Midterm Review

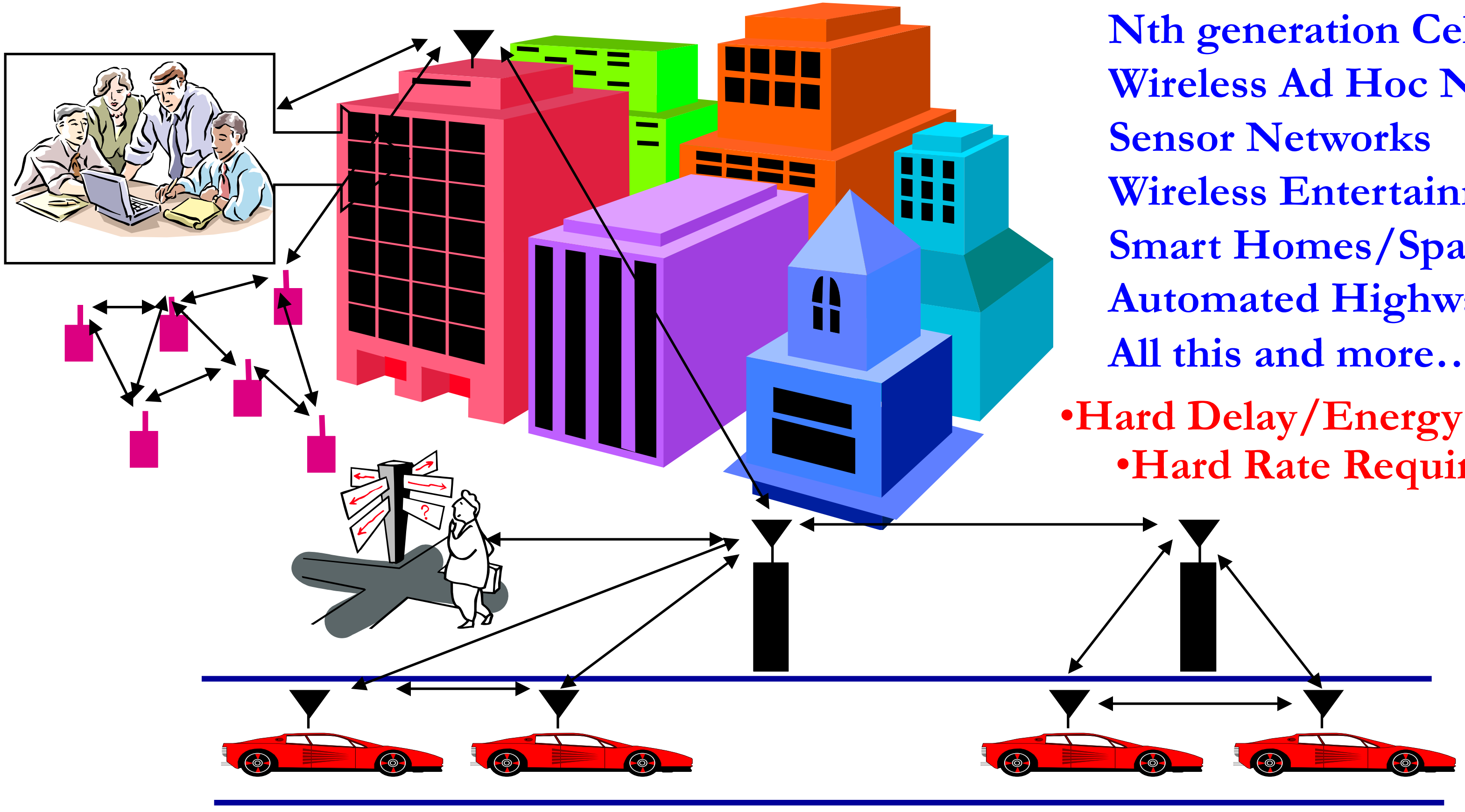
Midterm only covers material from lectures and HWs

- Overview of Wireless Systems:
  - Nothing from Chapter 1 is on the MT
- Signal Propagation and Channel Models
  - Chapter 2.1-2.4, 2.6-2.10
- Modulation and Performance Metrics
  - Chapter 3.1,3.2.1-3.2.2, 3.3
- Fundamental Capacity Limits
  - Chapter 4
- Impact of Channel on Performance
  - Chapter 6
- Diversity Techniques
  - Chapter 7.1,7.2.1-7.2.2,7.2.4,7.3.1,7.4.1

