

Assessing the System Value of Optimal Load Shifting

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Abstract—We analyze a competitive electricity market, where consumers exhibit optimal load shifting behavior to maximize utility and producers/suppliers maximize their profit under supply capacity constraints. The associated computationally tractable formulation can be used to inform market design or policy analysis in the context of increasing availability of the smart grid technologies that enable optimal load shifting. Through analytic and numeric assessment of the model, we assess the equilibrium value of optimal electricity load shifting, including how the value changes as more electricity consumers adopt associated technologies. For our illustrative numerical case, derived from the Current Trends scenario of the ERCOT Long Term System Assessment, the average energy arbitrage value per ERCOT customer of optimal load shifting technologies is estimated to be \$3 for the 2031 scenario year. We assess the sensitivity of this result to the flexibility of load, along with its relationship to the deployment of renewables. The model presented can also be a starting point for designing system operation infrastructure that communicates with the devices that schedule loads in response to price signals.

NOMENCLATURE

Superscripts

- i Electricity producers, $i = 1, \dots, m$
 j Electricity consumers, $j = 1, \dots, n$

Subscripts

- t Time periods, $t = 1, 2, \dots, T$

Decision Variables

- \mathbf{x}^i Electricity production vector from producer i , $\mathbf{x}^i \in \mathbf{R}_+^T$
 z^i Capacity investment by producer i , $z^i \in \mathbf{R}_+$
 \mathbf{w}^j Electricity consumption vector of consumer j , $\mathbf{w}^j \in \mathbf{R}_+^T$
 D^j Aggregated electricity demand across time periods for consumer j

Model Variables

- \mathbf{p} Equilibrium electricity price vector, $\mathbf{p}^i \in \mathbf{R}_+^T$
 \mathbf{y}^i Vector of dual variables associated with producer i 's physical constraints
 $\mathbf{v}^j, \lambda^j, \mathbf{w}^j$ Dual variables of constraints facing consumer j

Model Parameters

- g^i Capital cost of new capacity for producer i

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Manuscript received month dd, yyyy; revised month dd, yyyy.

- A^i Constraint matrix of producer i
 H^i Constraint matrix associated with new capacity of producer i
 \mathbf{b}^i Vector of physical limitations associated with constraint matrices A^i and H^i .
 C Circuit capacity
 \mathbf{e} Vector of all ones, $\mathbf{e} \in \mathbf{R}_+^T$
 \mathbf{e}^j Shiftable indicator vector for consumer j . For example, $\mathbf{e}^j = (1; 1; 1; 0; \dots; 0)$ implies that the electricity demand D^j could be met in any of time points 1, 2 or 3.

Functions

- $c^i(\cdot)$ Production cost function for producer i , $c^i(\cdot) : \mathbf{R}_+^T \rightarrow R$
 $w^j(\cdot)$ Continuous and continuously differentiable concave utility function $w^j(\cdot) : R_+ \rightarrow R$

I. INTRODUCTION

POWER systems are undergoing widespread change. On the demand side of the market, smart grid technologies provide the potential to automate the response of electricity demand to price signals. The electrification of various energy demands creates new types of load. Much work is ongoing in developing low cost storage technologies. On the supply side, variable renewable power sources are exhibiting exponential decreases in costs. Policies that support low carbon emission power sources are in place, or proposed, in many jurisdictions. Information technology enables non-traditional entities to participate in electricity markets.

This paper introduces a computationally tractable model of an electricity market that incorporates optimal load shifting capabilities, and subsequently uses the model to provide a high-level assessment of the value of such capabilities. By optimal load shifting capabilities, we refer to technologies that allow electricity demand to adjust automatically, by some programmed or learned rules, in response to a price signal, while maintaining the same quality of service for the electricity user. Examples of such technologies include behind-the-meter batteries, ‘smart’ dishwashers, automated pool pumps, the scheduling of electric vehicle charging, and systems that utilize the thermal inertia of a building when powering air conditioning systems. The unifying theme behind these technologies is the existence of an enhanced communication layer on top of the existing core power system.

Through an equilibrium representation of the electricity supply-demand system, this paper aims to be useful to both system analysts and power engineers. The computationally

tractable model that is introduced can be adapted to conduct a variety of market design and policy analyses, while the possibility of solving the model by decentralized algorithmic approaches may provide a starting point for designing system operation infrastructure. The paper also derives a term for the marginal value of shiftable electricity demand, and considers how this value changes as prices adjust with increasing deployment of load shifting technologies.

