

MIMO Fading Channel Capacity, Massive MIMO, Beamforming, Diversity vs. Multiplexing, RX Design

Lecture Outline

- MIMO Fading Channel Capacity
- Massive MIMO
- Beamforming
- MIMO Diversity/Multiplexing Tradeoffs
- MIMO RX Design

1. MIMO Fading Channel Capacity

- In fading, capacity with both transmitter and receiver knowledge is the average of the capacity for the static channel, with power allocated either by an instantaneous or average power constraint. Under the instantaneous constraint power is optimally allocated over the spatial dimension only. Under the average constraint it is allocated over both space and time.
- In fading, if the channel is unknown at transmitter, uniform power allocation is optimal, but this leads to an outage probability since the transmitter doesn't know what rate R to transmit at:

$$P_{out} = p \left(\mathbf{H} : B \log_2 \det \left[\mathbf{I}_{M_r} + \frac{\rho}{M_t} \mathbf{H} \mathbf{H}^H \right] < R \right).$$

2. Massive MIMO:

- When the number of TX (or RX) antennas is large, the channel becomes “static” due to the law of large numbers. Specifically

$$\lim_{M_t \rightarrow \infty} \frac{1}{M_t} \mathbf{H} \mathbf{H}^H = \mathbf{I}_{M_r}.$$

- The MIMO channel capacity then becomes

$$\lim_{M_t \rightarrow \infty} B \log_2 \det \left[\mathbf{I}_{M_r} + \frac{\rho}{M_t} \mathbf{H} \mathbf{H}^H \right] = B \log_2 \det [\mathbf{I}_{M_r} + \rho \mathbf{I}_{M_r}] = M_r B \log_2(1 + \rho).$$

- Defining $M = \min(M_t, M_r)$, this implies that as M_t grows large, the MIMO channel capacity in the absence of TX CSI approaches $C = MB \log_2(1 + \rho)$ for ρ the SNR, and hence grows linearly in M . This is also true for finite M_t and M_r but at large SNR.

3. MIMO Systems: Beamforming

- Beamforming sends the same symbol over each transmit antenna with a different scale factor.
- At the receiver, all received signals are coherently combined using a different scale factor.
- This produces a transmit/receiver diversity system, whose SNR can be maximized by optimizing the scale factors (MRC).
- Beamforming leads to a much higher SNR than on the individual channels in the parallel channel decomposition.
- Thus, there is a design tradeoff in MIMO systems between capacity and diversity.

4. Diversity versus Multiplexing in MIMO Systems

- Can exchange data rate for probability of error.
- Define rate scale factor $r = R/\log(SNR)$. Define diversity gain $d = \log(P_e)/\log(SNR)/r$.
- Can show (Zheng/Tse'02) that in high SNR regime, the optimal tradeoff is $d^*(r) = (M_t - r)(M_r - r)$.
- The optimal operating point on this tradeoff curve depends on the application.

5. MIMO Receiver Design (see supplemental notes)

- Optimal MIMO receiver is maximum-likelihood (ML) receiver. Finds input vector \mathbf{x} that minimizes $\|\mathbf{y} - \mathbf{H}\mathbf{x}\|_F^2$ for $\|\cdot\|_F$ the Frobenius (matrix) norm.
- This receiver is exponentially complex in the constellation size and number of transmitted data streams.
- Can reduce complexity through linear processing of input vector $\mathbf{A}\mathbf{x}$.
- Zero-forcing receiver forces all interference from other symbols to zero. This can result in significant noise enhancement.
- MMSE receiver: trades off cancellation of interference from other symbols for noise enhancement. Reduces to zero forcing in the absence of noise.
- Sphere decoder: uses upper triangular decomposition of H to reduce complexity. Finds constellation point within a sphere of a given radius. Provides near-ML performance with near-linear complexity.

Main Points

- With TX and RX CSI, capacity of MIMO channel uses waterfilling in space or space/time - leads to $\min(M_t, M_r)$ capacity gain.
- Without transmitter CSI, use outage as capacity metric.
- Capacity of massive MIMO or MIMO at high SNR scales as $\min(M_t, M_r)$.
- MIMO introduces diversity/multiplexing tradeoff: Optimal use of antennas depends on application.
- MIMO RX design trades complexity for performance. ML detector is optimal but exponentially complex. Linear decoders enhance noise. Sphere decoders allow performance vs. complexity tradeoff via radius; most common technique in practice.