Not on MT

Future Wireless Networks

Ubiquitous Communication Among People and Devices



Wireless Internet access
Nth generation Cellular
Wireless Ad Hoc Networks
Sensor Networks
Wireless Entertainment
Smart Homes/Spaces
Automated Highways
All this and more…

- Hard Delay/Energy Constraints
- Hard Rate Requirements

Not on MT

Design Challenges

- Wireless channels are a difficult and capacity-limited broadcast communications medium
- Traffic patterns, user locations, and network conditions are constantly changing
- Applications are heterogeneous with hard constraints that must be met by the network
- Energy, delay, and rate constraints change design principles across all layers of the protocol stack

Not on MT

Current/Futre Wireless Systems

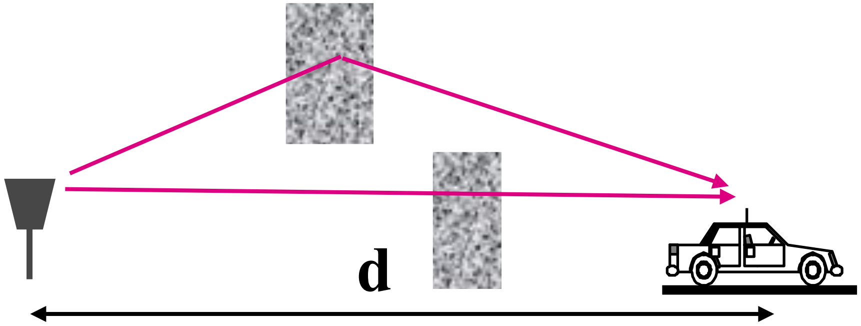
- Current Systems
  - 4G Cellular Systems
  - 802.11b/a/n/ac Wireless LANs
  - Satellite Systems
  - Paging Systems
  - Bluetooth
  - Zigbee radios

- Emerging Systems (Can cover in bonus lecture)
  - Ad hoc/mesh wireless networks
  - Cognitive radio networks
  - Wireless sensor networks
  - Energy-harvesting radios
  - Distributed control networks
  - Communications/SP in Health, Bio-medicine, and Neuroscience

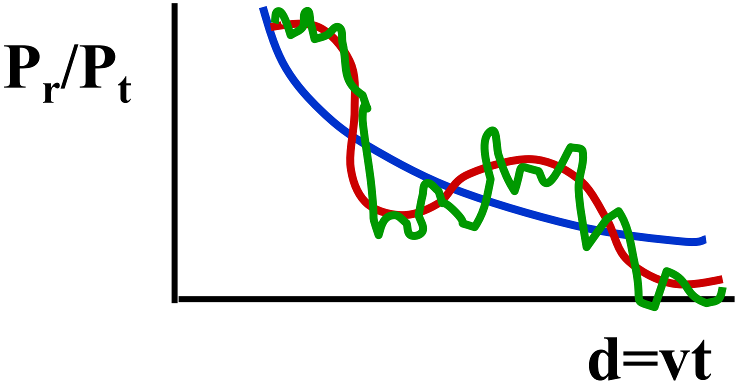
Signal Propagation

- Path Loss
  - Free space, 2-path,…
  - Simplified model

$$P_r = P_t K\left[\frac{d_0}{d}\right]^{\gamma},\ 2 \le \gamma \le 8$$

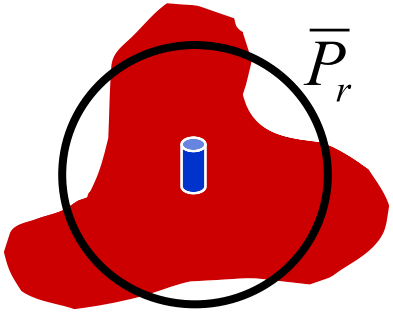


- Shadowing
  - dB value is Gaussian
  - Find path loss exponent and shadow STD by curve fitting