Our illustrative numerical case is derived from the ERCOT Long Term System Assessment. For this case, if 10-15% of each hour's demand is shiftable within a 12-24 hour window, the same benefits accrue as if all electricity demand could shift to any other hour of the year. Equivalently, 10-15% represents the point at which the arbitrage opportunities are saturated and no further installation of technologies that enable adjustable demand will be economical. For the example dataset the average energy arbitrage value of optimal load shifting is of the order of \$3 per customer for the 2031 scenario year. Only one of numerous potential sources of value, energy arbitrage value in this context is defined as the difference in total costs between operating the power system with optimal automatic load shifting in place relative to a system where no automatic load shifting occurs. Whether this amount of money is enough to cover the cost of implementing such a system is not assessed here. Additionally, while the total benefits are non-negative under the structure of the model, it is possible for either the consumers or producers to lose relative to the world where demand is non-shiftable.

Section II provides context for this paper in relation to previous work, Section III introduces our core model, Section IV analyzes how to solve the model, Section V contains examples and introduces our numerical findings, while Section VI discusses implications for the value of smart grid technologies, optimal generation capacity investment, market design, system operation, and policy analysis.

II. RELATION TO PREVIOUS WORK

In considering the system value of these relatively new technologies, this work draws on several strands of literature, including equilibrium modeling approaches, assessments of the value of energy storage, and algorithms for decentralised power system operation.

To our current knowledge, while equilibrium modeling approaches in the energy sector are long established, for example [1], [2], this paper is the first framing of the electricity supply-demand system in this particular equilibrium form. While generally designed for different applications than that of this paper, related model formulations in the literature include [3]–[10]. Differences in these models from this paper include the aggregate electricity supply sector of [5], [6], and the non-equivalence of the complementarity formulation to an optimization formulation in [4]. The favorable computational properties of the model in this paper derives from its alternate formulation of utility, with utility based on aggregate demand across time instead of the individual utility associated with demand in each time period.

Underlying this paper is the concept that autonomous demand response technologies will be available. Examples of

the enabling underlying algorithms include those presented in [11]. A mathematical formulation for conducting optimal load shifting given an exogenous price profile is presented by [12]. How the price profile, and subsequently the system value, change with increased deployment of smart grid technologies motivates the equilibrium price framework we consider here. Meanwhile, most models of the power sector from both an operation and capacity expansion point of view assume a fixed demand shape (for example see [13] or [14]). How shiftable electricity demand affects the valuation of electricity supply technologies also motivates an equilibrium approach.

For the purposes of this paper, energy storage technologies are considered a subset of the technologies that can shift electricity demand, and the related literature can inform the investigation into optimal load shifting. [15] provides a comprehensive treatment on assessing the value of large-scale electricity storage, whereas the arbitrage value of storage in a number of real markets has been assessed by [16]. A similar metric of valuation to that used by [16] is derived directly from our equilibrium framework. Further numerical analyses of the value of energy storage are [17]–[20], and are compared to our numerical findings below. Patterns across these works include low energy arbitrage values in many circumstances, and the dynamic of declining marginal value of energy storage as its capacity increases, all implying the import of additional revenue streams for energy storage to be valuable. Whereas findings of such numeric studies are system-specific, this paper additionally attempts to illustrate the mechanisms behind the numerical findings.

The welfare impacts of energy storage are discussed in [21] and [22], showing that the societal benefits are always positive in a perfectly competitive market, but not necessarily so otherwise. This paper will additionally consider the distribution of benefits amongst producers and consumers.

III. MODEL

In this study, we consider multiple independent electricity power consumers and multiple independent electricity suppliers / producers over a period of T discrete time points, where all of them are price-takers.

We will present a general case of our formulation, where both consumers and producers face convex optimization problems. The formulation is completed by a market clearing condition.

A. Producer's Convex Optimization Problem

Given the price vector, \mathbf{p} , each producer i , $i = 1, \dots, m$ (electricity producers such as thermal, hydro, renewable generators, or even discharging storage units) maximizes profit subject to physical or policy constraints.

$$\begin{aligned} & \text{maximize} && \mathbf{p}^T \mathbf{x}^i - c^i(\mathbf{x}^i) - g^i z^i \\ & \text{subject to} && A^i \mathbf{x}^i + H^i z^i \leq \mathbf{b}^i, \quad : \mathbf{y}^i \\ & && \mathbf{x}^i \geq \mathbf{0}, \\ & && z^i \geq 0; \end{aligned} \quad (1)$$

Note that some components of the generation vector, \mathbf{x}^i , could be negative if the supplier is a storage unit,¹ but, for simplicity, we assume here that \mathbf{x}^i is a nonnegative vector. The producer problem possesses standard linear constraints, but all our results are applicable when \mathbf{x}^i is constrained in a more general polyhedral set. We assume that $c^i(\cdot)$ is a continuous and differentiable convex cost function, that is, the marginal cost does not decrease as unit i increases production. In this general form of the model, a producer can choose to invest in new capacity, z^i , in order to sell more electricity.

