MGF $\overline{P}_s$ approach. Combined average error and outage Impact of delay spread and Doppler, Intro to diversity

Lecture Outline

- **Moment Generating Function Approach to compute $\overline{P}_s$**
- **Combined Outage and Average Probability of Error**
- **Delay Spread Effects on Error Probability**
- **Doppler Effects on Error Probability**
- **Introduction to Diversity**

1. **Moment Generating function technique for Average P_s**

   - The alternate Q function representation is

   $$Q(z) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left[\frac{-z^2}{2\sin^2\phi}\right] d\phi.$$

   - Using the alternate Q function representation, average error probability is

   $$\overline{P}_s = \alpha\int_0^{\infty} Q(\sqrt{\beta\gamma_s})p(\gamma_s)d\gamma_s = \alpha\int_0^{\infty}\frac{1}{\pi}\int_0^{\pi/2}\exp\left[\frac{-\beta\gamma_s}{2\sin^2\phi}\right] d\phi p(\gamma_s)d\gamma_s.$$

   - Since the inner integral is bounded, we can switch the order of integration, yielding

   $$\overline{P}_s = \frac{\alpha}{\pi}\int_0^{\pi/2}\mathcal{M}_{\gamma_s}\left(\frac{-.5\beta}{\sin^2\phi}\right)d\phi,$$

   where

   $$\mathcal{M}_{\gamma_s}(s) = \int_0^{\infty} e^{s\gamma_s}p(\gamma_s)d\gamma_s$$

   is a moment generating function (MGF) of the distribution $p(\gamma_s)$ and is in the form of a Laplace transform.

   - The MGF for any distribution of interest can be computed in closed-form using classical Laplace transforms. If Laplace transform is not in closed form, it is easily computed numerically for most fading distributions of interest.
   - The parameter $s = -.5\beta/\sin^2\phi$ of the moment generating function depends on the modulation via β.

2. **MGF for common fading distributions**

   - The MGF corresponding to the most common distributions are given as

   $$\text{Rayleigh}: \mathcal{M}_r\left(\frac{-.5\beta}{\sin^2\phi}\right) = \left(1 + \frac{.5\beta\,\overline{\gamma}_s}{\sin^2\phi}\right)^{-1}.$$

$$\text{Ricean with factor } K: \mathcal{M}_K\left(\frac{-.5\beta}{\sin^2\phi}\right) = \frac{(1+K)\ \sin^2\phi}{(1+K)\sin^2\phi + .5\beta\ \overline{\gamma}_s}\ \exp\left(\frac{-.5K\ \beta\ \overline{\gamma}_s}{(1+K)\sin^2\phi + .5\beta\ \overline{\gamma}_s}\right).$$

$$\text{Nakagami-}m: \mathcal{M}_m\left(\frac{-.5\beta}{\sin^2\phi}\right) = \left(1 + \frac{\beta\ \overline{\gamma}_s}{m\ \sin^2\phi}\right)^{-m}.$$

- All of these functions are simple trigonometrics and are therefore easy to integrate over a finite range.

3. **Average Probability of Error with MGF**

- To compute the average probability of error for BPSK modulation in Nakagami fading, we use the fact that for an AWGN channel BPSK has $P_b = Q(\sqrt{2\gamma_b})$, so $\alpha = 1$ and $g = 1$ in average error probability expression above.
- Using the moment generating function for Nakagami-m fading and substituting this into the average error probability integral with $\alpha = g = 1$ yields

$$\overline{P}_b = \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{\overline{\gamma}_b}{m\sin^2\phi}\right)^{-m} d\phi.$$

4. **Combined outage and average error probability:**

- Shadowing causes outage and flat-fading determines $\overline{P}_s$ during nonoutage
- $\overline{P}_s$ obtained in small region where $\overline{\gamma}_s$ approximately constant as $\overline{P}_s = \int P_s(\gamma_s)p(\gamma_s|\overline{\gamma}_s)$.
- A target $\overline{\gamma}_s$ is needed to obtain a target $\overline{P}_s$.
- Outage occurs when shadowing causes $\overline{\gamma}_s$ to fall below its target value.

5. **Delay Spread (ISI) Effects on Performance.**

- Delay spread exceeding a symbol time causes ISI (self-interference).
- ISI leads to an irreducible error floor. Approximated as $\overline{P}_{b,floor} \approx (\sigma_{T_m}/T_s)^2$.
- Without ISI compensation, avoid error floor by reducing data rate: $T_s >> T_m$ or $R \leq \log_2(M) \times \sqrt{\overline{P}_{b,floor}/\sigma^2_{T_m}}$.

6. **Doppler Spread Effects on Performance** *(not covered in lecture)*

- Doppler causes the channel phase to decorrelate.
- Doppler impacts coherent modulation if an accurate coherent phase reference cannot be obtained at the receiver. A noisy phase estimate leads to large errors.
- Phase decorrelation between symbols leads to an irreducible error floor for differential modulation.
- Error floor approximated by $P_{b_{\text{floor}}} \approx .5(\pi B_d T_b)^2$.

7. **Introduction to Diversity**

- Basic concept is to send same information over independent fading paths.
- Paths are combined to mitigate the effects of fading.

8. **Realization of Independent Fading Paths**

   - Space Diversity: Multiple antenna elements spaced apart by decorrelation distance.
   - Polarization Diversity: Two antennas, one horizontally polarized and one vertically polarized.
   - Frequency diversity: Multiple narrowband channels separated by channel ooherence bandwidth.
   - Time diversity: Multiple timeslots separated by channel coherence time.

Main Points

- Easy to compute average P_s using alternate Q function and MGF approach: becomes a simple finite range integral of the MGF for the fading distribution.
- In combined fast and slow fading, outage is determined by shadowing, and average probability of error computed by averaging over the fading distribution conditioned on a fixed path loss and shadowing.
- Fading greatly degrades performance.
- Need to find ways to combat flat fading (adaptive modulation, which adapts to fading, or diversity, which removes fading).
- ISI leads to an irreducible error floor at high data rates - much work on ISI mitigation in current systems.
- Diversity is a powerful technique to overcome the effects of flat fading by combining multiple independent fading paths
- Diversity typically entails some penalty in terms of rate, bandwidth, complexity, or size.