# Received Signals for Narrowband Channels. Signal Envelope Distribution. Wideband Channels.

## Lecture Outline

- **Mean, Autocorrelation and Crosscorrelation in Narrowband Fading.**
- **Correlation and PSD under Uniform Scattering.**
- **Envelope Distributions: Rayleigh, Rician, Nakagami.**
- **Wideband Channel Models and their Characterization**

1. **Mean, Autocorrelation, and Cross Correlation in Narrowband Fading**

   - Received signal $r(t) = r_I(t)\cos(2\pi f_c t + \phi_0) - r_Q(t)\sin(2\pi f_c t + \phi_0)$, where $r_I(t) = \sum_{i=0}^{N(t)} \alpha_i(t)\cos(\phi_i(t))$ and $r_Q(t) = \sum_{i=0}^{N(t)} \alpha_i(t)\sin(\phi_i(t))$, is a Gaussian random process under CLT.
   - Assuming $\phi_i(t)$ uniform, $E[r_I(t)] = E[r_Q(t)] = 0$ and $E[r_I(t)r_Q(t)] = 0$. Thus, $r_I(t)$ and $r_Q(t)$ are uncorrelated, hence independent. Moreover, $E[r(t)] = 0$
   - $A_{r_I}(t, t+\tau) = A_{r_I}(\tau) = A_{r_Q}(\tau) = .5\sum_{i=0}^{N-1} E[\alpha_i^2]E_{\theta_i}[\cos(2\pi v\tau/\lambda)\cos\theta_i]$; the processes are WSS and hence stationary.
   - By a similar analysis, $A_{r_I,r_Q}(t, t+\tau) = E[r_I(t)r_Q(t+\tau) = .5\sum_{i=0}^{N-1} E[\alpha_i^2]E_{\theta_i}[\sin(2\pi v\tau/\lambda)\cos\theta_n] = A_{r_I,r_Q}(\tau)$.
   - Using these derivations, we get the autocorrelation for the received signal as $A_r(t, t+\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) + A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau) = A_r(\tau)$. Since this only depends on $\tau$, the received signal is WSS and hence stationary.

2. **Correlation and PSD under Uniform Scattering**

   - Under uniform scattering the $\theta_i$s are uniformly distributed. We also assume all multipath components have the same expected power, so $E[\alpha_i^2] = 2P_r/N$ for $P_r$ the total received power and $N$ the number of multipath components. With these assumptions the in-phase and quadrature signal components have zero cross correlation and have autocorrelation $A_{r_I}(\tau) = A_{r_Q}(\tau) = P_r J_0(2\pi f_D\tau)$.
   - Thus the signal decorrelates over roughly one half of a wavelength, but later recorrelates.
   - Can obtain the PSD of the received signal by taking the Fourier transform of its autocorrelation.
   - This yields $S_r(f) = .25[S_{r_I}(f - f_c) + S_{r_I}(f + f_c)]$ where

   $$S_{r_I}(f) = S_{r_Q}(f) = \mathcal{F}[A_{r_I}(\tau)] = \begin{cases} \frac{2P_r}{\pi f_D}\frac{1}{\sqrt{1-(f/f_D)^2}} & |f| \leq f_D \\ 0 & \text{else} \end{cases}$$

   - The PSD is useful in simulating fading channels.

3. **Signal Envelope Distributions**

   - CLT approximation leads to Rayleigh distribution (in-phase and quadrature zero mean and jointly Gaussian): $p_Z(z) = \frac{2z}{P_r}\exp[-z^2/P_r] = \frac{z}{\sigma^2}\exp[-z^2/(2\sigma^2)],\ \ z \geq 0$.
   - Obtain power distribution by making the change of variables $z^2(t) = |r(t)|^2$ to obtain $p_{Z^2}(x) = \frac{1}{P_r}e^{-x/P_r} = \frac{1}{2\sigma^2}e^{-x/(2\sigma^2)},\ \ x \geq 0$.
   - A LOS component leads to a received signal with non-zero mean. The Rician distribution models signal envelope in this case, with $K$ factor dictating the relative power of the LOS component: $p_Z(z) = \frac{z}{\sigma^2}\exp\left[\frac{-(z^2+s^2)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right),\ \ z \geq 0$.
   - Experimental results support a Nakagami distribution for some environments. Similar to Rician, but can model "worse than Rayleigh." Model generally leads to closed-form expressions in BER and diversity analysis.
   - Distribution is $p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)P_r^m}\exp\left[\frac{-mz^2}{P_r}\right],\ \ m \geq .5$. By change of variables, power distribution is $p_{Z^2}(x) = \left(\frac{m}{P}\right)^m \frac{x^{m-1}}{\Gamma(m)}$.

4. **Wideband Channel Models**

   - In wideband multipath channels the individual multipath components can be resolved by the receiver. True if $T_m > 1/B$.
   - If the components can be resolved then they can be combined for diversity gain (e.g. using an equalizer).
   - Typically time-varying channel impulse response $c(\tau, t)$ is unknown, so its wideband model must be characterized statistically.
   - Since under our random model with a large number of scatterers, $c(\tau, t)$ is Gaussian, only need to characterize its mean and correlation, which we show is independent of time (wide-sense stationary or WSS). Similar to narrowband model, for $\phi_n$ uniformly distributed, $c(\tau, t)$ has mean zero.

## Main Points

- A narrowband fading model and the CLT lead to inphase, quadrature, and received signals that are stationary Gaussian processes with zero mean and an autocorrelation function that depends on the AOAs of the multipath components.
- Uniform scattering leads to in-phase and quadrature signal components that are uncorrelated, hence independent.
- The autocorrelation of these components under uniform scattering follows a Bessel function (decorrelates after half signal wavelength).
- The PSD has a bowel shape centered around the carrier frequency.
- The signal envelope under narrowband fading with uniform AOA is Rayleigh. Other common distribution are Ricean (when a LOS component exists) and Nakagami.
- Wideband channels have resolvable multipath. Channel response $c(\tau, t)$ is random and by CLT it is Gaussian, hence characterized by mean and autocorrelation.