B. Consumer's Convex Optimization Problem

We base our formulation of the consumer's problem on [12]. Given the price vector \mathbf{p} , the j th consumer's utility-maximization problem can be represented as the maximization of utility net of costs on electricity. Note that consumer j can represent not just an individual consumer, but an individual device on the system. This formulation requires that the utility function convert the consumption of total demand to a dollar value.

$$\begin{aligned} & \text{maximize} && w^j(D^j) - \mathbf{p}^T \mathbf{u}^j \\ & \text{subject to} && \mathbf{u}^j \geq \mathbf{d}^j, && : \mathbf{v}^j \\ & && (\mathbf{e}^j)^T \mathbf{u}^j - D^j \geq 0, && : \lambda^j \\ & && \mathbf{u}^j \leq C\mathbf{e}, && : \mathbf{w}^j \\ & && \mathbf{u}^j, D^j \geq \mathbf{0}; \end{aligned} \quad (2)$$

One may add multiple shiftable demand variables into the objective and constraints; but, for simplicity, we assume here that consumer j has one shiftable demand variable, D^j .

The first constraint requires consumption to be greater than non-shiftable demand in each time period, the second constraint requires the sum of consumption over the allowable time period to be at least the total required power consumption for the consumer/device in question, while the third constraint requires that power consumed in any period does not exceed any constraint on power inflow to the consumer.

C. Competitive Market Equilibrium

The market clearing condition requires that total supply equal total consumption at every time point, that is,

$$\sum_{i=1}^m \mathbf{x}^i = \sum_{j=1}^n \mathbf{u}^j. \quad (3)$$

IV. SOLVING THE MODEL

As noted in [3], if the KKT conditions of the producers' and consumers' optimization problems are gathered along with the market clearing constraint, then the resulting problem can be classified as a Mixed Complementarity Problem (MCP). The structure of our particular formulation however allows the problem to be equivalent to a convex optimization problem, generally more computationally tractable.

¹Storage technologies can be represented in either none or both the producer's and consumer's problem. In the consumer's problem they are represented implicitly as a behind-the-meter device that can shift electricity demand from one period to another.

A. Equivalence to a Convex Problem

It will likely not come as a surprise to the reader that this decentralized model can be solved as a social optimization problem.² For example, the model presents similar conditions to that of a Walrasian partial equilibrium where multiple goods (electricity in each hour in this case) are considered. These particular goods are a small subset of the economy as a whole and expenditure on these goods is a small subset of a consumer's total expenditure. Under these conditions, such a model has been shown to be equivalent to a social optimization problem.³ The following theorems show how the particular problem as formulated here can be solved as a 'social convex program'.

Theorem 1: The market equilibrium price \mathbf{p} , together with optimal production \mathbf{x}^i and optimal consumption \mathbf{u}^j , can be represented by a system of convex equality and inequality constraints on a convex set. More precisely, the set of conditions can be represented as:

$$\begin{aligned} \mathbf{p} - \nabla c^i(\mathbf{x}^i) & \leq (A^i)^T \mathbf{y}^i, && \forall i, \\ -g^i & \leq (H^i)^T \mathbf{y}^i, && \forall i, \\ A^i \mathbf{x}^i + (H^i)^T \mathbf{z}^i & \leq \mathbf{b}^i, && \forall i, \\ \mathbf{p} & \geq \mathbf{v}^j + \lambda^j \mathbf{e}^j - \mathbf{w}^j, && \forall j, \\ -u^j(D^j)' & \geq -\lambda^j, && \forall j, \\ \mathbf{u}^j & \geq \mathbf{d}^j, && \forall j, \\ (\mathbf{e}^j)^T \mathbf{u}^j - D^j & \geq 0, && \forall j, \\ \mathbf{u}^j & \leq C\mathbf{e}, && \forall j, \\ \sum_{i=1}^m \mathbf{x}^i & = \sum_{j=1}^n \mathbf{u}^j, && \forall j, \\ \sum_{i=1}^m (\nabla c^i(\mathbf{x}^i)^T (\mathbf{x}^i) + g^i z^i) & + (\mathbf{b}^i)^T \mathbf{y}^i \\ = \sum_{j=1}^n ((\mathbf{d}^j)^T \mathbf{v}^j + u^j(D^j)' \cdot D^j - C\mathbf{e}^T \mathbf{w}^j), &&& \\ \mathbf{x}^i, \mathbf{z}^i, \mathbf{y}^i, \mathbf{u}^j, \mathbf{v}^j, D^j, \lambda^j, \mathbf{w}^j & \geq \mathbf{0}, && \forall i, j. \end{aligned} \quad (4)$$

The associated proof is presented in Appendix A. Moreover, there is an aggregated social convex program representing equilibrium conditions (4).

