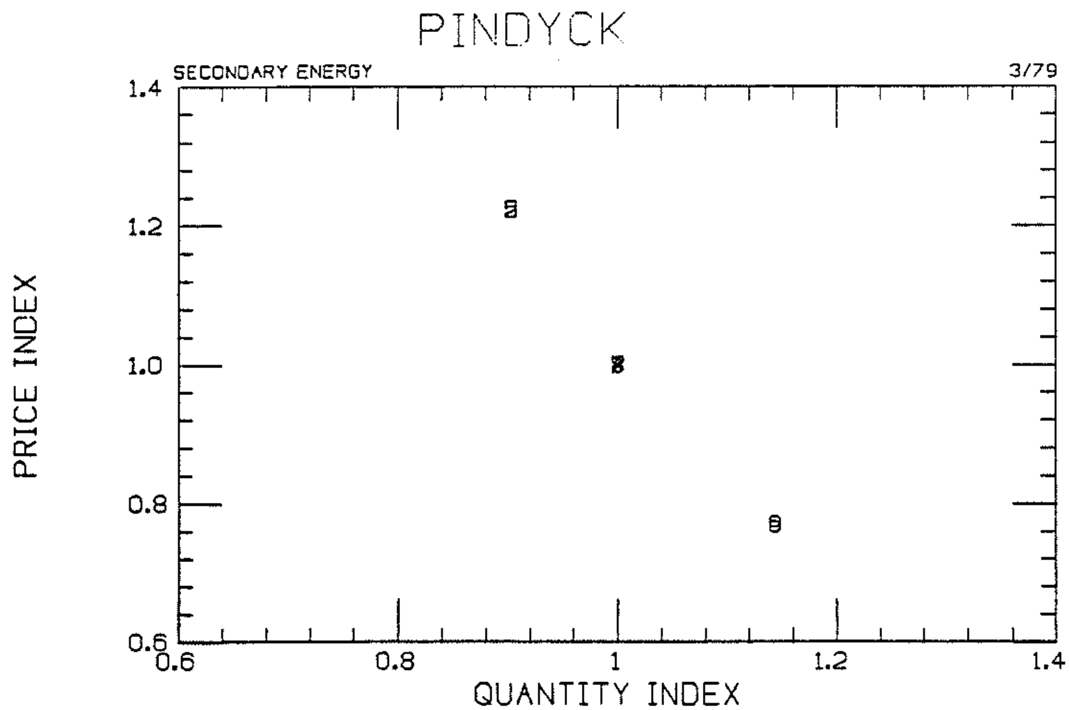


SECTOR: RESIDENTIAL
YEAR: 2000 (25-YEAR)



SECTOR: COMMERCIAL/INDUSTRIAL
YEAR: 2000 (25-YEAR)

Figures 7-52 and 7-53



SECTOR: TRANSPORTATION
YEAR: 2000 (25-YEAR)

Figure 7-54

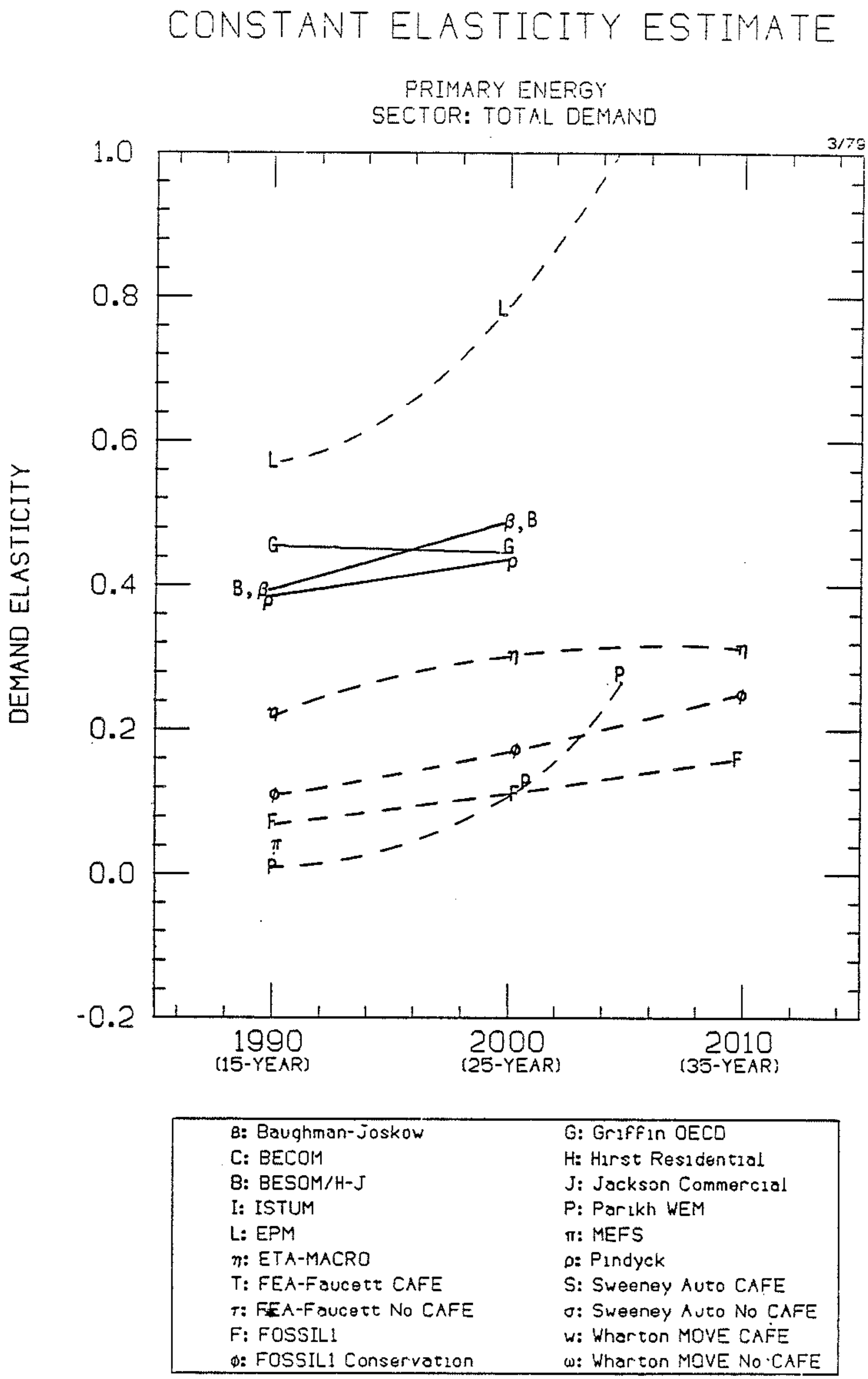
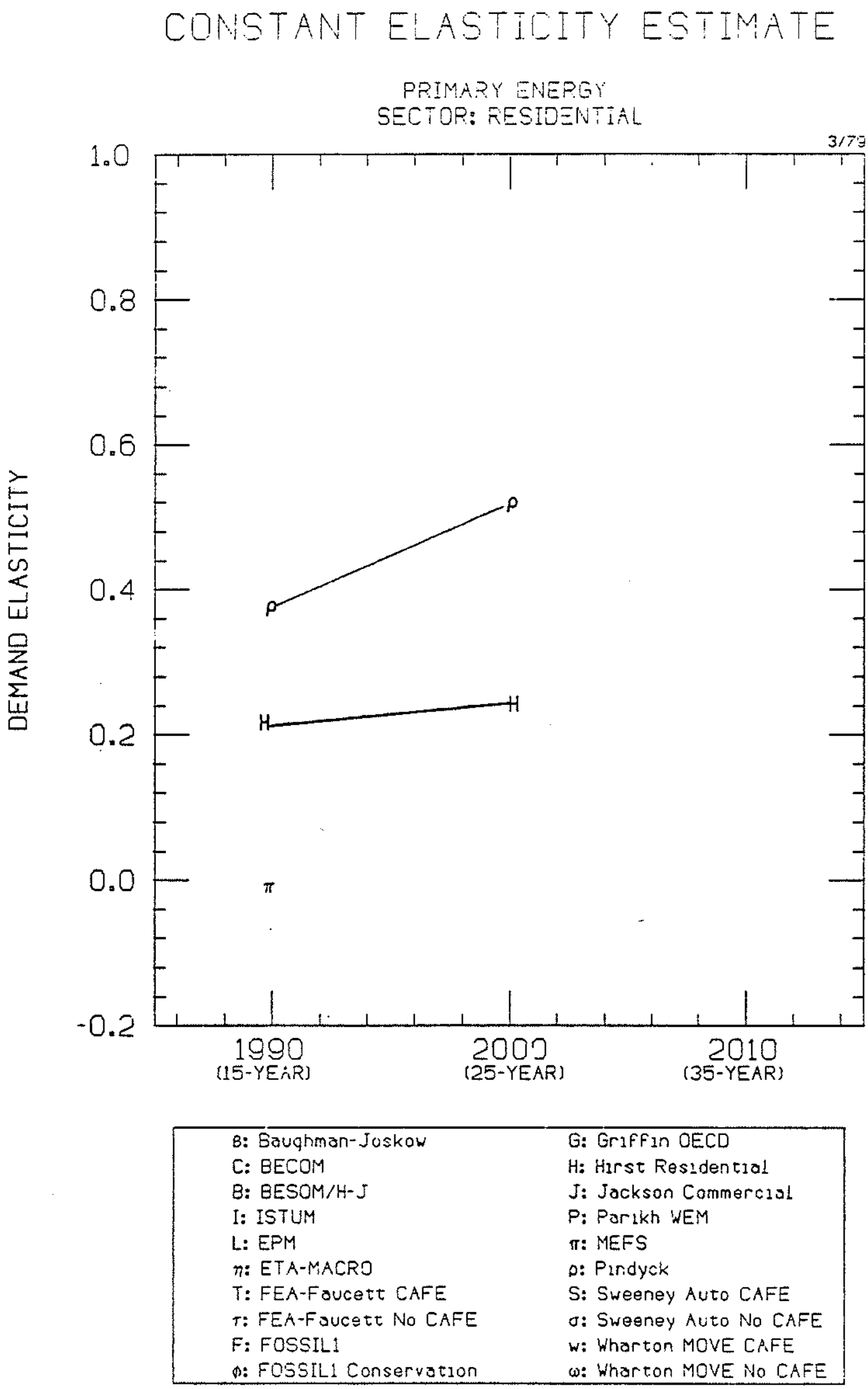
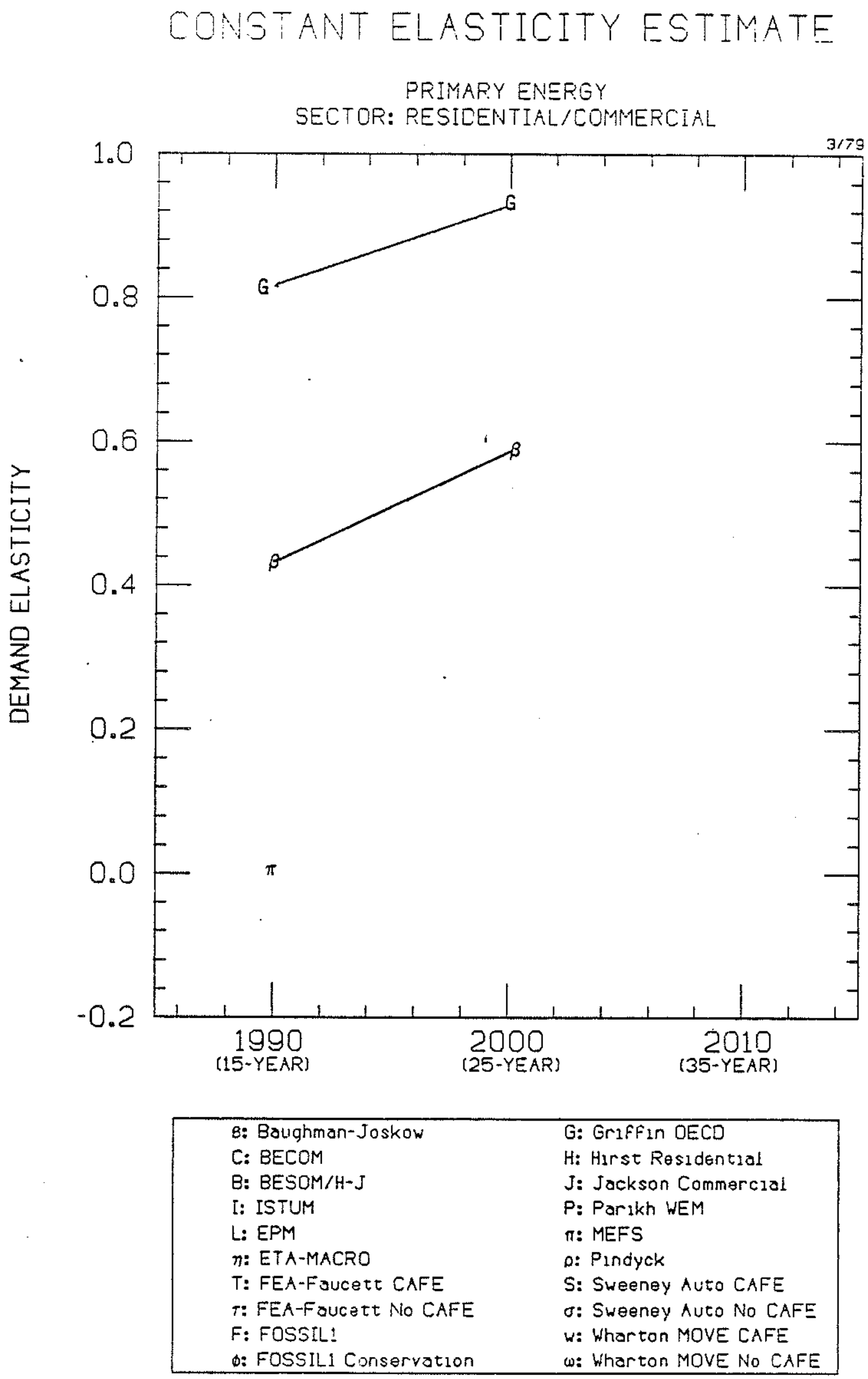

CONSTANT ELASTICITY ESTIMATE
PRIMARY ENERGY
SECTOR: TOTAL DEMAND
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
β: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
φ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto No CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE No CAFE

Figure 7-55



CONSTANT ELASTICITY ESTIMATE
PRIMARY ENERGY
SECTOR: RESIDENTIAL
3/79
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
DEMAND ELASTICITY
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
B: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
φ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto No CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE No CAFE

Figure 7-56



CONSTANT ELASTICITY ESTIMATE
PRIMARY ENERGY
SECTOR: RESIDENTIAL/COMMERCIAL
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
G
β
π
β: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
ϕ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto No CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE No CAFE

Figure 7-57

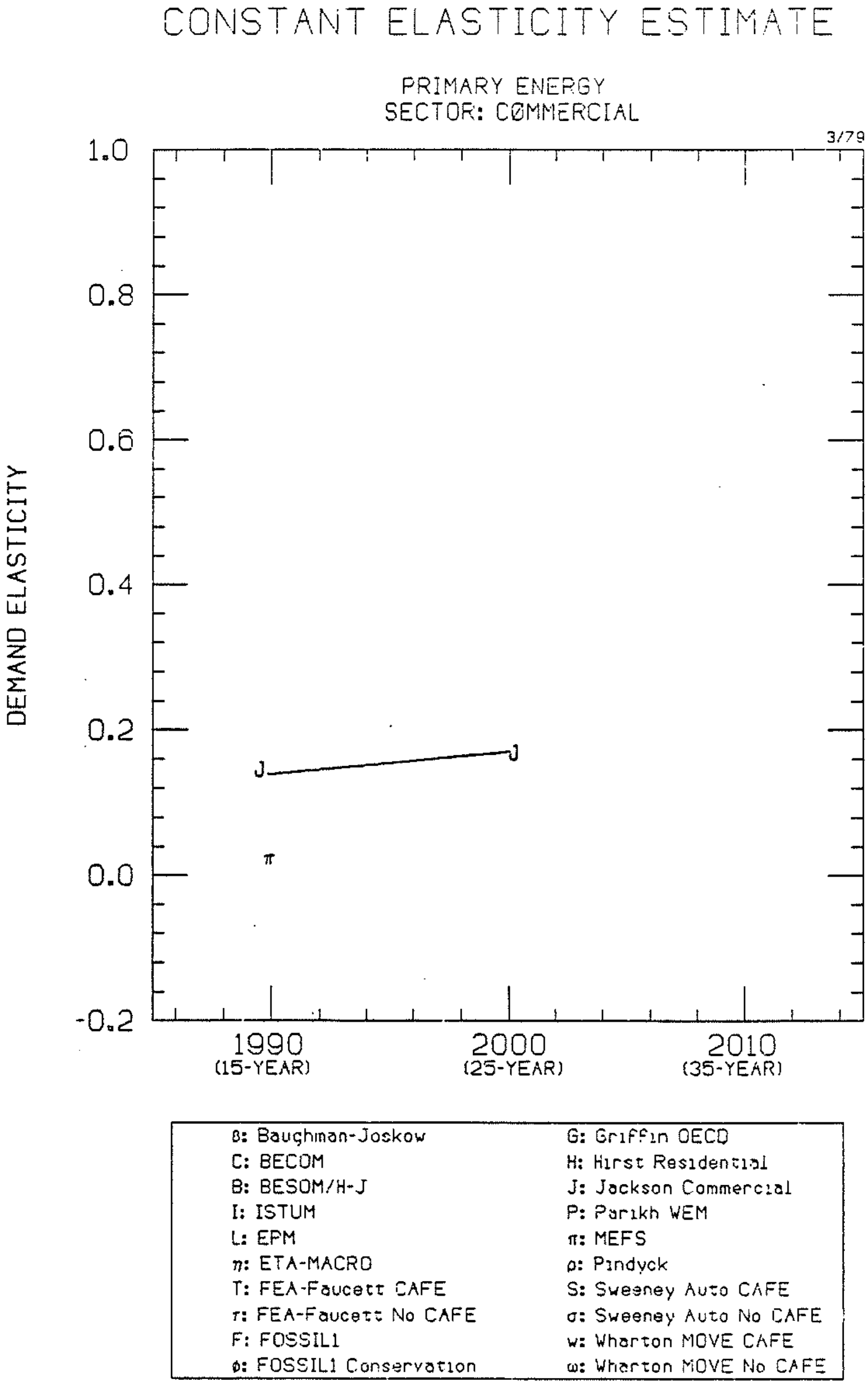
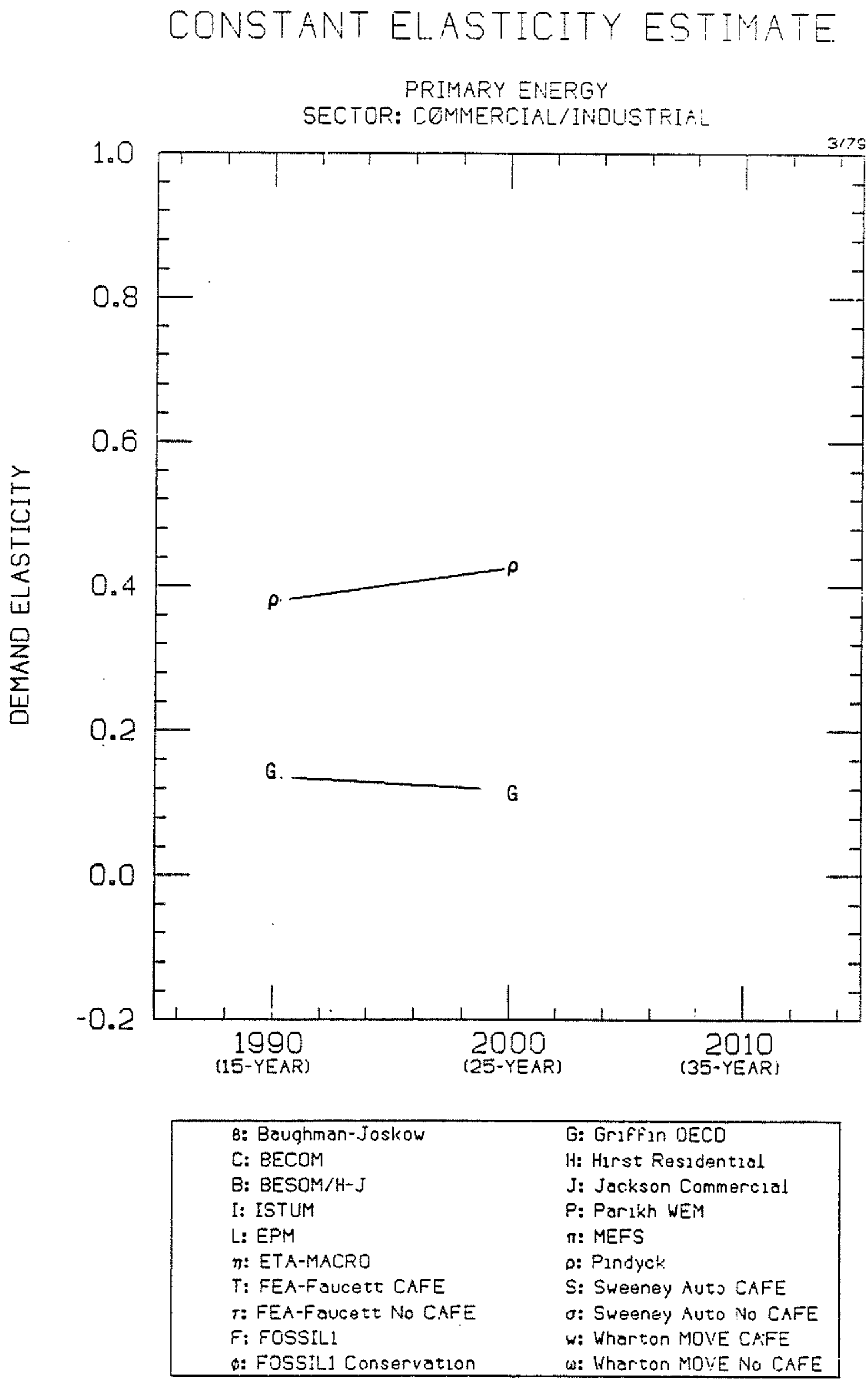

CONSTANT ELASTICITY ESTIMATE
PRIMARY ENERGY
SECTOR: CØMMERCIAL
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
ß: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
φ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto No CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE No CAFE

Figure 7-58



CONSTANT ELASTICITY ESTIMATE
PRIMARY ENERGY
SECTOR: COMMERCIAL/INDUSTRIAL
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
ρ
ρ
G
G
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
8: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
ϕ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto No CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE No CAFE

Figure 7-59

CONSTANT ELASTICITY ESTIMATE

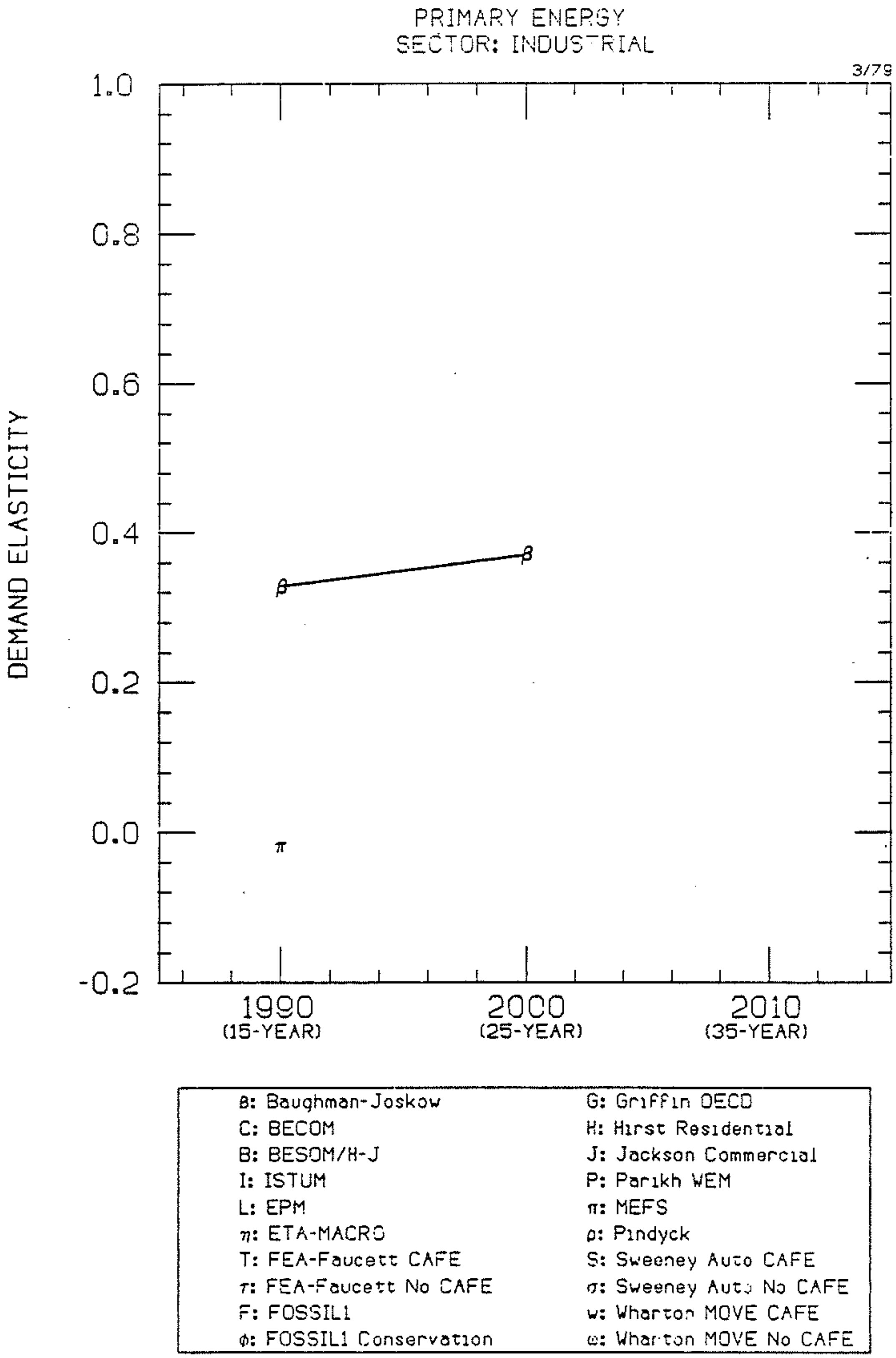


Figure 7-60

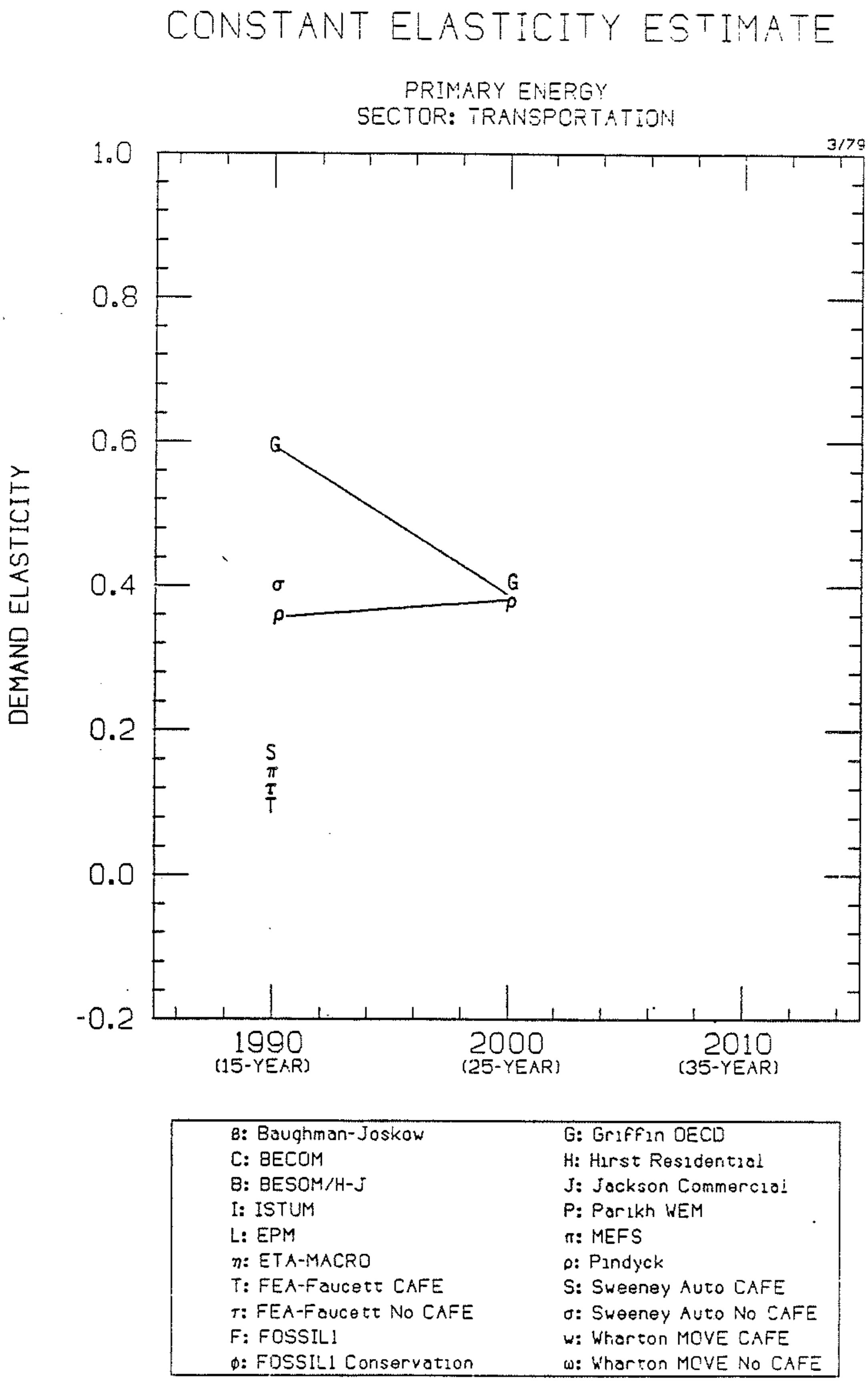
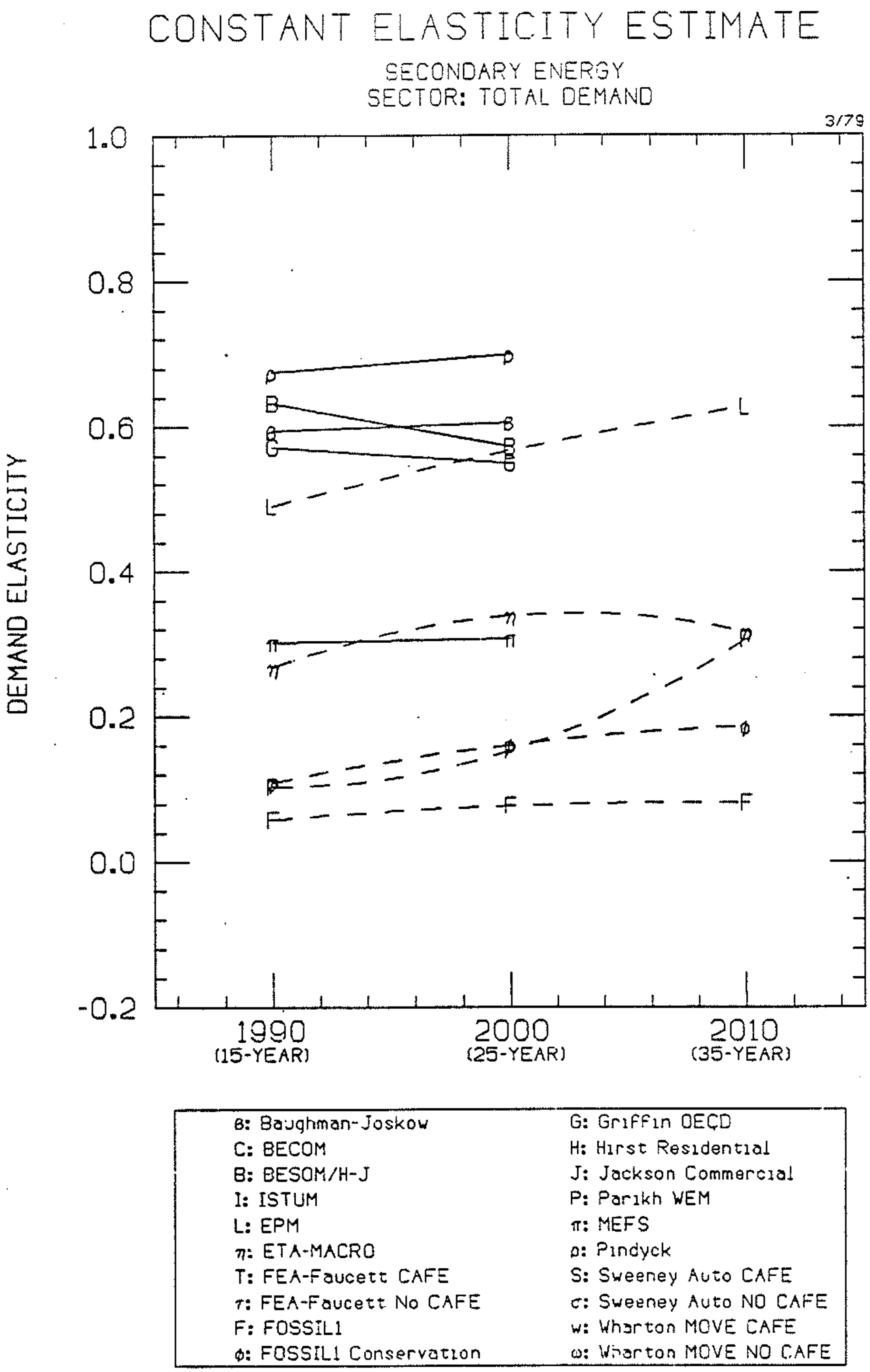

CONSTANT ELASTICITY ESTIMATE
PRIMARY ENERGY
SECTOR: TRANSPORTATION
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
G
G
σ
ρ
ρ
S
π
τ
T
β: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
ϕ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto No CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE No CAFE

Figure 7-61



CONSTANT ELASTICITY ESTIMATE
SECONDARY ENERGY
SECTOR: TOTAL DEMAND
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
ϐ: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
ϕ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
c: Sweeney Auto NO CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE NO CAFE

Figure 7-62

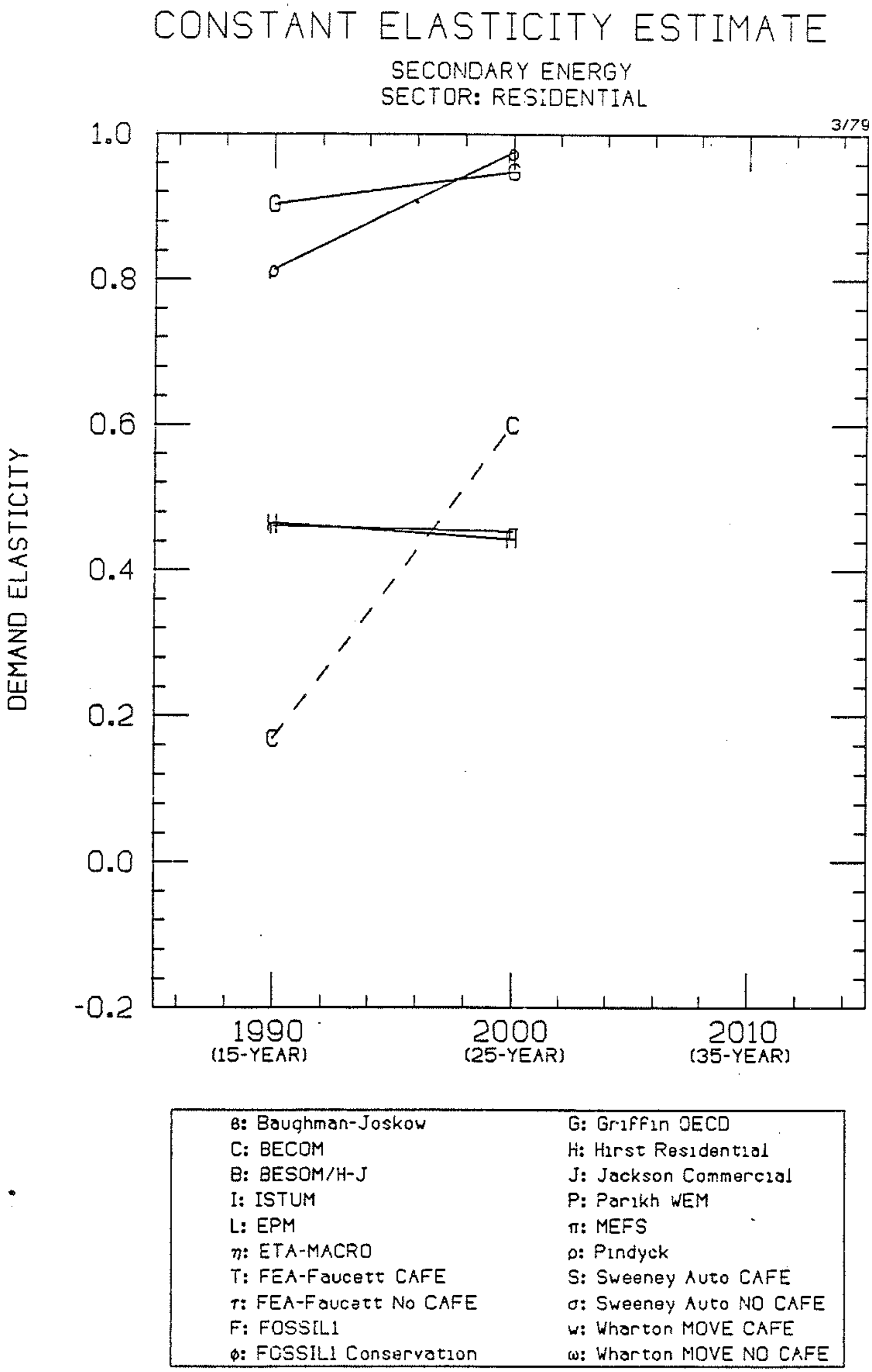

CONSTANT ELASTICITY ESTIMATE
SECONDARY ENERGY
SECTOR: RESIDENTIAL
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
β: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
φ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto NO CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE NO CAFE