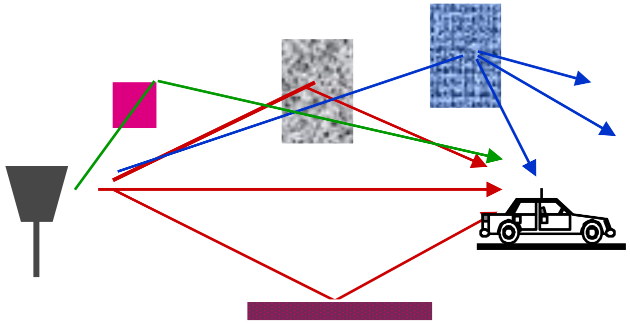
- Multipath
  - Ray tracing
  - Statistical model

Outage Probability and Cell Coverage Area



- Path loss: circular cells
- Path loss+shadowing: amoeba cells
  - Tradeoff between coverage and interference
- Outage probability
  - Probability received power below given minimum
- Cell coverage area
  - % of cell locations at desired power
  - Increases as shadowing variance decreases
  - Large % indicates interference to other cells

Statistical Multipath Model



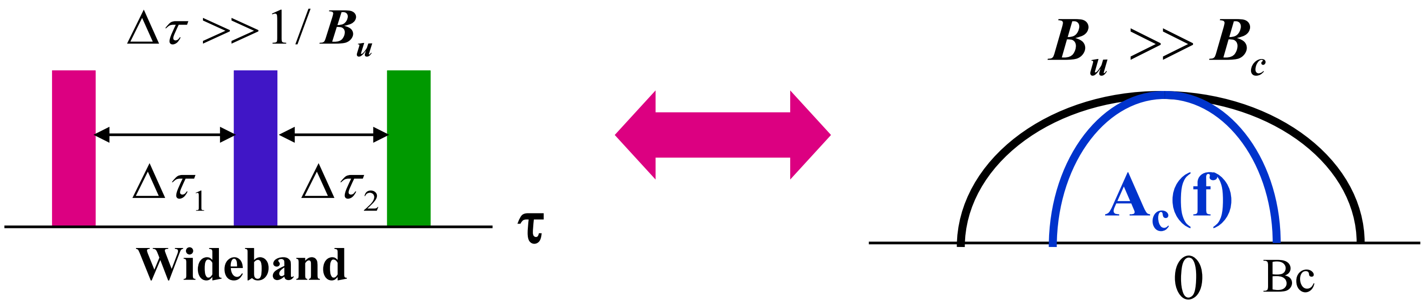
- Random # of multipath components, each with varying amplitude, phase, doppler, and delay
- Leads to time-varying channel impulse response

$$c(\tau,t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\varphi_n(t)}\delta(\tau-\tau_n(t))$$

- Narrowband channel
  - No signal distortion, just a complex amplitude gain
  - Signal amplitude varies randomly (Rayleigh, Ricean, Nakagami).
  - 2nd order statistics (Bessel function), Average fade duration

Wideband Channels

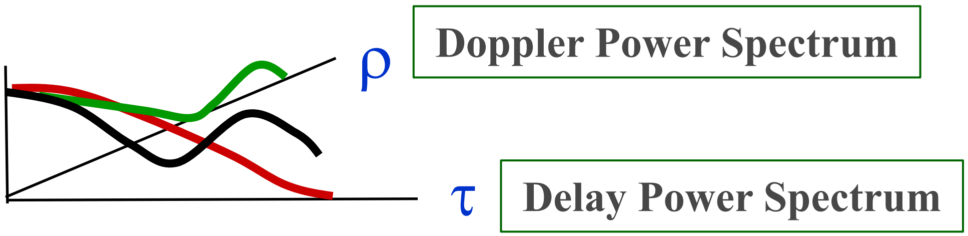
- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth



- Scattering function

  $s(\tau,\rho)=\mathcal{F}_{\Delta t}[A_c(\tau,\Delta t)]$

  - Yields delay spread/coherence BW ($\sigma_\tau \sim 1/B_c$)
  - Yields Doppler spread/coherence time ($B_d \sim 1/T_c$)



Capacity of Flat Fading Channels

- Channel Capacity
  - Maximum data rate that can be transmitted over a channel with arbitrarily small error
- Capacity of AWGN Channel: $B\log_2[1+\gamma]$ bps
  - $\gamma=P_r/(N_0B)$ is the receiver SNR
- Capacity of Flat-Fading Channels
  - Nothing known: capacity typically zero
  - Fading Statistics Known (few results)
  - Fading Known at RX (average capacity)

$$C=\int_0^\infty B\log_2(1+\gamma)p(\gamma)d\gamma \le B\log_2(1+\bar{\gamma})$$