Theorem 2: The convex program

$$\begin{aligned} & \text{minimize} && \sum_{i=1}^m (c^i(\mathbf{x}^i) - g^i z^i) - \sum_{j=1}^n w^j(D^j) \\ & \text{subject to} && -A^i \mathbf{x}^i - H^i \mathbf{z}^i \geq -\mathbf{b}^i, && \forall i, && : \mathbf{y}^i \\ & && \mathbf{u}^j \geq \mathbf{d}^j, && \forall j, && : \mathbf{v}^j \\ & && (\mathbf{e}^j)^T \mathbf{u}^j - D^j \geq 0, && \forall j, && : \lambda^j \\ & && -\mathbf{u}^j \geq -C\mathbf{e}, && \forall j, && : \mathbf{w}^j \\ & && \sum_{i=1}^m \mathbf{x}^i - \sum_{j=1}^n \mathbf{u}^j = \mathbf{0}, && && : \mathbf{p} \\ & && \mathbf{x}^i, \mathbf{u}^j, \mathbf{z}^i, D^j \geq \mathbf{0}, && \forall i, j \end{aligned} \quad (5)$$

produces (i) $(\mathbf{x}^i, \mathbf{u}^j, \mathbf{z}^i, D^j)$ satisfying (4), (ii) the equilibrium price vector \mathbf{p} , being the optimal Lagrange multipliers associated with the market clearing equality constraints, and (iii) the other optimal multipliers $(\mathbf{y}^i, \mathbf{v}^j, \lambda^j, \mathbf{w}^j)$.

The proof of the theorem is straightforward, comparing the KKT conditions of the social optimization problem (5) with (4). One can see that the social welfare objective of the social problem consists of two parts: the first is the total electricity

²The term 'a social formulation' or 'a social problem' is used throughout this paper to refer to the problem that a benevolent central agent with full control and full information would solve.

³See Chapter 10 of [23]. The conditional equivalence of MCP to optimization problems is also discussed in [3] and [7].

production cost and the second is the utility values of all consumers. Significantly, Theorem 2 shows that the market-clearing equilibrium price vector can be computed as a convex optimization problem, which makes the computation tractable.

B. Distributed Computation

Given the formulation of the model as a convex program, the model can be solved by a number of methods, including the Alternating Direction Method of Multipliers (ADMM). Adapting the ‘ADMM Iteration for Separable Problems’ scheme set out in [24] allows the producers’ and consumers’ problems to be solved in parallel. The adaptation is presented in Appendix C below. In particular, at each iteration, the Lagrangians of each problem, augmented by a penalty on an imbalance in the aggregate market clearing condition (3) are solved. The prices are then updated before the next iteration starts. The only information required by each decentralized program in each iteration is the updated price vector and the aggregate shortfall/excess of electricity in the previous iteration. Distributed computation potentially provides two advantages: a) when the number of agents modeled becomes large, parallel solution may reduce model solve time, and b) it provides a method that may aid real world operation, as discussed in Section VI-D.

C. Derivation of Marginal Value

From the model structure, the marginal value of the ability to shift electricity demand can be derived as follows, where \mathbf{p} is our equilibrium electricity price.

$$\text{marginal value} (\$/\text{MW}) = \sum_t |p_t - \text{median}(\mathbf{p})| \quad (6)$$

The derivation is presented in Appendix B and is conditional on a number of simplifying assumptions relative to the general model presented above.⁴ The metric as derived is a measure of the dispersion of the electricity prices, being the sum of the absolute differences between the prices in each time period and the median price. While this is not a surprise, it is perhaps useful to derive an intuitive identity from the equilibrium model, an identity that uses price alone as a sufficient statistic, avoiding the need to include technical considerations such as the underlying capacity mix or flexibility of demand in the system that generated the prices. Given real system price data, this metric allows the marginal value of adjustable demand to be calculated by simply assessing electricity prices, avoiding the need to run the full optimization model. A reduction in this metric indicates that fewer arbitrage opportunities are available, and implies a reduction in the energy arbitrage value of load shifting technologies. We can expect this to happen as more such technologies are installed.

⁴For example, the formula as presented relates to fully flexible electricity demand across the model’s full horizon. For those cases where the hours to which demand can move is restricted to within a certain window, the value is the cumulative application of the formula to prices within each window. Where the index w indicates a window: $\sum_w \sum_{t(w)} |p_{t(w)}^w - \text{median}(\mathbf{p}^w)|$.