Figure 7-63

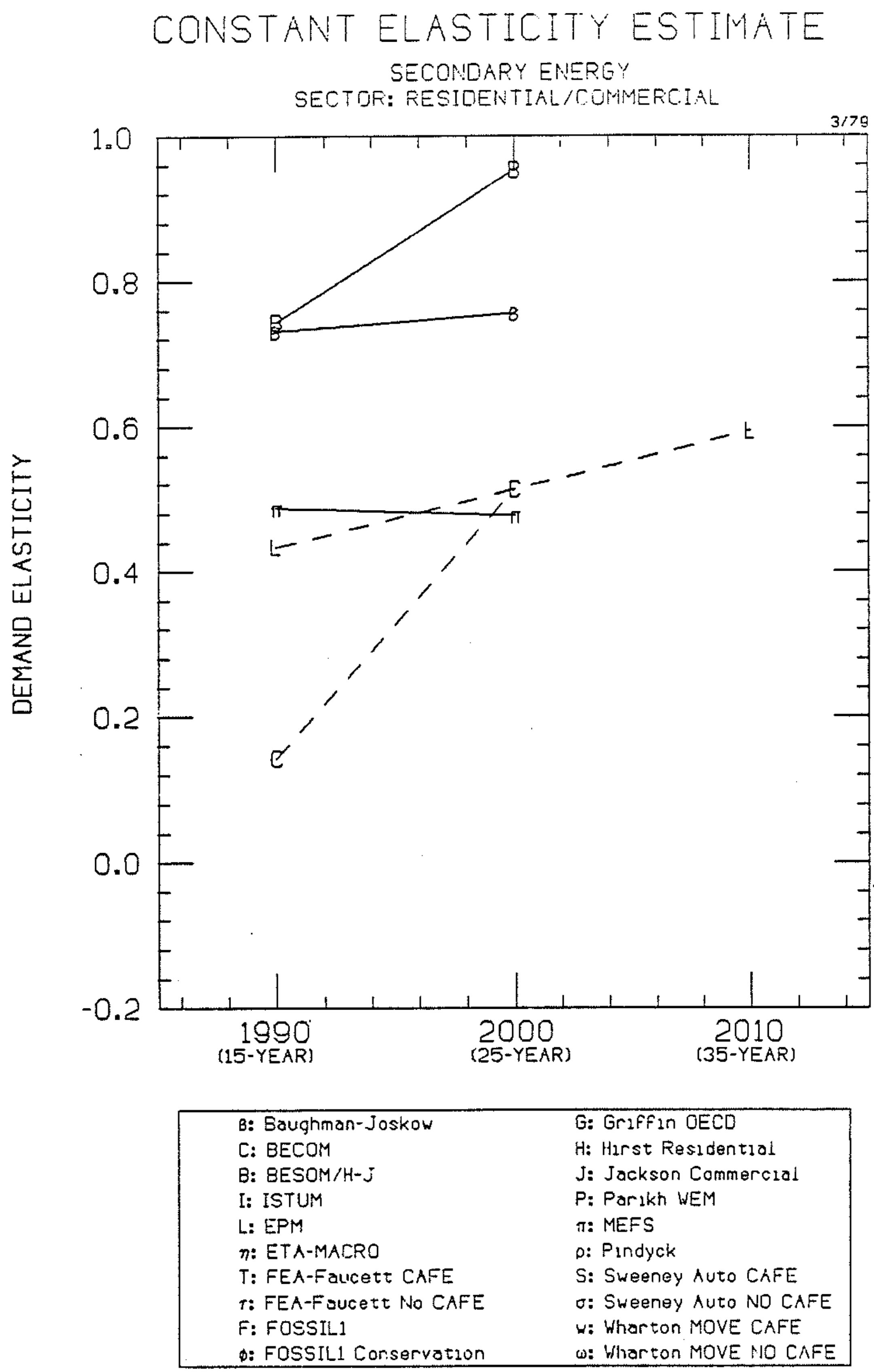

CONSTANT ELASTICITY ESTIMATE
SECONDARY ENERGY
SECTOR: RESIDENTIAL/COMMERCIAL
3/79
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
DEMAND ELASTICITY
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
Β: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
ϕ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto NO CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE NO CAFE

Figure 7-64

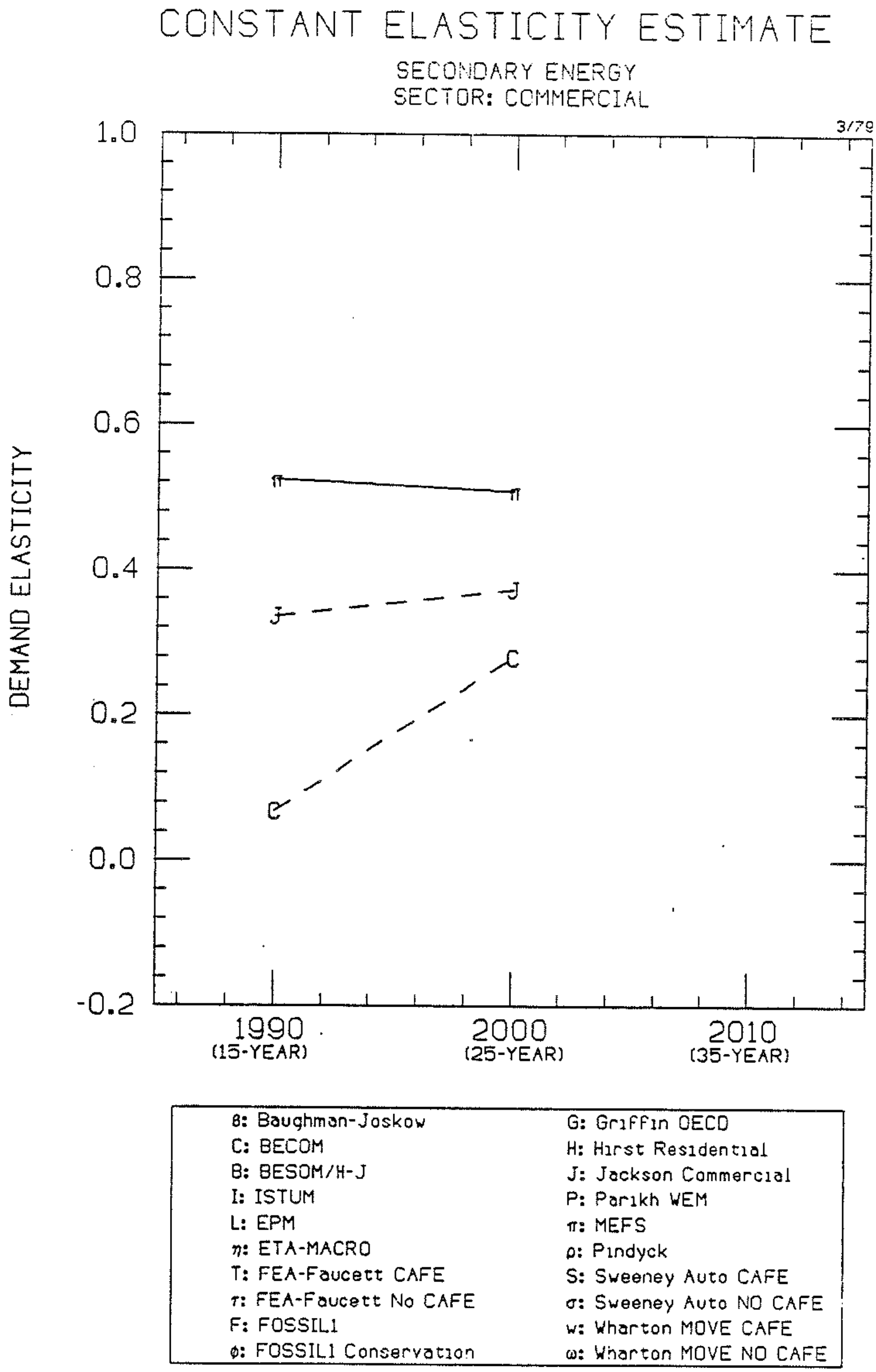
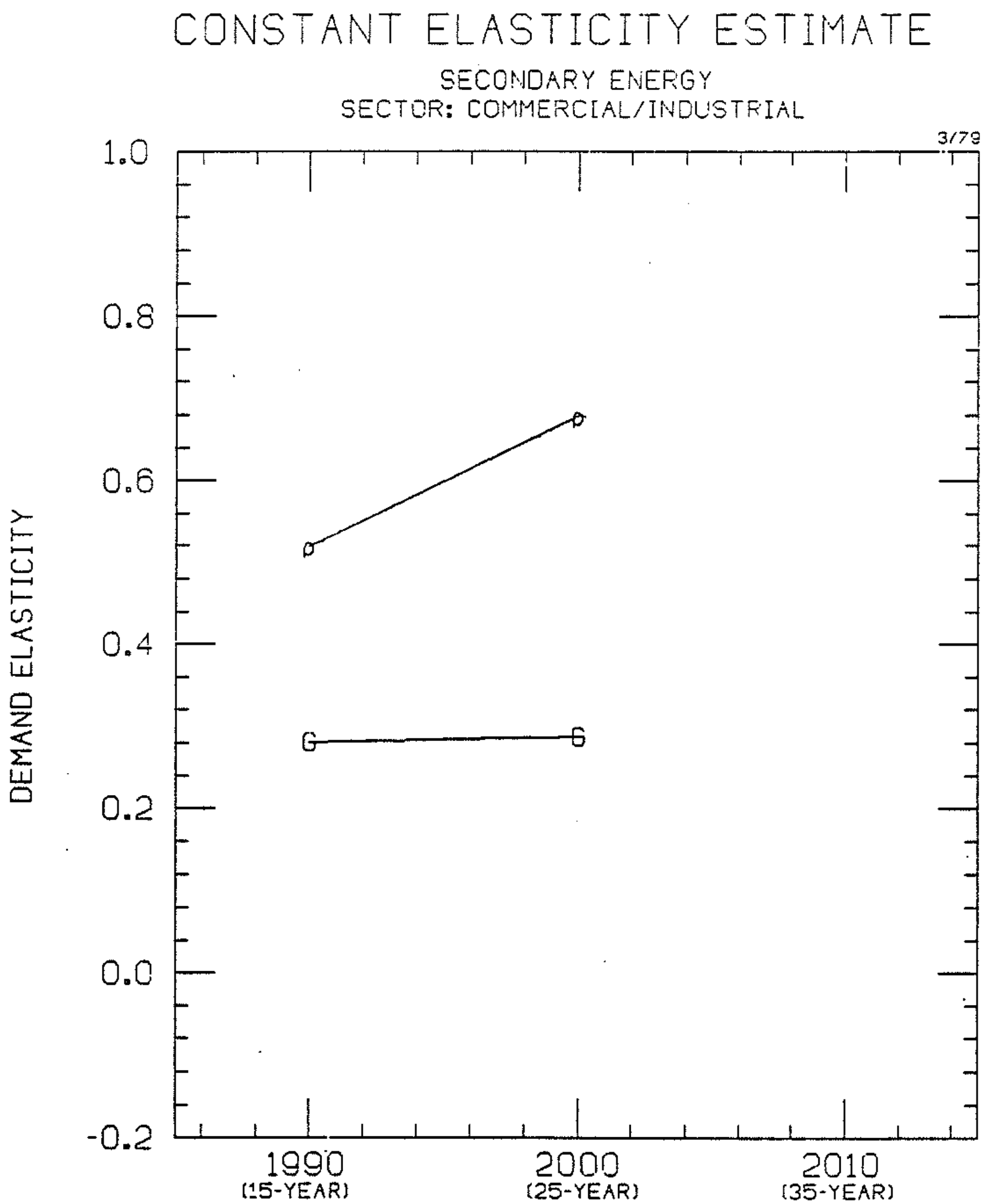
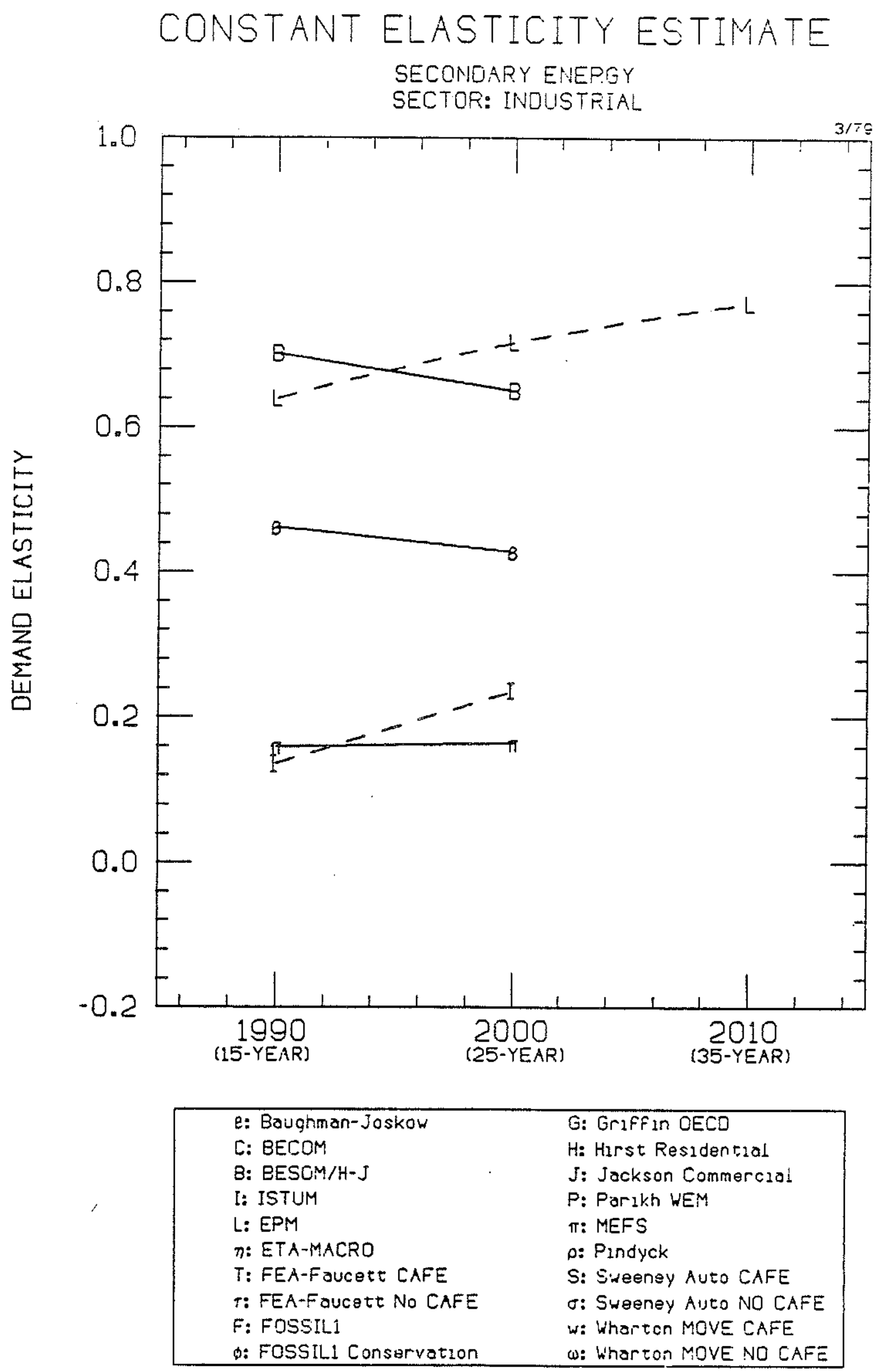

CONSTANT ELASTICITY ESTIMATE
SECONDARY ENERGY
SECTOR: COMMERCIAL
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
θ: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
φ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto NO CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE NO CAFE

Figure 7-65



β: Baughman-Joskow	G: Griffin OECD
C: BECOM	H: Hirst Residential
B: BESOM/H-J	J: Jackson Commercial
I: ISTUM	P: Parikh WEM
L: EPM	π: MEFS
η: ETA-MACRO	ρ: Pindyck
T: FEA-Faucett CAFE	S: Sweeney Auto CAFE
τ: FEA-Faucett No CAFE	σ: Sweeney Auto NO CAFE
F: FOSSIL1	w: Wharton MOVE CAFE
φ: FOSSIL1 Conservation	ω: Wharton MOVE NO CAFE

Figure 7-66



CONSTANT ELASTICITY ESTIMATE
SECONDARY ENERGY
SECTOR: INDUSTRIAL
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
ϐ: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
ϕ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto NO CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE NO CAFE

Figure 7-67

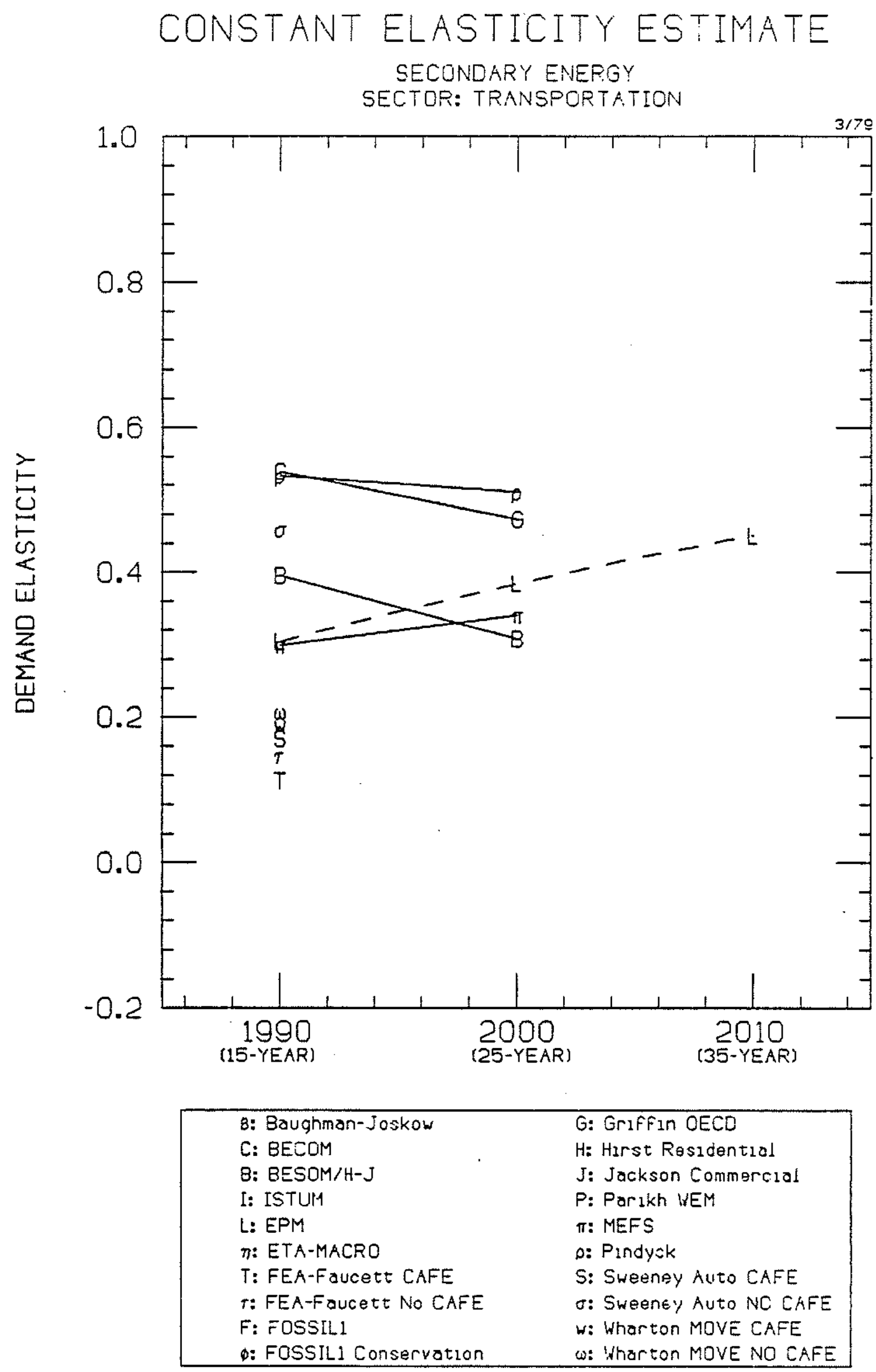
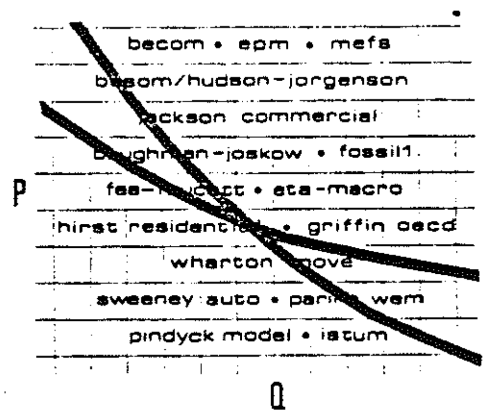

CONSTANT ELASTICITY ESTIMATE
SECONDARY ENERGY
SECTOR: TRANSPORTATION
3/79
DEMAND ELASTICITY
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1990
(15-YEAR)
2000
(25-YEAR)
2010
(35-YEAR)
B: Baughman-Joskow
C: BECOM
B: BESOM/H-J
I: ISTUM
L: EPM
η: ETA-MACRO
T: FEA-Faucett CAFE
τ: FEA-Faucett No CAFE
F: FOSSIL1
φ: FOSSIL1 Conservation
G: Griffin OECD
H: Hirst Residential
J: Jackson Commercial
P: Parikh WEM
π: MEFS
ρ: Pindyck
S: Sweeney Auto CAFE
σ: Sweeney Auto NC CAFE
w: Wharton MOVE CAFE
ω: Wharton MOVE NO CAFE

Figure 7-68

Chapter 8

EXPLAINING INTERMODEL DIFFERENCES*

James M. Griffin†
David O. Wood



*This chapter formerly was Working Paper EMF 4.11.

†The authors would like to thank Adam Borison, John Weyant, and Thomas Wilson for many helpful suggestions.

Chapter 8

EXPLAINING INTERMODEL DIFFERENCES

INTRODUCTION

Chapter 3 compares the structural characteristics and approaches to demand parameter estimation of the 16 models employed in the "Aggregate Elasticity of Energy Demand" study. The present chapter attempts to explain the intermodel differences in empirical aggregate elasticity estimates generated by the study using those a priori model comparisons. Thus, it supports and expands upon the major intermodel comparison conclusions drawn in Chapter 1. First, those characteristics of the models' structure and method of parameter estimation that seemed likely to result in differences in aggregate elasticities are reviewed briefly. The expected differences are then compared with the observed results. Differences in the estimates of the aggregate elasticity for the nine models that include all energy consuming sectors are analyzed first. Then, the sectoral elasticity estimates available from these comprehensive models are contrasted with those from the seven sectoral models.

COMPARISON OF AGGREGATE ELASTICITIES FOR TOTAL ENERGY DEMAND

The elasticity estimates can be interpreted in light of the characteristics of the various models. Some of the most important characteristics are noted in Chapter 3. Two aspects of the models are of particular interest, their structures and approaches to parameter estimation. As far as the structure of the models is concerned, our a priori expectation is that models allowing the greatest degree of flexibility in response to price changes will have the largest elasticities. Such models incorporate energy-economy feedback, explicit interfactor substitution, and dynamic adjustment. At the other extreme are static energy system models with interfactor substitution that is nonexistent or implicit. There is no expectation as far as the method of parameter estimation is concerned.

In assessing the overall price elasticity of energy demand, the confidence one attaches to any estimate should depend on certain structural characteristics of the model. Five structural attributes appear potentially important in this regard. First, due to the inherent technical differences in energy use among sectors, it is desirable to provide a sectoral disaggregation of energy usage in industrial, transportation, residential and commercial, and electricity generation sectors. Failure to provide a sectoral disaggregation can bias the aggregate estimate as sectors experience different growth rates.

A second potentially important structural characteristic is the degree to which the model describes interfuel substitution. Prices for specific energy forms seldom change proportionally implying differing relative fuel prices. In turn, the new relative fuel prices will induce changes in the mix of energy which can affect total energy use as well.

A third structural aspect is the care taken in capturing energy/nonenergy substitution. In a trivial sense, any model which implies a nonzero price elasticity of demand implicitly assumes some input factor is being substituted for energy. Nevertheless, in many instances it is important to identify the precise channels through which this substitution is to occur. For example, in the industrial sector, is energy substituted primarily against labor, materials, or capital? In some instances, it may be justifiable to restrict the range of energy/nonenergy substitution changes if there are scientific reasons to do so. However, when it is not justified either by statistical test or valid a priori reasoning, the resulting energy/nonenergy substitution response can be seriously biased.

A fourth important structural characteristic is the inclusion of dynamics in the model so as to describe the long-run price response over the whole adjustment period. Much of the demand for energy is embodied in the capital stock; each new vintage has technical efficiency characteristics. A model should reflect the gradual adjustment of the economies capital stock to changing prices explicitly or implicitly, yielding dynamic price elasticities.

Fifth, most price elasticity estimates take as given the level of economic activity and the price of other goods. Yet in the case of energy, there is reason to believe there are substantial feedbacks from the macroeconomy to the energy sector. These feedbacks can affect the level of economic activity and most certainly will affect the relative prices of nonenergy factors--both of which will affect the price elasticity calculation.

Given the structure of a model, there remains an important choice as to the method of parameter measurement used to implement the model's underlying structure. The present group of models uses two different methodologies. First, econometric techniques applied to historical time-series or cross-sectional data can be employed to provide statistical estimates of the parameters. The Baughman-Joskow, Griffin OECD, Pindyck, BESOM/H-J, and MEFS models rely on statistically estimated parameters. A second source for parameter values is informed judgment which is used in FOSSIL1, ETA-MACRO, EPM, and Parikh WEM. Estimates based on informed judgment may be conditioned on statistical and engineering parameters used in other studies, but the linkage is informal in most cases.

Table 8-1 categorizes nine of the models by parameter measurement approach and structural characteristic, including for comparison the aggregate energy demand elasticity estimates. The data do not particularly bear out our a priori expectations. First, the parameter estimation method appears to be the dominant factor in explaining model differences. Generally, models that rely on an engineering or judgmental approach to parameter estimation imply elasticities that are significantly lower than those that rely on a statistical approach. Secondly, there appears to be no clear relationship between the degree of flexibility in the model and the reported elasticity. Apparently, the comprehensive models are sufficiently different in other ways to prohibit generalizations such as "structural flexibility" from being very important.

The major distinguishing feature is the influence upon the results of the approach to parameter measurement. The range of estimates from the models that employ statistically estimated parameters is from 0.3 to 0.7, while the range for four of the models employing judgmentally estimated parameters is from 0.1 to 0.2, with the fifth being 0.6. In some cases, an attempt was made to calibrate the judgmental demand parameters to those employed in the statistical models. The results of this study illustrate the imprecision of those ad hoc calibrations. In other cases, the modelers employing judgmental parameter estimates are skeptical of the statistically estimated values because they perceive the data base used to estimate them as not being particularly rich and the level of uncertainty inherent in them correspondingly high.

SECTORAL COMPARISONS

Chapter 3 compares the differences in the structure and approach to parameter measurement employed in the sectoral models. This section investigates the relationship between the structure and approach to parameter measurement employed in the models and the aggregate elasticites generated. The sectoral elasticity estimates from the models that include all consuming sectors are also included in these comparisons.

Table 8-1

MODEL CLASSIFICATION BY PARAMETER MEASUREMENT APPROACH AND STRUCTURAL CHARACTERISTICS

Parameter Estimation Approach	Structural Characteristics					25-Year Aggregate Elasticity Estimate
	Sectoral	Interfuel Substitution	Interfactor Substitution	Dynamics	Energy-Economy Interaction	
Statistical						
Griffin OECD	x	x	x	x		0.58
Pindyck	x	x	x			0.71
Baughman-Joskow	a	x	x	x		0.59
BESOM/H-J	x	x	x		x	0.53
MEFS	x	x		x		0.33[b]
Judgmental						
FOSSIL1		x		x		0.28
FOSSIL1 Conservation		x		x		0.20
ETA-MACRO		x	c	x		0.23
Parikh WEM	x	x	c	x	x	0.13
EPM	x	x	x	x	x	0.58

[a]Excludes transportation.

[b]Estimates for 1990.

[c]Judgmental.

The structures of the sectoral models can be compared on a number of levels. All of the models contain three basic components: (1) a component in which exogenous variables, such as demographic and supply forecasts, are calculated and fed into the rest of the model; (2) a component which computes energy demand from those forecasts and from the characteristics and amount of fuel consuming equipment; and (3) a dynamic component which revises the amount and characteristics of that consuming equipment over time. The second component is represented in two different ways. One approach separately estimates the total stock of consuming units, the market shares of classes of units (those with identical technical characteristics), and the intensity with which each class is used. The second approach assumes cost minimization to estimate demand. Additionally, a number of methods are used to update the composition of consuming units. Some models employ exogenous decay rates, while others endogenously represent the equipment retirement process. Some models also permit increasing fuel prices to induce substitution of capital for energy, and thus permit higher technical efficiencies among newly installed consuming equipment. Finally, one model, ISTUM, attempts to represent the delaying effect technological uncertainty has upon the date at which a new technology will be introduced.

Table 8-2 presents a comparison of 25-year secondary demand elasticities across sectors and parameter estimation approaches. (For a graphic presentation, please refer to Chapter 7, Figures 7-62 through 7-68.)

Several parameter estimation methods are used to determine the sectoral model equations: statistical, statistical/engineering, engineering, and engineering/judgmental.

Statistical methods are used in the Sweeney Auto, FEA-Faucett, and Wharton MOVE models. Each of those representations includes detailed mathematical descriptions of automotive demand for gasoline. Functional forms for the relationships between variables are assumed, and, most importantly, all parameters are estimated econometrically.

The Hirst Residential and Jackson Commercial models are detailed descriptions of the residential and commercial sectors, respectively, but not all parameters are estimated statistically. Instead, engineering studies were used to make estimates of parameters for which statistical information was inadequate (for example, the equations describing the tradeoff between capital and energy in these models were derived from engineering data). For that reason, the parametric estimation process used in the Hirst Residential and Jackson Commercial models can be viewed as an amalgam of statistical and engineering methods.

In the BECOM and ISTUM systems, few statistically estimated parameters are used. There, because of the approach taken by those two models--cost minimization--variables are related on the basis of engineering data. ISTUM, however, uses a combination of engineering data, such as its cost curves, and highly subjective information, such as its estimates of the response of market shares to relative price changes. Accordingly, ISTUM can be viewed as an engineering-judgmental model.

Table 8-2

25-YEAR SECONDARY DEMAND
ELASTICITIES, BY SECTOR
(Paasche Index)

Sector	Statistical		Engineering		Judgmental	
Residential			Hirst Residential[a]	0.4		
	Griffin OECD	0.9	BECOM	0.6		
	MEFS	0.5				
	Pindyck	1.0				
Residential/ Commercial	Baughman-Joskow	0.8	BECOM	0.5	EPM	0.5
	BESOM/H-J	0.7				
	MEFS	0.5				
Commercial	MEFS	0.5	BECOM	0.3		
			Jackson Commercial[a]	0.4		
Commercial/ Industrial	Griffin OECD	0.3				
	Pindyck	0.7				
Industrial	Baughman-Joskow	0.4	ISTUM	0.2	EPM	0.7
	BESOM/H-J	0.5				
	MEFS	0.2				
Transportation[b]	BESOM/H-J	0.2			EPM	0.4
	FEA-Faucett	0.1				
	Griffin OECD	0.5				
	MEFS	0.3				
	Pindyck	0.5				
	Sweeney Auto	0.5				
	Wharton MOVE	0.2				
All Sectors	Baughman-Joskow[c]	0.6			EPM	0.6
					ETA-MACRO	0.2
	BESOM/H-J	0.4			FOSSIL1	0.1
	Griffin OECD	0.5			FOSSIL1[d]	0.2
	MEFS	0.3			Parikh WEM	0.1
	Pindyck	0.7				

[a]Combines both the engineering and statistical approach.

[b]The FEA-Faucett, Sweeney Auto, and Wharton MOVE results are for automobile gasoline only. These are 15-year elasticities. All runs exclude the new car fuel efficiency standards.

[c]Excludes the transportation sector.

[d]FOSSIL1 Conservation.

Much of the difference in elasticity estimates shown in Table 8-2 can be attributed to the fact that the models consider different sectors of the economy. However, as can be seen from intrasector comparisons, the structural characteristics of the models also strongly affect their outputs. It appears that the most important of these structural differences are: (1) whether the possibility of fuel efficiency changes are considered, (2) whether price-induced changes in the rate of technological development are considered, (3) whether the model allows for changes in the demand for energy services, (4) differences in the preprocessing of GNP growth rates into growth rates in other demographic variables, and (5) the manner in which equipment retirements are considered.

Of the five structural factors, the first two constitute important mechanisms of factor substitution and may be the most important. The Sweeney Auto, Hirst Residential, and Jackson Commercial models all attempt to represent the tradeoff between capital and energy and to show the effects that fuel price variations have on interfactor substitution. Price increases can lead to substitutions using existing technology or stimulate efficiency, thereby enhancing technological progress. In all three models, consumers are shown to switch to more fuel-efficient units as the prices of fuels increase. However, in the FEA-Faucett and Wharton MOVE models, the technical characteristics of consuming units are assumed to be fixed and such substitutions are precluded. Models which represent such tradeoffs will generate higher elasticity estimates than those which do not. These five models generally have similar structures (BECOM and ISTUM are cost-minimization models and explicitly represent more fuel-efficient energy utilizing technologies). However, the three which permit interfactor substitution (Hirst Residential, Jackson Commercial, and Sweeney Auto) have obtained much higher estimates than Wharton MOVE and FEA-Faucett. Accordingly, it might be deduced that allowing interfactor substitution is the crucial factor which differentiates between the models.

Price increases can lead to factor substitutions via existing technologies, but they can also stimulate energy efficiency-enhancing technological progress. The differential effects of higher prices on the rate of technology development is considered only in the Sweeney Auto model.

Stronger statements can be made about models which use exogenously determined energy services consumption forecasts. Both ISTUM and BECOM fall into that category and, because they use such forecasts, their elasticity estimates can be expected to be lower than the actual elasticity values (assuming that all other attributes of the model would tend to produce accurate elasticity figures).

This effect can be seen from the workings of ISTUM. There, the total demand for "useful energy" (the amount of energy that is demanded after conversion losses are subtacted) within each subsector of industry is determined exogenously, while the market share for each different technology which is capable of meeting that demand is explicitly modeled. Thus, the consumption of useful energy was made independent of fuel price while conversion losses, and thus a portion of energy demand, were allowed to respond to fuel price variations. This failure to completely relate demand to fuel prices may explain, at least in part, ISTUM's relatively low elasticity estimate (0.2 as compared to a midrange value of about 0.4 for all industrial sector models).

Differences in the values that are assigned to demographic variables may have also affected model results. The design of the EMF experiment included GNP growth-rate assumptions. Demographic variables, such as population and income, are often calculated from the GNP projections using preprocessing routines and then fed into the rest of the system. In at least some of the models, the system equations are formulated in such a way that relatively small differences in demographic variable values could produce large differences in elasticity estimates. It should be emphasized that such effects are far from certain at this point. For example, in the FEA-Faucett model, vehicle miles, a key variable in the determination of gasoline demand, are estimated on a per household basis, where the total number of households is determined exogenously. Moreover, it appears possible that, in the long-run, small changes in estimated household growth could produce significant variations in estimates of vehicle miles traveled.

The effects of differences in representations of equipment retirement is also important. Most of the models reviewed here use decay functions that are price independent, but FEA-Faucett and Wharton MOVE relate fuel prices to scrappage and usage rates of consuming units (e.g., automobiles). Thus, through their decay functions, these two models account more fully for fuel price variations than other models. However, it is not known whether this inclusion of price-related decay functions will tend to increase or decrease elasticity estimates.