- **Capacity in Flat-Fading: γ known at TX/RX**

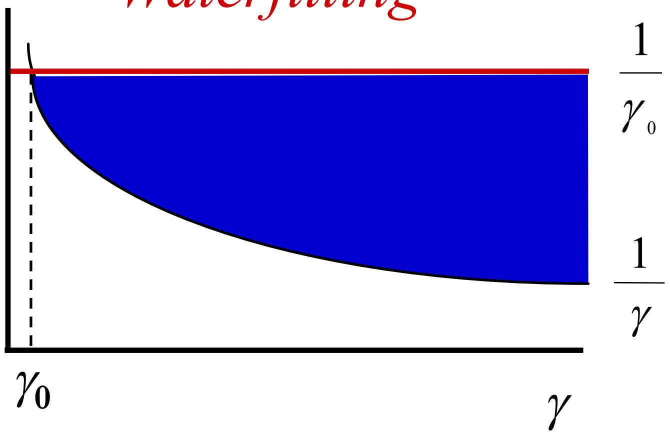
$$C = \max_{P(\gamma):E[P(\gamma)]=\overline{P}} \int_0^\infty B\log_2\left(1+\frac{\gamma P(\gamma)}{\overline{P}}\right)p(\gamma)d\gamma$$

- **Optimal Rate and Power Adaptation**

$$\frac{P(\gamma)}{\overline{P}} = \begin{cases} \frac{1}{\gamma_0}-\frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \textbf{else} \end{cases}$$

$$\frac{C}{B} = \int_{\gamma_0}^\infty \log_2\left(\frac{\gamma}{\gamma_0}\right)p(\gamma)d\gamma.$$

Waterfilling



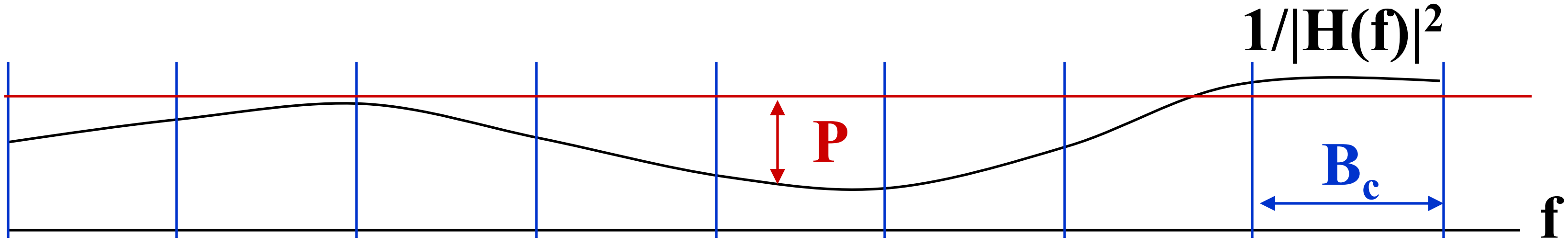
- **The instantaneous power/rate only depend on p(γ) through γ_0**

Channel Inversion

- Fading inverted to maintain constant SNR
- Simplifies design (fixed rate)
- Greatly reduces capacity
  - Capacity is zero in Rayleigh fading
- Truncated inversion
  - Invert channel above cutoff fade depth
  - Constant SNR (fixed rate) above cutoff
  - Cutoff greatly increases capacity
    - Close to optimal

