EE359 – Lecture 4 Outline

- **Announcements:**
  - 1st HW due Friday 4pm: HW can be submitted in hardcopy (box outside Dash's office) or upload to canvas.
- Review of Last Lecture
- Random Multipath Model
- Time Varying Channel Impulse Response
- Narrowband Fading Model
- In-Phase and Quad Signal Components
- Auto and Crosscorrelation of received signal

Review of Last Lecture

- Shadowing: Log-normal random variable based on **CLT** applied to many attenuating objects
- Combined Path Loss and Shadowing

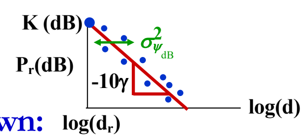
$$\frac{P_r}{P_t}(dB) = \underbrace{10\log_{10} K}_{K_{dB}} - 10\gamma\log_{10}\left(\frac{d}{d_r}\right) - \psi_{dB},\ \psi_{dB} \sim N(\mu_{\psi_{dB}}, \sigma^2_{\psi_{dB}})$$

$\mu_{\psi_{dB}}$=0 when average shadowing incorporated into K and γ, else $\mu_{\psi_{dB}}$>0

- **Outage probability:** $p_{out}(P_{min}, d) = p(P_r(d) < P_{min})$
- **For log-normal shadowing model** E[$P_r(d)$]

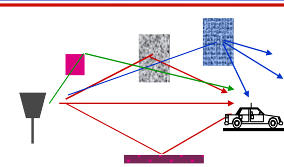
$$p(P_r(d) \le P_{min}) = 1 - Q\left(\frac{P_{min} - (P_t + K_{\text{dB}} - 10\gamma\log_{10}(d/d_r))}{\sigma_{\psi_{dB}}}\right)$$

Review continued



- Fit model to data
- Path loss (K,γ), d_r known:
  - "Best fit" line through dB data:
  - 1D (γ) or 2D (γ,K) minimization of the MSE
  - For 1D, K obtained from free-space or at d_r.
  - Captures mean due to shadowing
- Shadowing variance
  - Variance of data relative to path loss model (straight line) with MMSE estimate for γ

Statistical Multipath Model



- Random # of multipath components, each with
  - Random amplitude
  - Random phase
  - Random Doppler shift
  - Random delay
- Random components change with time
- Leads to time-varying channel impulse response

Time Varying Impulse Response

- Response of channel at t to impulse at t-τ:

$$c(\tau,t)=\sum_{i=0}^{N(t)}\alpha_i(t)e^{-j\phi_i(t)}\delta(\tau-\tau_i(t))$$

  - t is time when impulse response is observed
  - $t-\tau$ is time when impulse put into the channel
  - τ is how long ago impulse was put into the channel for the current observation
    - path delay for MP component currently observed

5

Received Signal Characteristics

- Received signal consists of many multipath components
- Amplitudes change slowly
- Phases change rapidly
  - Constructive and destructive addition of signal components
  - Amplitude fading of received signal (both wideband and narrowband signals)

6

Narrowband Model

- Assume delay spread $\max_{m,n}|\tau_n(t)-\tau_m(t)|<<1/B$
- Then $u(t)\approx u(t-\tau)$.
- Received signal given by

$$r(t)=\mathrm{Re}\left\{u(t)e^{j2\pi f_c t}\left(\sum_i \alpha_i(t)e^{-j\phi_i(t)}\right)\right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Characterize scale factor by setting $u(t)=e^{j\phi_0}$:

$$s(t)=\mathrm{Re}\{e^{j(2\pi f_c t+\phi_0)}\}=\cos(2\pi f_c t+\phi_0)$$

7

In-Phase and Quadrature under CLT Approximation

- In phase and quadrature signal components:

$$r_I(t)=\sum_{i=0}^{N(t)}\alpha_i(t)\cos\phi_i(t),$$

$$\phi_i(t)=2\pi f_c\tau_i(t)-\phi_{D_i}(t)-\phi_0$$

$$r_Q(t)=\sum_{i=0}^{N(t)}\alpha_i(t)\sin\phi_i(t)$$

- For $N(t)$ large, $r_I(t)$ and $r_Q(t)$ jointly Gaussian by CLT (sum of large # of random vars).
- Received signal characterized by its mean, autocorrelation, and cross correlation.
- If $\varphi_i(t)$ uniform, the in-phase/quad components are mean zero, independent, and stationary.

8

Auto and Cross Correlation

- Assume ϕ_i~U[0,2π]
- Recall that θ_i is the multipath arrival angle
- Autocorrelation of inphase/quad signal is

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = .5\sum_i \mathbf{E}[\alpha_i^2]\cos\left(\frac{2\pi v\tau}{\lambda}\cos\theta_i\right)$$

- Cross Correlation of inphase/quad signal is

$$A_{r_I,r_Q}(\tau) = -A_{r_Q,r_I}(\tau) = .5\sum_i \mathbf{E}[\alpha_i^2]\sin\left(\frac{2\pi v\tau}{\lambda}\cos\theta_i\right)$$

- Autocorrelation of received signal is

$$A_r(\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) + A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$$

9

Main Points

- Statistical multipath model leads to a time-varying channel impulse response
- Resulting received signal has rapidly varying amplitude due to constructive and destructive multipath combining
- Narrowband model has in-phase and quad. comps that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on AOAs of multipath

10