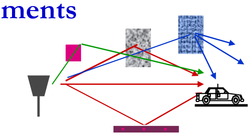
EE359 – Lecture 5 Outline

- Announcements:
  - HW posted, due Friday 4pm
  - Background on random processes in Appendix B
- Review of Last Lecture: Narrowband Fading
- Auto and Cross Correlation of In-Phase and Quadrature Signal Components
- Correlation and PSD in uniform scattering
- Signal Envelope Distributions
- Wideband Channels and their Characterization

Review of Last Lecture



- Model Parameters from Measurements
- Random Multipath Model
- Channel Impulse Response

$$c(\tau,t)=\sum_{i=1}^{N}\alpha_i(t)e^{-j\varphi_i(t)}\delta(\tau-\tau_i(t))$$

  - Many multipath components, Amplitudes change slowly, Phases change rapidly
- For delay spread max$|\tau_n(t)-\tau_m(t)| \ll 1/B$, $u(t)\approx u(t-\tau)$.
  - Received signal given by

$$r(t)=\Re\left\{u(t)e^{j2\pi f_c t}\boxed{\left[\sum_{i=0}^{N(t)}\alpha_i(t)e^{-j\phi_i(t)}\right]}\right\}$$

- No signal distortion in time
- Multipath yields complex scale factor in brackets

Review Continued:

Narrowband Model

- For $u(t)=e^{j\phi_0}$, $r(t)=r_I(t)\cos(2\pi f_c t+\phi_0)-r_Q(t)\sin(2\pi f_c t+\phi_0)$
- In-phase and quadrature signal components:

$$\boxed{r_I(t)=\sum_{i=0}^{N(t)}\alpha_i(t)e^{-j\phi_i(t)}\cos(2\pi f_c t),\ r_Q(t)=\sum_{i=0}^{N(t)}\alpha_i(t)e^{-j\phi_i(t)}\sin(2\pi f_c t)}$$

$$\boxed{\phi_i(t)=2\pi f_c\tau_i(t)-\phi_{D_i}(t)-\phi_0}$$

- For $N(t)$ large, $r_I(t)$ & $r_Q(t)$ jointly Gaussian by CLT
- Received signal characterized by its mean, autocorrelation, and cross correlation. Let $\phi_i \sim U[0,2\pi]$

$$\Rightarrow\quad A_{r_I}(\tau)=A_{r_Q}(\tau)=.5\sum_i \mathbf{E}[\alpha_i^2]\,\mathbf{E}_{\theta_i}\left[\cos\left(\frac{2\pi v\tau}{\lambda}\cos\theta_i\right)\right]$$

- If $\varphi_i(t)$ uniform, in-phase/quad components are mean zero, independent, and stationary (WSS)

Cross Correlation

$$r_I(t)=\sum_{i=0}^{N(t)}\alpha_i(t)e^{-j\phi_i(t)}\cos(2\pi f_c t),\ r_Q(t)=\sum_{i=0}^{N(t)}\alpha_i(t)e^{-j\phi_i(t)}\sin(2\pi f_c t),\quad \phi_i \sim U[0,2\pi]$$

- Cross Correlation of inphase/quad signal is

$$A_{r_I,r_Q}(\tau)=-A_{r_Q,r_I}(\tau)=.5\sum_i \mathbf{E}[\alpha_i^2]\,\mathbf{E}_{\theta_i}\left[\sin\left(\frac{2\pi v\tau}{\lambda}\cos\theta_i\right)\right]$$

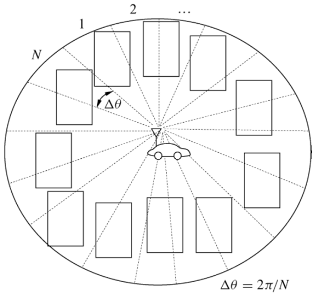
  - Thus, $A_{r_I,r_Q}(0)=0$, so $r_I(t)$ and $r_Q(t)$ independent
- Autocorrelation of received signal is

$$A_r(\tau)=A_{r_I}(\tau)\cos(2\pi f_c\tau)-A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$$