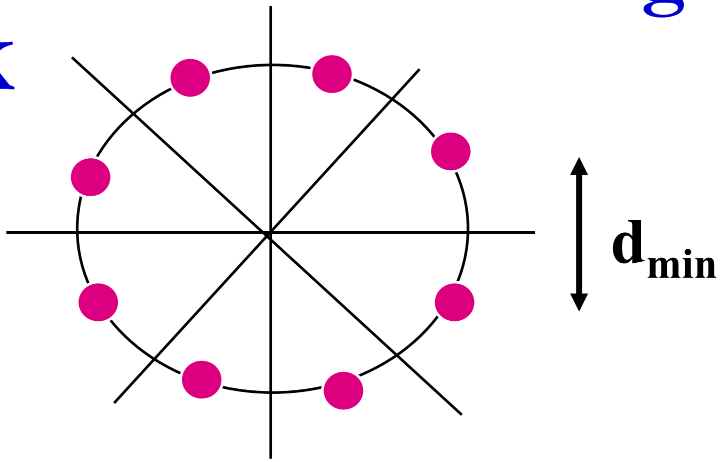
Frequency Selective Fading Channels

- For time-invariant channels, capacity achieved by water-filling in frequency
- Capacity of time-varying channel unknown
- Approximate by dividing into subbands
  - Each subband has width B_c
  - Independent fading in each subband
  - Capacity is the sum of subband capacities



Linear Modulation in AWGN: MPSK and MQAM

- ML detection induces decision regions
  - Example: 8PSK

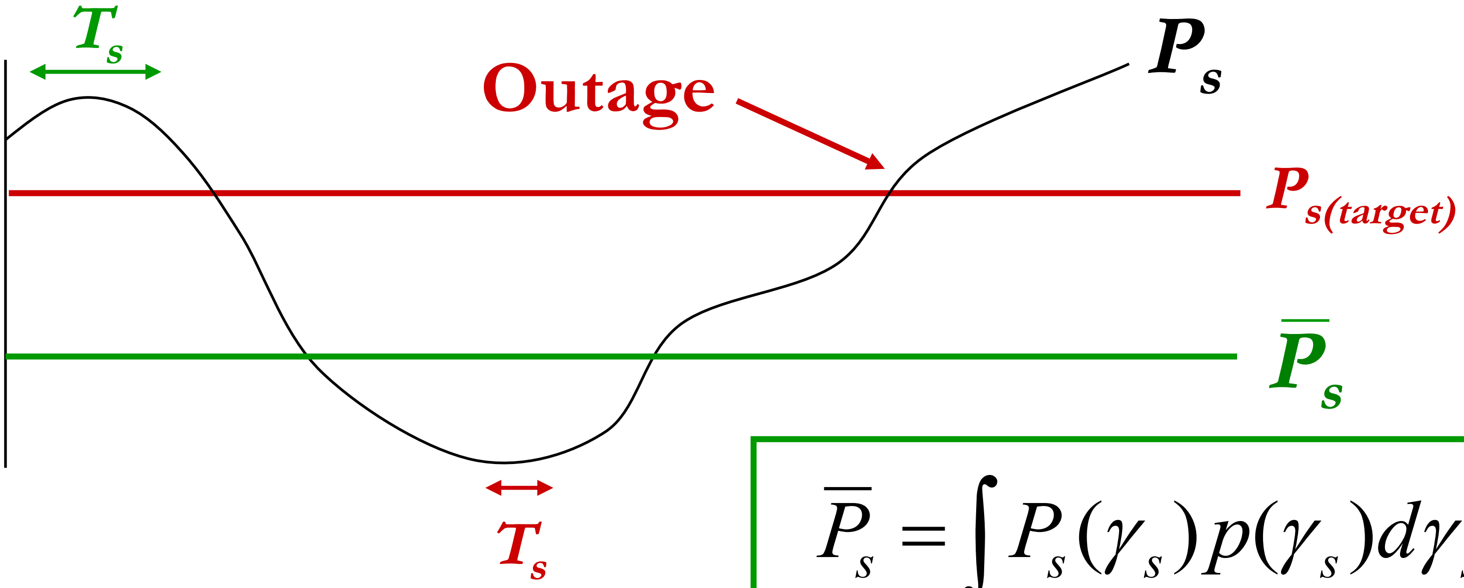


- P_s depends on
  - # of nearest neighbors
  - Minimum distance *d_{min} (depends on γ_s)*
  - Approximate expression

$$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$$

Linear Modulation in Fading

- In fading γ_s and therefore P_s random
- Metrics: outage, average P_s , combined outage and average.



$$\overline{P}_s = \int P_s(\gamma_s)p(\gamma_s)d\gamma_s$$

Moment Generating Function Approach

- Simplifies average P_s calculation
- Uses alternate Q function representation
- $\overline{P_s}$ reduces to MGF of γ_s distribution
- Closed form or simple numerical calculation for general fading distributions
- Fading greatly increases average P_s .

