EE359 – Lecture 16 Outline

- **Announcements:**
  - HW due Fri, new HW to be posted today
  - End-of-Quarter schedule and possible bonus lecture
    - No lecture March 5, makeup lecture Monday March 2, 10:30-11:50, Gates B03
    - Advanced topics lecture? Could extend last class March 12 to 3:30 (i.e. 1:30-3:30)
    - Final exam: Tues March 17, 3:30-6:30pm, here. More details next week.
- **MIMO Receiver Design**
  - Linear Receivers, Sphere Decoder
- **Other MIMO Design Issues**
  - Space-time coding, adaptive techniques, limited feedback
- **ISI Countermeasures**
- **Multicarrier Modulation**

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Review of last lecture
Capacity of Fading & Massive MIMO Systems

- Static channel with perfect TX and RX CSI: water-fill over space
- In fading waterfill over space (based on instantaneous power constraint P)

$$C = \mathrm{E}_{\mathbf{H}}\left[\max_{\mathbf{R_x}:\mathrm{Tr}(\mathbf{R_x})=\rho} B\log_2\det\left[\mathbf{I}_{\mathbf{M_r}} + \mathbf{H}\mathbf{R_x}\mathbf{H}^{\mathbf{H}}\right]\right] = \mathbf{E}_{\mathbf{H}}\left[\max_{P_i:\Sigma_i P_i\le \bar{P}} \sum_i B\log_2\left(1+\frac{P_i\gamma_i}{\bar{P}}\right)\right]$$

  - Or over space-time (with average power constraint P)

$$C = \max_{P_H : E_H[P_H]\le \bar{P}} E_H\left[\max_{P_i:\sum_i P_i \le P_H} \sum_i B\log_2\left(1+\frac{P_i\gamma_i}{P_H}\right)\right]$$

- Without transmitter channel knowledge, capacity metric is based on an outage probability with respect to transmitted rate C:

$$P_{\rm out} = p\left(\mathbf{H} : B\log_2\det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right] < R\right)$$

- Massive MIMO: As $M_t \rightarrow \infty$, by random matrix theory

$$\lim_{M_t\to\infty} B\log_2\det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right] = B\log_2\det\left[\mathbf{I}_{M_r} + \rho\mathbf{I}_{M_r}\right] = M_r B\log_2(1+\rho).$$

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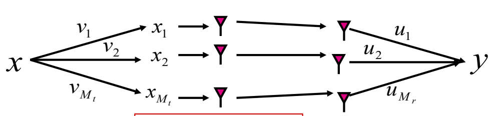
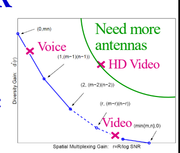
Review of Last Lecture (Cont'd)

- Beamforming: Scalar transmission
  - Principle vectors of U and V are weights: maximizes SNR



$y = u^H H v x + u^H n$

- Diversity-Multiplexing Tradeoff: high SNR
  - Can use some antennas for diversity, some for capacity gain: $\mathbf{d^*(r) = (M_t - r)(M_r - r)}$
  - How antennas used depends on system metric
  - If requirements unmet, need more antennas



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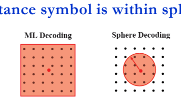
MIMO RX Design

- Optimal Receiver is ML: finds input vector x most likely to have resulted in $y=Hx+n$, exponentially complex in M_t
- Linear Receivers: performs linear equalization $\tilde{x} = Ay$ then quantizes $\tilde{x}$ to nearest constellation point $x \in \mathcal{X}^{M_t}$
  - Zero-Forcing ($A = \mathbf{H}^\dagger$, the Moore-Penrose pseudo inverse of H): (if H invertible, equals inverse, else $\mathbf{H}^\dagger = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$); forces off-diagonal terms to zero ($\tilde{x}_i = x_i + \tilde{n}_i$; $\tilde{n} = \mathbf{H}^\dagger n$, enhances noise)
  - Minimum Mean Square Error ($A = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \lambda\mathbf{I})^{-1}$): $\lambda \propto 1/SNR$ Balances zero forcing against noise enhancement
- Sphere Decoder: Uses QR decomposition of H
  - Considers possibilities within sphere of transformed received symbol.
    - If minimum distance symbol is within sphere, optimal, otherwise null is returned

$\hat{x} = \arg\min |y - Hx|^2$

$Hx+n$



$\hat{x} = \arg\min_{x:|Q^H y - Rx|<r} |Q^H y - Rx|^2$

$Q^H y = Rx + Q^H n$

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Other MIMO Design Issues

Not covered in lecture/HW/exams

- Space-time coding:
  - Map symbols to both space and time via space-time block and convolutional codes.
  - For OFDM systems, codes are also mapped over frequency tones.
- Adaptive techniques:
  - Need fast and accurate channel estimation
  - Adapt the use of transmit/receive antennas
  - Adapting modulation and coding.
- Limited feedback transmit precoding:
  - Partial CSI introduces interference in parallel decomp: can use interference cancellation at RX
  - TX codebook design for quantized channel

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ISI Countermeasures

- Equalization
  - Signal processing at receiver to eliminate ISI
  - Complex at high data rates, performs poorly in fast-fading
  - Not used in state-of-the-art wireless systems
- Multicarrier Modulation
  - Break data stream into lower-rate substreams modulated onto narrowband flat-fading subchannels
- Spread spectrum
  - Superimpose a fast (wideband) spreading sequence on top of data sequence, allows resolution for combining or attenuation of multipath components.
- Antenna techniques (Massive MIMO)
  - (Highly) directional antennas reduce delay spread/ISI

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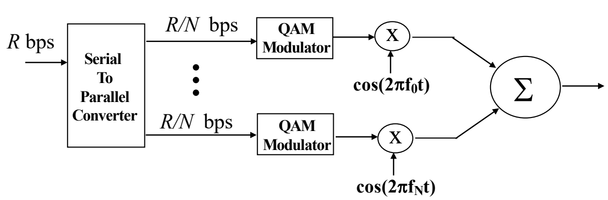
Multicarrier Modulation



- Breaks data into N substreams
- Substream modulated onto separate carriers
  - Substream passband BW is B/N for B total BW
  - $B/N<B_c$ implies flat fading on each subcarrier (no ISI)

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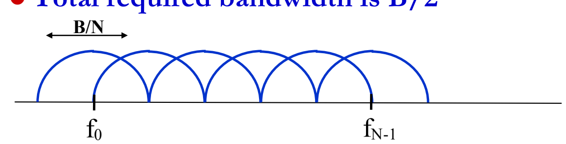
Overlapping Substreams

- Can have completely separate subchannels
  - Required passband bandwidth is B.
- OFDM overlaps substreams
  - Substreams (symbol time T_N) separated in RX
  - Minimum substream separation is $1/T_N$ for rectangular pulses
  - Total required bandwidth is B/2



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Main Points

- MIMO RX design trades complexity for performance
  - ML detector optimal - exponentially complex
  - DF receivers prone to error propagation
  - Sphere decoders allow performance tradeoff via radius
- Other MIMO design issues include space-time coding, adaptation, codebooks for limited feedback
- ISI mitigated through equalization, multicarrier modulation (MCM) or spread spectrum
  - Today, equalizers often too complex or can't track channel.
  - MCM splits channel into NB flat fading subchannels
  - Can overlap subcarriers to preserve bandwidth

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