- Thus, r(t) is stationary (WSS)

Uniform Scattering

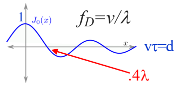
- Multipath comes uniformly from all directions
- Power in each component is the same: $\mathbf{E}[\alpha_i^2] = 2P_r/N$



Autocorrelation and PSD under uniform scattering

- Under uniform scattering, in phase and quad comps have no cross correlation and autocorrelation is

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = P_r J_0(2\pi f_D \tau)$$

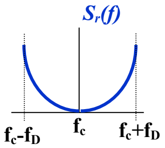


Decorrelates over roughly half a wavelength

- The PSD of received signal is

$$S_r(f) = .25[S_{r_I}(f - f_c) + S_{r_I}(f + f_c)]$$

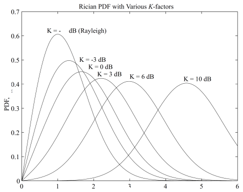
$$S_{r_I}(f) = \mathcal{F}[P_r J_0(2\pi f_D \tau)]$$

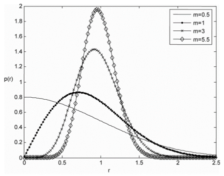


Used to generate simulation values

Signal Envelope Distribution

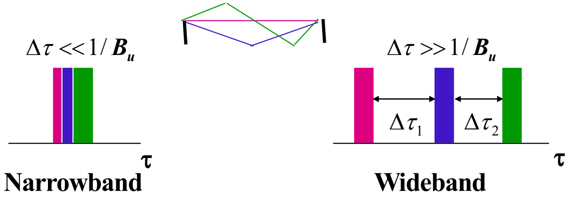
- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models "worse than Rayleigh"
  - Lends itself better to closed form BER expressions

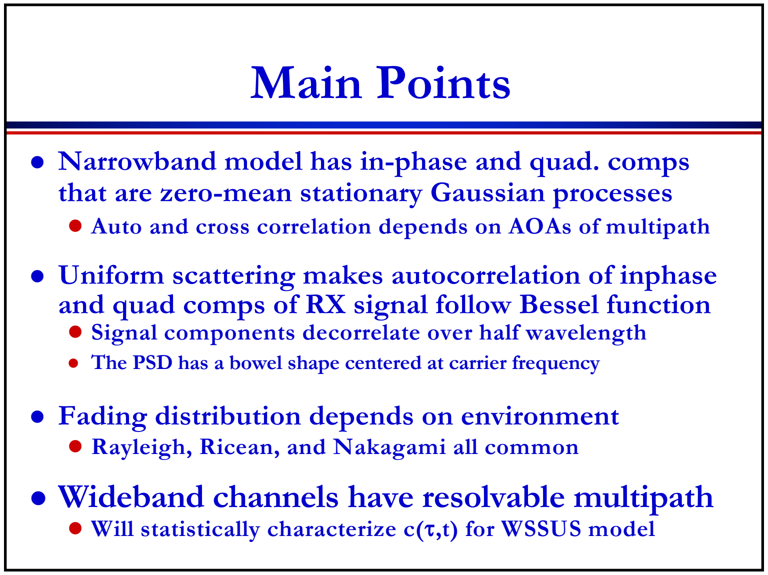




Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth
- Requires statistical characterization of c(τ,t)
  - Assume CLT, stationarity and uncorrelated scattering
  - Leads to simplification of its autocorrelation function





Main Points

- Narrowband model has in-phase and quad. comps that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on AOAs of multipath
- Uniform scattering makes autocorrelation of inphase and quad comps of RX signal follow Bessel function
  - Signal components decorrelate over half wavelength
  - The PSD has a bowel shape centered at carrier frequency
- Fading distribution depends on environment
  - Rayleigh, Ricean, and Nakagami all common
- Wideband channels have resolvable multipath
  - Will statistically characterize c(τ,t) for WSSUS model