V. APPLICATION

For clarity of exposition, the set of examples will be a simpler version of our general model, namely with constant costs of production given fixed capacity constraints and a utility of total electricity demand such that total demand remains fixed. In such cases, our problem reduces to a linear programming problem.

A. Toy Example

Here we consider an interval with three time periods $t = 1, 2, 3$. There are two producers and two consumers. The producers comprise a thermal resource with constant availability, denoted by superscript th , and a renewable resource with variable availability, denoted by superscript r . The production problem is as follows for the thermal generator:

$$\begin{aligned} &\text{maximize} && \mathbf{p}^T \mathbf{x}^{th} - 7e^T \mathbf{x}^{th} \\ &\text{subject to} && x_i^{th} \leq 16, \quad i = 1, 2, 3, \\ &&& x_i^{th} \geq 0, \quad i = 1, 2, 3; \end{aligned}$$

And the variable renewable generator’s production problem:

$$\begin{aligned} &\text{maximize} && \mathbf{p}^T \mathbf{x}^r - 0e^T \mathbf{x}^r \\ &\text{subject to} && x_1^r \leq 2 \\ &&& x_2^r \leq 7 \\ &&& x_3^r \leq 9 \\ &&& x_i^r \geq 0, \quad i = 1, 2, 3; \end{aligned}$$

that is, the thermal producer has a capacity of 16 available every period at a variable cost of 7, while the renewable producer has a varying available capacity at zero variable cost. The problems faced by the two consumers, a and b , are as follows:

$$\begin{aligned} &\text{minimize} && \mathbf{p}^T \mathbf{u}^a && \mathbf{p}^T \mathbf{u}^b \\ &\text{subject to} && u_1^a \geq 8, && u_1^b \geq 3, \\ &&& u_2^a \geq 13, && u_2^b \geq 3, \\ &&& u_3^a \geq 3, && u_3^b \geq 2, \\ &&& u_1^a + u_2^a + u_3^a \geq 28, && u_1^b + u_2^b + u_3^b \geq 9, \\ &&& u_1^a, u_2^a, u_3^a \geq 0; && u_1^b, u_2^b, u_3^b \geq 0; \end{aligned}$$

where the constraints indicate that up to 4 units of electric energy can be shifted amongst the time periods by consumer a and up to 1 unit is available to be shifted by consumer b .

The social linear program is:

$$\begin{aligned} &\text{minimize} && 7e^T \mathbf{x}^{th} \\ &\text{subject to} && -x_i^{th} \geq -16, \quad i = 1, 2, 3, \\ &&& -x_1^r \leq -2, \\ &&& -x_2^r \leq -7, \\ &&& -x_3^r \leq -9, \\ &&& u_1^a \leq 8, \\ &&& u_2^a \leq 13, \\ &&& u_3^a \leq 3, \\ &&& u_i^b \leq 3, \quad i = 1, 2, \\ &&& u_3^b \leq 2, \\ &&& u_1^a + u_2^a + u_3^a \leq 28, \\ &&& u_1^b + u_2^b + u_3^b \leq 9, \\ &&& x_i^{th} + x_i^r = u_i^a + u_i^b, \quad i = 1, 2, 3, \\ &&& x_i^{th}, x_i^r, u_i^a, u_i^b \geq 0, \quad i = 1, 2, 3; \end{aligned}$$

The equilibrium prices are $(7; 7; 7)$ and the \mathbf{u} vectors are $(8; 13; 7)$ and $(3; 3; 3)$ for a and b respectively. The cost of production is 133, and the net profit of the producers is 0 and 126 respectively. We see that the flexible units of demand were used in period 3, when there was highest availability of the zero variable cost renewable resource. If this demand was required, for example, in the first period and not flexible to move, the more expensive thermal supply resource would have met this demand, with an associated increase in total system costs. This would also have implied a lower electricity price in period 3. These dynamics will be discussed further with the next example.

B. ERCOT 2031 Application

To illustrate the model further, we now implement the problem (5) with some aggregated data from the ERCOT power system in Texas. In so doing, we will undertake an assessment of the value of optimal load shifting enabled by smart grid technologies by comparing model outcomes when load shifting is available, and when it is not.