CONCLUSIONS

Table 8-3 summarizes the elasticity comparisons produced by the study along two key dimensions: the sectoral coverage of the model and its approach to parameter estimation. The parameter estimation approach is the most important determinant of

Table 8-3

SECTORAL MODEL CLASSIFICATION BY PARAMETER MEASUREMENT APPROACH AND STRUCTURAL CHARACTERISTICS

SECTOR: MODEL	Parameter Measurement[a]	Interfuel Substitution[b]	Explicit Interfactor Substitution[b]	Dynamics[b]	Energy-Economy Feedback[b]	25-year Aggregate Elasticity Estimate
Residential						
Griffin OECD	S	x	x	x	o	0.9
Hirst Residential	S/E	x	x	x	o	0.4
MEFS	S	x	o	x	o	0.5
Pindyck	S	x	x	o	o	1.0
Residential/Commercial						
Baughman-Joskow	S	x	o	x	o	0.8
BESOM/H-J	S	x	x	o	x	0.8
BECOM	E					
Commercial						
Jackson Commercial	S/E	x	x	x	o	0.4
MEFS	S	x	o	x	o	0.5
Commercial/Industrial						
Griffin OECD	S	x	x	x	o	0.3
Pindyck	S	x	x	o	o	0.7
Parikh WEM	J	x	x	x	x	0.1
Industrial						
Baughman-Joskow	S	x	o	x	o	0.4
ISTUM	E/J	x	x	x	o	0.2
BESOM/H-J	S	x	x	o	x	0.5
MEFS	S	x	o	x	o	0.6
Transportation						
FEA-Faucett	S	o	x	x	o	0.1
Griffin OECD	S	o	x	x	o	0.5
BESOM/H-J	S	o	x	o	x	0.2
MEFS	S	o	x	x	o	0.3
Sweeney Auto	S	o	x	x	o	0.5
Pindyck	S	x	x	o	o	0.5

[a] S = Statistical, E = engineering, J = judgmental.

[b] x = yes, o = no.

the aggregate elasticity of those models which estimate total energy demand. The models that include all energy consuming sectors and employ statistically estimated parameters yield considerably higher elasticity estimates than those from the models with judgmentally estimated parameters.

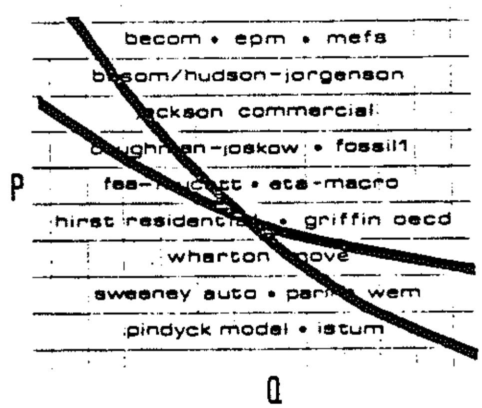
The comparison of sectoral elasticity estimates is more complex. The sectoral models employ both engineering and statistical approaches to demand parameter estimation. But, more importantly, in their representations of energy demand, many of them include restrictions on the ability of consumers to respond to higher energy prices. In ISTUM and BECOM, end-use service demands (e.g., industrial process heat, residential space heat) are not responsive to price changes. This omits consideration of important mechanisms through which reductions in energy demand can be accomplished. For example, industrial firms can switch to production processes requiring less process heat or to the production of less energy intensive products and building owners can turn down their thermostats or install microprocessors for temperature control. Consequently, it is not surprising that the sectoral elasticity estimates from BECOM and ISTUM are lower than the other residential/commercial and industrial estimates, respectively. Additionally, the FEA-Faucett and Wharton MOVE automobile models do not include the possibility of fuel efficiency improvements within automobile size classes. This may lead them to underestimate the elasticity of automobile gasoline demand. The comparison with the results from the other models tends to support this conclusion.

Of the sectoral models included in the study, only the Hirst Residential, Jackson Commercial, and Sweeney Auto models do not place a priori restrictions on the response of sectoral demand to higher prices. The Hirst Residential and Jackson Commercial models combine statistical and engineering approaches to parameter estimations and it is noted that these models produce somewhat lower elasticity estimates than those for the corresponding sectors of the statistically estimated models that consider all consuming sectors.

Chapter 9

PRICE AND QUANTITY CHANGE DECOMPOSITION FOR AGGREGATED COMMODITIES*

James L. Sweeney†



*This chapter formerly was Working Paper EMF 4.7.

†The author would like to thank, without implicating, Thomas Wilson and Steven Duvall for their extensive work, including conducting all the estimations herein, and Adam Borison, William Hogan, James Griffin, Lawrence Lau, Lester Taylor, John Weyant, and David Wood for their ideas, comments, and criticisms.

ABSTRACT

For many analytical purposes, it is often desirable to aggregate a set of commodities into a single composite commodity or to define an aggregate quantity index, along with a price index. However, a demand function for this aggregate does not necessarily exist, even if the demand mapping for the individual commodities is single-valued and smooth. Then the notion of a demand correspondence or region may replace the notion of a demand function. If changes in the price vector map linearly into changes in the commodity vector, the demand region mapped out by all possible unit magnitude price changes will be an ellipse characterized by two fundamental parameters: the average slope and the width. The width is zero if the demand mapping is generated by a cost-minimizing, separable, homothetic production function. Thus, the demand region width provides a measure of the degree to which these assumptions may be violated. The width also provides a measure of how dependable a single average aggregate price elasticity or demand derivative can be used to predict how the actual aggregate will vary for alternative directions of price changes.

Chapter 9

PRICE AND QUANTITY CHANGE DECOMPOSITION FOR AGGREGATED COMMODITIES

INTRODUCTION

For a single commodity, a given price generally will be mapped into a unique quantity demanded. As prices increase, the quantity demanded decreases, thus tracing out a demand function which is single-valued throughout its entire range. Similarly, for a class of commodities each having individual prices, as those prices change the quantities demanded change. The demand function for a class of n commodities is a single-valued mapping from the n-dimensional space of prices to the n-dimensional space of quantities.

For some purposes, it is useful to aggregate the n commodities into a single composite commodity or quantity index. In the fourth Energy Modeling Forum study, "Aggregate Elasticity of Energy Demand," the goal was to estimate the aggregate elasticity of energy demand implicit in the 16 models examined. This required aggregating the demands for oil, gas, coal, and other energy commodities into some index of energy consumed. One such index commonly used in energy studies is the Btu-weighted. Others include the Paasche, Tornquist, or Laspeyres indexes. For statistical or econometric purposes, other commodities also are often aggregated. For example, the demand for housing is often aggregated by number of units, total value, or some measure of aggregate housing "services." Similar aggregates include autos, food, clothing, or educational services. In fact, almost all data used for economic studies consist of quantity or price aggregations of commodities that are to some extent heterogeneous.

If an aggregated class of commodities is being considered, the demand function for this aggregate no longer necessarily is single-valued, even if the demand functions for each of the n commodities of the aggregated class is single-valued. For example, it can occur that one price increases while another decreases, so as to keep the aggregate price constant. These price changes will alter the individual demand quantities and may change the aggregate quantity. If the aggregate quantity is changed while the aggregate price is not, the mapping from aggregate price to aggregate quantity is not single-valued. The notion of an aggregate single-valued demand function must be replaced with the notion of a demand correspondence or region.

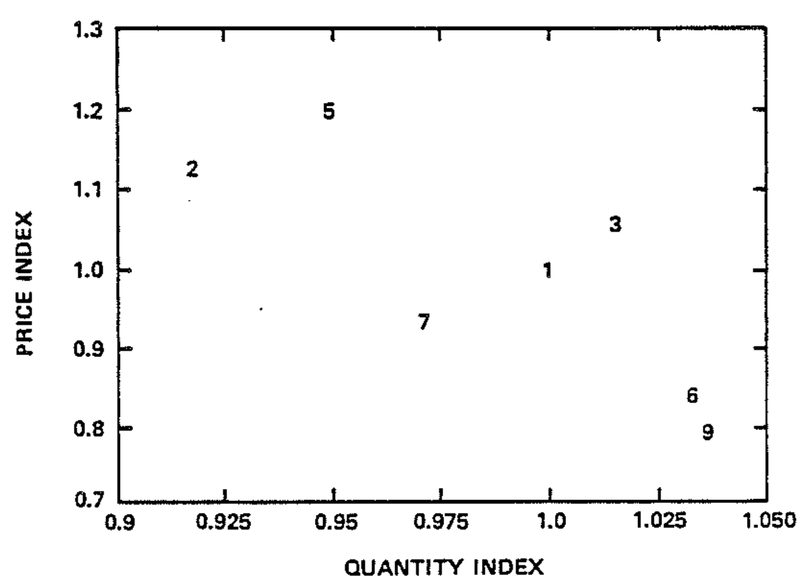
As an example, Figure 9-1 shows changes in aggregate quantity and price indexes (Paasche) resulting from the experiments with the ISTUM model, experiments in which each of three primary energy prices were independently altered. Note that it is impossible to construct a single-valued monotonic demand function through these points.

The remainder of this chapter develops a decomposition approach in order to describe the shape of a demand region and to provide a method of fitting such a region to pseudo-data generated by a model. Particular attention is paid to the two-commodity case before the approach is generalized to a general n-dimensional case.

It will be shown under linearity assumptions that the region mapped out by all possible unit magnitude price changes has a particular form. The frontier of this region (the entire region for the two-dimensional case) has an elliptical form:

$$\Delta\pi = D \cos \phi \qquad (9\text{-}1)$$

$$\Delta\theta = B \cos \phi + A \sin \phi \quad , \qquad (9\text{-}2)$$



Note: The numbers refer to price cases. The Paasche index was used to calculate these estimates.

Figure 9-1 25-Year Industrial Secondary Energy Index Values for the ISTUM Model where $\Delta\pi$ and $\Delta\theta$ are changes in the price and quantity indexes respectively, ϕ is an angle of price change varying between 0° and 360°, and A, B, and D are constants. This is diagrammed in Figure 9-2.

It will be further shown that in some cases this region is reduced to a single-valued demand function, as would occur when $A = 0$. This will be the case when the demand function is generated by a firm, or group of firms, minimizing cost to produce a given output, and having a homothetic production function separable into an energy subaggregator and other inputs. This result will provide a test of how well these assumptions are implicitly satisfied by a process which has generated a set of price and quantity vectors. Finally, the procedure will be applied to data generated by the EMF "Aggregate Elasticity of Energy Demand" experiment.

THE DECOMPOSITION APPROACH

Each price change can be decomposed into components: one parallel to $\nabla\pi$, the gradient of the price index, and the others perpendicular to $\nabla\pi$. The latter set of components will each lead to zero changes in the price index. If the model and the price and quantity indexes are linear, analysis of the quantity index changes associated with each possible component of a price change will allow analysis of all possible price changes.

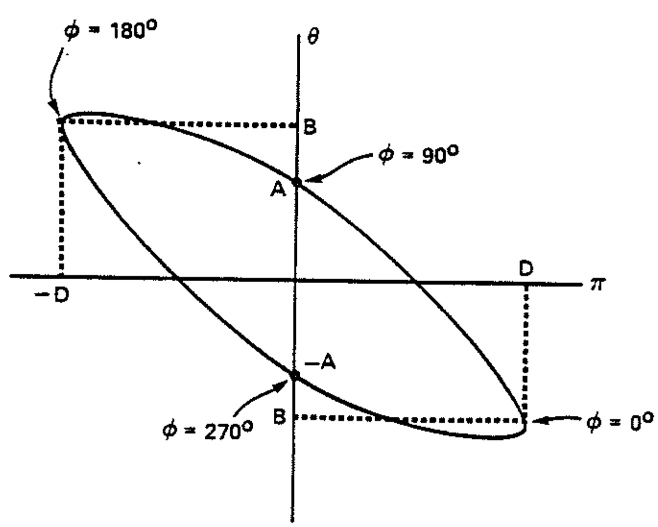


Figure 9-2 Aggregate Price and Quantity Changes from a Unit Magnitude Price Change Vector as ϕ Varies Over its Range

Define a set of n unit price change vectors,* each having unit magnitude: $P_0,\ldots,P_{n-1}$. Each unit price change vector will be orthogonal to each other. The first, P_0, will be parallel to $\nabla\pi$, the latter n-1 vectors will be orthogonal to $\nabla\pi$. These vectors can always be defined. Then any price change can be decomposed into components, each some multiple of one of the unit price change vectors:

$$\Delta P = \beta P_0 + \sum_{i=1}^{n-1} \alpha_i P_i \quad , \tag{9-3}$$

where

$$\beta = \Delta P \cdot P_0 \quad , \qquad \text{and} \tag{9-4}$$

$$\alpha_i = \Delta P \cdot P_i \quad . \tag{9-5}$$

These various vectors are illustrated in Figure 9-3 for the two-commodity case.

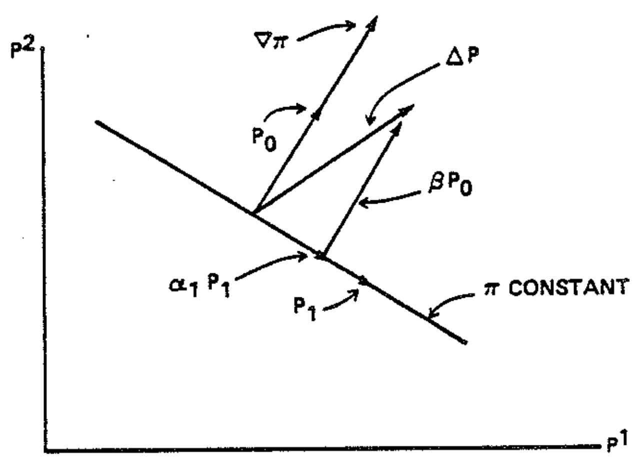


Figure 9-3 Decomposition of Price Change

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*Note: P_i is a vector, not the component of a vector.

The linearity assumption can be formalized.*

- Linearity Assumption: The model exhibits a linear relationship between price changes and quantity changes. The price index is linear in price; the quantity index is linear in quantities.

By the linearity assumption, changes in quantity vector ΔQ can be related to changes in the price vector ΔP by a matrix M of demand derivatives:

$$\Delta Q = M \cdot \Delta P \quad . \tag{9-6}$$

The matrix M may not be observable or identifiable from available data. By linearity, changes in the quantity index change can be determined:

$$\Delta \theta = \nabla \theta \cdot M \cdot \Delta P \quad . \tag{9-7}$$

Equations 9-3 and 9-7 can be combined to give:

$$\Delta \theta = \beta[\nabla \theta \cdot M \cdot P_0] + \sum_{i=1}^{n-1} \alpha_i [\nabla \theta \cdot M \cdot P_i] \quad . \tag{9-8}$$

This equation is fundamental to the decomposition approach because each expression in brackets, although appearing complicated, is simply a scalar. Therefore Eq. 9-3 can be written

$$\Delta \theta = \beta B + \sum_{i=1}^{n-1} \alpha_i A_i \quad , \tag{9-9}$$

where B and the A_i's are defined by the corresponding terms between the brackets in Eq. 9-8.

*While the linearity assumption will be necessary for the mathematical development, this assumption is less restrictive than it appears. First, in practice, it is required that changes be small enough for approximate linearity. Second, by appropriate interpretation, other smooth relationships are admissible. For example, a log-linear relationship between prices and quantities can be assumed. In this case, each P_i or Q_i will be interpreted as the logarithm of a price or a quantity. θ and π will be logarithms of the quantity and price index, and so on. This generalization will remain applicable except in the limiting case (explained in the section entitled "A Limiting Case"), where each symbol will always refer to the untransformed variables.

The corresponding price index change can be calculated easily using Eq. 9-3, since $\nabla\pi \cdot P_i = 0$, for $i \neq 0$:

$$\Delta\pi = \nabla\pi \cdot \Delta P = \beta[\nabla\pi \cdot P_0] .$$

Since the term in brackets is a constant, which can be denoted as D, this equation becomes:

$$\Delta\pi = \beta D . \tag{9-10}$$

In order to estimate values of the A_i's, B, and D, Eqs. 9-9 and 9-10 can be estimated directly from data on price and quantity changes. The variables α_i and β can be calculated directly using Eqs. 9-4 and 9-5.

Two-Commodity Case

The properties of Eq. 9-9 can be explored more fully by first restricting attention to the two-commodity case. Then the general n-commodity case can be considered.

In the case of two commodities, Eq. 9-9 reduces to

$$\Delta\theta = \beta B + \alpha A , \tag{9-11}$$

where the subscripts have been dropped from α and A. The first term on the right-hand side is associated with the component of ΔP that leads to changes in the price index, the second term is associated with that component that does not.

Consider now a price change of unit magnitude and angle ϕ from $\nabla\pi$; see Figure 9-4. Then by Eqs. 9-4 and 9-5, $\beta = \cos\phi$ and $\alpha = \sin\phi$. Therefore, Eqs. 9-1 and 9-11 become

$$\Delta\theta = B \cos\phi + A \sin\phi , \tag{9-12}$$

and

$$\Delta\pi = D \cos\phi . \tag{9-13}$$

These expressions are identical to Eqs. 9-1 and 9-2. These equations define, in parametric form, an elliptical region of price and quantity index changes resulting from a unit magnitude price change. This is the region illustrated in Figure 9-2.

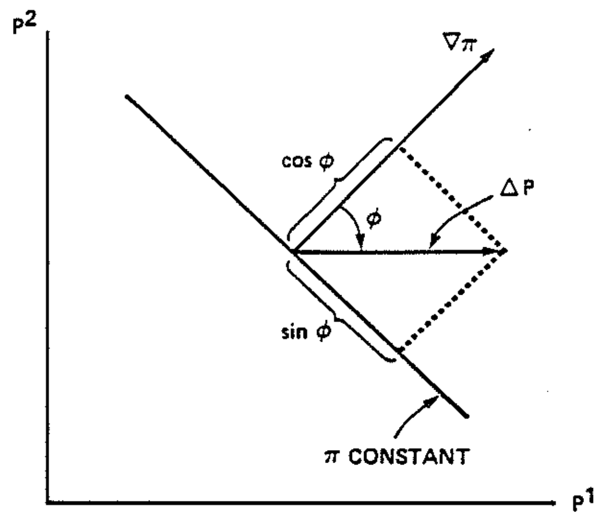


Figure 9-4 Unit Magnitude Price Change in the Two-Commodity Case

The derivative of the quantity index with respect to the price index can be calculated as

$$\frac{\Delta\theta}{\Delta\pi} = \sigma + \tau \tan\phi \quad , \tag{9-14}$$

where $\sigma = B/D$ and $\tau = A/D$.

Equation 9-14 indicates that the demand derivative can be decomposed into two terms. The first term is a constant and independent of the direction of the price change vector. The second is directly proportional to the ratio of the perpendicular price change magnitude to the parallel price change magnitude. Therefore, this term is dependent upon the direction of the price change. The first component will be referred to as the "price level" demand derivative while the second will be referred to as the "relative price" demand derivative.

Equation 9-14 provides an analytical answer to the basic question: Can one define, for a given model, an aggregate demand elasticity applicable to a wide range of price changes? By Eq. 9-14, if $A = 0$, the answer is yes. For any small price change, the aggregate demand derivative is independent of the direction of price change. Conversely, if $A \neq 0$, the price elasticity can vary with the direction of price change: as various directions of price change are examined, $\tan\phi$ can vary between $-\infty$ and $+\infty$. Thus, the aggregate demand derivative and the aggregate demand elasticity can vary between $-\infty$ and $+\infty$, depending upon the direction of price changes considered.

The parameter σ can be defined as the average aggregate demand derivative when all possible directions of change are considered. Consider a price change of unit magnitude but of angle ϕ varying with equal frequency between 0° and 360°. The average of $\Delta\theta/\Delta\pi$ is the integral of the right-hand side of Eq. 9-14 as ϕ varies. Since $\tan \phi = -\tan(-\phi)$, the second term integrates to zero, leaving only the first term. Thus the average aggregate demand is equal to σ, the price level demand derivative.

It should be noted that σ will not be the average of the derivatives obtained by an experiment which asymetrically selects price directions, unless $A = 0$. σ will be the average of the price derivatives derived from an experiment if and only if the term $\tan \phi$ averages to zero over all variations chosen.

In addition, it should be noted that σ will not be the derivative obtained in response to a proportional increase of all prices, unless $\nabla\pi$ is proportional to P, since only in the latter case does $\tan \phi$ equal zero. For commonly used price indexes (e.g., Paasche, Laspeyres, etc.), $\nabla\pi$ is proportional to Q. Thus σ will be the derivative obtained in response to a proportional increase in all prices if and only if the vector of quantities is proportional to the price vector or if $A = 0$.

Although the demand derivative can vary with the direction of price change, for small values of τ/σ most possible price changes will lead to derivatives approximately equal to σ. The smaller the ratio τ/σ (or equivalently, A/B), the wider the range of price change directions in which the demand derivative is closely approximated by σ.

Figure 9-4 could also illustrate, for a given value of τ/σ, the values of ϕ for which the demand derivative is within a given percentage of σ. The areas of close approximation symmetrically surround the price index gradient and its negative, since $\tan(-\phi) = -\tan \phi$. The magnitude of this region of close approximation can be calculated. Assume that a fractional error of E is acceptable. Then Eq. 9-14 allows a calculation of the ranges of $\tan \phi$ in which the demand elasticity is within a fraction of $\pm E$ of σ. This range is defined by

$$|\tan \phi| \leq E/|A/B| = E/|\tau/\sigma| \quad .$$

Table 9-1 presents values of ϕ at which the above inequality becomes an equality. These are then maximum values for which the demand derivative falls within a range of $\sigma(1 - E)$ and $\sigma(1 + E)$. Table 9-1 also presents the fraction of the direc-

Table 9-1

MAXIMUM VALUES OF ϕ AND PERCENT OF DIRECTIONS FOR WHICH DEMAND DERIVATIVE REMAINS WITHIN ERROR[a] OF $\pm E$: TWO-COMMODITY CASE

τ/σ or A/B	E							
	0.10		0.20		0.5		1.0	
	ϕ	%	ϕ	%	ϕ	%	ϕ	%
0.01	±84°	94	±87°	97	±89°	98.7	±89°	99.4
0.05	±63°	70	±76°	84	±84°	94	±87°	97
0.2	±27°	30	±45°	50	±68°	76	±79°	87
0.5	±11°	13	±22°	24	±45°	50	±63°	70
1.0	± 6°	6	±11°	13	±27°	30	±45°	50

[a]The demand derivative remains within an error of $\pm E$ for all values of ϕ within the limits indicated plus all values obtained by adding 180° to the limits. Thus for $E = 0.10$ and $\tau/\sigma = 0.01$, the demand derivative remains between $\sigma(1-E)$ and $\sigma(1+E)$ if $-84° < \phi < 84°$ or if $96° < \phi < 264°$.

tions of price change for which the demand derivative error is within a fraction $\pm E$ of σ. For example, for τ/σ equal to 0.05, and an allowable 20% error, the maximum value of ϕ is ±76°. Eighty-four percent of the price directions lead to aggregate demand derivatives which fall within ±20% of the price level demand derivative.

Assume now, as an example, that the value of the price level demand derivative is known, that one would like to estimate aggregate demand change for a number of different price change directions, and that a 20% error is acceptable. To calculate aggregate demand change, the price level demand derivative is applied to the aggregate price change. Then if τ/σ were as small as 0.01, 97% of the price directions would give actual aggregate changes within ±20% of the estimated value. Three percent of the directions--those most nearly perpendicular to the gradient of the price index--would lead to more than 20% error. However, if τ/σ equaled 0.5, only 24% of the directions would give estimates with no more than 20% error while 76% would give a greater error.

The error fraction of 1 has a particular meaning. If the error fraction is smaller than 1, the sign of the demand derivative is always the same as the estimated sign. However, for error fractions greater than 1, the sign will be in error for one half of the directions whose error fractions exceed unity. Thus, if $\tau/\sigma = 0.5$, for 15%

of the directions the estimated sign of the demand derivative will be the opposite from the actual sign, while if $\tau/\sigma = 1$, the sign will be in error for 25% of the directions.

The n-Commodity Case

For the n-commodity case (with n greater than 2), the basic principles for decomposition are similar to those developed for the two-commodity case. Equation 9-9 can be used to examine the shape of the feasible region for the general n-dimensional case. It will be shown that the frontier of the region occurring in response to a unit magnitude price change will have an elliptical form identical to that described by Eqs. 9-12 and 9-13. However, for $n > 3$, the entire interior of the ellipse will also be feasible.

One can generalize to the n-dimensional case the idea of an angle (ϕ) between the direction of the price change vector and the price index gradient vector.* Then for a given $\Delta\theta$, there will exist a range of possible values of $\Delta\theta$, with those values obtaining for various directions of price changes within the hyperplane of price index constant. The maximum and minimum values of $\Delta\theta$, for a unit magnitude price change, will depend in a predictable manner on ϕ. All values between the maximum and minimum will be feasible.

For a price change of unit magnitude, the sum of the squares of the component magnitudes† likewise must equal unity. By Eq. 9-3,

$$\beta^2 + \sum_{i=1}^{n-1} (\alpha_i)^2 = 1 \quad , \qquad (9\text{-}15)$$

where the α_i's and β are defined by Eqs. 9-5 and 9-4, respectively.

Now ϕ can be defined

$$\cos\phi = \beta \quad . \qquad (9\text{-}16)$$

*The angle can be constructed by defining a plane which contains the vectors $\nabla\pi$ and ΔP. Then ϕ will be the angle between $\nabla\pi$ and ΔP.

†Equation 9-15 depends upon the previous requirement that all unit price change vectors are orthogonal to one another.

Equivalently, by Eq. 9-15

$$\sin \phi = \left[\sum_{i=1}^{n-1} (\alpha_i)^2 \right]^{1/2} . \tag{9-17}$$

For the two-dimensional case, Eqs. 9-16 and 9-17, combined with Eq. 9-9, reduce to Eq. 9-12. Equation 9-16, combined with Eq. 9-10 gives Eq. 9-1 for the n-dimensional case, as well as for the 2-dimensional case. Thus, no further analysis is needed to calculate the price index change.

An additional equation for $\Delta\theta$ can be determined. Equations 9-9 and 9-16 imply that

$$\Delta\theta = B \cos \phi + \sum \alpha_i A_i . \tag{9-18}$$

By Eq. 9-18, $\Delta\theta$ depends upon the precise values of the α_i's. However, one can characterize the frontier of the demand region by solving for those α_i's which either maximize or minimize $\Delta\theta$ under the constraint of Eq. 9-17. The maximum, or minimum, occurs if

$$\alpha_i = \pm \frac{A_i \sin \phi}{[\sum (A_i)^2]^{1/2}} ,$$

for all i. Inserting the above α_i's into Eq. 9-18 gives an equation for the demand region frontier values of $\Delta\theta$:

$$\Delta\theta = B \cos \phi \pm [\sum (A_i)^2]^{1/2} \sin \phi . \tag{9-19}$$

Now define A as

$$A = [\sum_i (A_i)^2]^{1/2} . \tag{9-20}$$

Then Eq. 9-19 reduces to the following:

$$\Delta\theta = B \cos \phi + A \sin \phi , \tag{9-21}$$

an expression identical to Eq. 9-2.

The equations describing the frontier of the n-commodity demand region thus correspond precisely to Eqs. 9-1 and 9-2. The frontier of the region traced out by a unit magnitude price change continues to be an ellipse, even for the n-dimensional case! However, for $n > 2$, all points in the interior of the ellipse are traced out, while in the case of $n = 2$, only the frontier is feasible.

Equation 9-9 can be interpreted somewhat differently by a specific choice of unit price change vectors. It will always be possible* to choose n-2 orthogonal directions which are perpendicular to $\nabla\pi$, such that the corresponding quantity changes are perpendicular to $\nabla\theta$. Let $P_2 \ldots P_{n-1}$ be so chosen. In this case

$$A_2 = A_3 = \ldots A_{n-1} = 0 ,$$

and Eq. 9-9 reduces to Eq. 9-11.

Under this choice of unit price change vectors, two unit vectors will be particularly relevant: P_0 and P_1. Only those components of a price change parallel to these two unit vectors will lead to changes in either θ or π. All other price change components lead to $\Delta\pi = \Delta\theta = 0$. Therefore, examination of the values of $\Delta\pi$ and $\Delta\theta$ reduces to examination of the projection of ΔP onto a plane spanned by the vectors P_0 and P_1. All components of ΔP perpendicular to this plane are irrelevant to $\Delta\pi$ and $\Delta\theta$.

Under this interpretation, Ω can be defined to measure the angle of ΔP from this plane. This angle (Ω) is implicitly defined by:

$$\sin\phi \cos\Omega = \alpha_1 = \Delta P \cdot P_1 . \qquad (9\text{-}22)$$

For $\Omega = 0°$, ΔP lies in the plane spanned by P_0 and P_1; for $\Omega = 90°$, ΔP is perpendicular to P_1.

*Given any initial set of orthogonal unit price vectors, at least n-2 linear combinations can be chosen that give vectors which are perpendicular to one another and which have zero change in the quantity index.

Figure 9-5 diagrams the angles ϕ and Ω for the case of $n = 3$. For the case $n > 3$, the interpretation of ϕ is straightforward, as indicated above, but the interpretation of Ω follows from a generalization of geometric ideas into spaces of greater than three dimensions and thus cannot be diagrammed simply.

For a unit magnitude price change, under the above choice of unit price change vectors, 9-9 reduces to*

$$\Delta\theta = B \cos\phi + A \sin\phi \cos\Omega \quad , \tag{9-23}$$

and the aggregate demand derivative becomes

$$\frac{\Delta\theta}{\Delta\pi} = \sigma + \tau \tan\phi \cos\Omega \quad , \tag{9-24}$$

where $\sigma = B/D$ and $\tau = A/D$.

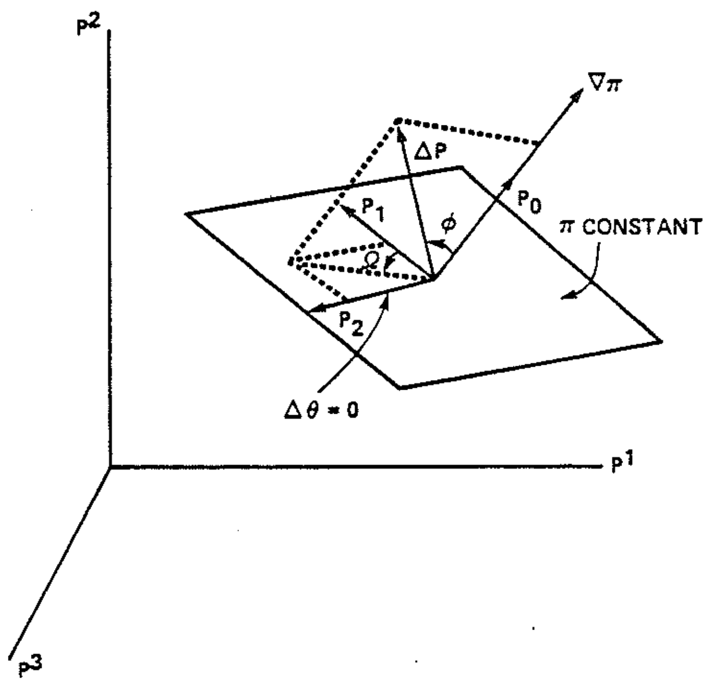


Figure 9-5 Price Changes in the Three-Commodity Case

*Note that, by Eq. 9-20, in this case $A = A_1$.

This equation should be compared to Eq. 9-14. For $\Omega = 0°$ or $180°$, $\cos \Omega$ becomes 1 or -1 respectively, its maximum magnitude. In this case, Eq. 9-24 reduces to Eq. 9-14. For Ω fixed and ϕ varying, $\tau \cos \Omega$ takes a fixed value, smaller in magnitude than τ. Thus, for Ω fixed, Eq. 9-24 will have the same form as Eq. 9-14.

As in the two-dimensional case, the demand derivative can be decomposed into two terms: the price level demand derivative (σ) and the relative price demand derivative ($\tau \tan \phi \cos \Omega$). The first component is a constant, independent of direction of price change, while the latter component may vary between $+\infty$ and $-\infty$ as the direction of price change varies.

The price level demand derivative can also be interpreted as the average price derivative, just as is possible for the two-commodity case. As all possible directions of price change are spanned, the second term in Eq. 9-24 integrates to zero, leaving the average overall possible directions of price change of the values of $\Delta\theta/\Delta\pi$ equal to σ.

The parameter τ will be referred to as the directional demand derivative. High values of τ imply that the variable component of the demand derivative is frequently large, while small values of τ imply that the variable component is usually small.

Under this choice of unit price vectors, the price and quantity index changes can be plotted for a unit magnitude price change as ϕ varies over its range. Figure 9-6 illustrates the resulting demand region for the n-commodity case. Note that this diagram is identical to Figure 9-5 except that the entire area within the ellipse would be spanned as Ω and ϕ vary. For $\Omega = 90°$ or $270°$, the locus of $\Delta\pi$ and $\Delta\theta$ is a single line, having slope σ. As Ω varies, while ϕ remains constant, $\Delta\theta$ will change but $\Delta\pi$ will not. This leads to vertical changes on the figure, as illustrated.

The parameter τ can be interpreted as a measure of the thickness of the demand region in Figure 9-2. The maximum vertical thickness of the demand region occurs where $\Delta\pi = 0$; here the thickness is 2A. The maximum price index variation is 2D. Thus, the ratio of the maximum vertical thickness to the maximum price index variation is $A/D = \tau$. Hence, τ can be interpreted as the demand region thickness relative to the maximum price index variation.

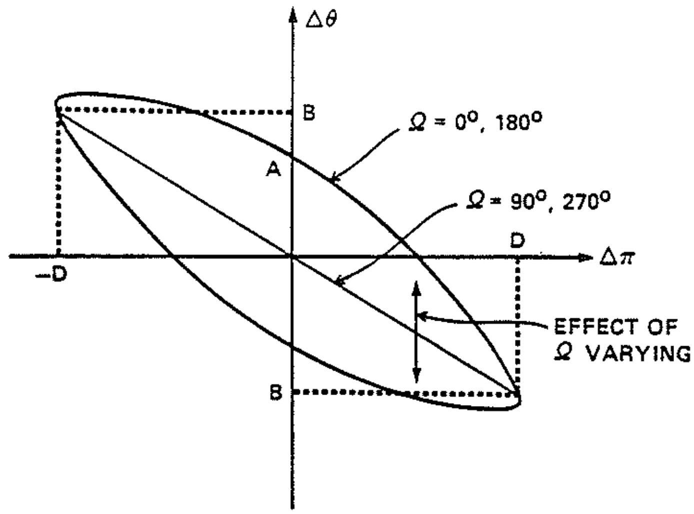


Figure 9-6 Aggregate Price and Quantity Changes from a Unit Magnitude Price Change Vector as ϕ Varies Over its Range ($n > 2$)

A third parameter, the shape parameter ν, can be defined to measure the relative thickness of the demand region. Let $\nu = A/B = \tau/\sigma$. Then ν is the ratio of two quantity index changes, one evaluated at the minimum absolute price index change ($\Delta\pi = 0$) and the other evaluated at the maximum absolute price index change. ν is a measure of the relative contributions of the variable and fixed demand derivative components, and so it provides information about the shape of the demand region. A large value of ν indicates that the demand is highly sensitive to the direction of the price change and the region will be fairly thick. A small value of ν indicates that the demand is relatively insensitive to the direction of price change and the region will be more elongated. In this last case, the demand region can be closely approximated by a single-valued demand function with slope σ.

The range of applicability of the demand derivative estimate, σ, can be estimated as ϕ and Ω vary over their ranges. Assume that a fractional error of E is acceptable. Then the actual demand derivative falls within a range of $\sigma(1-E)$ and $\sigma(1+E)$ whenever

$$|\tan \phi \sin \Omega| \leq E/|\nu| \quad .$$

Thus, the shape parameter ν provides a measure of the fraction of price directions for which σ is a close approximation to the price derivative. For $n = 3$, Table 9-2 presents the fraction of the directions for which the inequality holds. For

Table 9-2

PERCENT OF DIRECTIONS FOR WHICH DEMAND DERIVATIVE REMAINS WITHIN ERROR OF E: THREE-COMMODITY CASE

τ/σ or A/B	E			
	0.1	0.2	0.5	1.0
0.05	0.81	0.90	0.96	0.98
0.1	0.66	0.81	0.92	0.96
0.2	0.47	0.66	0.84	0.92
0.3	0.37	0.55	0.77	0.88
0.4	0.31	0.47	0.71	0.84
0.5	0.27	0.41	0.66	0.81
0.6	0.23	0.37	0.61	0.77
0.7	0.21	0.33	0.57	0.74
0.8	0.19	0.31	0.53	0.71
0.9	0.17	0.29	0.50	0.68
1.0	0.16	0.27	0.47	0.66
1.5	0.12	0.20	0.37	0.55
2.0	0.09	0.16	0.31	0.47

example, if ν equals 0.5 and an error of 20% is allowable, then 90% of the price directions lead to demand derivations which fall within 20% of σ.

As in the two-commodity case, an average demand derivative can be calculated by letting ϕ and Ω vary over their ranges with equal probability of any direction. Then again ν is the average demand derivative in the three-commodity case.

A LIMITING CASE

Although in general the decomposition approach has been designed to help identify regions of price change and quantity change, in a limiting case, the region could have zero thickness. In this case, the relative price demand derivative is zero; a single-valued demand function exists.

