Linear Digital Modulation and its Performance in AWGN and in Fading

Lecture Outline

- **Performance of Linear Modulation in AWGN**
- **Performance Metrics in Flat Fading**
- **Outage Probability**
- **Average Probability of Error**

1. **Performance of Linear Modulation in AWGN:**
   - ML detection corresponds to decision regions.
   - For coherent modulation, probability of symbol error P_s depends on the number of nearest neighbors α_M, and the ratio of their distance d_{min} to the square root $\sqrt{N_0}$ of the noise power spectral density (this ratio is a function of the SNR γ_s).
   - P_s approximated by $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$, where α_M and β_M depend on the constellation size and modulation type (MPSK vs. MQAM).
   - Alternate Q function representation $Q(z) = \frac{1}{\pi}\int_0^{.5\pi} \exp[-z^2/(2\sin^2\phi]d\phi$ leads to closed form expression for error probability of PSK in AWGN and, more importantly, greatly simplifies fading/diversity analysis.

2. **Performance of Linear Modulation in Fading:**
   - In fading γ_s and therefore P_s are random variables.
   - Three performance metrics to characterize the random P_s.
   - Outage: $p(P_s > P_{target} = p(\gamma < \gamma_{target})$
   - Average P_s ($\overline{P}_s = \int P_s(\gamma)p(\gamma)d\gamma$).
   - Combined outage and average P_s.

3. **Outage Probability:** $p(P_s > P_{s_{\text{target}}}) = p(\gamma_s < \gamma_{s_{\text{target}}})$.
   - Outage probability used when fade duration long compared to a symbol time.
   - Obtained directly from fading distribution and target γ_s.
   - Can obtain simple formulas for outage in log-normal shadowing or in Rayleigh fading.

4. **Average P_s:** $\overline{P}_s = \int P_s(\gamma_s)p(\gamma_s)d\gamma_s$.
   - Rarely leads to close form expressions for general $p(\gamma_s)$ distributions.
   - Can be hard to evaluate numerically.
   - Can obtain closed form expressions for general linear modulation in Rayleigh fading (using approximation $P_s \approx \alpha Q(\sqrt{\beta\gamma_s})$ in AWGN).

5. **Moment Generating function technique for Average P_s**

- The alternate Q function representation is

$$Q(z) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left[\frac{-z^2}{2\sin^2\phi}\right] d\phi.$$

- Using this alternate Q function representation, the average error probability is given by

$$\overline{P}_s = \alpha\int_0^{\infty} Q(\sqrt{\beta\gamma_s})p(\gamma_s)d\gamma_s = \alpha\int_0^{\infty}\frac{1}{\pi}\int_0^{\pi/2}\exp\left[\frac{-\beta\gamma_s}{2\sin^2\phi}\right] d\phi p(\gamma_s)d\gamma_s.$$

- Since the inner integral is bounded, we can switch the order of integration, yielding

$$\overline{P}_s = \frac{\alpha}{\pi}\int_0^{\pi/2}\mathcal{M}_{\gamma_s}\left(\frac{-.5\beta}{\sin^2\phi}\right)d\phi,$$

  where

$$\mathcal{M}_{\gamma_s}(s) = \int_0^{\infty} e^{s\gamma_s}p(\gamma_s)d\gamma_s$$

  is a moment generating function (MGF) of the distribution $p(\gamma_s)$ and is in the form of a Laplace transform.

- The MGF for any distribution of interest can be computed in closed-form using classical Laplace transforms.
- If Laplace transform is not in closed form, it is easily computed numerically for most fading distributions of interest.
- The parameter $s = -.5\beta/\sin^2\phi$ of the moment generating function depends on the modulation via β.

Main Points

- Can approximate symbol error probability P_s of MPSK and MQAM in AWGN using simple formula: $P_s \approx \alpha_M Q(\sqrt{\beta_M\gamma_s})$, with standard or alternate Q function representation.
- In fading, P_s is a random variable, characterized by average value, outage, or combined outage and average.
- Outage probability based on target SNR in AWGN.
- Closed-form expressions for average P_s of BPSK and DPSK in Rayleigh fading decrease as $1/\overline{\gamma_b}$.
- Average P_s obtained by integrating P_s in AWGN over fading distribution. In general hard to compute for standard Q function since double-integral diverges analytically and numerically.
- Moment generating function approach for average P_s makes computation easy.
- Fading leads to large outage probability and greatly increased average P_s.