1) *Boundary conditions of analysis:* The boundary conditions of this application of the model are important for interpreting the results. Boundary conditions include:

- a) Throughout this particular exercise we assume a utility of total demand by consumer j such that total demand remains fixed, allowing the utility term to be dropped from the consumer's problem.
- b) Electricity supply capacity is fixed in this example, no investment or retirements are allowed.
- c) As capacity is fixed, fixed operation and maintenance costs are not included as they are relatively small and will cancel each other out when comparing scenarios.
- d) Forecasting error is not included.
- e) This assessment only considers one of the potential sources of value for load shifting capability, energy arbitrage.
- f) The application assumes consumers face fully dynamic real time pricing.
- g) In this application, there is only one aggregate consumer.
- h) The findings extend to one numerical case. However, we will attempt to point out some general principles in the discussion of the results.

We will discuss the implications of these boundary conditions further in Section VI.

2) *Data:* The data used to conduct this exercise is from the 'Current Trends' scenario of the ERCOT Long Term System Assessment process [25]. The data includes capacity mix, fuel costs, variable O&M costs, and hourly load, wind and solar availability profiles for the 2031 scenario year. Capacity mix and short run marginal costs used in the problem are displayed in Table I.

3) *Abstract representation of optimal load shifting:* Rather than an explicit representation of optimal load shifting technologies and associated capabilities, we consider a more abstract representation. In particular, we assess the impact of changing a) the percentage of benchmark load in every hour that is flexible, and b) the duration of the window in which

TABLE I
CAPACITY MIX IN EXAMPLE

Technology	Capacity (GW)	Short-Run Marginal Cost (\$/MWh)
Solar	21.7	0
Wind	21.5	0
Nuclear	5.2	11.4
Gas CC	37.3	48.7
Gas CT	12.1	72.6
Gas Steam	8.7	79.8
Coal	10.2	34.3

load can shift. For example, a central scenario below is the case where 15% of load in each hour can shift to any other hour in the associated 24-hour window. The idea is that this high level approach can provide insight into what characteristics drive the value of technologies that enable optimal load shifting.

4) *Results:* Table II compares the welfare losses and gains associated with two cases, one where 15% of the reference demand in any hour is shiftable within each 24-hour window, and one where no demand is shiftable.

TABLE II
DISTRIBUTION OF THE BENEFITS OF OPTIMAL LOAD SHIFTING BETWEEN CONSUMERS AND PRODUCERS IN NUMERIC EXAMPLE (MILLION \$)

	No shiftable demand	Shiftable demand	Δ
Consumer Cost	22,434	21,780	-654
Producer Profit	10,575	9,992	-583
Welfare			+71

Figure 1 provides some graphical intuition behind these results, and helps illustrate how system prices adjust under optimal load shifting. The consumers save due to the decline in peak prices as demand is shifted from peak pricing hours to lower pricing hours. Similarly, profits that inframarginal generators such as nuclear and coal were making during peak pricing hours decline as the number of such hours declines, leading to a decline in aggregate producer profit. Comparing the cases, there is also an increase in price in the low price hours in the optimal load shifting case, as demand shifts to those hours. When the structure of the system is such that this effect outweighs the peak pricing effect, total consumer costs can actually increase in the presence of optimal load shifting. The earlier toy example was such a system.

Table II and Figure 1 are based on an assumption that at most 15% of load could shift forward or backward in time within each predefined 24-hour window. Figure 2 displays the sensitivity of the result to the duration of this window. If demand can shift from any hour of the year to any other, we do not get any additional benefits than if load can shift within a 24 hour period. Additionally, we can see the value is halved if the window is 12 hours.

Figure 3 illustrates the sensitivity of the arbitrage value of optimal load shifting to the fraction of demand that is shiftable in any given hour. Most of the value comes from being able to shift 10-15% of demand in any given hour. Equivalently, it is at this point that the benefits of technologies that enable load shifting saturate.

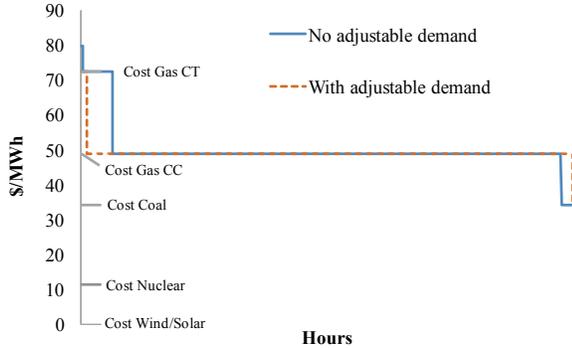


Fig. 1. Price duration curve for ERCOT 2031 Current Trends scenario. In adjustable demand case, 15% of demand is flexible within each 24-hour window.