The limiting case will occur under a set of assumptions. First, assume that there exists a production function, using a set of commodities as inputs to produce some output. The commodity demands are derived from minimizing the cost required to obtain a given level of output. The production function is assumed to be separable in that one group of commodities can be aggregated into a single aggregate commodity. For concreteness, these commodities will be assumed to be various energy carriers. Thus the production function will be assumed separable into an energy subaggregator, E(Q), and a vector of other inputs. This subaggregator is assumed to be homothetic. These can be summarized as follows:

- Assumption: Separable, homothetic subaggregator. Energy demand will be assumed to be derived by minimizing cost of obtaining a given level of output, based upon a separable production function, with a homothetic energy subaggregator.

The price index and the quantity index will be assumed to have the following gradients for all changes:

$$\nabla\pi = Q \tag{9-25}$$

$$\nabla\theta = P \;. \tag{9-26}$$

This assumption is exactly satisfied for infinitesimal changes around the reference price and quantity vectors and is approximately satisfied for small price and quantity changes for the Paasche, Laspeyres, Ideal, and Tornquist indexes.

The following proposition can be established:

- Proposition: Under the assumption of a separable, homothetic energy subaggregator, with price and quantity indexes satisfying Eqs. 9-25 and 9-26, any price vector change which leaves the price index unchanged will be associated with a quantity vector change which leaves the quantity index unchanged.

Proof: Cost minimization implies that

$$P = \lambda\nabla E \;,$$

where E is the gradient of the energy subaggregator and λ is a scalar. By Eq. 9-26, this implies

$$\nabla\theta = \lambda\nabla E \;. \tag{9-27}$$

The gradient of the quantity index will be parallel to the gradient of the energy subaggregator.

Now choose an infinitesimal δP, such that $\nabla\pi \cdot \delta P = 0$. Any finite price change which does not change the price index can be made up of a set of such infinitesimal changes.

By Eq. 9-25, for this price change

$$Q \cdot \delta P = 0 \quad . \tag{9-28}$$

Equation 9-28 implies that the minimized cost of obtaining the given quantity of the energy aggregate is unchanged. Under the homotheticity assumption, the marginal cost of purchasing the given energy quantity is also unchanged. Hence, the optimal quantity of the energy aggregate remains unchanged:

$$0 = \Delta E = \nabla E \cdot \delta Q \quad , \tag{9-29}$$

where δQ is the resultant change in the vector of energy commodities.

Using Eq. 9-27 with 9-29 gives

$$\Delta\theta = \nabla\theta \cdot \delta Q = 0 \quad . \tag{9-30}$$

The quantity index change must be zero.

For finite price changes, the logic can be applied to infinitesimal price and quantity changes, and the result integrated to give the result stated. Q.E.D.

This result can be used as a test of whether data generated by a model or an economic process is consistent with the existence of cost minimization under a separable, homothetic energy subaggregator. If the demand region derived from unit magnitude price changes is thick, the separability or cost minimizing assumptions will be poor approximations. If the region is thin, these assumptions may provide good approximations to the process generating the data.

The width of the elliptical demand region can be interpreted as a measure of the degree to which the separability assumption is valid. Such an interpretation includes a maintained hypothesis that the data is consistent with cost minimizing behavior. Since the demand region thickness provides a measure of the degree of failure of the separability assumption, it goes beyond current tests which only indicate whether or not separability may hold.

RESULTS FOR THE "AGGREGATE ELASTICITY OF ENERGY DEMAND" STUDY

The decomposition methodology was applied to several sets of data generated for this study. These energy price and quantity data, generated for up to three fuels, were aggregated using four indexes, the Paasche, Laspeyres, Tornquist, and Btu-weighted. Demand regions were then fitted to Eqs. 9-13 and 9-18 using ordinary least squares (OLS) and the frontiers were plotted using Eqs. 9-13 and 9-21. The results are presented in Tables 9-3 and 9-4. Table 9-3 contains primary demand elasticity estimates while Table 9-4 provides secondary demand elasticity estimates.

The parameters σ, τ, and ν have been estimated based upon the constants A, B, and D as described above in Eq. 9-24. Since the price and quantity indexes equal unity for the Reference case, the demand derivative is algebraically equal to the demand elasticity for small changes in price and quantity around the Reference case.* Thus, σ and τ can be interpreted as components of the demand elasticity.

An examination of Table 9-4 shows that the elasticities estimated for the Paasche, Laspeyres, and Tornquist indexes are consistently close in value, whereas those estimated for the Btu-weighted index often differ significantly from the others. This was to be expected since the Btu-weighted index, in contrast to the others, is not based upon the assumption of optimizing behavior. The aggregate elasticities and, hence, the decomposition parameters computed using the Btu-weighted index, will be different from the others unless all energy prices are identical.

A comparison of Tables 9-3 and 9-4 supports the observation discussed in Chapter 1: elasticities measured at the primary level are less dependable than those measured at the secondary level. For almost all models, values of ν calculated at the primary level are larger in absolute value than for the equivalent statistics calculated at the secondary level. Hence, at the secondary level, the average elasticities closely match the actual elasticities for a greater fraction of the price direction changes than occurs at the primary level.

Representative demand regions based upon the secondary fuels data are displayed in Figures 9-7 through 9-11. A complete set of demand region graphs is included in Appendix B. The $\Delta\pi$'s and $\Delta\theta$'s computed from the basic data and adjusted to correspond to unit magnitude price changes were plotted as x's in these figures. The

*The sign convention for elasticities used here is the opposite from that used in Chapter 1. Here a downward sloping demand function will have a negative elasticity.

Table 9-3

DECOMPOSITION RESULTS FOR PRIMARY ENERGY

Model	Sector	Index											
		Paasche			Laspeyres			Tornquist			Btu-weighted		
		σ	τ	ν	σ	τ	ν	σ	τ	ν	σ	τ	ν
Baughman/Joskow	Total Demand	0.95	1.15	1.21	0.11	0.60	5.41	-0.31	0.43	-1.40	-0.36	0.16	-0.45
	Residential/ Commercial[a]	0.49	1.09	2.20	-0.10	0.62	-6.14	-0.46	0.46	-1.01	-0.49	0.10	-0.21
	Industrial	1.39	1.19	0.86	0.28	0.58	1.96	-0.18	0.39	-2.20	-0.26	0.20	-0.76
BESOM/H-J	Total Demand	-0.57	0.17	-0.29	-0.59	0.23	-0.39	-0.61	0.24	-0.39	-0.38	0.18	-0.47
EPM[b]	Total Demand	0.56	1.23	2.22	0.53	1.53	2.88	0.52	1.36	2.63	0.20	0.87	4.39
ETA-MACRO	Total Demand	-0.57	0.95	-1.66	-0.28	0.30	-1.08	-0.41	0.13	-0.30	-0.11	0.08	-0.74
FOSSIL1	Total Demand	-0.03	0.13	-4.12	-0.06	0.09	-1.53	-0.10	0.03	-0.26	-0.07	0.07	-0.97
FOSSIL1 Conservation	Total Demand	-0.07	0.15	-2.15	-0.10	0.11	-1.15	-0.14	0.06	-0.41	-0.10	0.03	-0.33
Griffin OECD	Total Demand	-0.32	0.37	-1.15	-0.28	0.43	-1.53	-0.27	0.45	-1.64	0.26	0.15	-0.59
	Residential/ Commercial[a]	-0.33	0.84	-2.51	-0.19	0.92	-4.76	-0.18	0.90	-4.97	-0.36	0.11	-0.29
	Industrial	-0.03	0.20	-7.45	0.04	0.24	6.37	0.04	0.31	6.97	-0.11	0.26	-2.37
Hirst Residenital[b]	Residential	-0.23	-0.05	0.21	-0.27	-0.02	0.07	0.04	-0.33	-9.22	-0.32	0.24	-0.75
Jackson Commercial[b]	Commercial	-0.17	-0.02	0.14	-0.20	0.00	-0.01	0.06	-0.30	-5.24	-0.27	0.12	-0.46
MEFS	Total Demand	-0.07	0.24	-3.45	-0.07	0.30	-3.98	-0.10	0.44	-4.52	-0.06	0.31	5.25
	Residential[a]	0.02	0.05	2.45	-0.03	0.19	-6.27	-0.01	0.10	-7.68	-0.06	0.39	-6.32
	Commercial[a]	-0.03	0.02	-0.71	0.04	0.24	6.37	-0.05	0.09	-1.65	-0.02	0.02	-0.89
	Industrial	-0.01	0.18	-3.15	0.00	0.19	-7.43	-0.02	0.18	-7.44	0.00	0.20	8.67
Parikh WEM[b]	Total Demand	6.48	26.7	4.12	6.52	26.9	4.12	6.50	26.8	4.12	5.76	23.9	4.14
Pindyck	Total Demand	-0.47	0.03	-0.07	-0.48	0.03	-0.06	-0.48	0.06	-0.13	-0.44	0.38	-0.86
	Residential[a]	-0.52	-0.19	0.36	-0.56	-0.17	0.31	-0.52	-0.22	0.42	-0.64	0.09	-0.15
	Industrial	-0.32	0.05	-0.15	-0.75	0.27	-0.36	-0.64	0.18	-0.28	-1.25	0.69	-0.56

σ - average elasticity (slope).
τ - directional elasticity (thickness).
ν - shape factor.

[a]Less than three fuels reported.

[b]Less than eight scenarios reported.

Table 9-4

DECOMPOSITION RESULTS FOR SECONDARY ENERGY

Model	Sector	Paasche σ	Paasche τ	Paasche ν	Laspeyres σ	Laspeyres τ	Laspeyres ν	Tornquist σ	Tornquist τ	Tornquist ν	Btu-weighted σ	Btu-weighted τ	Btu-weighted ν
Baughman-Joskow	Total Demand	-0.46	0.53	-1.14	-0.50	0.56	-1.11	-0.48	0.55	-1.14	-0.47	0.45	-0.95
	Residential/ Commercial[a]	-0.40	-1.00	2.47	-0.28	-1.31	4.61	-0.35	-1.13	3.19	-0.24	-1.36	5.56
	Industrial	-0.17	0.69	-4.04	-0.23	0.79	-3.48	-0.20	0.76	-3.87	-0.31	0.43	-1.42
BECOM	Residential[a]	5.88	-6.55	-1.11	8.31	-9.08	-1.09	7.05	-7.76	-1.10	14.98	-16.31	-1.09
	Residential/ Commercial[a]	4.08	-3.86	-0.94	6.16	-5.65	-0.92	5.09	-4.72	-0.93	11.42	-10.44	-0.91
	Commercial[a]	-0.50	0.12	-0.25	0.33	-0.35	-1.05	-0.06	-0.12	1.99	0.03	-0.31	-9.83
BESOM/H-J	Total	-0.52	0.42	-0.80	-0.50	0.46	-0.92	-0.51	0.44	-0.87	-0.65	0.50	-0.77
	Residential/ Commercial[a]	-1.83	0.60	-0.33	-1.78	0.58	-0.32	-1.81	0.59	-0.33	-2.70	0.88	-0.33
	Industrial	-0.57	0.29	-0.51	-0.62	0.38	-0.61	-0.60	0.34	-0.57	-0.57	0.39	-0.68
EPM	Total	-0.45	3.22	-7.17	-0.45	3.19	-7.05	-0.45	3.22	-7.15	-0.46	3.26	-7.07
	Residential/ Commercial	-0.16	0.86	-5.23	-0.07	1.08	-15.70	-0.12	0.97	-8.33	-0.13	1.24	-9.59
	Industrial	-0.22	0.65	-3.01	-0.22	0.65	-2.96	-0.22	0.68	-3.13	-0.20	0.63	-3.07
ETA-MACRO	Total Demand	-0.22	0.05	-0.25	-0.25	0.12	-0.46	-0.24	0.08	-0.35	-0.44	0.29	-0.66
FOSSIL1	Total Demand	-0.07	0.05	-0.68	-0.08	0.05	-0.61	-0.08	0.05	-0.65	-0.14	0.04	-0.29
FOSSIL1 Conservation	Total Demand	-0.15	0.06	-0.36	-0.17	0.05	-0.30	-0.16	0.05	-0.33	-0.23	0.05	-0.23
Griffin OECD	Total Demand	-0.67	1.58	-2.34	-0.92	4.94	-5.35	-0.80	3.27	-4.09	-1.08	5.88	-5.43
	Residential	-0.84	-0.45	0.53	-0.88	-0.47	0.54	-0.86	-0.46	0.53	-0.96	-0.80	0.83
	Commercial/ Industrial	-0.30	1.84	-6.15	-0.38	3.45	-9.07	-0.34	2.61	-7.77	-0.52	2.62	-5.04
Hirst Residential	Residential[a,b]	-0.31	0.24	-0.78	-0.34	0.22	-0.65	-0.32	0.23	-0.72	-0.65	-0.24	0.37
ISTUM	Industrial[b]	3.74	7.76	2.07	1.18	2.80	2.38	2.47	5.30	2.14	-0.91	1.75	-1.93
Jackson Commercial	Commercial[a,b]	-0.32	0.09	-0.28	-0.33	0.09	-0.26	-0.33	0.09	-0.27	-0.40	-0.08	0.19

Table 9-4 (continued)

Model	Sector	Paasche σ	Paasche τ	Paasche ν	Laspeyres σ	Laspeyres τ	Laspeyres ν	Tornquist σ	Tornquist τ	Tornquist ν	Btu-weighted σ	Btu-weighted τ	Btu-weighted ν
MEFS	Total Demand	-0.29	0.03	-0.11	-0.32	0.02	-0.08	-0.31	0.03	-0.08	-0.43	0.08	-0.18
	Residential[a]	-0.43	0.05	-0.11	-0.47	0.03	-0.06	-0.45	0.04	-0.08	-0.71	0.19	-0.27
	Residential/ Commercial[a]	-0.47	0.03	-0.07	-0.51	0.02	-0.03	-0.49	0.03	-0.05	-0.76	0.22	-0.29
	Commercial[a]	-0.41	0.00	0.00	-0.44	-0.02	0.05	-0.43	-0.01	0.03	-0.68	-0.29	0.42
	Industrial	-0.14	0.04	-0.30	-0.16	0.03	-0.20	-0.15	0.03	-0.20	-0.27	0.10	-0.36
Parikh WEM	Total Demand[b]	-0.15	0.92	-6.23	-0.14	0.88	-6.18	-0.14	0.90	-6.21	-0.14	0.68	-4.88
Pindyck Model	Total Demand	-0.68	0.04	-0.06	-0.71	0.04	-0.06	-0.70	0.03	-0.05	-0.72	0.52	-0.72
	Residential[a]	-1.17	-0.19	0.16	-1.24	-0.26	0.21	-1.21	-0.22	0.18	-1.52	-0.73	0.48
	Commercial/ Industrial	-0.37	-0.13	-0.36	-0.90	0.07	-0.08	-0.51	0.09	-0.18	-2.47	0.89	-0.36

σ - average elasticity (slope).
τ - directional elasticity (thickness).
ν - shape factor.

[a]Less than three fuels reported.

[b]Less than eight scenarios reported.

solid line in each figure is the frontier of the demand region obtained by fitting Eqs. 9-13 and 9-20 to the adjusted price and quantity indexes. The dashed line is the locus of points for which the directional component of the demand derivative is zero; its slope is σ.

The squares and stars are the index points adjusted to correspond to price change directions consistent with the frontier of the demand region. These latter points were included to help interpret and explain the observation that, for many of the models, the frontier of the fitted demand region did not correspond to the index points (i.e., the x's). The adjustment was made by computing an error term for each point as the difference between the value of $\Delta\theta$ computed from the basic data and the value of $\Delta\theta$ computed from Eq. 9-23, with the values of A and B obtained from the least squares regression. $\Delta\theta$ was then adjusted to correspond to the situation of $\cos \Omega = 1$, i.e., to Eq. 9-21. The error term was then added to the calculated value of $\Delta\theta$ corresponding to the frontier of the demand region. The squares correspond to data points lying above the central axis of the ellipse; the stars correspond to the data points lying below this axis.

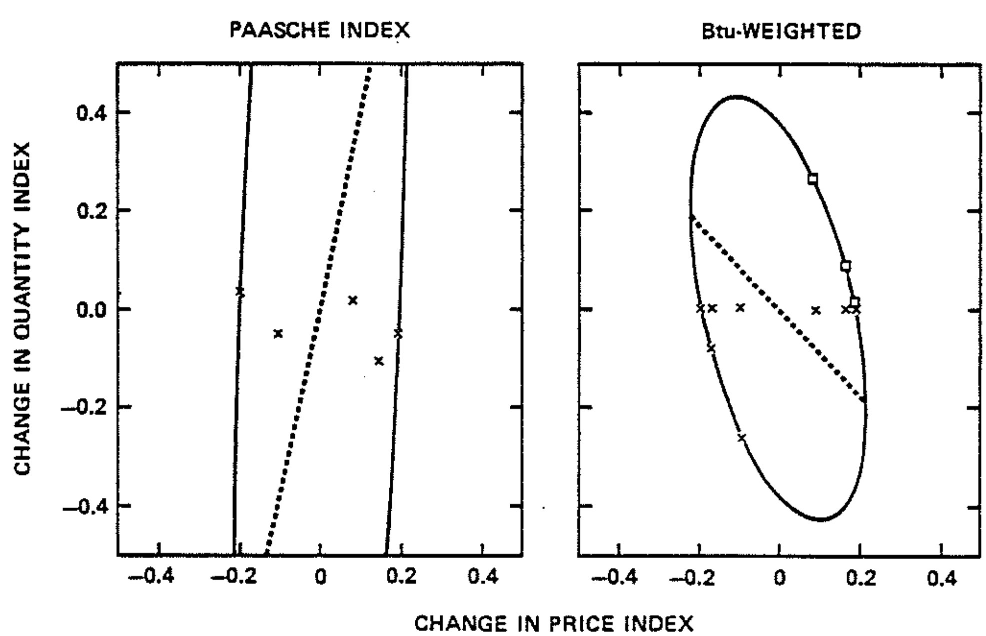


Figure 9-7 25-Year Fitted Demand Regions for the ISTUM Model (Industrial Sector)

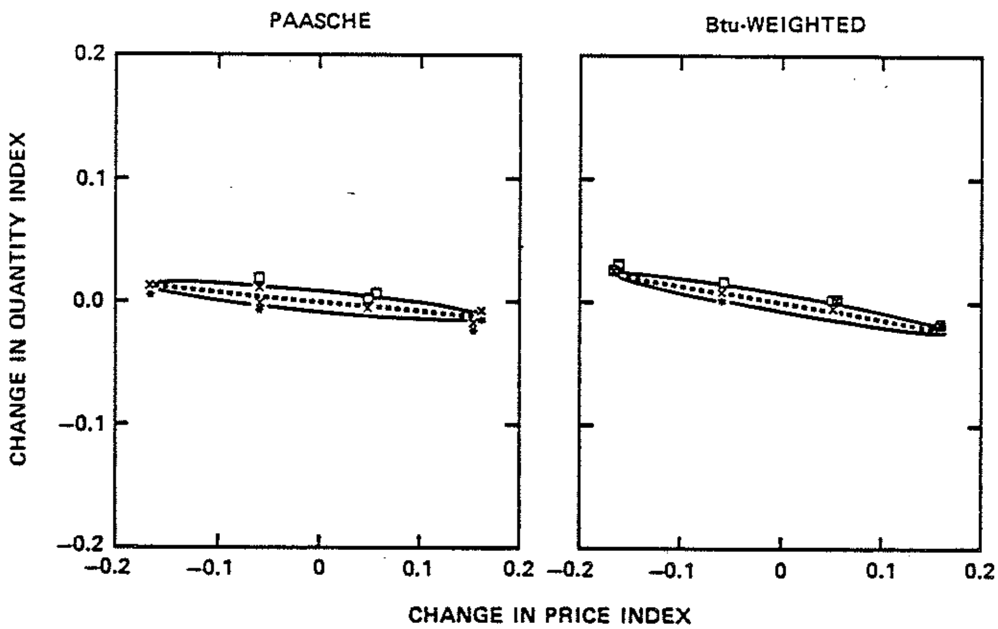


Figure 9-8 25-Year Fitted Demand Regions for the FOSSIL1 Model (Total Demand)

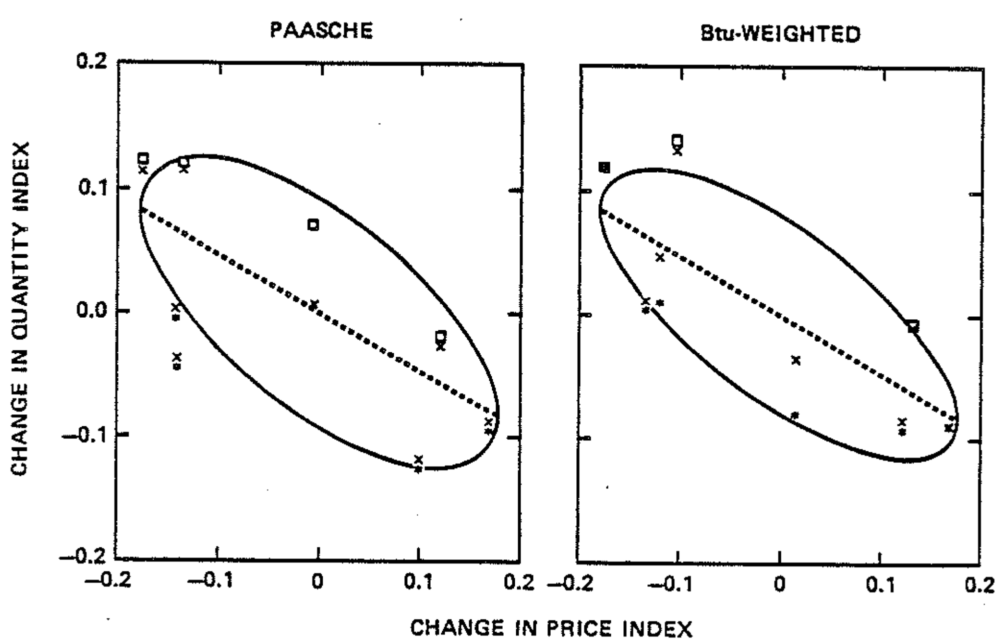


Figure 9-9 25-Year Fitted Demand Regions for the Baughman-Joskow Model (Total Demand)

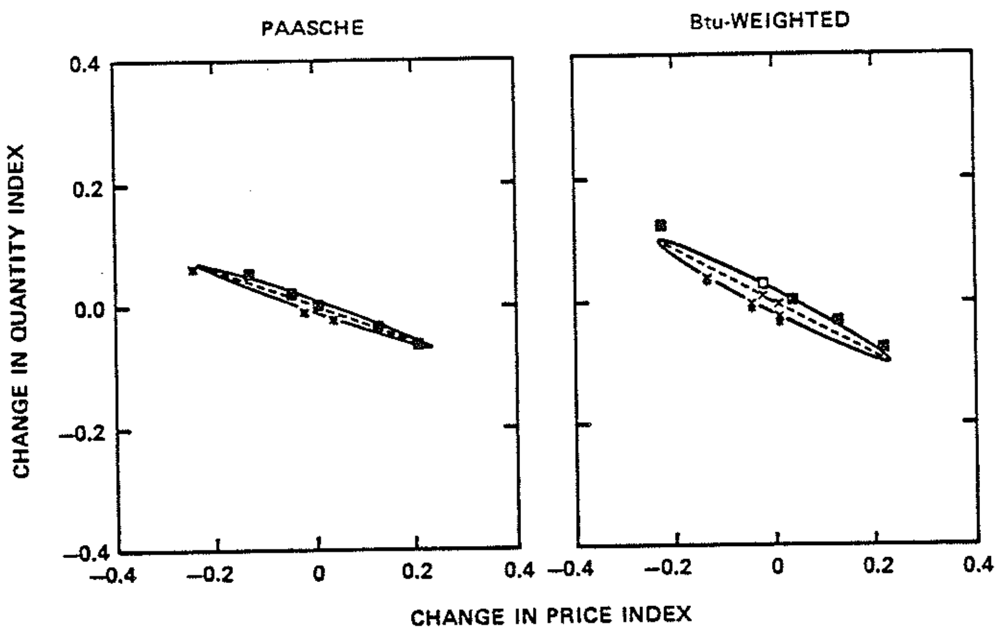


Figure 9-10 25-Year Fitted Demand Regions for the MEFS Model (Total Demand)

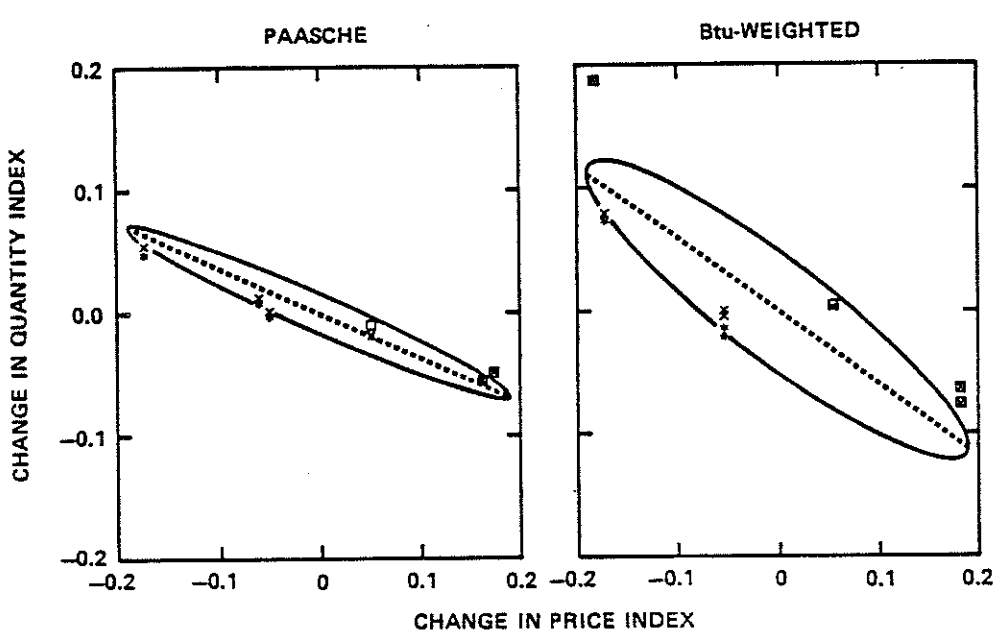


Figure 9-11 25-Year Fitted Demand Regions for the ETA-MACRO Model (Total Demand)

A comparison of these figures illustrates the effect of variations in the parameters. In particular, consider the demand regions of Baughman-Joskow, Figure 9-9 and MEFS, Figure 9-10. With the Paasche index, the value of the shape parameter for the Baughman-Joskow model is relatively large, -4.04, and the region is fairly thick, indicating that the demand is highly dependent upon the direction of the price change. In contrast, the value of ν of the MEFS model is relatively small, -0.34, and the region is thin, indicating that the demand is relatively independent of the direction of price change.

SUMMARY AND CONCLUSIONS

Econometric studies based upon an aggregated class of commodities may face severe problems associated with aggregation. Even the normal concept of a demand curve may not be valid for the aggregates. Rather, a demand region may be required to describe the quantity index changes occurring in response to price changes of a given magnitude. The problem could raise havoc with econometric studies since most historical data available is, to a significant extent, composed of aggregates of many somewhat heterogeneous commodities.

This chapter has provided a theory for econometric modeling of aggregated commodity classes, a theory strictly applicable only under restrictive linearity assumptions. The basic shape of a demand region has been identified for the linear case. Parameters have been identified which could, in principle, be estimated in order to calculate average price elasticities and to examine the extent to which demand functions can closely approximate the demand regions.

The thickness of the demand region has been related to the concept of separability. If the commodities are inputs to a separable production function, embodying a homothetic subaggregator for the commodity class, and if cost minimizing behavior is occurring, then the demand region is of zero thickness: a demand function does exist. Therefore, a thick region implies the violation of at least one of the above assumptions. Parameters can be defined to describe the relative thickness of the demand region and these parameters can indicate the degree to which the assumption set is violated. Thus, if cost minimization is a maintained hypothesis, the degree to which separability provides a close approximation is testable in principle.

The basic theory has been applied to a body of pseudodata generated for the "Aggregate Elasticity of Energy Demand" study. Tests with this data set confirm that even for such clean data, the aggregation problems are highly significant and that the normal concept of a demand function must be interpreted with extreme caution.

Appendix A

SUMMARY OF THE COMPUTATIONS USED IN THE DECOMPOSITION EXPERIMENT

by

Steven Duvall

The equations used to compute the price and quantity indexes are displayed in Table 9A-1. These equations were defined so that each index has a value of unity at the Reference case.

Table 9A-1

PRICE AND QUANTITY INDEXES

Index	Price	Quantity
Paasche	$\pi = \dfrac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^1}$	$\theta = \dfrac{\sum_{i=1}^{n} p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^1 q_i^0}$
Laspeyres	$\pi = \dfrac{\sum_{i=1}^{n} p_i^1 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$	$\theta = \dfrac{\sum_{i=1}^{n} p_i^0 q_i^1}{\sum_{i=1}^{n} p_i^0 q_i^0}$
Tornquist	$\pi = \exp \quad 1/2 \sum_{i=1}^{n} \left[\dfrac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} + \dfrac{p_i^1 q_i^1}{\sum_{i=1}^{n} p_i^1 q_i^1} \right]$	$\theta = \dfrac{\sum_{i=1}^{n} p_i^1 q_i^1}{\pi \cdot \sum_{i=1}^{n} p_i^0 q_i^0}$
Btu-weighted	$\pi = \dfrac{\sum_{i=1}^{n} p_0^1 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$	$\theta = \dfrac{\sum_{i=1}^{n} q_i^1}{\sum_{i=1}^{n} q_i^0}$

As an aid to the comparison of the decomposition results for the various models, the computations were based upon unit magnitude price changes. That is, for each scenario, the price change vector was normalized by dividing it by its magnitude*: $\Delta P/||\Delta P||$. Consistent with this approach, the $\Delta\theta$'s and $\Delta\pi$'s were computed using

$$P^0 + \frac{P^1 - P^0}{||P^1 - P^0||} ,$$

as the price vector for the specific scenario and,

$$Q^0 + \frac{Q^1 - Q^0}{||P^1 - P^0||} ,$$

as the quantity vector for the specific scenario.

The demand regions corresponding to unit magnitude price changes are described by equations (9A-1) and (9A-2):

$$\Delta\pi = D \cos \phi \tag{9A-1}$$

$$\Delta\theta = \beta B + \alpha A_1 + \gamma A_2 \tag{9A-2}$$

where

ϕ = the angle between $\nabla\pi$ and ΔP ,

$\beta = \Delta P \cdot P_0 = \cos \phi$,

$\alpha = \Delta P \cdot P_1$, and

$\gamma = \Delta P \cdot P_2$.

The vectors P_0, P_1, and P_2 form an orthogonal basis in price space with P_0 parallel to $\Delta\pi$.

P_1 and P_2 were computed by applying the Gram-Schmidt orthogonalization method to $P_0 = \nabla\pi/||\nabla\pi|| = Q^0/||Q^0||$ and two arbitrary unit vectors. The basic matrix thus obtained, before normalization of the columns, is

*$||\cdot||$ will denote the norm of the vector X; $||\cdot|| = (X \cdot X)^{1/2}$.

$$M' = \begin{bmatrix} q_1^0 & 1 & \frac{q_1^0}{q_1^0} \\ q_2^0 & 0 & -\frac{(q_1^0)^2 + (q_3^0)^2}{q_1^0 q_3^0} \\ q_3^0 & -\frac{q_1^0}{q_3^0} & 1 \end{bmatrix} .$$

Defining M as the matrix obtained by normalizing the columns of M', the data vector (β, α, γ) in Eq. 9A-2 was computed using Eq. 9A-3:

$$(\beta, \alpha, \gamma) = (\Delta P)^T M \quad . \tag{9A-3}$$

The parameters A_1, A_2, and B in Eq. 9A-2 were estimated by applying ordinary least squares (OLS) to Eq. 9A-2 with the data vector estimated using Eq. 9A-3 and $\Delta\theta$ adjusted to correspond to a unit price change.

The parameter D in Eq. 9A-1 could be estimated in a similar fashion using OLS. However, under the assumption that $\nabla\pi = p^0/q^0 \cdot p^0$, D can be calculated directly from the basic data. By definition

$$D = \nabla\pi \cdot P_0 \quad . \tag{9A-4}$$

But, $P_0 = \nabla\pi/||\nabla\pi||$. Therefore,

$$D = \nabla\pi \cdot \nabla\pi/||\nabla\pi|| = ||\nabla\pi|| \quad .$$

So, setting $\nabla\pi = \dfrac{q_0}{q_0 P_0}$,

$$D = \frac{||q^0||}{|q^0 \cdot P^0|} \quad . \tag{9A-5}$$

The correspondence between the fitted demand region and the data points can be judged by adjusting $\Delta\theta$ so that each data point corresponds to the frontier of the demand region. That is, an error term is computed for each data points as the vertical distance between $\Delta\theta$ and the fitted demand region. Thus,

$$\varepsilon = \Delta\theta - \hat{B}\cos\phi - \alpha\hat{A}_1 - \gamma\hat{A}_2 \quad , \tag{9A-6}$$

where $\hat{B}$, $\hat{A}_1$, and $\hat{A}_2$ are the values of the parameters B, A_1, and A_2 obtained from the regression. It should be clear that ε can be either positive or negative. The value of $\Delta\theta$ adjusted to correspond to the frontier of the demand region is then calculated by adding ε to the equation for the frontier:

$$\Delta\theta^* = B\cos\phi + A\sin\phi + \varepsilon \quad , \tag{9A-7}$$

where $A = \sqrt{A_1^2 + A_2^2}$.

Alternatively, when sufficient data are available to estimate the matrix of demand derivatives,

$$G = \begin{bmatrix} \partial q_1/\partial P_1 & \partial q_1/\partial P_2 & \partial q_1/\partial P_3 \\ \partial q_2/\partial P_1 & \partial q_2/\partial P_2 & \partial q_2/\partial P_3 \\ \partial q_3/\partial P_1 & \partial q_3/\partial P_2 & \partial q_3/\partial P_3 \end{bmatrix} ,$$

a more direct approach for estimating the parameters of the demand regions is available. Assuming G is known, the parameters in Eq. 9A-2 are given by Eq. 9A-8.

$$\begin{aligned} B &= (\nabla\theta, GP_0) \\ A_1 &= (\nabla\theta, GP_1) \\ A_2 &= (\nabla\theta, GP_2) \ . \end{aligned} \tag{9A-8}$$

The decomposition method was compared to this latter approach by applying both methods to a set of pseudodata generated using the FOSSIL1 model under 27 different scenarios. As is apparent in Table 9A-2 and Figures 9A-1 and 9A-2, the two methods yielded relatively consistent results, though the demand region estimated using decomposition was slightly thicker.

Table 9A-2

COMPARISON OF DECOMPOSITION RESULTS WITH DIRECT ESTIMATION

Estimation Approach	σ	τ	ν
Direct Estimation	-0.05	0.04	-0.68
Decomposition			
Paasche Index	-0.07	0.07	-0.93
Laspeyres Index	-0.08	0.06	-0.76

σ = average elasticity (slope).
τ = directional elasticity (thickness).
ν = shape factor.

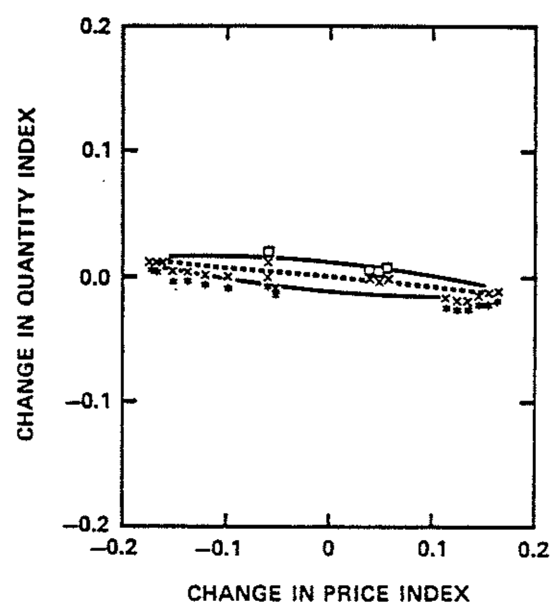


Figure 9A-1 25-Year Direct Approach Estimates for the FOSSIL1 Model (Total Demand, Paasche Index)

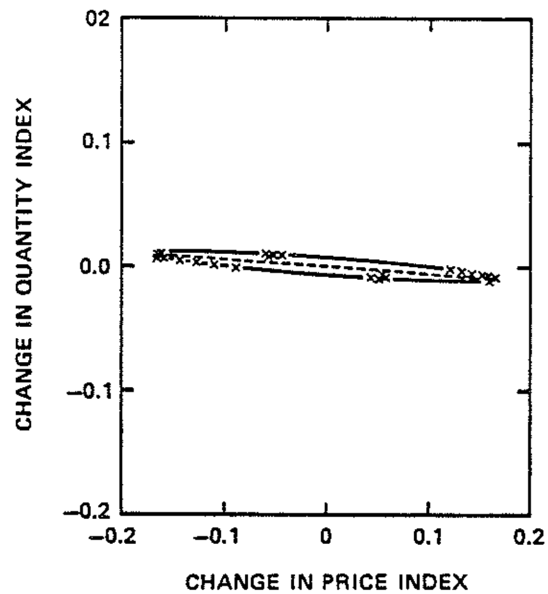


Figure 9A-2 25-Year Decomposition Estimates for the FOSSIL1 Model (Total Demand, Paasche Index)

A second experiment using the same FOSSIL1 data was designed to evaluate the dependence of the decomposition results upon the particular sample used. Demand regions were fitted to nine different subsets of the 27 scenarios and compared with the demand regions obtained from the original nine EMF 4 scenarios and the full set of 27 scenarios. Each of the nine subsets were chosen to reflect the effect of changing one of the underlying assumptions of fuel prices. For example, the first case consists of all scenarios based upon low oil and gas price. As seen in Table 9A-3, with the exception of the nominal coal price case, the results are fairly consistent, indicting that the decomposition estimates are fairly robust.