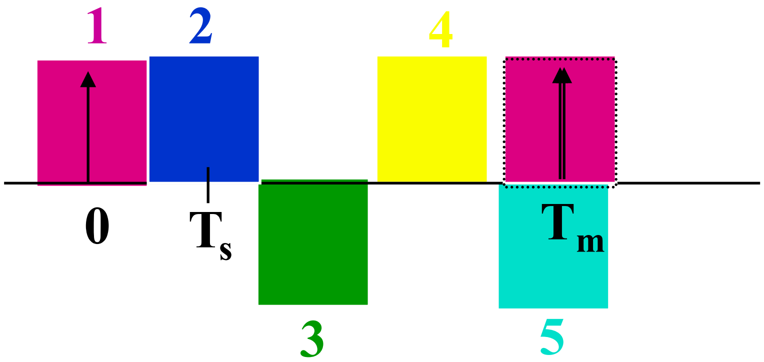
Doppler Effects

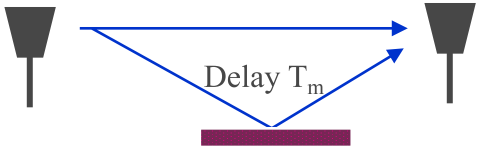
- High doppler causes channel phase to decorrelate between symbols
- Leads to an irreducible error floor for differential modulation
  - Increasing power does not reduce error
- Error floor depends on $f_D T_b$ as

$$P_{floor} = \frac{1 - J_0(2\pi f_D T_b)}{2} \approx .5(\pi f_D T_b)^2$$

Delay Spread (ISI) Effects

- Delay spread exceeding a symbol time causes ISI (self interference).





- ISI leads to irreducible error floor: $\overline{P}_{b,floor} \approx (\sigma_{T_m}/T_s)^2$
  - Increasing signal power increases ISI power
- ISI imposes data rate constraint: $T_s >> T_m$ ($R_s << B_c$)

$$R \leq \log_2(M) \times \sqrt{\overline{P}_{b,floor}/\sigma^2_{T_m}}$$

Diversity

- Send bits over independent fading paths
  - Combine paths to mitigate fading effects.
- Independent fading paths
  - Space, time, frequency, polarization diversity.
- Combining techniques
  - Selection combining (SC)
  - Maximal ratio combining (MRC)
- Can have diversity at TX or RX
  - In TX diversity, weights constrained by TX power

Selection Combining

- Selects the path with the highest gain
- Combiner SNR is the maximum of the branch SNRs.
- CDF easy to obtain ($\Pi_i p(\gamma_i < \gamma_{thr})$), pdf found by differentiating the CDF
- P_{out} obtained from CDF. Average P_s typically found numerically
- Diminishing returns with number of antennas.
- Can get up to about 20 dB of gain.

MRC and its Performance

- With MRC, $\gamma_\Sigma=\Sigma\gamma_i$ for branch SNRs γ_i
  - Optimal technique to maximize output SNR
  - Yields 20-40 dB performance gains
  - Distribution of γ_Σ hard to obtain
- Standard average BER calculation

$$\overline{P}_S = \int P_S(\gamma_\Sigma)p(\gamma_\Sigma)d\gamma_\Sigma = \int\int\ldots\int P_S(\gamma_\Sigma)p(\gamma_1)*p(\gamma_2)*\ldots*p(\gamma_M)d\gamma_1 d\gamma_2\ldots d\gamma_M$$

  - Hard to obtain in closed form
  - Integral often diverges
- MGF Approach: $\overline{P}_s = \frac{\alpha_M}{\pi}\int_0^{\pi/2}\prod_{i=1}^{M}\mathcal{M}_{\gamma_i}\left[\frac{-.5\beta_M}{\sin^2\phi}\right]d\phi$
- TX diversity gain with CSI same as RX diversity

Main Points

- Wireless channels introduce path-loss, shadowing and multipath fading
  - Shadowing introduced outage
  - Flat-fading causes large power fluctuations
  - ISI causes self-interference
- Performance of digital communications in wireless channels random
  - Characterized by outage probability and average probability of error in flat-fading
  - Characterized by irreducible error floors in ISI/Doppler
- Need mechanisms to compensate for multipath
- Diversity compensates for effects of flat fading.