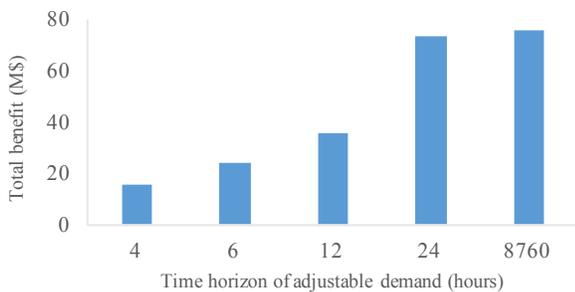


Fig. 2. Sensitivity of the value of load shifting to the duration of the window within which load can shift (assuming 15% of load flexible)

Our earlier derived identity for marginal value, (6) in Section IV-C above, can be applied to assess the rate at which the value is saturated, graphed in Figure 4. For this case, we see that the marginal value declines rapidly from its initial value of \$400/kW to the order of \$10/kW until the value is saturated in the 10-15% range.

VI. IMPLICATIONS

A. Economic Implications

These results indicate an average value of system-wide optimal load shifting in this case of approximately \$3 per year

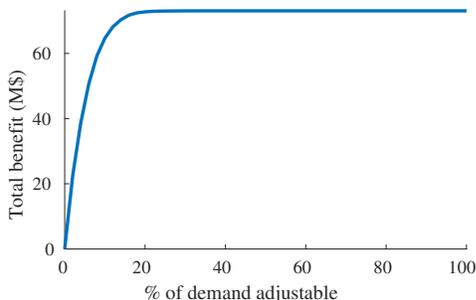


Fig. 3. Sensitivity of the value of load shifting to the fraction of demand that can shift within each hour (assuming 24 hour window).

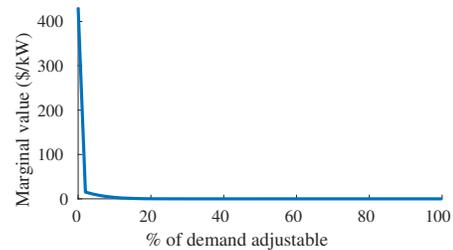


Fig. 4. Marginal Value of an additional kW of adjustable electricity demand in ERCOT 2031 Current Trends example

per customer.⁵ This figure assumes 15% of demand in each hour is available for shifting within each 24 hour period. If the window within which load can be shifted is 12 hours, the value is approximately \$1.50 per year. Whether this amount of money is enough to cover the cost of implementing such a system is not assessed here.

These findings correspond with the patterns reported by [17], [18], [20] in their illustration of limited energy arbitrage value in the context of energy storage and the decline in marginal value. For example, [17] cite a range of energy arbitrage value from a variety of studies from \$25/kW for a 4 hour storage device in California to \$240/kW for a device in New York City.

There are undoubtedly aspects of the benefits of the system not included in this example such as avoided capacity investments, avoiding power plant startup and shutdown costs, grid stability, meeting short term fluctuations, more transparency, and control for consumers. Additionally, the base case prices may not be as flat as presented here and more arbitrage opportunities may be available if the thermal units of the same class are not as homogenous in characteristic as modeled, and also if there are local pockets of more variable prices due to transmission constraints. On the other hand, this assessment did not consider rate structures, which typically exhibit less variability across an hourly basis than wholesale market prices. The purpose of this exercise, however, was to illustrate the use of the model. If this exercise showed either huge value or no value to optimal load shifting technologies, more detailed studies may not be necessary. The result from this example is somewhere in the middle, and this modeling approach could aid more detailed investigations.

Also for this case study, total welfare increases with the introduction of shiftable electricity demand, but producer surplus declines in this case while consumer surplus increases. As [22] shows, we expect the net welfare to be non-negative under the competitive market setup. The distributional results have not, however, been discussed extensively in the literature.

B. Optimal Capacity Mix

While included in the presentation of the model, our numerical case did not include endogenous capacity investments and retirements. [17] discuss findings in the energy storage context

⁵\$76 million divided by an assumed 24 million customers served within the ERCOT area equals \$3.04 per customer.

where the capacity value was an important component of the system value. Load shifting capabilities could be expected to change the optimal capacity mix, particularly in scenarios with tight carbon constraints and/or with significant ongoing declines in costs of wind and solar technologies. While we will not explore this numerically, the following section is relevant to this topic analytically.

C. Relationship to Value of Renewables

Increased deployment of variable wind and solar resources can be expected to increase the dispersion of electricity prices and thus increase the value of load shifting technologies [19]. In parallel, increased flexibility in load can alter the economics of investment in wind and solar generators. Using the marginal value of an investment in a wind or solar power generator, as first shown by [26], as the denominator, we can introduce the ratio between the marginal value of shiftable demand and investment in a wind or solar power generator.

$$\alpha = \frac{\sum_t |p_t - \text{median}(\mathbf{p})|}{|T|[E(\mathbf{p}) \cdot E(\mathbf{a}_g) + \text{Cov}(\mathbf{p}, \mathbf{a}_g)]} \quad (7)$$

Where \mathbf{a}_g is the availability of generator g , and $E()$ is the time-weighted expectation. The essence of the formula is:

$$\alpha = \frac{\text{dispersion of prices}}{\text{correlation of solar/wind \& price}} \quad (8)$$

α , the relative marginal value metric, can frame how shiftable demand and renewable energy affect each other, with the former benefitting from the latter's dispersion of prices, and the latter benefitting from the potentially increased correlation between prices and renewables availability.