Table 9A-3

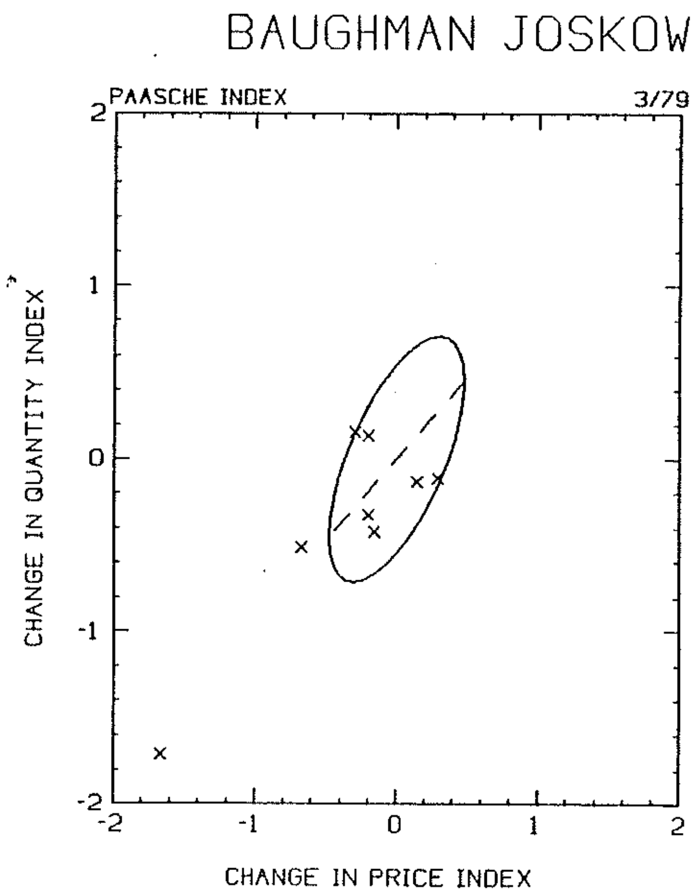
SAMPLE DEPENDENCE OF THE DECOMPOSITION RESULTS
(Paasche Index)

Price Case	σ	τ	ν
Low Oil and Gas Prices	-0.05	0.05	-0.85
Nominal Oil and Gas Prices	-0.23	0.24	-1.01
High Oil and Gas Prices	-0.09	0.10	-1.04
Low Coal Prices	-0.09	0.12	-1.40
Nominal Coal Prices	-0.38	11.99	32.00
High Coal Prices	-0.06	0.05	-0.87
Low Electricity Prices	-0.08	0.06	-0.73
Nominal Electricity Prices	-0.11	0.38	-3.44
High Electricity Prices	-0.07	0.09	-1.28
EMF 4 Scenarios	-0.07	0.05	-0.68
All Scenarios (27)	-0.07	0.07	-0.93

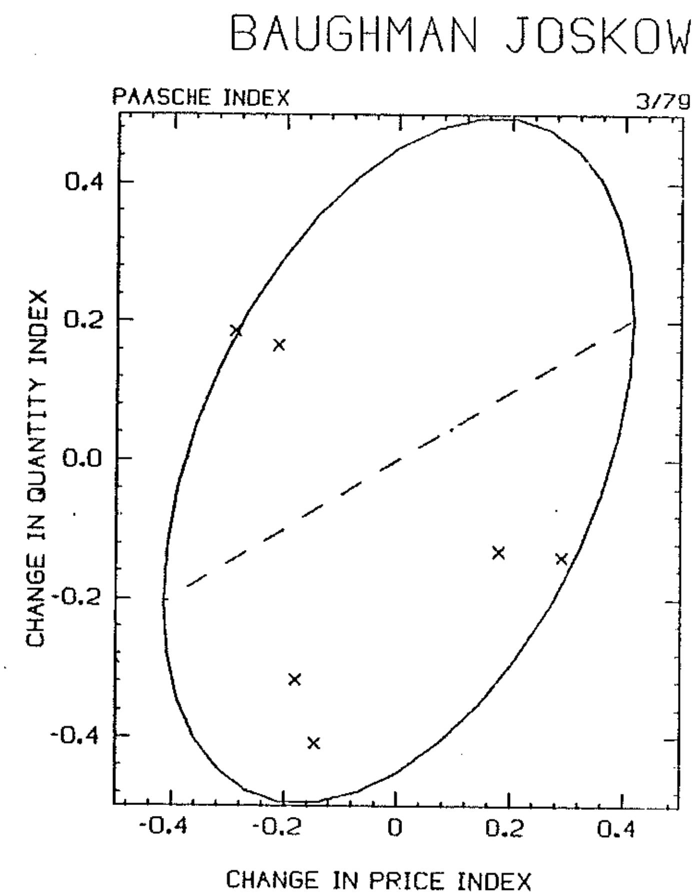
σ = average elasticity (slope).
τ = directional elasticity (thickness).
ν = shape factor.

Appendix B

MODEL/SECTOR DEMAND REGIONS FOR PRIMARY AND SECONDARY ENERGY

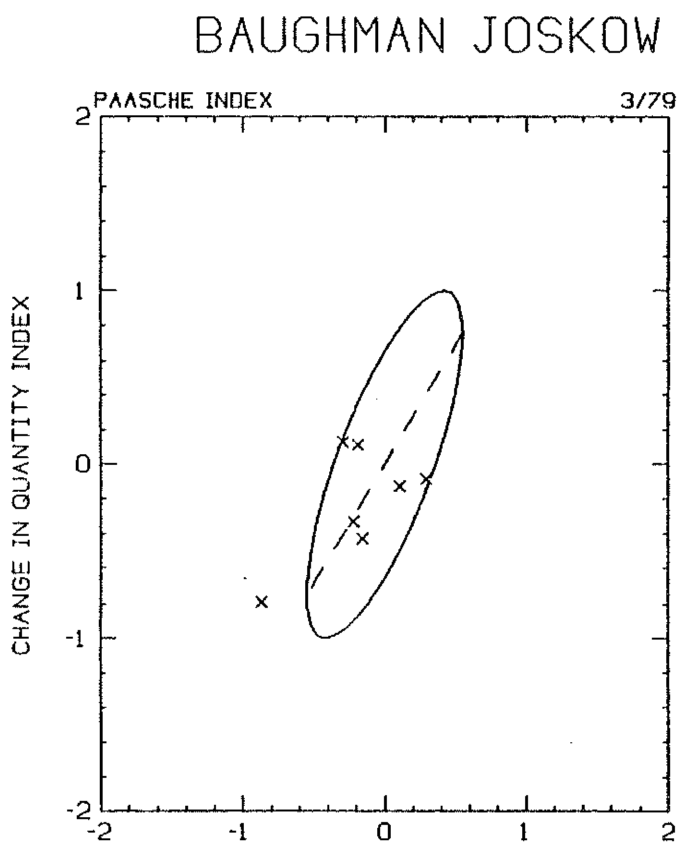


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Primary Energy

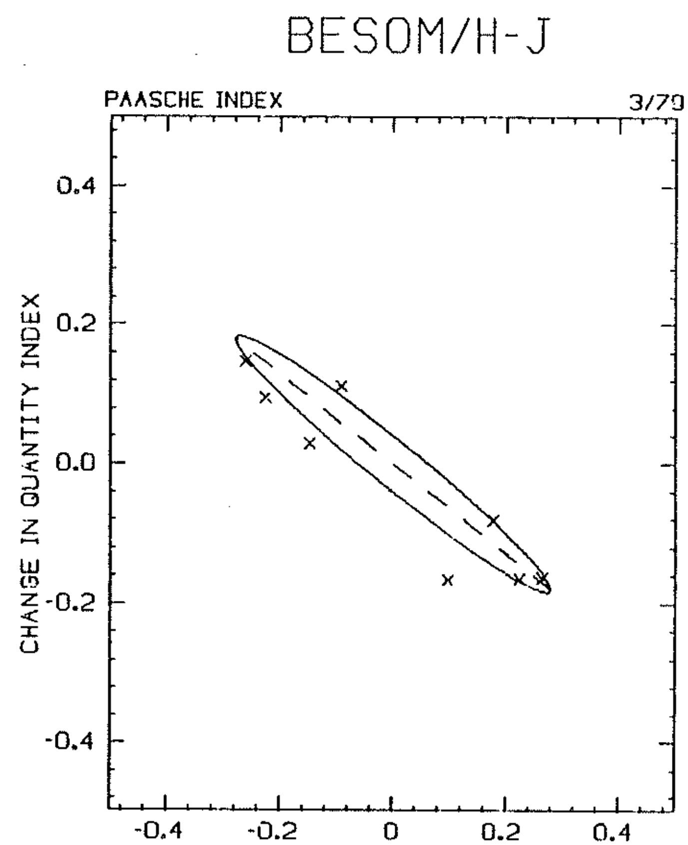


SECTOR: RESIDENTIAL/COMMERCIAL
YEAR: 2000 (25-YEAR)
Primary Energy

Figures 9B-1 and 9B-2

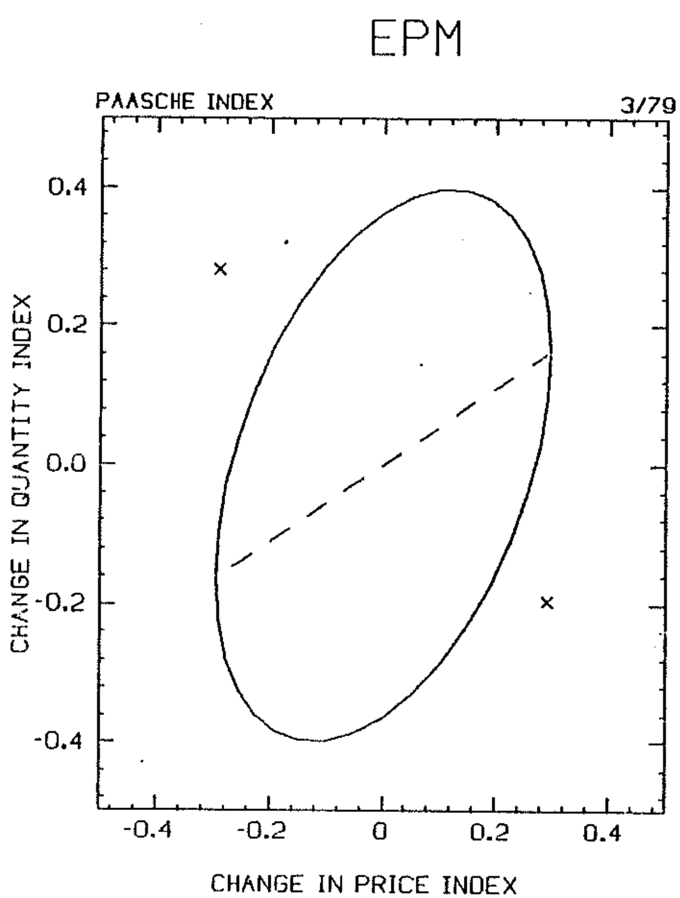


SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)
Primary Energy

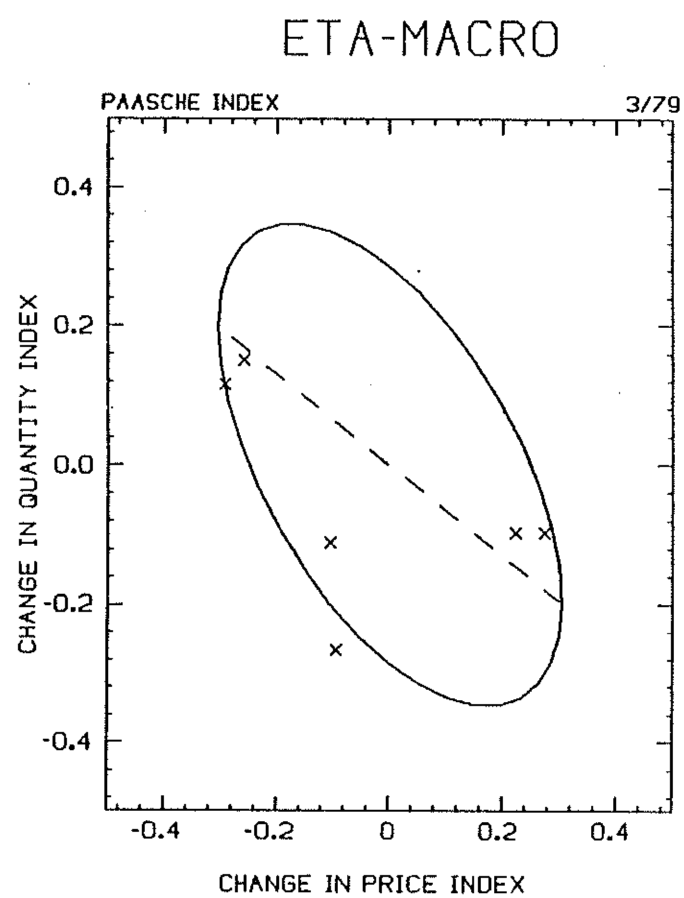


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Primary Energy

Figures 9B-3 and 9B-4

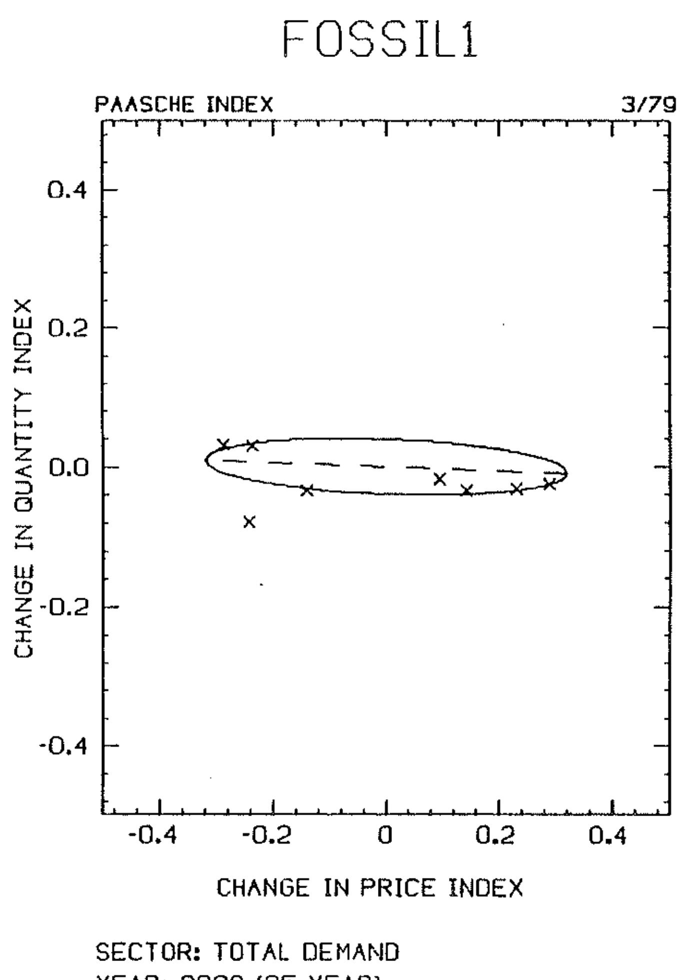


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Primary Energy

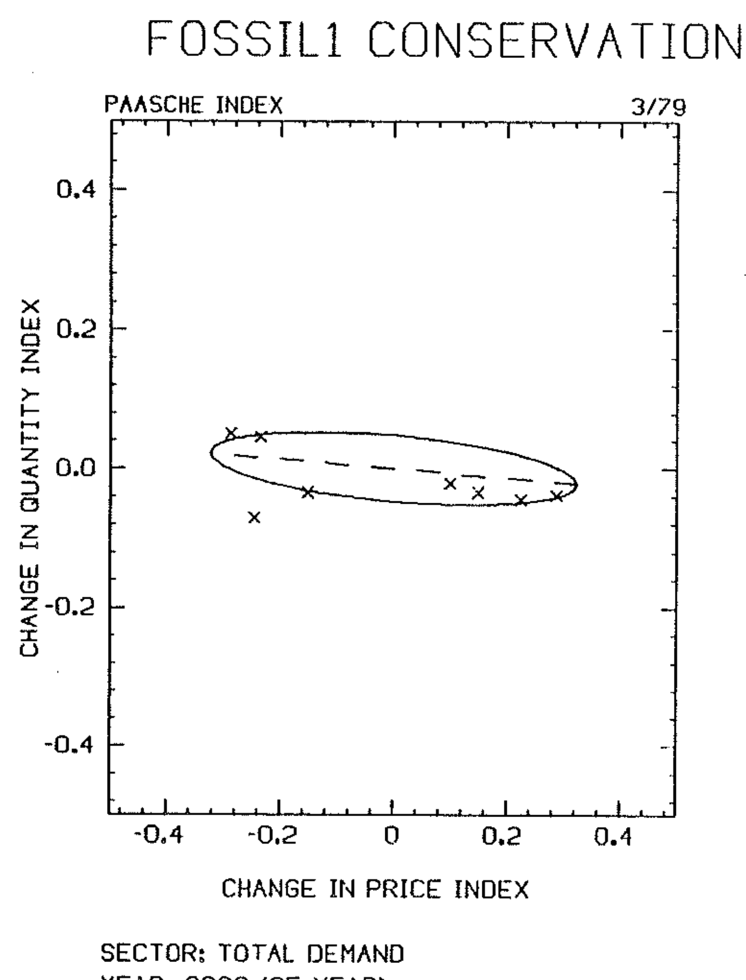


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Primary Energy

Figures 9B-5 and 9B-6



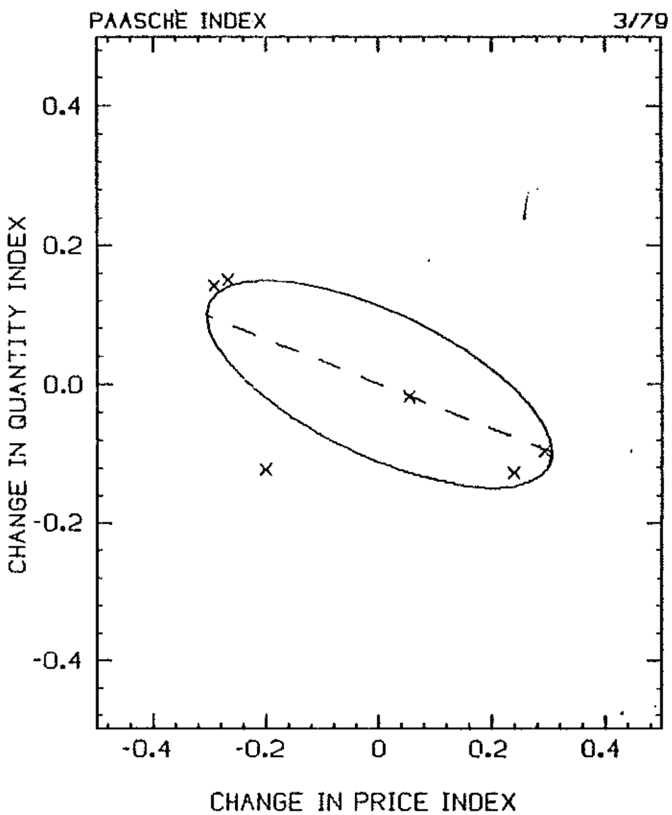
Primary Energy



Primary Energy

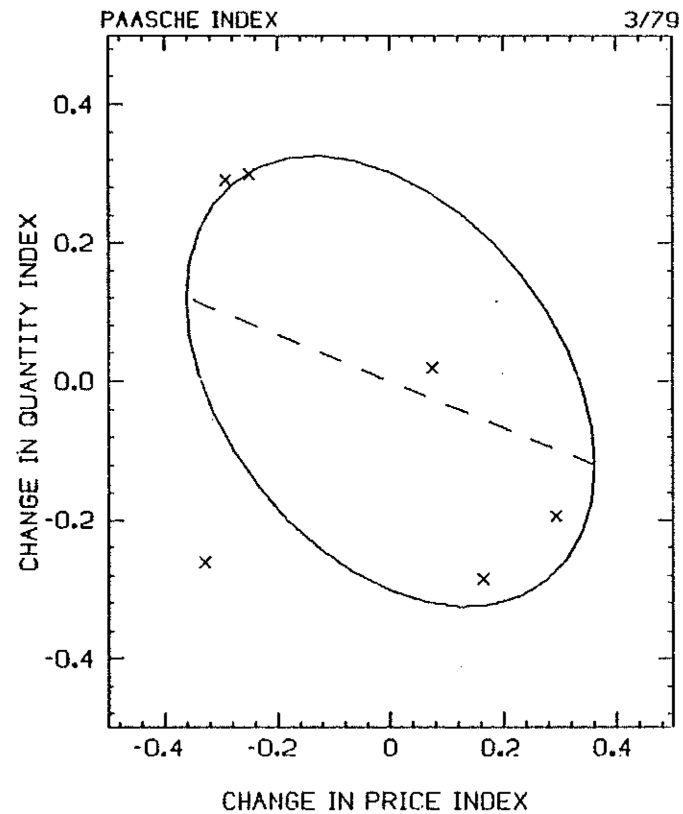
Figures 9B-7 and 9B-8

GRIFFIN OECD



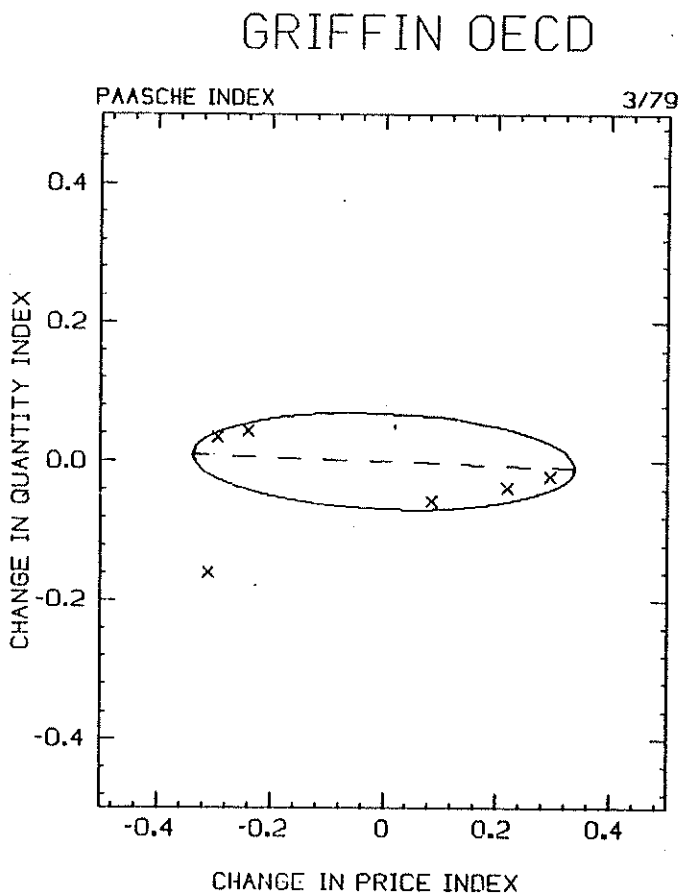
SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Primary Energy

GRIFFIN OECD

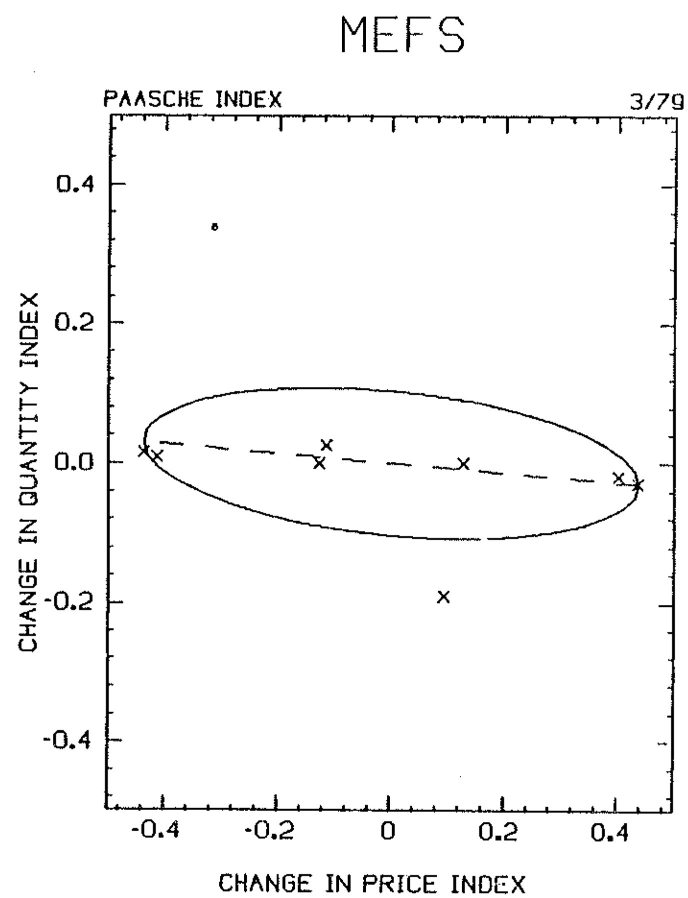


SECTOR: RESIDENTIAL/COMMERCIAL
YEAR: 2000 (25-YEAR)
Primary Energy

Figures 9B-9 and 9B-10

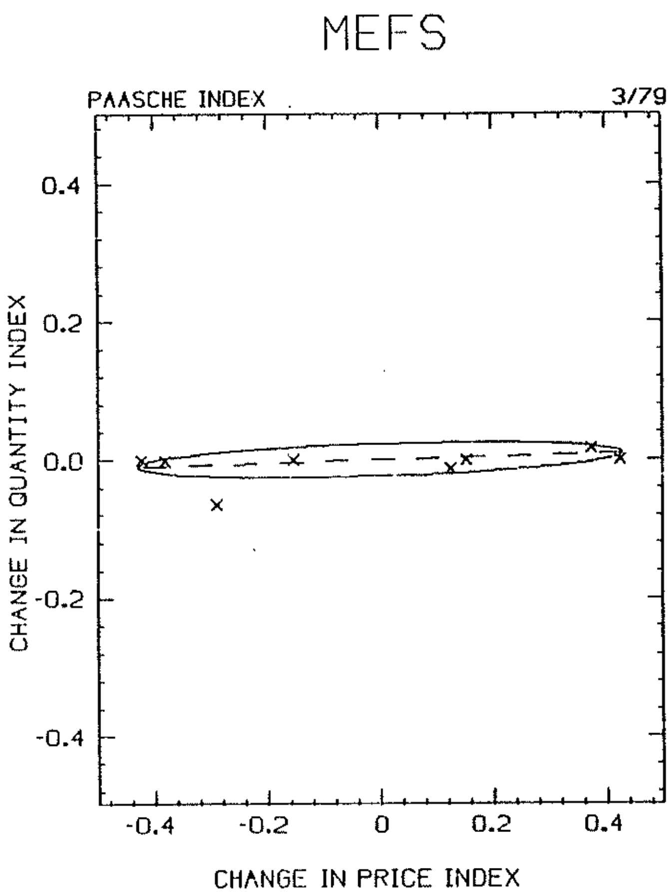


SECTOR: COMMERCIAL/INDUSTRIAL
YEAR: 2000 (25-YEAR)
Primary Energy

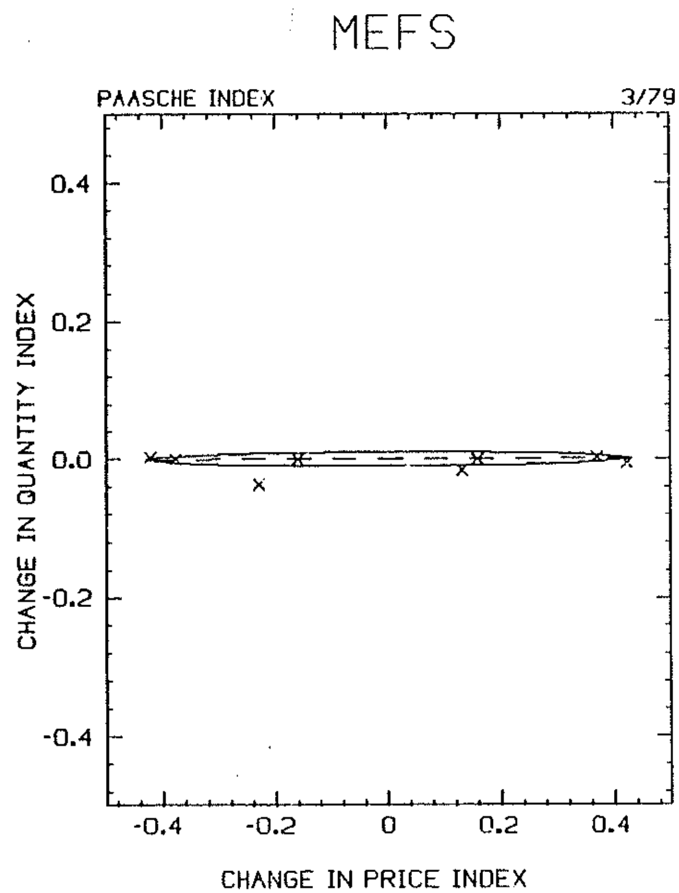


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Primary Energy

Figures 9B-11 and 9B-12

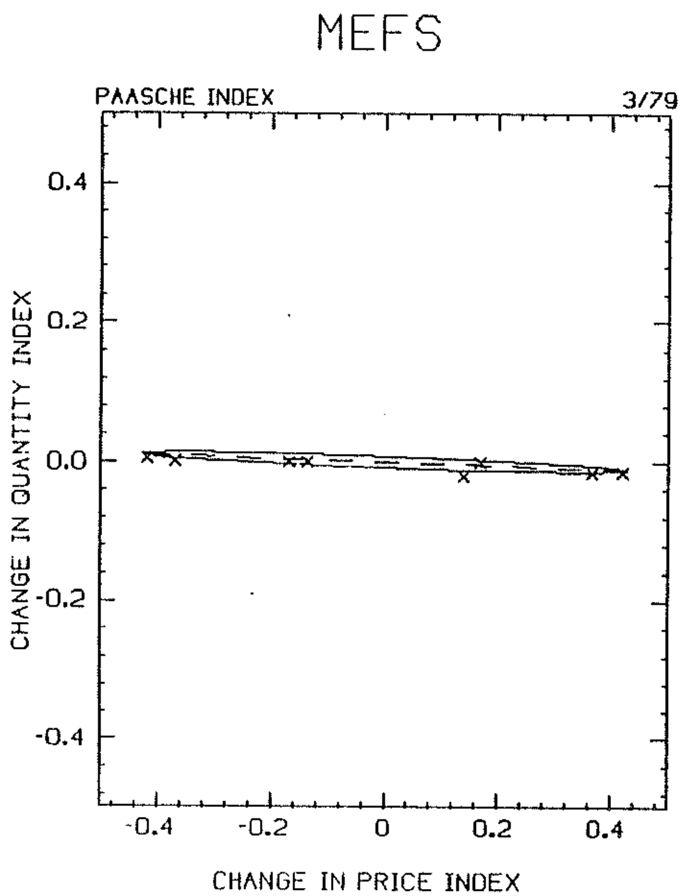


SECTOR: RESIDENTIAL
YEAR: 2000 (25-YEAR)
Primary Energy

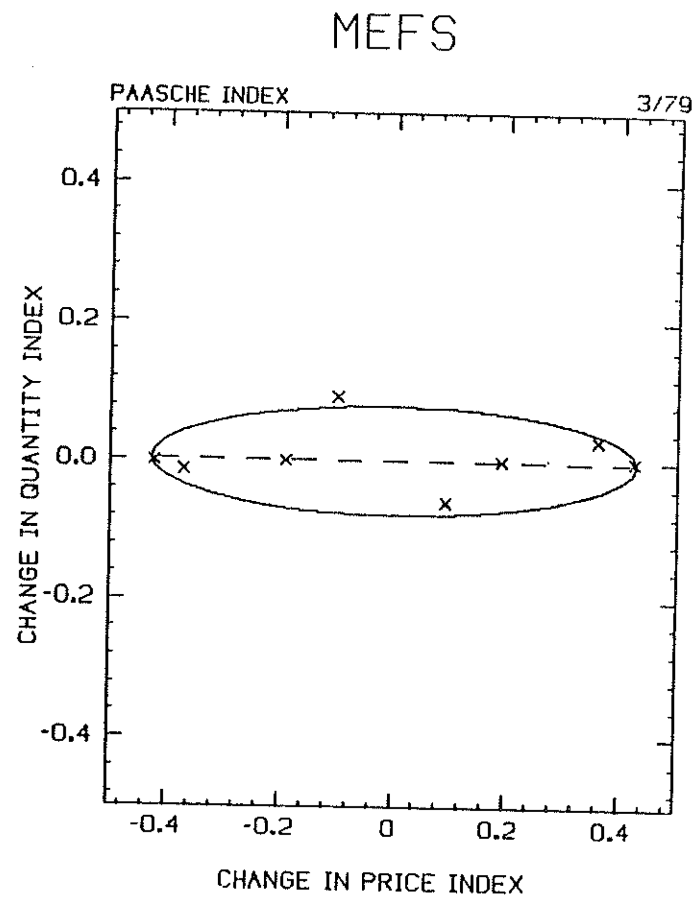


SECTOR: RESIDENTIAL/COMMERCIAL
YEAR: 2000 (25-YEAR)
Primary Energy

Figures 9B-13 and 9B-14

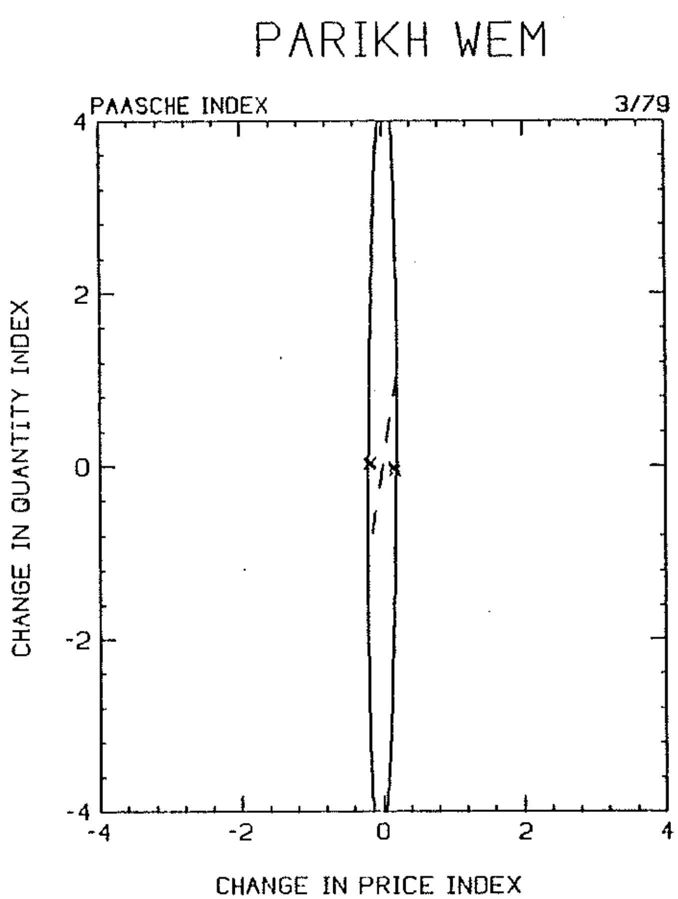


SECTOR: COMMERCIAL
YEAR: 2000 (25-YEAR)
Primary Energy

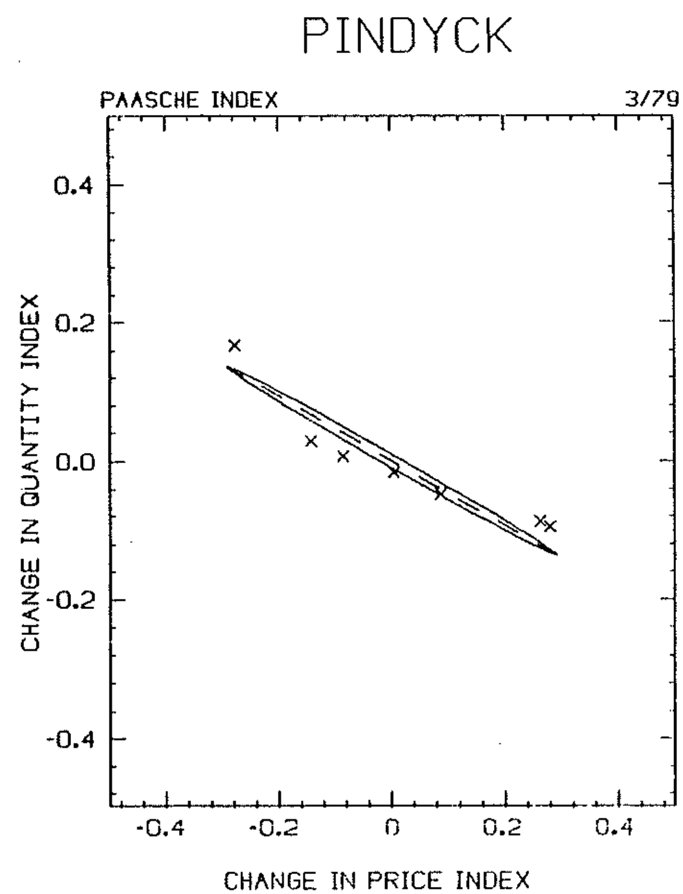


SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)
Primary Energy

Figures 9B-15 and 9B-16



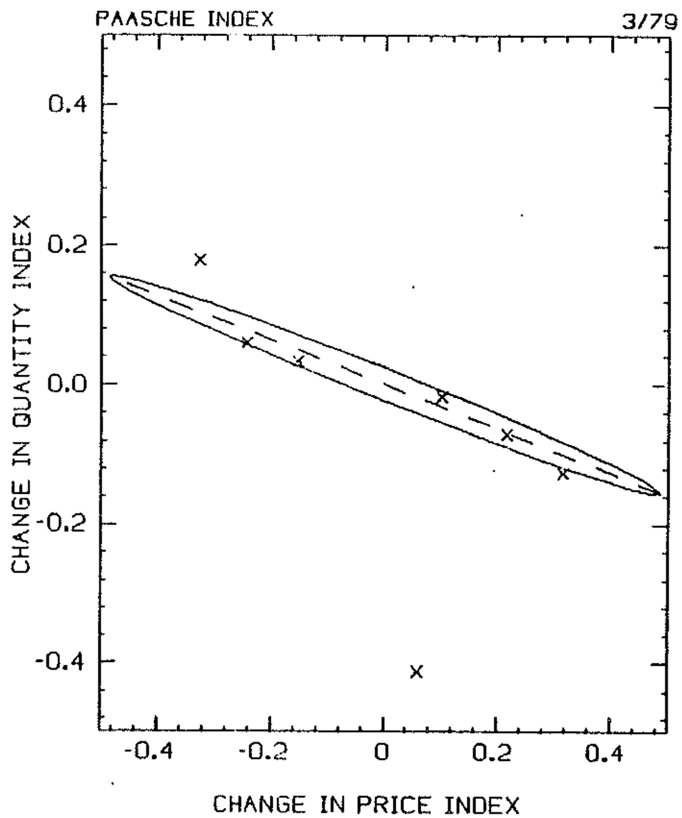
SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Primary Energy



SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Primary Energy

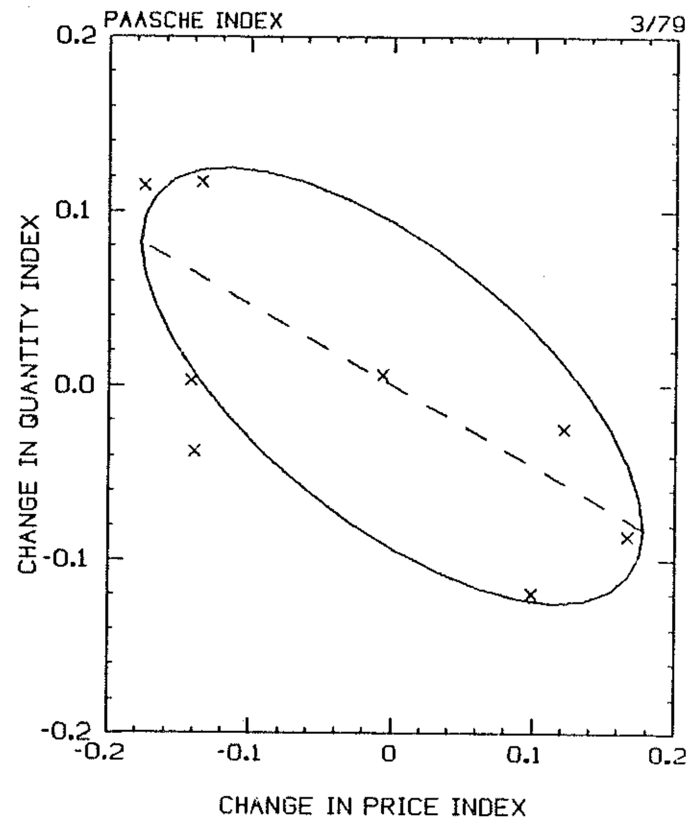
Figures 9B-17 and 9B-18

PINDYCK



SECTOR: COMMERCIAL/INDUSTRIAL
YEAR: 2000 (25-YEAR)
Primary Energy

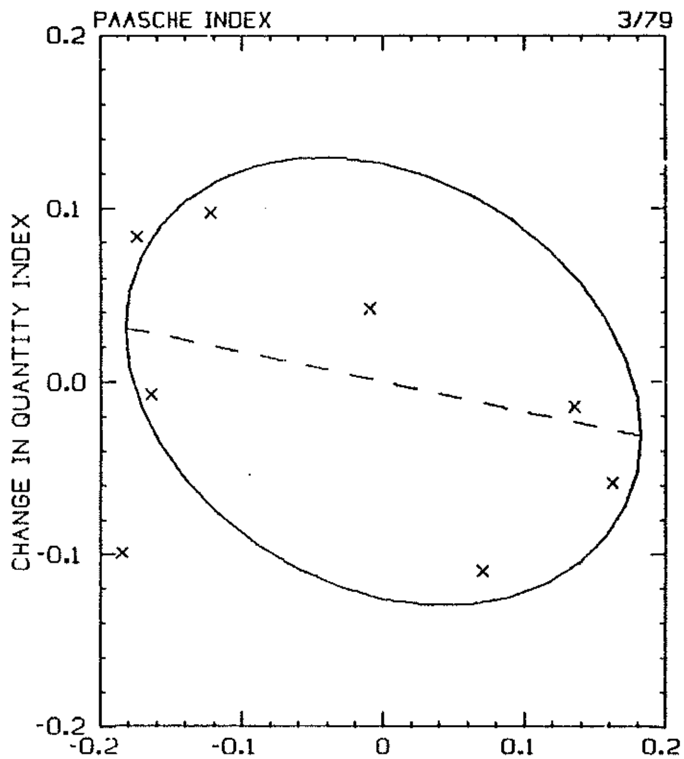
BAUGHMAN-JOSKOW



SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy

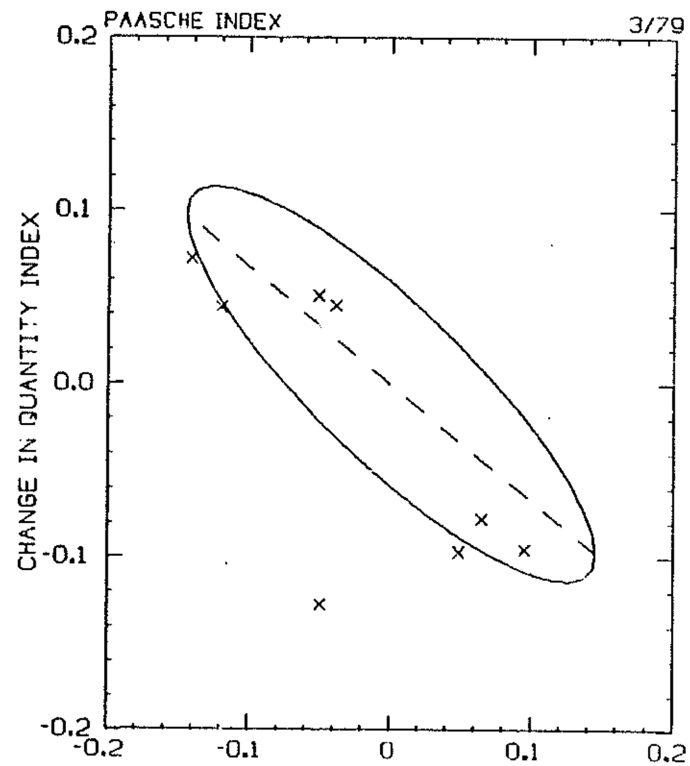
Figures 9B-19 and 9B-20

BAUGHMAN-JOSKOW



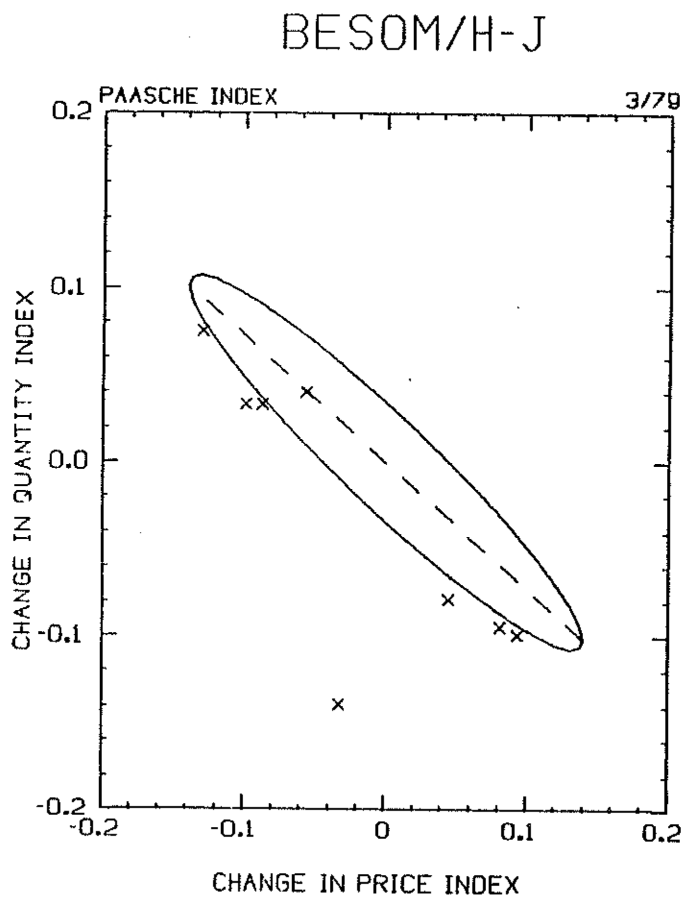
SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)
Secondary Energy

BESOM/H-J

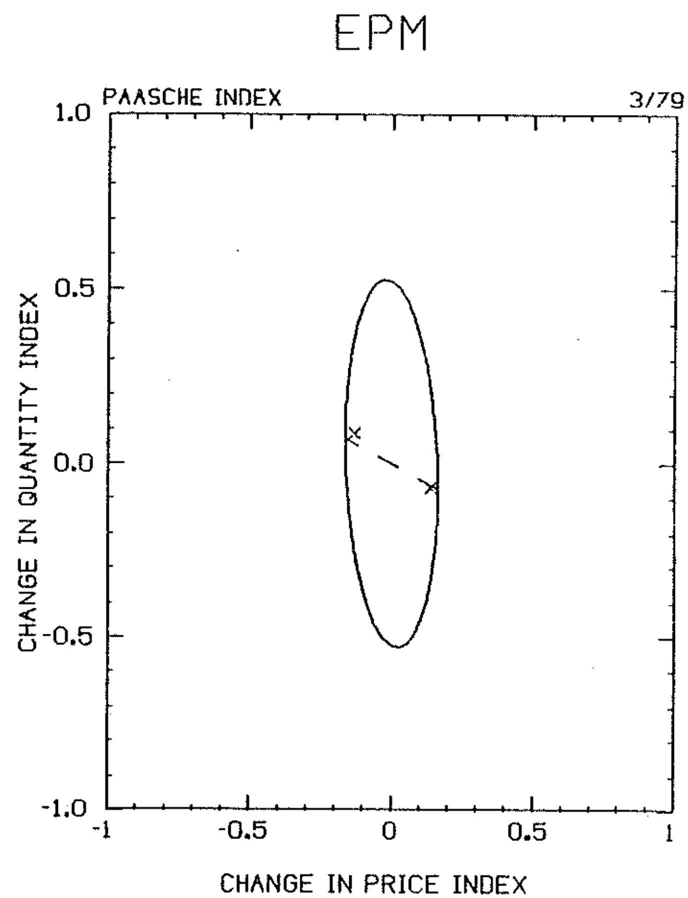


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy

Figures 9B-21 and 9B-22



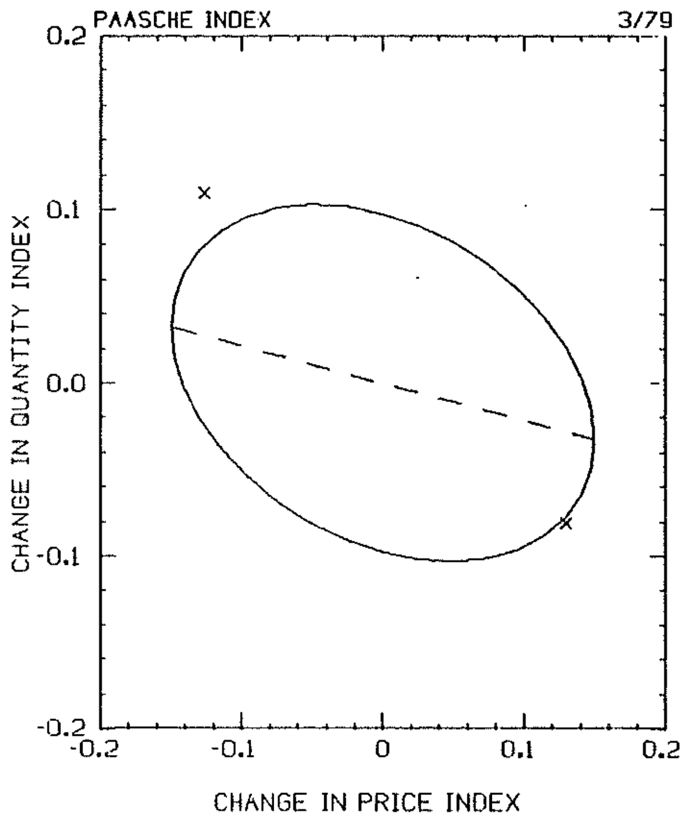
SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)
Secondary Energy



SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy

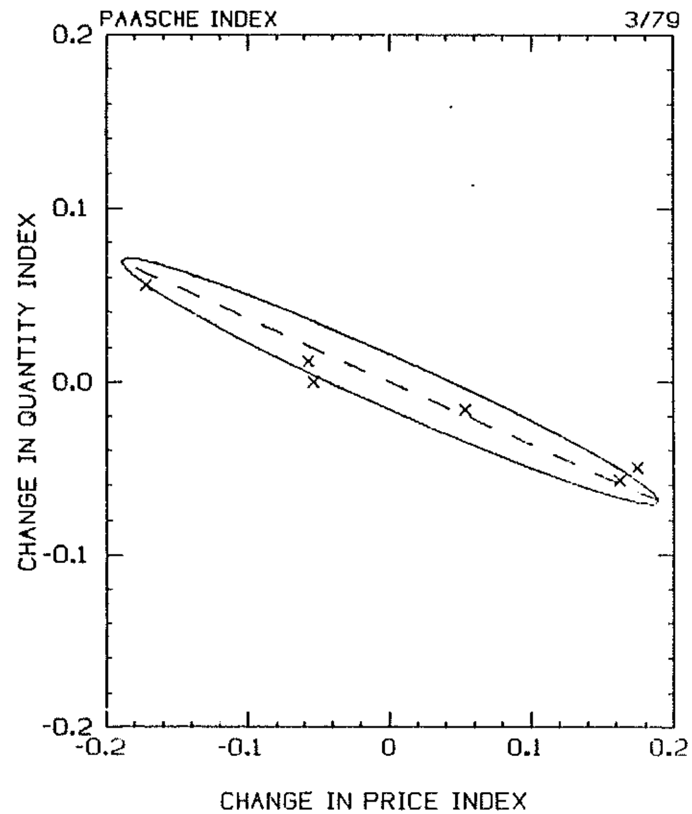
Figures 9B-23 and 9B-24

EPM



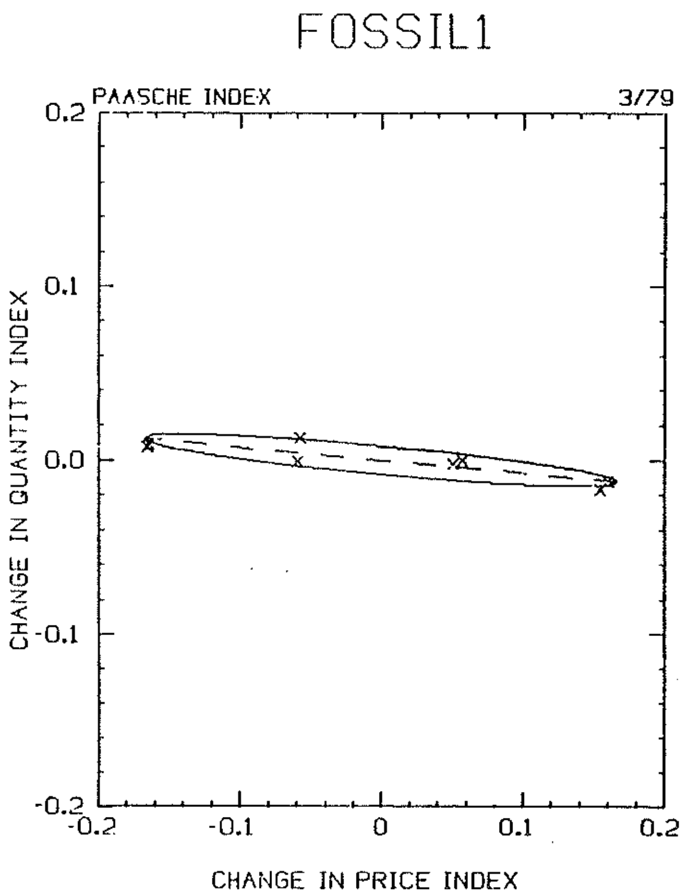
SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)
Secondary Energy

ETA-MACRO

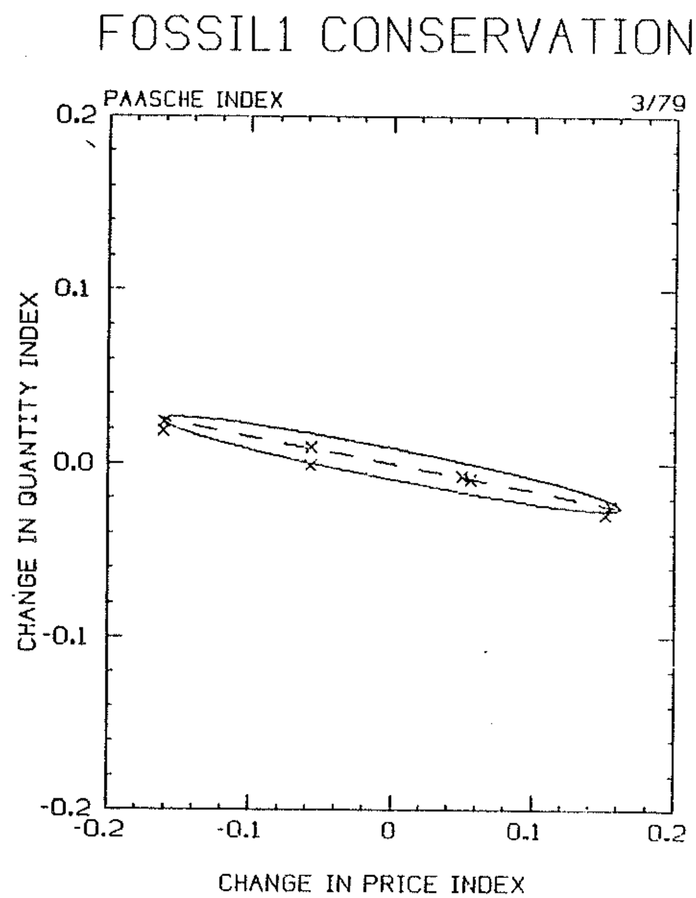


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy

Figures 9B-25 and 9B-26

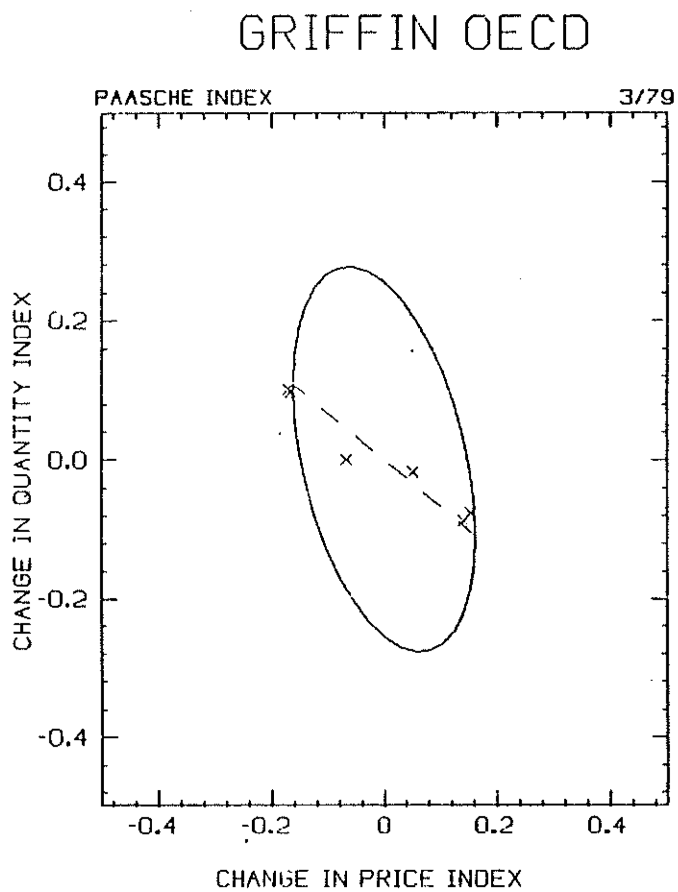


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy

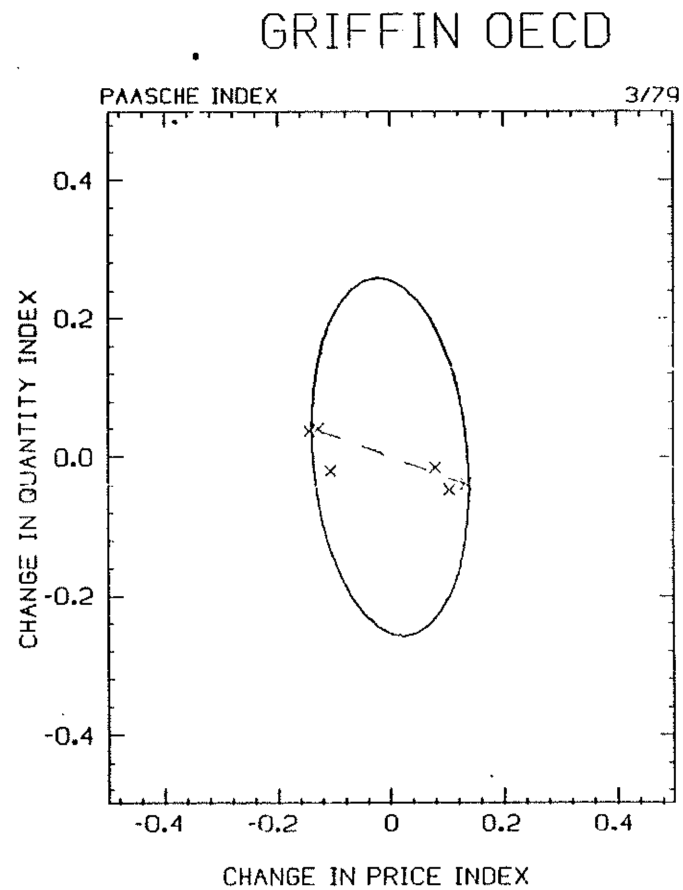


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy

Figures 9B-27 and 9B-28

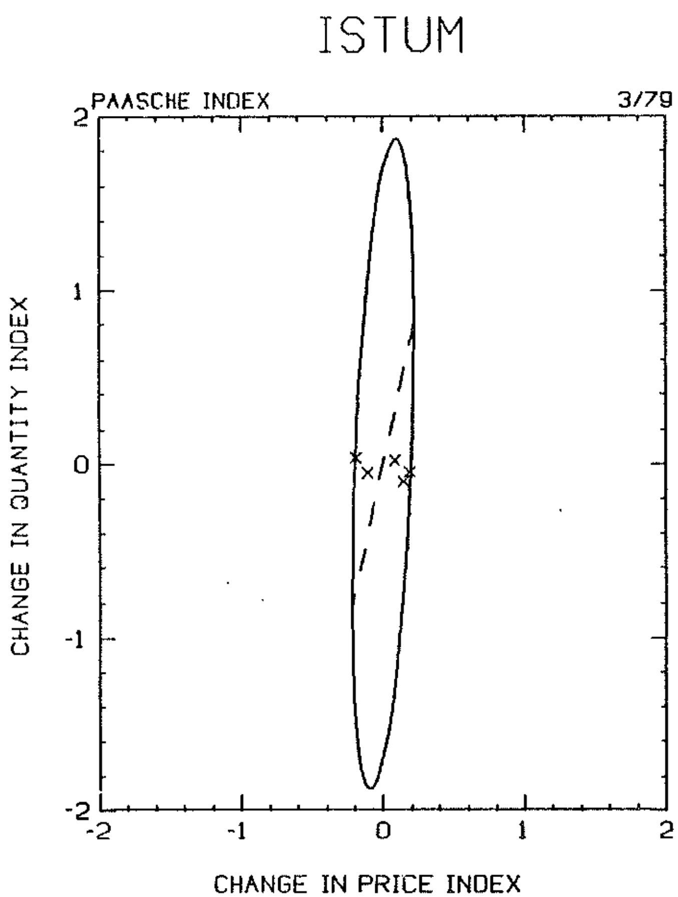


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy

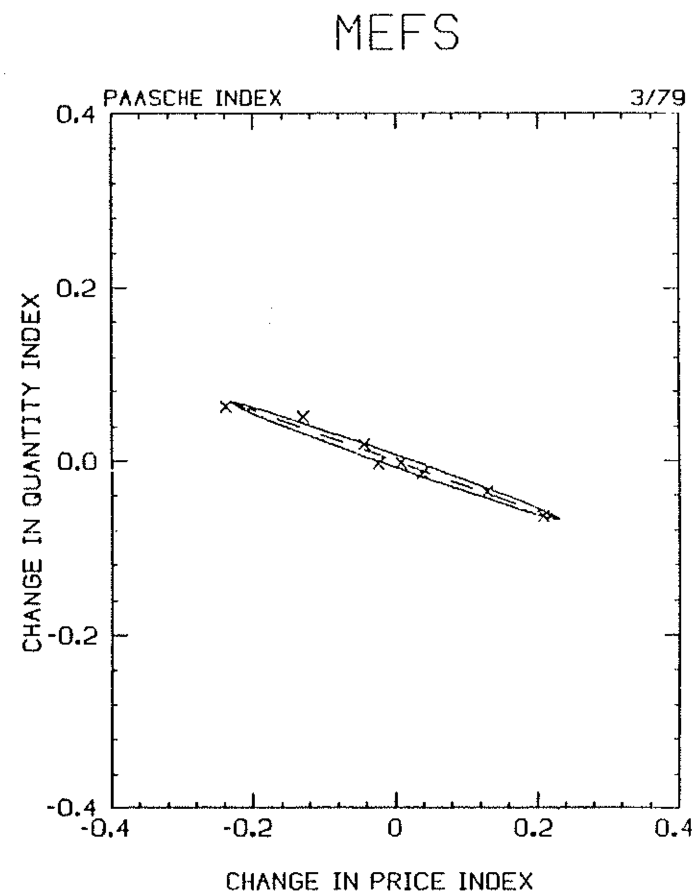


SECTOR: COMMERCIAL/INDUSTRIAL
YEAR: 2000 (25-YEAR)
Secondary Energy

Figures 9B-29 and 9B-30

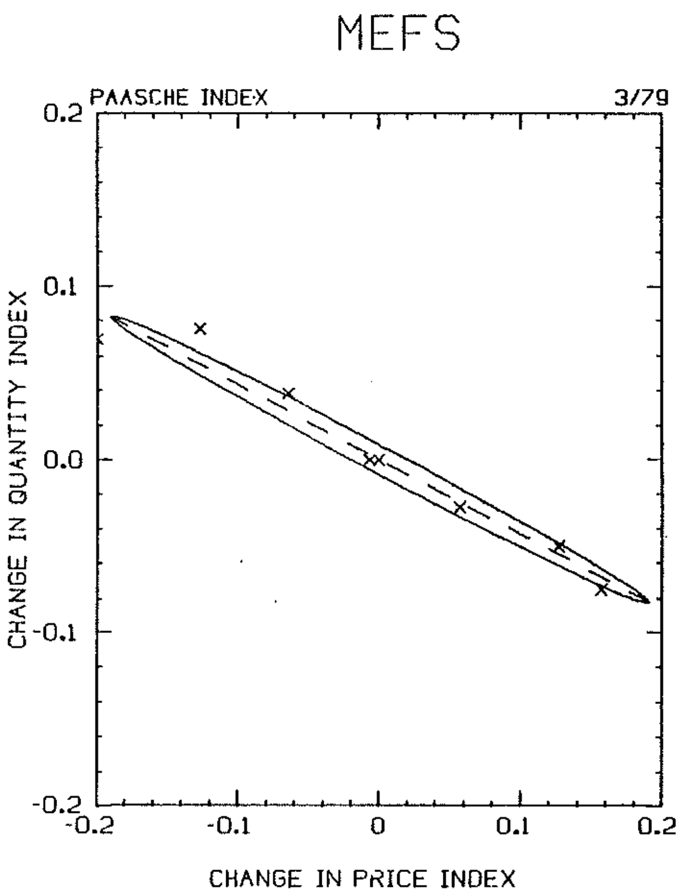


SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)
Secondary Energy

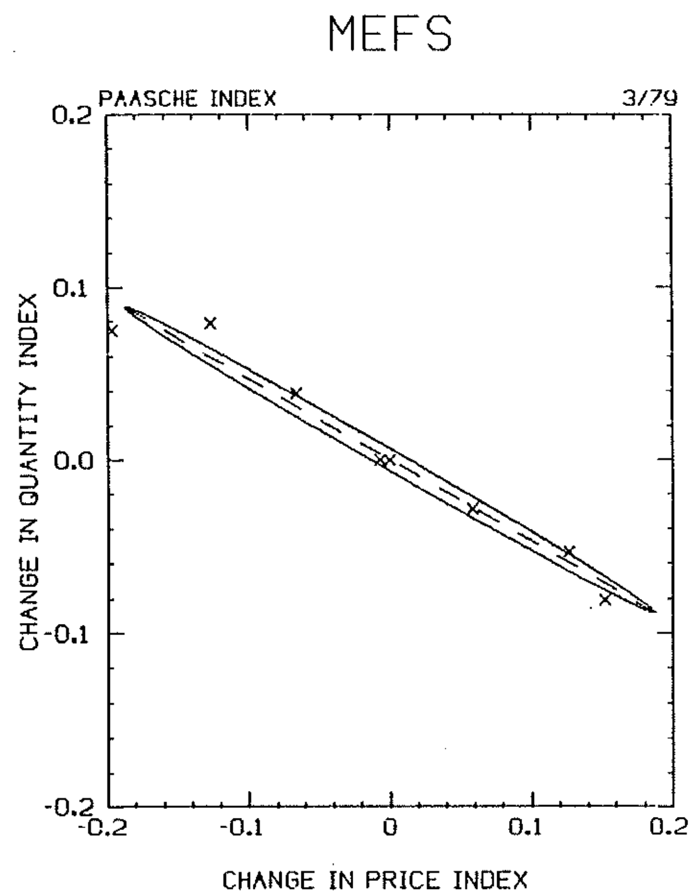


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy

Figures 9B-31 and 9B-32



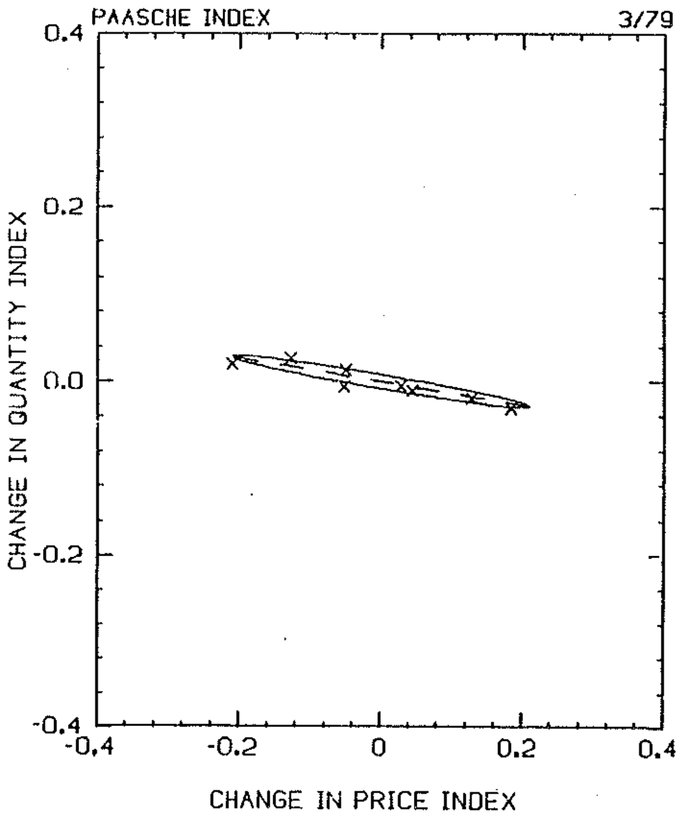
SECTOR: RESIDENTIAL
YEAR: 2000 (25-YEAR)
Secondary Energy



SECTOR: RESIDENTIAL/COMMERCIAL
YEAR: 2000 (25-YEAR)
Secondary Energy

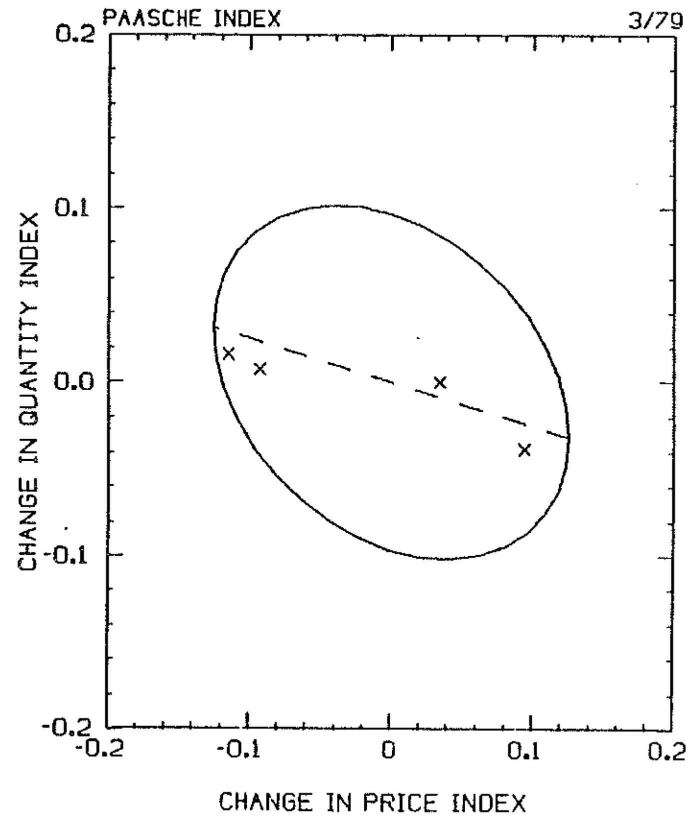
Figures 9B-33 and 9B-34

MEFS



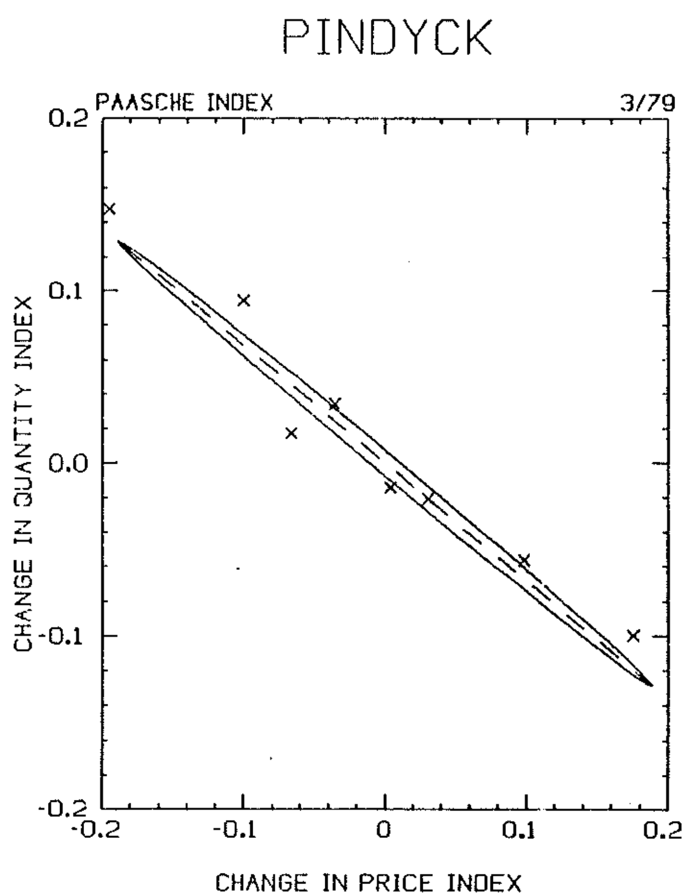
SECTOR: INDUSTRIAL
YEAR: 2000 (25-YEAR)
Secondary Energy

PARIKH WEM

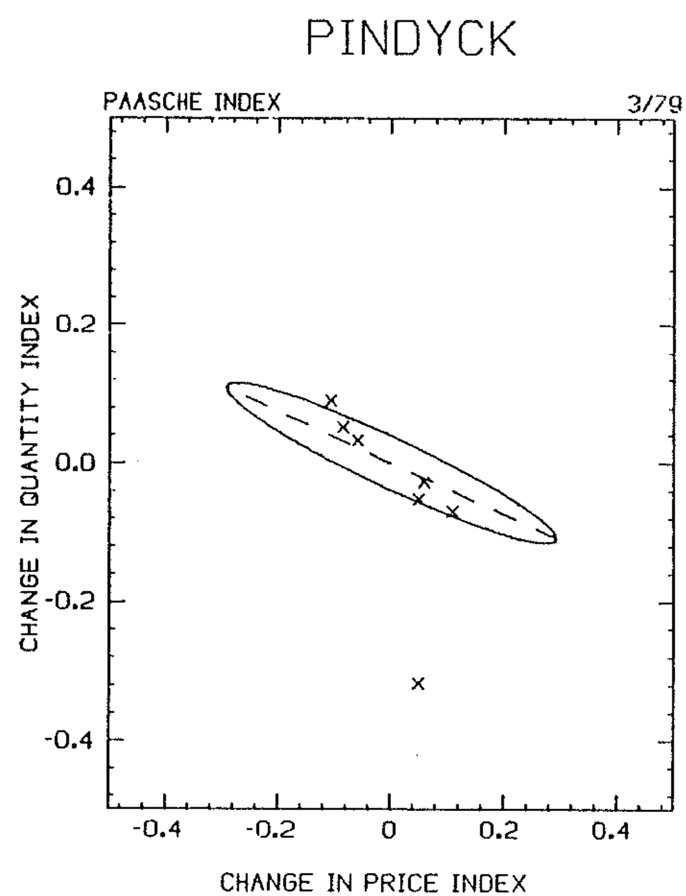


SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy

Figures 9B-35 and 9B-36



SECTOR: TOTAL DEMAND
YEAR: 2000 (25-YEAR)
Secondary Energy



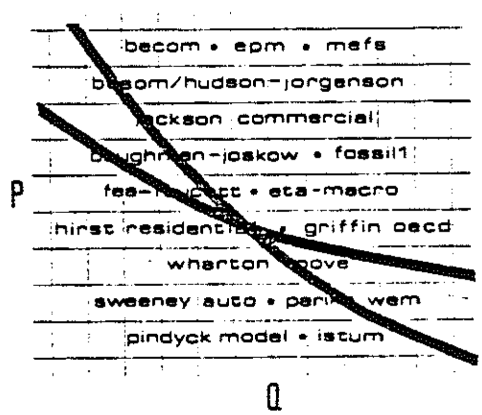
SECTOR: COMMERCIAL/INDUSTRIAL
YEAR: 2000 (25-YEAR)
Secondary Energy

Figures 9B-37 and 9B-38

Chapter 10

QUANTIFICATION OF UNCERTAINTY IN ELASTICITY ESTIMATES OBTAINED FROM THE ECONOMETRIC MODELS*

Lawrence J. Lau



*This chapter formerly was Working Paper EMF 4.5.

Chapter 10

QUANTIFICATION OF UNCERTAINTY IN ELASTICITY ESTIMATES OBTAINED FROM THE ECONOMETRIC MODELS

INTRODUCTION

There is a range of uncertainty associated with any demand elasticity estimate: The actual elasticity could be greater than or less than any of the point estimates presented in the "Aggregate Elasticity of Energy Demand" report. However, limitations in the current state of the art either preclude calculation of explicit uncertainty measures or make calculation extremely costly. Furthermore, the objective measures of uncertainty which could be conceptually calculated can never be expected to capture the subjective uncertainties in the minds of many potential model users or modelers. For these reasons, the goal of developing a satisfactory measure has not been reached within the present study. This chapter summarizes the difficulties involved, suggests a path for future research, and describes an appropriate estimation methodology.

DIRECT APPROACH

Let y be a scalar function of the vector of predetermined variables, X, and the vector of parameters, β, so that

$$y = f(X,\beta) , \qquad (10\text{-}1)$$

where f may be a nonlinear function of β for given X. It is desired to obtain an estimate of y, say $\hat{y}$, by substituting for β an estimator of β, say $\hat{\beta}$. Suppose that $\hat{\beta}$ is consistent and asymptotically normal with a known variance, then the variance of $\hat{y}$ may be approximated by:

$$v(\hat{y}) = \frac{\partial f'}{\partial \beta} v(\hat{\beta}) \frac{\partial f}{\partial \beta} , \qquad (10\text{-}2)$$

where $\frac{\partial f}{\partial \beta}(X,\beta)$ is evaluated at $\beta = \hat{\beta}$ and $v(\hat{\beta})$ is the variance of $\hat{\beta}$. Note however, that in general, $E\hat{y} \neq f(X,E\beta)$ unless $f(X,\beta)$ is linear in β.

Now suppose that $f(X,\beta)$ is, in fact, the formula for the demand elasticity; then its asymptotic variance can be readily estimated. In principle, this can be done for every model in the present study which provides estimates of the variance-covariance matrix of its parameters.

The difficulty of implementing such an approach lies in the fact that, except for very simple models, it is not possible to express the demand elasticity as a function of only the predetermined variables and parameters in closed form. One can still estimate the variance numerically, but the computation is likely to be elaborate.

However, it may be worthwhile to examine the models included in the "Aggregate Elasticity of Energy Demand" study to see if they are amenable to this direct approach.

STOCHASTIC SIMULATION

If the direct approach outlined above proves computationally intractable, a simpler but computer-time intensive method called stochcastic simulation can be used.

Let the system of equations that defines the model be written as

$$F(Y, X, \beta) = 0 \ , \tag{10-3}$$

where F is a vector-valued function and Y is a vector of the jointly determined variables of the model. Typically, the demand elasticity can be calculated as some scalar function of Y for given X and β, say

$$\varepsilon = f(Y, X, \beta) \ . \tag{10-4}$$

Given the estimator $\hat{\beta}$ of β and an estimate of $v(\hat{\beta})$, and assuming that $\hat{\beta}$ is asymptotically normal, one can make random drawings of $\hat{\beta}$ from its underlying multivariate normal distribution. Given any such drawing, one can solve Eq. 10-3 for Y, and given X and Y, can solve for a value of the demand elasticity. One can repeat this process to obtain a sequence of estimates for the demand elasticity, ε. Given this sequence of estimates, one can compute an estimate of the mean and variance of the demand elasticity. If the number of drawings is sufficiently large, this method should yield an estimate of the variance that is close to the true variance. In other words, one can do a Monte Carlo study to estimate the error properties.

If the model is large and complex, this may be the only feasible method to obtain any answer at all. This method need not be particularly expensive, but it does require running the models over and over again for different sets of parameter values that are drawn from the underlying distribution.

PARAMETER ESTIMATION ERROR VERSUS FORECAST ERROR

Although the two are related, one should not confuse the error in the estimate of a parameter, such as the demand elasticity, with the error in a conditional forecast, such as the quantity of energy demanded if the price of energy is X. The error in a conditional forecast comes from two separate sources--the error in the estimation of the parameters β and the intrinsic stochastic error in equations of the model. In order to estimate the standard error of a conditional forecast of energy demand, it is better to approach the problem directly, rather than through the demand elasticity. In general, one may write

$$\hat{y}_o = f(X, \hat{\beta}, \varepsilon_o) \quad , \qquad (10\text{-}5)$$

where $\hat{y}_o$ is a forecast and ε_o is a vector of stochastic disturbances. Then

$$v(\hat{y}_o) = \begin{bmatrix} \frac{\partial f}{\partial \beta} \\ \\ \frac{\partial f}{\partial \varepsilon_o} \end{bmatrix}' v\begin{pmatrix} \hat{\beta} \\ \varepsilon_o \end{pmatrix} \begin{bmatrix} \frac{\partial f}{\partial \beta} \\ \\ \frac{\partial f}{\partial \varepsilon_o} \end{bmatrix} ,$$

where $\frac{\partial f}{\partial \beta}$ and $\frac{\partial f}{\partial \varepsilon_o}$ are evaluated at $\beta = \hat{\beta}$ and $\varepsilon_o = 0$. It should be noted that, in general, $c(\hat{\beta},\varepsilon_o)$ is assumed to be zero, since $\hat{\beta}$ is presumably based on past observations.

It may be difficult to have a closed form representation for Eq. 10-5, so that stochastic simulation may again be the only tractable way to attack this problem. Thus, starting from

$$F(Y, X, \beta, \varepsilon_o) = 0 \quad , \qquad (10\text{-}6)$$

one can draw $\binom{\beta}{\varepsilon_o}$ from the underlying distributions. We note that in the form of Eq. 10-6, most models can be written as

$$F^*(Y, X, \beta) + \varepsilon_o = 0 \quad . \tag{10-7}$$

Equation 10-7 is a substantial simplification of Eq. 10-6.

From forecasts of Y and its estimated standard errors, one can construct a forecast of y_o and its estimated standard error.

LINEARIZATION APPROACH

A low cost alternative that may work well is to linearize the complete model in β and ε around the equilibrium solution values. One can then proceed as if the model were linear; and computation of the estimates of the error variance becomes quite straightforward. There is, however, no real assurance that such estimates will be close to the true values, although limited experience with stochastic simulation of other models suggests that the procedure should work reasonably well. Moreover, this procedure is likely to be relatively inexpensive, and also should take relatively little time to implement. This may be the way to obtain some preliminary idea of the degree of uncertainty in estimates of demand elasticities while efforts along the lines indicated above are pursued.

Chapter 11

THE DYNAMICS OF ELASTICITY ESTIMATION

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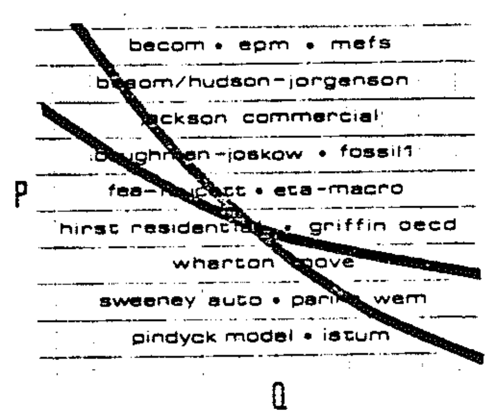

Chapter 11

THE DYNAMICS OF ELASTICITY ESTIMATION

INTRODUCTION

The "Aggregate Elasticity of Energy Demand" study attempted to measure the aggregate price elasticity of energy demand implicit in 16 energy demand and energy-economy models by using data generated through controlled runs of these models. The experiment was set up whereby individual prices were varied, and both these prices and resulting fuel demands were aggregated and compared across scenarios to determine energy demand elasticities. In this chapter, we discuss some of the problems encountered when estimating elasticities in a dynamic context, and we describe a few alternative approaches to the problem.

In static models, the demand elasticity is a well defined concept, being the percentage change in demand in response to a percentage change in price. In dynamic models, however, the picture is a bit more complicated; a vector or time sequence of prices results in a time sequence of demands. Demand responses depend on the nature of the price path, as well as the terminal price level. When we speak of a price change, therefore, we need to specify which elements of a price sequence are being changed.

Because of this ambiguity, a price elasticity is normally thought of as the limiting percentage change in demand in response to a change in the terminal steady-state level of price relative to the initial level of price. Most analysts usually assume that demand will fully adjust to new price levels within 20 years. The elasticity measured in this manner is referred to as the long-run elasticity of demand.

In this chapter, we show that estimates of long-run demand elasticities depend on both the experiment run to generate data, i.e., the sequence of prices chosen, and the procedure used to estimate the elasticities. We also show that, even if it is known exactly, the long-run demand elasticity is useful only when prices remain constant for an extended time after a shift in prices. We use empirical results obtained using the Hirst Residential model [1] to illustrate the ideas we discuss.

Our major conclusion is that the long-run demand elasticity by itself is an interesting measure, but has limited applicability in time frames under 20 or 25 years. It is more informative to account for the dynamics of energy demand explicitly, i.e., to consider both short- and long-run elasticities, when prices are expected to change over time.

MEASUREMENT OF THE LONG-RUN ELASTICITY IN A DYNAMIC CONTEXT

The Experiment

Because the choice of experiment is by no means unique, a crucial question arises as to what extent our measure of aggregate demand elasticities depends upon our choice of experiment. We wish to design experiments that can yield maximum information about demand response over time.

Here we define an experiment in terms of a price path for energy or energy fuels. We assume all other model parameters are fixed at some base level. We also assume that all model runs start from the same point (E_0,P_0) and that the price P is changed in period 1 and in all periods thereafter.

Three general price paths are commonly used. They can be described as:

- a one time step change in prices with prices held constant thereafter;
- a percentage jump over some set base price path (the "Aggregate Elasticity of Energy Demand" experiment); and
- a variation in the rate of growth of prices (called the rate experiment).

Static Estimation Models

Many energy demand models include dynamics in one form or another. A simple dynamic model is used to analyze the use of a static model as the basis for estimating long-run demand elasticities. We assume that the underlying dynamic process is correctly described by a constant geometric lag

$$E_t = KE_{t-1}^{\lambda} P_t^{\beta} \quad . \tag{11-1}$$

In logarithmic form this is

$$\ell n\ E_t = \ell n\ K + \lambda \ell n\ E_{t-1} + \beta \ell n\ P_t \quad ,$$

where λ and β are constants independent of the price.

Three parameters are of interest: the lag parameter λ ; the short-run demand elasticity β ; and the long-run demand elasticity $\sigma = \beta/(1-\lambda)$. Assume that estimates $(\hat{\beta},\hat{\sigma})$ are made of β and σ , and that an estimate $\hat{\lambda}$ of the lag parameter λ is determined as

$$\lambda = 1 - \frac{\hat{\beta}}{\hat{\sigma}} \ . \qquad (11\text{-}2)$$

We examine both short-run (1 period) elasticity estimates $\hat{\beta}$ and long-run estimates $\hat{\sigma}$, and use elasticity estimates made after 20 and 25 years as approximations of the long-run elasticity. Equation 11-1 can now be expressed in closed final form as

$$E_t = E_0^{\lambda^t} K^{\left[\frac{1-\lambda^t}{1-\lambda}\right]} P_t^{\beta} P_{t-1}^{\beta\lambda}, \ldots, P_1^{\beta\lambda^{t-1}} \qquad (11\text{-}3)$$

$$\equiv K_t P_t^{\beta} P_{t-1}^{\beta}, \ldots, P_1^{\beta\lambda^{t-1}}$$

<u>The Estimators</u>. We assume elasticity estimates are made on a pairwise basis, i.e., two runs are made of the model, each with a different set of inputs. The results of these runs are then used to compute the elasticities. We use two estimators referred to as the EMF estimator and the arc elasticity estimator. Both estimators are based on a static model of demand.

The EMF estimators of elasticities are based on the equation

$$\ell n\ E_t = A_t + \sigma_t\ \ell n\ P_{Et} \ , \qquad (11\text{-}4)$$

where t denotes the time period. The EMF estimators are defined as

$$\hat{\beta} = \frac{\ell n\ (E_1/E_1)}{\ell n\ (P_1^1/P_1^2)} \qquad (11\text{-}5a)$$

$$\hat{\sigma} = \frac{\ell n\ (E_t^1/E_t^2)}{\ell n\ (P_t^1/P_t^2)} \qquad (11\text{-}5b)$$

where $t = 20$ or 25 , and

$$\hat{\lambda} = 1 - \frac{\hat{\beta}}{\hat{\sigma}} , \qquad (11\text{-}5c)$$

where

E_t^i = energy demand in period t for scenario i ,

P_t^i = energy price in period t for scenario i ,

$\hat{\beta}$ = estimate of the short-run (1 period) price elasticity,

$\hat{\sigma}$ = estimate of the long-run price elasticity, and

$\hat{\lambda}$ = estimate of the lag parameter.

The arc elasticity estimators are defined as

$$\varepsilon_{arc} = \frac{E^2 - E^1}{P^2 - P^1} \cdot \frac{P^1 + P^2}{E^1 + E^2} , \qquad (11\text{-}6)$$

where the one-period estimate is taken as $\hat{\beta}$ and periods $t = 20$ and $t = 25$ are taken as estimates of σ . $\hat{\lambda}$ is defined as in the EMF case.

Note that Eq. 11-4, which is used as the basis for the EMF estimators, is the long-run limit of the dynamic model, Eq. 11-1, when $E_t = E_{t-1}$ and $\sigma = \beta/(1-\lambda)$. Use of Eq. 11-4 is usually justified by assuming that 20 or 25 years is a long enough time period to closely approximate the full adjustment of demand to price changes. We show that this assumption is not always valid.

Biases in Static Model Estimators. We examine approximation errors incurred in estimating β , λ , and σ from both the EMF estimator, Eq. 11-5, and the arc elasticity estimator, Eq. 11-6. Appendix A contains details of our calculations. The results are summarized in Table 11-1. Table 11-2 lists the percentage errors between estimated long-run elasticities, using the EMF estimator, and the actual values.

Although each experiment always underestimates the elasticity response of the true model, both the step perturbation experiment and the EMF 4 experiment provide a close approximation when λ is less than 0.9 and $t = 20$ or 25. When $\lambda = 0.95$, the elasticity measurement would have to be made at $t = 45$ or greater to have less than a 10% error. On the other hand, estimates obtained from the rate-of-growth experiment are significantly less than the elasticity of the true model for any realistic time frame.

Table 11-1

SUMMARY OF ESTIMATOR BIASES

Elasticity Estimate[a]	Experiment	Short-Run	Long-Run
EMF	Step change	Exact	$\sigma(1 - \lambda^t)$
	EMF	Exact	$\sigma(1 - \lambda^t)$
	Rate change	Exact	$\sigma[1 - \frac{\lambda}{t}(\frac{1 - \lambda^t}{1 - \lambda})]$
Arc	Step change	Close, depends on interval	$\lvert\sigma_{EMF}\rvert < \lvert\sigma_{arc}\rvert < \lvert\sigma\rvert$
	EMF	Close, depends on interval	$\lvert\sigma_{EMF}\rvert < \lvert\sigma_{arc}\rvert < \lvert\sigma\rvert$
	Rate change	Very close	very close to EMF estimator

[a]See Appendix A for exact calculations.

Table 11-2

PERCENTAGE ERROR IN LONG-RUN ELASTICITY ESTIMATES USING THE EMF ESTIMATOR

λ	Step Perturbation and EMF 4 Experiments		Rate-of-Growth Change Experiment	
	t = 20	t = 25	t = 20	t = 25
0.8	- 1	--	-20	-16
0.9	-12	- 7	-40	-33
0.95	-36	-28	-61	-55

<u>Discussion of Static Model Estimators</u>. Our major conclusion is that measurement of the demand elasticity using estimators based on static models is dependent upon the experiment performed (in this case, the time path of energy prices). For each of the estimators considered, the step perturbation and EMF 4 experiments allow better estimates of true parameter values than does the rate-of-growth

variation experiment. Estimators obtained with the rate-of-growth experiment underestimate true parameter values.

The accuracy of the EMF and arc estimators depends on the parameter being estimated. For the short-run elasticity β, the EMF estimator is exact in each experiment while the arc elasticity is slightly larger than the true value. For the long-run elasticity estimates, the arc elasticity estimate gives a value less than the true value but larger than the EMF estimator.

Analysts sometimes use estimators measured at point t rather than the long-run elasticity to evaluate changes in demand corresponding to different price levels at time t. This practice suffers from the same difficulty as encountered with long-run elasticities: namely the elasticity estimate will depend on the time path of prices and not just the price level at the point of measurement. See the use of the long-run demand elasticity section for further discussion of this point.

This raises a question about our ability to measure elasticity parameters exactly. When we know the exact parameters to Eq. 11-1, we can predict exact demand responses for any price path or price level. Exact measurement of the parameters is possible when we use Eq. 11-1 rather than the static model, Eq. 11-4, as the basis of our estimation procedure. Since there are only two independent parameters out of the three of interest, β, λ, and σ, we only estimate two of them. We show in Appendix B that we obtain more accurate results if we estimate β and λ rather than β and σ. Presented in the next section are two general dynamic estimation procedures that can be used to estimate β and λ.

A final question is whether the results obtained generalize to models with more than one exogenous variable. In Appendix C, we describe two ways in which income can be incorporated in the model, concluding that, for these cases, the problem is analogous to the simple one-variable model.

<u>Dynamic Estimation Models</u>

As seen in the previous section, the selection of price paths for an experiment is extremely important in determining our ability to estimate long-run elasticities of demand. Many models contain some representations of dynamics and will respond differently to alternative price paths. A natural next step is to treat dynamic behavior explicitly by using Eq. 11-1 as the basis of our estimates. Use of this equation can overcome many of the shortcomings resulting from the use of a single elasticity measure to represent demand behavior over all time.

We introduce two methods of estimation based on Eq. 11-1. We estimate the short-run elasticity β and the lag parameter λ and compute the long-run elasticity estimate $\hat{\sigma}$ as

$$\hat{\sigma} = \frac{\hat{\beta}}{1 - \hat{\lambda}} \quad . \tag{11-7}$$

For this dynamic estimation process, we make no explicit assumptions about the true underlying model. We wish to determine parameters of our simple model that allow the best characterization of the output of the actual model for all time periods. Define

$$e_t = \ell n\, E_t$$

and

$$P_t = \ell n\, P_t \quad .$$

Equation 11-1 can be expressed as

$$e_t = \lambda e_{t-1} + \beta P_t \quad . \tag{11-8}$$

We can obtain parameter estimates $\hat{\lambda}$ and $\hat{\beta}$ using one of two methods if we are given a number of runs of the actual model and a number of observations of the sequences $[(P_t, e_t)]$. These two methods are referred to as the structural form method and the final form method, respectively. Derived estimates of e_t can be represented as $\hat{e}_t$.

The structural form method emphasizes the single period response of a model. This method estimates period t's demand as a function of the price in period t and the actual demand of period t-1 . Thus we have the expression

$$\hat{e}_t = \hat{\lambda} e_{t-1} + \hat{\beta} P_t \quad . \tag{11-9}$$

Our objective is to find estimates $\hat{\lambda}$ and $\hat{\beta}$ which allow the best characterization of Eq. 11-8. This objective requires that we minimize the goodness of fit, which is defined as

$$L_s = \sum_{t=1}^{T} (e_t - \hat{\lambda} e_{t-1} - \hat{\beta} P_t)^2 \quad . \tag{11-10}$$

This is a standard least-squares regression problem and $\hat{\lambda}$ and $\hat{\beta}$ can be determined by solving the simultaneous equations

$$\hat{\lambda} = \frac{\Sigma e_t e_{t-1} - \hat{\beta} \Sigma P_t e_{t-1}}{\Sigma e_{t-1}^2} \tag{11-11a}$$

and

$$\hat{\beta} = \frac{\Sigma e_t P_t - \hat{\lambda} \Sigma e_{t-1} P_t}{\Sigma P_t^2} \quad . \tag{11-11b}$$

In many applications, we wish to consider the demand approximation in not only one period, but all periods. The final form method takes this objective into account explicitly.

The final form estimator is based on the equation

$$\hat{e}_t = \hat{\lambda} \hat{e}_{t-1} + \hat{\beta} P_t \quad , \tag{11-12}$$

where we assume that e_0 and $(P_1, P_2, \ldots, P_T)$ are given. This form estimates period t's demand as a function of the current period price and the estimate of the previous period's demand. Equation 11-12 can be expressed in terms of all exogenous inputs by writing it in the final form

$$\hat{e}_t = \hat{\lambda}^t e_0 + \sum_{i=1}^{t} \hat{\lambda}^{t-i} \hat{\beta} P_i \quad . \tag{11-13}$$

Again, our objective is to find estimates $\hat{\lambda}$ and $\hat{\beta}$ that minimize the goodness of fit. For the final form, the goodness of fit is expressed as

$$L_F = \sum_{t=1}^{T} (e_t - \hat{\lambda}^t e_0 - \sum_{i=1}^{T} \hat{\lambda}^{t-i} \hat{\beta} P_i)^2 \quad . \tag{11-14}$$

A model giving the best global approximation is determined by minimizing the goodness-of-fit function, Eq. 11-14, rather than the one in Eq. 11-10. Unfortunately, minimizing L_F involves equations which are nonlinear in $\hat{\lambda}$. Appendix D outlines the algorithm we use to estimate λ and β. It also explains modifications necessary when data for each year is not available from the true model's output.

Because we make no assumptions about the structure of the models being characterized, no general statement can be made about biases in the parameter estimates. If Eq. 11-1 accurately characterizes the dynamic structure of the true model, both structural and final form estimators give exact estimates. The final form-estimated model will always give better estimates than the structural form-estimated model. However, we are unable to make any a priori statements as to which experiment leads to better estimates.

USE OF THE LONG-RUN DEMAND ELASTICITY IN A DYNAMIC CONTEXT

The purpose of measuring the demand elasticity implicit in the energy demand and energy-economy models is to allow us to easily characterize how energy demand projections of the models will change given a change in the price of energy.

This elasticity measure, combined with the simple model

$$E^t = a_t P_t^{\sigma} , \qquad (11\text{-}15)$$

allows quick, back-of-the-envelope calculations of demand changes for policy discussions or for energy-economic analysis.

This type of back-of-the-envelope analysis ignores how prices get to the level of interest. For example, if we want to examine a doubling in energy prices by the year 2000, do prices increase at a constant rate between now and then, do they increase today and stay constant thereafter, or do they follow some other time path? We examine the effects of alternative price paths by assuming (as discussed above) that the underlying dynamic process is correctly described by the constant geometric lag model Eq. 11-1.*

Appendix D shows the calculations for both the step-change price path and the rate-change price path. Tables 11-3 and 11-4 show errors in predictions of demand

*We assume that quantities and prices are scaled so that the constant K in Eq. 11-1 is unity.

Table 11-3

DEMAND PREDICTIONS GIVEN A STEP CHANGE IN PRICES
(percentage difference from actual)

σ	λ 0.8 (%)	λ 0.9 (%)	λ 0.95 (%)
-0.2	0	-2	- 5
-0.4	0	-3	-10
-0.6	0	-5	-14
-0.8	-1	-6	-18

Table 11-4

DEMAND PREDICTIONS GIVEN A RATE CHANGE IN PRICES
(percentage difference from actual)

σ	λ 0.8 (%)	λ 0.9 (%)	λ 0.95 (%)
-0.2	- 3	- 5	- 8
-0.4	- 5	-10	-15
-0.6	- 8	-15	-22
-0.8	-10	-19	-28

changes resulting from a doubling of prices for various combinations of long-run elasticity and lag values. Table 11-3 shows the error when prices double today and remain constant thereafter; Table 11-4 shows the errors when prices increase at a constant rate (3.5%) over 20 years.

It can be seen from the tables that the errors are larger for both longer lags and higher long-run elasticities. All predictions overstate the decrease in energy demand. (For price increases, they would overstate the increase in energy demand.) The prediction error from the rate-change-in-price scenario is always significantly larger than the prediction errors from the step-change-in-price scenario.

NUMERICAL EXAMPLE

Estimation methodologies outlined in the earlier sections were applied to runs of the Hirst Residential energy demand model. We estimated elasticities for each fuel (electricity, gas, and oil), and not for aggregate energy. The major observations should be the same for either case.

Two data sets were used. Each set consisted of 56 runs of the Hirst Residential model. The first set was generated by using step changes in the price variables beginning in 1985. The second set was generated by introducing various rates of growth for each of the prices, again starting in 1985. Prices in the year 2000 are the same for corresponding step-change and rate-of-growth-change scenarios.

Table 11-5 presents the elasticity and lag estimates generated by each estimation method for the Hirst Residential model. The EMF estimation procedure uses the static model, Eq. 11-4, with least-squares regressions to estimate the σ_t values. To obtain parameter estimates, the 56 model runs in each set are used rather than the pairwise procedure outlined above.

Table 11-5

ELASTICITY AND LAG ESTIMATES[a]

Fuel	Structural[b]	Final Form[b]	EMF[c] 1995	EMF[d] 2000
Electricity				
Step Scenarios	-0.52 (0.81)	-0.50 (9.78)	-0.45	-0.51
Rate Scenarios	-0.58 (0.83)	-0.51 (0.77)	-0.36	-0.40
Gas				
Step Scenarios	-0.68 (0.84)	-0.62 (0.79)	-0.56	-0.63
Rate Scenarios	-0.79 (0.86)	-0.61 (0.74)	-0.46	-0.51
Oil				
Step Scenarios	-1.10 (0.87)	-0.99 (0.83)	-0.83	-0.96
Rate Scenarios	-1.31 (0.89)	-0.98 (0.80)	-0.65	-0.74

[a]Elasticities are estimates of the long-run elasticity.

[b]Values in parentheses are estimates of the lag values.

[c]This is an observation after 11 years.

[d]This is an observation after 16 years.

From Table 11-5 we see that elasticity estimates from the EMF 4 procedure are about 20% lower for rate-of-growth perturbation scenarios than step-change scenarios, as predicted in the Static Model Estimator section above. Elasticity and lag estimates from the structural and final form procedures are similar to each other, with estimates from the structural form procedure being slightly higher. However, the difference is not enough to be considered significant.

SUMMARY AND CONCLUSIONS

When dealing with dynamic models, it is preferable to represent actual model outputs using simple dynamic models rather than simple static models. A dynamic model, if estimated with the structural or final form, enables one to determine the response of the true model to any price path, not just the one used for estimation. Parameter estimates of static models, on the other hand, are dependent on the type of experiment used to generate data.

Measurements of the long-run elasticity based on the step-change (or EMF) scenario are considerably more accurate than those based on the rate-change scenario. Application of the long-run elasticity is fairly accurate when there is a step change in prices, but less so when applied to rate changes in prices. More generally, failure to take account of the price path can lead to serious misestimates of actual demand.

REFERENCES

1. E. Hirst and J. Carney. "The ORNL Residential Energy Use Model." Land Economics, Volume 55, No. 3, August 1979. pp. 319-333.

Appendix A

BIASES IN ELASTICITY ESTIMATES

THE EMF ESTIMATOR

Experiment 1: One-Time Step Perturbation of Inputs

We set prices so that

$$P_t^1 = P_0 \quad \text{for all } t \geq 1 \quad ,$$

and

$$P_t^2 = sP_0 \quad \text{for all } t \geq 1 \quad .$$

Short-Run Elasticity Estimator. We have by assumption

$$E_1^1 = KE_0^1 P_0^\beta \quad ,$$

and

$$E_1^2 = KE_0^2 s^\beta P_0^\beta \quad .$$

Since $E_0^1 = E_0^2$, using the EMF estimator to determine $\hat{\beta}$ leads to

$$\hat{\beta} = \frac{\ln(E_1^1/E_1^2)}{\ln(P_0/sP_0)} = \frac{-\beta \ln s}{-\ln s} = \beta \quad .$$

Thus, the short-run EMF estimator is exact. We note that this applies only if the first period demands E_1^1 and E_1^2 are used. If the first available demand output E_t^i is for $t = 5$, the EMF estimator no longer has the exactness property. See Appendix B for further discussion of this case.

Long-Run Elasticity Estimator. For a step change in prices, Eq. 11-3 implies that

$$E_t^1 = K_t P_0^{\beta\left[\frac{1-\lambda^t}{1-\lambda}\right]}$$

$$E_t^2 = K_t s^{\beta\left[\frac{1-\lambda^t}{1-\lambda}\right]} P_0^{\beta\left[\frac{1-\lambda^t}{1-\lambda}\right]}$$

Using the EMF estimator, we obtain

$$\hat{\eta} = \frac{\ln(E_t^1/E_t^2)}{\ln(P_t^1/P_t^2)}$$

$$= \frac{-\beta\left[\frac{1 - \lambda^t}{1 - \lambda}\right] \ln s}{-\ln s}$$

$$= \frac{\beta}{1 - \lambda} [1 - \lambda^t] \quad .$$

The estimate $\hat{\eta}$ will be less than the true η [$\equiv \beta/(1-\lambda)$] by a multiplicative factor. See Table 11-2 for computations of the magnitude of this difference. Note that if λ were known, one could eliminate this bias by adjusting the multiplicative factor. Appendix B contains further discussion of this topic.

<u>Lag Estimator</u>.

$$\hat{\lambda} = 1 - \frac{\hat{\beta}}{\hat{\eta}}$$

$$= \lambda \frac{1 - \lambda^{t-1}}{1 - \lambda^t} \quad .$$

See Table 11A-1 for calculations given typical values of λ . Analogous to the case for $\hat{\eta}$, $\hat{\lambda}$ will always underestimate the true λ for $\lambda < 1$.

<u>Experiment 2: Rate-of-Growth Perturbation of Inputs</u>

For this experiment we set prices so that

$$P_t^1 = (1 + r_1)^t P_0$$

and

$$P_t^2 = (1 + r_2)^t P_0 \quad .$$

<u>Short-Run Elasticity Estimators</u>. We obtain first-year demand projections

$$E_1^1 = K_1(1 + r_1)^\beta P_0^\beta$$

and

$$E_1^2 = K_1(1 + r_2)^\beta P_0^\beta \quad .$$

Table 11A-1

ERROR IN LAG ESTIMATES USING THE EMF ESTIMATOR OF λ

Actual λ	Experiment 1 (Step Perturbation)		Experiment 2 (Rate-of-Growth Changes)	
	t = 20	t = 25	t = 20	t = 25
0.75	0.75	0.75	0.71	0.72
0.80	0.80	0.80	0.75	0.76
0.85	0.84	0.85	0.79	0.81
0.90	0.89	0.89	0.84	0.85
0.95	0.92	0.93	0.87	0.89

Using the EMF estimator, we obtain the short-run elasticity estimate

$$\hat{\beta} = \frac{\beta \ \ell n \left[\frac{1 + r_1}{1 + r_2}\right]}{\ell n \left[\frac{1 + r_1}{1 + r_2}\right]}$$

$$= \beta \ .$$

One can see that the short-run EMF estimator is exact. Again note that this is true only if the first period demand observations E_1^1 and E_1^2 are used. If the first available observation E_t^i is for $t = 5$, the EMF estimator no longer has the exactness property. See Appendix B.

Long-Run Elasticity Estimator

By applying Eq. 11-3 for later years, we obtain

$$E_t^1 = K_t P_0^{\beta\left[\frac{1 - \lambda^t}{1 - \lambda}\right]} (1 + r_1)^{\beta \sum_{i=0}^{t-1} (t - 1)\lambda^i}$$

and

$$E_t^2 = K_t P_0^{\beta\left[\frac{1 - \lambda^t}{1 - \lambda}\right]} (1 + r_2)^{\beta \sum_{i=0}^{t-1} (t - 1)\lambda^i} .$$

The EMF estimator of the long-run elasticity is

$$\hat{\eta} = \frac{\beta\left(\sum_{i=0}^{t-1} (t - i)\lambda^i\right) \ln\left[\frac{1 + r_1}{1 + r_2}\right]}{t \ln\left[\frac{1 + r_1}{1 + r_2}\right]}$$

$$= \frac{\beta}{t}\left[\sum_{i=0}^{t-1} (t - i)\lambda^i\right]$$

$$= \frac{\beta}{1 - \lambda}\left(1 - \frac{\lambda}{t}\left[\frac{1 - \lambda^t}{1 - \lambda}\right]\right) ,$$

indicating that there is a downward bias to the estimate. Again, see Table 11-2 for a tabulation of the magnitude of the error implied by the above equation for typical values of λ .

<u>Lag Estimator</u>.

$$\hat{\lambda} = 1 - \frac{\hat{\beta}}{\hat{\eta}}$$

$$= \lambda\left[\frac{t(1 - \lambda) - (1 - \lambda^t)}{t(1 - \lambda) - \lambda(1 - \lambda^t)}\right]$$

$$= \lambda\frac{1 - \frac{1}{t}\left(\frac{1 - \lambda^t}{1 - \lambda}\right)}{1 - \frac{\lambda}{t}\left(\frac{1 - \lambda^t}{1 - \lambda}\right)}$$

This estimate underestimates λ for $\lambda < 1$. See Table 11A-1.

Experiment 3: The EMF 4 Experiment

The EMF 4 experiment can be shown to give exactly the same results as the step perturbation experiment. In the EMF 4 experiment, prices in period t are defined as

$$P_t^1 = (1 + r_1)^t P_0$$

and

$$P_t^2 = (1 + r_2)(1 + r_1)^t P_0 \quad ,$$

where r_1 and r_2 are percentages. If $s = (1 + r_2)$, we have

$$P_t^2 = (1 + r_1)^t s P_0$$

It is easy to show that estimators based on this formulation give the same results as estimators based on the step perturbation experiment.

The rate-of-growth perturbations always lead to estimates of the lags that are smaller than both the step-change based estimates and the actual values. This would imply that models using lags based on the rate-of-growth scenarios will show both a faster initial response to price changes and a smaller total long-run response than would actually occur.

THE ARC ELASTICITY ESTIMATOR

An alternative method of calculating estimates of the elasticities is to compute arc elasticities. These are defined as

$$\eta_t = \frac{E_t^2 - E_t^1}{E_t^2 + E_t^1} \cdot \frac{P_2^t + P_1^t}{P_2^t - P_1^t} \quad .$$

Using this estimate, again look at each of the three experiments.

Experiment 1: One-Time Step Perturbation of Inputs

Short-Run Elasticity Estimator. Given that $P_t^1 = P_0$ for all $t \geq 1$, and $P_t^2 = sP_0$ for all $t \geq 1$, we obtain the estimate

$$\hat{\beta} = \frac{(s - 1)\ (s + 1)}{(s + 1)\ (s - 1)} \quad .$$

The ability of this formula to approximate the true β depends on the value of s. As indicated in Table 11A-2, the estimation error diminishes to 0 as s approaches 1.

Table 11A-2

ARC ESTIMATES OF SHORT-RUN ELASTICITIES
(step perturbation)

True Value β	S Value							
	0.5	.075	0.95	1.05	1.25	1.75	2.0	3.0
-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.06
-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.11
-0.15	-0.16	-0.15	-0.15	-0.15	-0.15	-0.15	-0.16	-0.16
-0.20	-0.21	-0.20	-0.20	-0.20	-0.20	-0.21	-0.21	-0.22

Long-Run Elasticity Estimator. We obtain the estimate

$$\hat{\pi}_t = \frac{(s^{\beta\left[\frac{1-\lambda^t}{1-\lambda}\right]} - 1)}{(s^{\beta\left[\frac{1-\lambda^t}{1-\lambda}\right]} + 1)} \cdot \frac{(s + 1)}{(s - 1)} \quad .$$

The estimation error can be broken into two parts here. Table 11A-2 indicates the error in estimating the value of $\beta[(1 - \lambda^t)/(1 - \lambda)]$, and Table 11-2 indicates the error arising from the term $(1 - \lambda^t)$. Comparison of the two tables indicates that the estimate of $[(1 - \lambda^t)/(1 - \lambda)]$ is larger than the actual value, and that $\beta[(1 - \lambda^t)/(1 - \lambda)]$ is less than the actual value of $\beta/(1 - \lambda)$. The $(1 - \lambda^t)$ term is dominant, which leads to an estimate that is lower than the actual value of η but higher than the EMF estimator when using the step-perturbation experiment.

Lag Estimator.

$$\hat{\lambda} = 1 - \frac{\hat{\beta}}{\hat{\eta}}$$

$$= \frac{2s^{\beta\left[\frac{1 - \lambda^t}{1 - \lambda}\right]} - s^{\beta}}{\left(s^{\beta\left[\frac{1 - \lambda^t}{1 - \lambda}\right]} - 1\right)(s^{\beta} + 1)}$$

Experiment 2: Rate-of-Growth Perturbation of Inputs

Short-Run Elasticity Estimator. Given that $P_t^1 = (1 + r_1)^t P_0$ and $P_t^2 = (1 + r_2)^t P_0$, we obtain

$$\hat{\beta} = \frac{(1 + r_1)^{\beta} - (1 + r_2)^{\beta}}{(1 + r_1)^{\beta} + (1 + r_2)^{\beta}} \cdot \frac{(1 + r_1) + (1 + r_2)}{(1 + r_1) - (1 + r_2)} .$$

This can be rewritten as

$$\hat{\beta} = \frac{1 - \frac{(1 + r_2)^{\beta}}{(1 + r_1)}}{1 + \frac{(1 + r_2)^{\beta}}{(1 - r_1)}} \cdot \frac{1 + \frac{(1 + r_2)}{(1 + r_1)}}{1 - \frac{(1 + r_2)}{(1 + r_1)}}$$

$$= \frac{(s*^{\beta} - 1)}{(s*^{\beta} + 1)} \cdot \frac{(s* + 1)}{(s* - 1)} ,$$

where

$$s^* = \left[\frac{1 + r_2}{1 + r_1}\right] .$$

This expression has the same form as our result using the step-perturbation experiment and so Table 11A-2 can be used for evaluating the accuracy of this estimate. Note that s^* is generally close to unity, a value which Table 11A-2 indicates will give very good approximations to the true β .

Long-Run Elasticity Estimator.

$$\hat{\eta} = \left[\frac{(1 + r_1)^{\beta \sum_{i=0}^{t-1} (t-i)\lambda^i} - (1 + r_2)^{\beta \sum_{i=0}^{t-1} (t-i)\lambda^i}}{(1 + r_1)^{\beta \sum_{i=0}^{t-1} (t-i)\lambda^i} + (1 + r_2)^{\beta \sum_{i=0}^{t-1} (t-i)\lambda^i}}\right] \left[\frac{(1 + r_1)^t + (1 + r_2)^t}{(1 + r_1)^t - (1 + r_2)^t}\right]$$

$$= \left[\frac{s^{*\,\beta \Sigma_{i=0}^{t-1} (t-i)\lambda^i} - 1}{s^{*\,\beta \Sigma_{i=0}^{t-1} (t-i)\lambda^i} + 1}\right] \left[\frac{s^* + 1}{s^* - 1}\right] ,$$

where

$$s^* = \left[\frac{1 + r_2}{1 + r_1}\right] .$$

As with the step-perturbation experiment, Table 11-3 indicates the error in estimating the value of

$$\beta\left[\sum_{i=0}^{t-1} (t - i)\lambda^i\right] ,$$

which is equivalent to the EMF estimator based on the rate-of-growth experiment. Since s^* is close to unity, the Table 11A-2 errors are small and the degree of approximation to the true β is indicated by Table 11-2.

Lag Estimator. This is computed by employing the usual formula.

$$\lambda = 1 - \frac{\hat{\beta}}{\hat{\eta}} .$$

Appendix B

EXACT ESTIMATION BY EXPANDING THE MODEL

If we use the model given in Eq. 11-1,

$$E_t = K_t E_{t-1}^{\lambda} P_t^{\beta} ,$$

as the basis for our estimators rather than the static model,

$$E_t = \ell n\ A_1 + E_t\ \ell n\ P_t ,$$

we can estimate β and λ, and use these parameters to derive a long-run elasticity estimate as

$$\eta = \frac{\beta}{1 - \lambda} .$$

In solving for λ, one would ideally use data on a year-by-year basis. Many models, however, report results on a five-year increment or some other interval. Although more complicated, it is still possible to solve for λ even in these cases. To estimate β, we still use the EMF estimator since it gives exact results.

ESTIMATION OF λ WHEN YEARLY DATA ARE AVAILABLE

Experiment 1: One-Time Step Perturbation in Inputs

One may solve for λ using

$$\frac{E_t}{E_{t-1}} = \frac{E_{t-1}^{\lambda}}{E_{t-2}^{\lambda}} \cdot \frac{s^{\beta} P_0^{\beta}}{s^{\beta} P_0^{\beta}}$$

or

$$\lambda = \frac{\ln \left(\frac{E_t}{E_{t-1}}\right)}{\ln \left(\frac{E_{t-1}}{E_{t-2}}\right)} .$$

<u>Experiment 2: Rate-of-Growth Perturbation in Inputs</u>

One may solve for λ using

$$\frac{E_t}{E_{t-1}} = \frac{E_{t-1}^{\lambda}(1 + r)^{t\beta}P_0^{\beta}}{E_{t-2}^{\lambda}(1 + r)^{(t-1)\beta}P_0^{\beta}}$$

$$= \left[\frac{E_{t-1}}{E_{t-2}}\right]^{\lambda} (1 + r)^{\beta}$$

or

$$\lambda = \frac{\ln \left[\frac{E_{t-1}}{E_{t-1}}\right] - \beta \ln (1 + r)}{\ln \left[\frac{E_{t-1}}{E_{t-2}}\right]} .$$

ESTIMATION OF λ WHEN DATA ARE AVAILABLE AT FIVE-YEAR INTERVALS

<u>Experiment 1: One-Time Step Perturbation in Inputs</u>

$$\frac{E_t}{E_{t-5}} = \frac{E_{t-5}^{\lambda^5}(sP_0)^{\beta+\lambda\beta+\ldots+\lambda^4\beta}}{E_{t-10}^{\lambda^5}(sP_0)^{\beta+\lambda\beta+\ldots+\lambda^4\beta}}$$

or

$$\lambda = \left[\frac{\ln (E_t/E_{t-5})}{\ln (E_{t-5}/E_{t-10})}\right] .$$

Experiment 2: Rate-of-Growth Perturbation of Inputs

$$\frac{E_t}{E_{t-5}} = \frac{E_{t-5}^{\lambda^5} P_0^{\sum_{i=1}^{4} \beta\lambda^i} (1 + r)^{\beta \sum_{i=0}^{4} (t-i)\lambda^i}}{E_{t=10}^{\lambda^5} P_0^{\sum_{i=0}^{4} \beta\lambda^i} (1 + r)^{\beta \sum_{i=5}^{4} (t-i)\lambda^{i-5}}} .$$

This equation cannot be easily solved for λ , but it can be reduced to the form

$$\ell n \, (E_t/E_{t-5}) = \lambda^5 \, \ell n \, (E_{t-5}/E_{t-10}) + 5\beta\left[\frac{1 - \lambda^5}{1 - \lambda}\right] \ell n \, (1 + r) .$$

This later expression can be used as the basis for obtaining numerical estimates of λ .

Appendix C

EXPANDING THE MODEL

Of particular interest is whether the results obtained here apply when the model has been expanded to include additional parameters and variables. We examine the inclusion of income in the original model as an example.

First, consider a situation where income effects have the same lag structure as prices. Our model becomes

$$E_t = aE_{t-1}^{\lambda} P_{Et}^{-\beta} Y_t^{\varepsilon} \quad .$$

The corresponding static representation becomes

$$E = aP_e^{\sigma} Y^{\eta} \quad ,$$

where

$$\sigma = \frac{\beta}{1 - \lambda} \quad \text{and}$$

$$\eta = \frac{\varepsilon}{1 - \lambda} \quad .$$

Application of the EMF estimator to obtain estimates of σ and η gives

$$\hat{\sigma}_t = \frac{\ln(E_t^1/E_t^2)}{\ln(P_{Et}^1/P_{Et}^2)} \quad ,$$

with Y held constant, and

$$\hat{\eta} = \frac{\ln(E_t^1/E_t^2)}{\ln(Y_t^1/Y_t^2)} \quad ,$$

with P_E held constant.

The estimate of σ is independent of Y and η. Likewise, the estimate of η is independent of P_E and σ. Thus, the results derived earlier for single variables apply to the estimation of $\hat{\sigma}$ and $\hat{\eta}$ without modification.

Alternatively, consider the case where there is instantaneous response in energy demand to changes in income but a lagged response to changes in price. This model can be expressed as

$$\frac{E_t}{Y_t^\theta} = aP_t^\beta \left[\frac{E_{t-1}}{Y_{t-1}^\theta}\right]^\lambda .$$

The corresponding static representation becomes

$$E = aP_E^\sigma Y^\theta$$

where $\sigma = \beta/(1 - \lambda)$.

In this case, estimation of $\hat{\sigma}$ is the same as for the single variable case. The estimation of θ is comparable to the single variable short-run elasticity estimation and is exact using the EMF estimator.

Appendix D

THE FINAL FORM ESTIMATION PROCEDURE

STANDARD METHOD

We seek estimates $\hat{\lambda}$ and $\hat{\beta}$ that minimize the goodness-of-fit criterion

$$L_F = \sum_{k=1}^{K} \sum_{t=1}^{T} (e_{kt} - \hat{\lambda}^t e_0 - \sum_{i=1}^{t} \hat{\lambda}^{t-i} \beta P_{ki})^2 .$$

We solve for the optimal $\hat{\beta}$, given $\hat{\lambda}$, and search over the domain of $\lambda \varepsilon (0,1)$ for the value of $\hat{\lambda}$ giving the smallest L_F . To solve for the best value of $\hat{\beta}$ given $\hat{\lambda}$, define auxiliary variables $\bar{e}_{kt|\hat{\lambda}}$ and $\bar{P}_{kt|\hat{\lambda}}$ as

$$\bar{e}_{kt|\hat{\lambda}} = e_{kt} - \hat{\lambda}^t e_0$$

and

$$\bar{P}_{kt|\hat{\lambda}} = \sum_{i=1}^{t} \hat{\lambda}^{t-i} P_i .$$

L_F can then be written as

$$L_{F|\hat{\lambda}} = \sum_{k=1}^{K} \sum_{t=1}^{T} (e_{kt|\hat{\lambda}} - \hat{\beta} P_{kt|\hat{\lambda}})^2 ,$$

which is a standard least-squares problem. We select the $\hat{\lambda}$ giving the smallest value of $L_{F|\hat{\lambda}}$.

Missing Data

Many models do not report their output on a yearly basis. Models such as Jackson Commercial and ISTUM, for example, use a time period of five years. With minor modifications, it is possible to estimate a final form model for a yearly time period.

If S represents the set of years for which we have model observations, our goodness-of-fit criterion can be written

$$L_F = \sum_{k=1}^{K} \sum_{t\epsilon S} (e_{kt} - \hat{\lambda}^t e_0 - \sum_{i=1}^{t} \hat{\lambda}^{t-i} \beta P_i)^2 \quad .$$

If we have available the yearly sequence of prices (P_t) , the initial demand e_0 , and the demand output of the model, it is possible to modify the standard method. We need only use the points $t \epsilon S$ rather than $t = 1,2,3,\ldots,T$.

Appendix E

ERRORS IN THE USE OF LONG-RUN DEMAND ELASTICITIES

DEMAND PREDICTION IN RESPONSE TO A STEP CHANGE IN PRICES

Let

$$\sigma = \frac{\beta}{1 - \lambda}$$

and

$$E_0 = P_0^{\sigma} .$$

If P is the new level of price, then the demand in period t is

$$E_t = E_{t-1}^{\lambda} P_t^{\beta} \qquad (11E\text{-}1)$$

or

$$E_t = E_0^{\lambda^t} P_t^{\sigma(1-\lambda^t)}$$

or

$$E_t = P_t^{\sigma} \left(\frac{P_t}{P_0}\right)^{-\sigma\lambda^t} .$$

The static approximation is Eq. 11-15

$$\hat{E}_t = P_t^{\sigma} .$$

Define the ratio of estimated demand to actual model output as

$$\varepsilon_t = \frac{\hat{E}_t}{E_t} .$$

Then for the step change in prices we have

$$\varepsilon_t = \left(\frac{P_t}{P_0}\right)^{\sigma\lambda^t} .$$

Calculations for $(P_t/P_0) = 2$ are shown in Table 11-3.

DEMAND PREDICTION IN RESPONSE TO A RATE CHANGE IN PRICES

Let

$$\sigma = \frac{\beta}{1 - \lambda}$$

and

$$E_0 = P^{\sigma} .$$

Suppose that prices increase at the rate r beginning in period 1 so that

$$P_t = (1 + r)^t P_0 .$$

Then the actual model demand is

$$E_t = E_0^{\lambda^t} P_0^{\sigma(1-\lambda^t)} (1 + r)^{\beta \sum_{i=1}^{t-1} (t-i)\lambda^i}$$

or

$$E_t = P_0^{\sigma}(1 + r)^{\sigma(t-\lambda[\frac{1-\lambda^t}{1-\lambda}])} .$$

If we approximate this by

$$E_t = P_0^{\sigma}(1 + r)^{t\sigma} ,$$

the ratio of estimated demand to actual model output is

$$\varepsilon_t = (1 + r)^{\sigma\lambda[\frac{1-\lambda^t}{1-\lambda}]} .$$

Calculations for $(P_t/P_0) = 2$ for $t = 20$, or $r = 0.035$ are shown in Table 11-4.

Chapter 12

THE ECONOMETRIC EXPERIMENT OF 1973*

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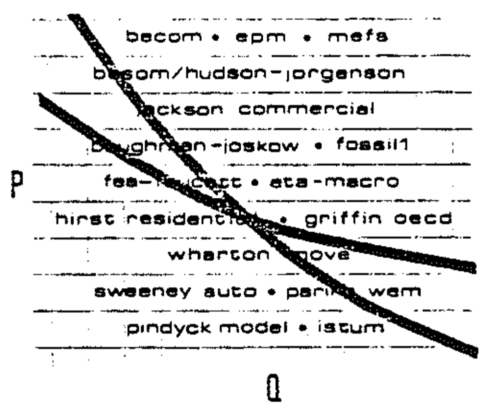
Thomas F. Wilson†



*This chapter formerly was Working Paper EMF 4.5.

†The authors are indebted to James Griffin for helpful comments on an earlier draft.

Chapter 12

THE ECONOMETRIC EXPERIMENT OF 1973

INTRODUCTION AND SUMMARY

In 1973, OPEC performed a unique experiment. Unlike most econometric time series, this is a case in which there were dramatic changes in relative prices, but comparatively small changes in the GNP (gross national product). Between 1972 and 1978, U.S. crude oil prices rose by 140% (measured in dollars of constant purchasing power). See Figure 12-1 and Table 12-1. During this same period, the energy-GNP ratio dropped by 8%. For the longer term, year 2000 and beyond, what are we to infer from this six-year experience? Many competing hypotheses are available, but the three most plausible ones would appear to be the following:

- the price elasticity of demand for primary energy is low,
- adjustment lags are long, or
- energy price elasticities are highly uncertain.

Each of these interpretations leads to a rather different policy position. If there is a low price elasticity of demand, our best bet is to proceed rapidly toward commercialization of expensive, and possibly unsafe, supply technologies. If the adjustment lags are long, we might do best to rely upon energy consumers' gradual responses to increasing prices. If the price elasticities are highly uncertain, we would do well to adopt a hedging strategy, that is, to develop new supply and conservation technologies, but not to attempt immediate large-scale commercialization. The "Aggregate Elasticity of Energy Demand" study could have a significant influence upon the climate of expert opinion in the U.S., and it is essential that we not overstate the degree of confidence to be placed in our current estimates of demand elasticities.

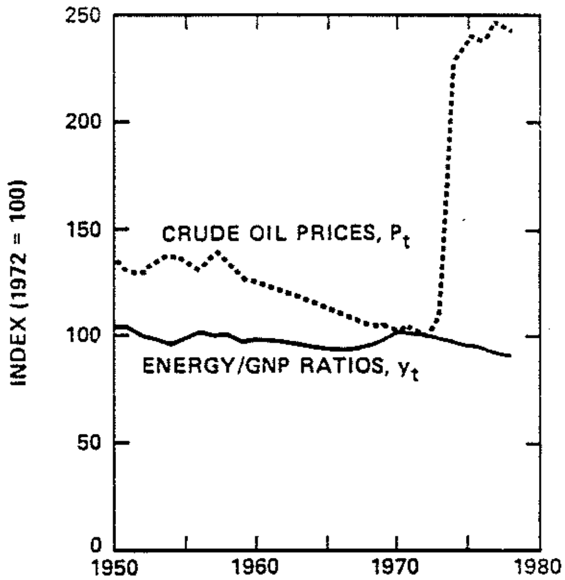


Figure 12-1 Crude Oil Prices and Energy-GNP Ratios

A LAGGED ADJUSTMENT MODEL

The essence of this problem is that it takes time to adjust to higher energy prices. Let y_t denote the energy-GNP ratio* in year t . Let p_t denote the price of primary energy (measured by the wellhead price or the refiners' acquisition cost of crude oil). And let λ denote the fraction of the long-run adjustment to the price p_t that takes place within a single year. Except for minor changes, our estimating equation is identical to that employed in simulation experiments by Hogan [5]. This is a first-order Koyck distributed lag model:

$$y_t = \alpha p_t^{-\lambda\varepsilon} y_{t-1}^{(1-\lambda)} \quad . \qquad (12\text{-}1)$$

*Instead of total primary energy consumption, we have also experimented with a "value-weighted" measure, counting each quad of coal at 70% the economic value of one quad of oil and gas. This leads to significant changes in the index y_t for the years 1920-50, but not for the period since 1950. Accordingly, we shall omit a detailed account of this value-weighted index of energy consumption.

Table 12-1

BASIC TIME-SERIES DATA

Year	Crude Oil Prices ($/10^6 Btu) Current Prices[a]	Crude Oil Prices ($/10^6 Btu) 1972$[b]	Total Primary Energy Consumption (10^{15} Btu)	GNP (10^9 1972$)
1950	0.43	0.81	33.99	533.50
1951	0.44	0.76	36.78	576.50
1952	0.44	0.75	36.46	598.50
1953	0.46	0.79	37.59	621.80
1954	0.48	0.80	36.26	613.70
1955	0.48	0.78	39.70	654.80
1956	0.48	0.77	41.70	668.80
1957	0.53	0.82	41.71	680.90
1958	0.52	0.79	41.70	679.50
1959	0.50	0.74	43.14	720.40
1960	0.50	0.72	44.57	736.80
1961	0.50	0.72	45.32	755.30
1962	0.50	0.71	47.42	799.10
1963	0.50	0.70	49.31	830.70
1964	0.50	0.68	51.24	874.40
1965	0.49	0.66	53.34	925.90
1966	0.50	0.65	56.41	981.00
1967	0.50	0.64	58.27	1007.70
1968	0.51	0.61	61.76	1051.80
1969	0.53	0.62	64.98	1078.80
1970	0.55	0.60	67.14	1075.30
1971	0.58	0.61	68.70	1107.50
1972	0.58	0.58	71.95	1171.10
1973	0.67	0.63	74.74	1233.40
1974	1.56	1.34	73.04	1210.70
1975	1.79	1.41	70.67	1202.30
1976	1.88	1.40	74.41	1271.00
1977	2.05	1.45	76.25	1332.70
1978	2.15	1.41	78.01	1385.70

[a]Domestic wellhead prices through 1973; refiners' acquisition costs thereafter.

[b]Based on GNP deflator.

Sources: crude oil prices (current prices): 1950-1973 [1], 1974-1977 [2, January 1978], 1978 [2, March 1979]; crude oil prices (1972 $): 1950-1974 [1], 1975-1976 [3, September 1977], 1977 [3, March 1978], 1978 [4]; total primary energy consumption: 1950-1974 [1], 1975-1977 [2, November 1978], 1978 [2, March 1979]; GNP: 1950-1974 [1], 1975-1978 [4].

Taking logarithms in Eq. 12-1, we get the following linear equation:

$$\ln y_t = \ln \alpha - \lambda\varepsilon \ln p_t + (1-\lambda) \ln y_{t-1} \quad . \qquad (12\text{-}2)$$

We employ two distinct approaches to estimate the long-run primary energy price elasticity ε . The first approach is to estimate α, ε, and λ directly from the time series data, e.g., apply ordinary least squares (OLS) to Eq. 12-2. There are theoretical problems with such an approach. Alternatively, we can assign a plausible value to the adjustment parameter λ, substitute it in Eq. 12-2, and calculate α and ε from the time-series data. This estimation is much sounder theoretically, given that λ is chosen "correctly."

The lag parameter λ cannot be estimated with high reliability from historical data. If the lags are due to social and psychological inertia to lifestyle changes, λ may indeed be quite low. If consumers anticipate future increases in energy prices, λ may be high. And if the lags are related to the expansion and replacement of physical capital, they will be of the same order as the ratio of gross investment to the capital stock: 5-10% per year. This is the range that would be suggested by the "putty-clay" hypothesis on adjustments to price changes.

Equation 12-1 is highly aggregated. It is unclear whether the price elasticity effects have been over- or underestimated by using crude oil prices as a proxy for all forms of primary energy. There would have been difficulties, however, if we had attempted to measure nationwide natural gas and coal prices. Prior to 1973, there were three-to-one regional variations in the prices of each of these fuels. Moreover, natural gas has been subject to price control and has been allocated to consumers by regulatory agencies during the past decade.

Before proceeding with numerical estimation, it is worth noting some possible errors in the specification of Eq. 12-1. Each of these errors would tend to overestimate the impact of price elasticites upon U.S. energy demands since 1972.

- In effect, we are assuming a unitary elasticity of energy demand with respect to the GNP. If the GNP elasticity were less than unity, this in itself would have led to a reduction in the energy-GNP ratio.
- We are neglecting all forms of nonprice conservation: speed limits, efficiency standards, appliance labeling, moral suasion, etc. These could also account for a portion of the 8% decline in the energy-GNP ratio since 1972.

- We are assuming that energy consumers adjust their capital equipment only to current prices, and that they do not anticipate future price increases.

PRICE ELASTICITIES AND ADJUSTMENT LAGS--JOINT ESTIMATION

Joint estimation of α, ε, and λ using Eq. 12-2 raises two basic problems. Ordinary least squares (OLS) estimates will be biased because of the lagged dependent variable y_{t-1} . If, in addition, the disturbance terms are serially correlated, then OLS gives inconsistent estimators. We used both OLS and a Cochrane-Orcutt iterative scheme which attempts to eliminate first order autoregressive correlation in the errors.

For the 1950-1978 data series, OLS yields 0.11 as the "best estimate" of the long-run price elasticity ε and 0.24 as the best estimate of the adjustment parameter λ . With 80% confidence, the one-year adjustment rate lies between 9 and 38%. Using the Cochrane-Orcutt procedure, we obtain best estimates of $\varepsilon = 0.07$ and $\lambda = 0.38$. The confidence interval for λ is even wider in this case.

The results for the 1968-1978 data are almost identical to those above. The major difference is the slightly wider confidence bands for all of the parameters. For the balance of this chapter, we shall employ an exogenous value for λ .

PRICE ELASTICITY ESTIMATES--EXOGENOUS ADJUSTMENT LAGS

Three sets of primary energy elasticity estimates are summarized by Figures 12-2 through 12-4. These refer, respectively, to the years before 1968, after 1968, and the entire period 1950-1978. According to Figure 12-2, there is virtually no information on primary energy price elasticities contained in the 1950-1967 time series.* There was a downward trend in energy prices, but virtually no change in

*If one is attempting to pool international cross-section and time-series data, the OECD (Organization of Economic Cooperation and Development) publication Energy Statistics is perhaps the most convenient official source of energy price data. Unfortunately, these data are in the public domain only for the years since 1968. If the U.S. is at all indicative, however, the earlier years would provide little information on price elasticities in individual countries.

International data are perhaps best employed by pooling time-series with cross-section evidence. There then arises the difficult problem of whether or not to employ "dummy" variables to adjust for inter-country differences. Nordhaus [6, p. 571] has demonstrated how these dummy variables may have a large effect upon price elasticities estimated through international cross-section data.

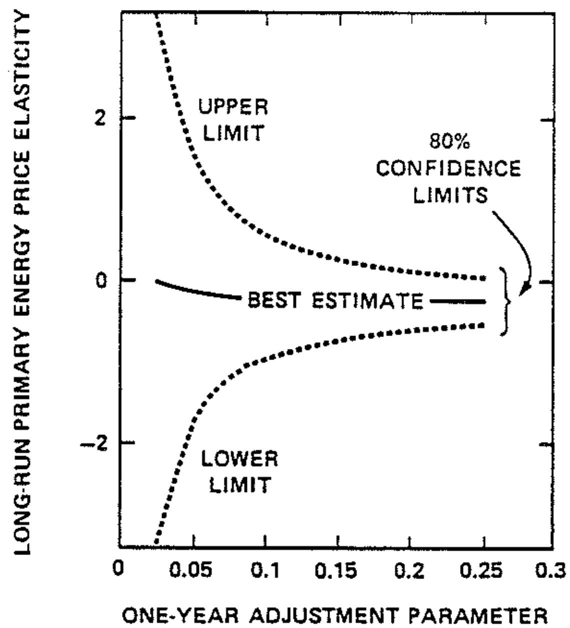


Figure 12-2 Dependence of Long-Run Elasticity Estimates on Assumed One-Year Adjustment Parameter: 1950-1967 Aggregate Data

the energy-GNP ratio during those years. Neither the magnitude nor the sign of the long-run price elasticity can be inferred reliably from this sample period through the distributed lag estimating Eq. 12-2.

Equation 12-2, however, performs reasonably well when applied to the 1968-1978 time period, including the OPEC "experiment" of 1973. From Figure 12-3, note that there is virtually a hyperbolic relationship between the adjustment parameter λ and the long-run primary energy price elasticity ε. If $\lambda = 0.05$, that is, if only 5% of the long-run price adjustments take place in any one year, 0.57 is our best estimate of the price elasticity ε. If, however, $\lambda = 0.10$, the estimated value of the price elasticity drops to 0.30. In any case, note the width of the error bands. For example, with $\lambda = 0.075$, the 80% confidence interval for ε extends from 0.19 to 0.59. For policy purposes, it is inadequate to report only the best estimate of 0.39.

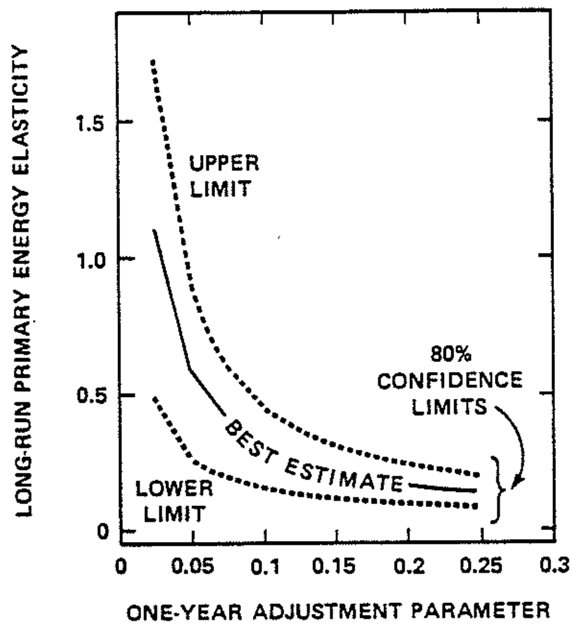
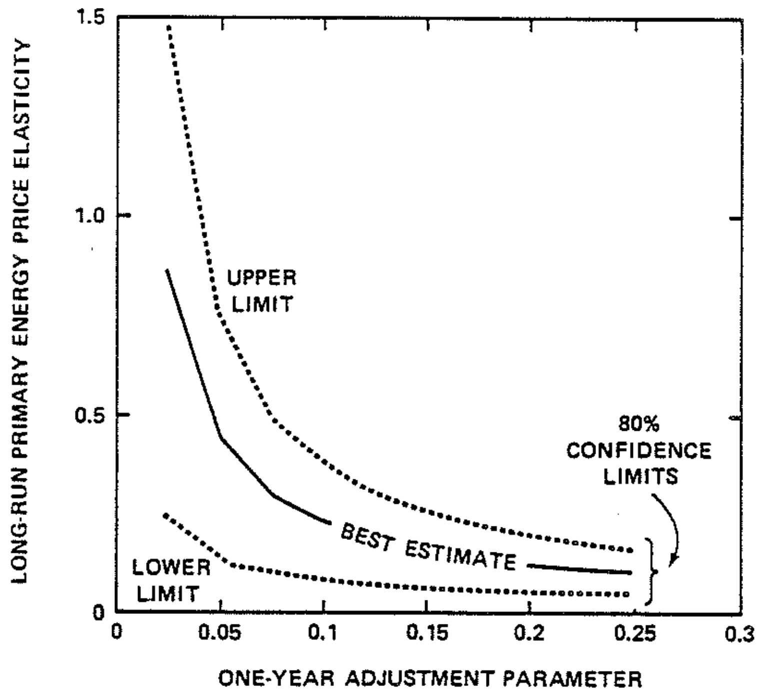


Figure 12-3 Dependence of Long-Run Elasticity Estimates on Assumed One-Year Adjustment Parameter: 1968-1978 Aggregate Data

Figure 12-4 shows the consequences of pooling the earlier period with the latter one. Note that the price elasticities are somewhat lower, but that the width of the error band remains virtually the same. Moreover, there is again a virtually hyperbolic relationship between the lag parameter λ and the estimated price elasticity ε .

ADDITIONAL RESULTS

The results of the least squares estimation of Eq. 12-2 are displayed in Table 12-2 for the three time periods: 1950-1967, 1968-1978, and 1950-1978. In every instance, the Durbin-Watson statistic implies that the successive residuals in Eq. 12-2 are uncorrelated. If positive serial correlation had been present, the estimated standard errors would have been too small. Fortunately, this does not appear to be the case.



Note: These are direct estimates from 1950–1978 aggregate data.

Figure 12-4 Dependence of Long-Run Elasticity Estimates on Assumed One-Year Adjustment Parameter: 1950-1978 Aggregate Data

The adjusted R^2 (here indicated by $\bar{R}^2$) values are so low for the 1950-1967 series that the mean of the energy-GNP ratio for the 18-year period is as good an estimate as that provided by the least squares regression equations. The results are somewhat more satisfactory for the 1968-1978 and combined series. These both have positive $\bar{R}^2$, but no $\bar{R}^2$ is larger than 0.5. This means, roughly, that no regression explains more than 50% of the variation in the dependent variable. But in the context of policy decisions, the real issue is accuracy in prediction, not necessarily how well the past data can be fit. And it is easy to see that $\bar{R}^2$ is not always a good criterion for judging forecasting ability. For instance, when Eq. 12-2 is used to estimate ε and λ jointly, the results (for the 1950-1978 series) are $\hat{\varepsilon} = 0.11$, $\hat{\lambda} = 0.24$, and $\bar{R}^2 = 0.68$. From Table 12-2, when we assumed $\lambda = 0.24$, the results were $\hat{\varepsilon} = 0.11$ and $\bar{R}^2 = 0.15$. The difference in $\bar{R}^2$ is due to the smaller variance of the dependent variable when the λ value is assumed. Since the predictive ability of Eq. 12-2 is the same in either case, $\bar{R}^2$ does not appear to be a useful measure here.

Table 12-2

U.S. LONG-RUN PRIMARY ENERGY PRICE ELASTICITY ESTIMATES

Adjustment Parameter λ	1950-1967 Data				1968-1978 Data				1950-1978 Data			
	Estimated Long-Run Price Elasticity ε	Estimated Standard Error of ε	Adjusted R^2	Durbin-Watson Statistic	Estimated Long-Run Price Elasticity ε	Estimated Standard Error of ε	Adjusted R^2	Durbin-Watson Statistic	Estimated Long-Run Price Elasticity ε	Estimated Standard Error of ε	Adjusted R^2	Durbin-Watson Statistic
0.03	-0.01	2.42	-0.07	2.21	1.10	0.46	0.32	1.46	0.86	0.46	0.08	1.83
0.05	-0.14	1.19	-0.07	2.23	0.57	0.23	0.35	1.50	0.44	0.23	0.09	1.84
0.08	-0.19	0.78	-0.06	2.24	0.39	0.15	0.37	1.54	0.30	0.15	0.10	1.84
0.10	-0.21	0.58	-0.06	2.26	0.30	0.11	0.40	1.57	0.23	0.11	0.11	1.84
0.13	-0.22	0.46	-0.05	2.27	0.25	0.09	0.42	1.61	0.19	0.09	0.12	1.84
0.15	-0.23	0.38	-0.04	2.29	0.21	0.07	0.44	1.65	0.16	0.07	0.12	1.82
0.18	-0.24	0.32	-0.03	2.30	0.18	0.06	0.46	1.68	0.14	0.06	0.16	1.81
0.20	-0.24	0.28	-0.01	2.30	0.16	0.05	0.48	1.71	0.13	0.05	0.14	1.78
0.23	-0.25	0.24	0.00	2.31	0.15	0.05	0.50	1.73	0.11	0.05	0.14	1.75
0.25	-0.25	0.22	0.02	2.31	0.14	0.04	0.52	1.75	0.10	0.04	0.15	1.72

The 1950-1967 time series contains little information about the long-run primary energy price elasticity. This is evident from the large standard errors when the series is considered alone. This is also why the 1968-1978 data dominate the results from the pooled data. The elasticity estimates for the pooled period, 1950-1978, are approximately four-fifths the magnitude of the 1968-1978 estimates. They are influenced very little by the earlier period. Therefore, it may be best to consider only the more recent decade, and to avoid using the inconclusive evidence of the 1950-1967 period.

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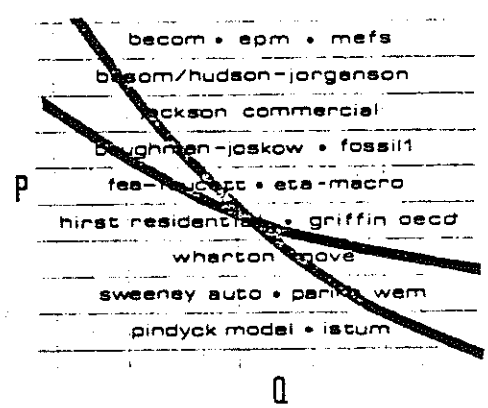
Chapter 13

DELIVERED ENERGY DEMAND ELASTICITIES: EVIDENCE FROM RECENT TRENDS*

William W. Hogan



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