D. Market Design / System Operation

The optimal load shifting model has a number of implications for electricity market design and system operations. We have presented a system where each supplier of electricity may submit their marginal cost of electricity production in each hour (the optimal bid in a competitive market setting), and where each consumer (or even each device) may submit their demand for electricity in each hour, along with their constraints on load shifting. The system operator then solves the tractable convex program to find the equilibrium solution. The associated price could then be released to the market participants. Each participant selfishly optimizing in a decentralized manner given this price signal will then lead to the socially optimum outcome, implying a zero 'price of anarchy' [27]. Note that centralized control of load would not be required, nor would a demand aggregator be required to achieve the theoretical optimal market outcome, potentially mitigating market power concerns.⁶

Alternatively, the convex program could be solved in a decomposed manner, with the distributed computing approach of Section IV-B implemented such that the producers' and consumers' themselves solve the distributed computation through

⁶In contrast, Section 6.4 of [28] discusses the value and role of aggregation and aggregators in evolving power systems, with numerous value streams derived from sources not included in this model.

information iterating back and forth with the system operator. Similar distributed schemes are discussed by [5], [6], [29]. Such an approach allows the problem to be solved with lower information requirements placed on the system operator.

Important issues for further research in the adoption of such an approach include non-competitive or strategic price settings, strategy-proof bidding systems, treatment of uncertainty including forecast error, and latency in communication signals.

E. Policy Analysis

The work here can support the design of models for policy analysis, as the inclusion of load shifting capability can potentially change the policy-relevant insights produced by the model. In addition to the modelling tool, the identities in Section IV-C and VI-C can provide simple rules for understanding the policy implications of shiftable electricity demand.

VII. CONCLUSION

The electric power system is undergoing significant change on both the demand side and the supply side. This study has made two contributions to discussions relating to the evolving power system - a) a computationally tractable model that admits flexible functional forms is derived that can be used for policy analysis or market design/operation purposes, and b) using the model, a high level assessment of the value of optimal automated load shifting is carried out.

Numeric analysis for a 2031 ERCOT scenario indicates that to capture the majority of the energy arbitrage value, an unrealistic 100% of demand is not required to be flexible, but a more achievable, yet far from trivial, 10-15% is. The majority of the value does not require adjustments across weeks or months, but again a more achievable yet far from trivial 12-24 hours. For this particular dataset and this particular methodology, the arbitrage value, an important but not sole source of value, unlocked by installing such a system is limited to a few dollars per year per customer. How this value would increase under greater deployment of variable renewables is shown analytically.

The associated high level model and perspectives may be useful for those designing the smart grid technologies and associated regulatory frameworks that enable automated optimal load shifting. There is much further research work to be undertaken to bring these perspectives and ideas to real power systems.

APPENDIX A PROOF OF THEOREM 1

Proof: As (4) contains the feasibility conditions associated with (1), for any \mathbf{p} and $((\mathbf{x}^i, z^i), \mathbf{y}^i)$ satisfying (4), $((\mathbf{x}^i, z^i), \mathbf{y}^i)$ is a feasible primal-dual pair of production maximization problem (1). Then, from the weak duality theorem, we have

$$\mathbf{p}^T \mathbf{x}^i - \nabla c^i(\mathbf{x}^i)^T \mathbf{x}^i - g^i z^i \leq (\mathbf{b}^i)^T \mathbf{y}^i, \quad \forall i,$$

or

$$\mathbf{p}^T \mathbf{x}^i \leq \nabla c^i(\mathbf{x}^i)^T \mathbf{x}^i + g^i z^i + (\mathbf{b}^i)^T \mathbf{y}^i, \quad \forall i.$$

Similarly, for any \mathbf{p} and $(\mathbf{u}^j, D^j, \mathbf{v}^j, \lambda^j, \mathbf{w}^j)$ satisfying (4), $((\mathbf{u}^j, D^j), (\mathbf{v}^j, \lambda^j, \mathbf{w}^j))$ is a feasible primal-dual pair of consumer problem (2). Again from the weak duality theorem, we have

$$\mathbf{p}^T \mathbf{u}^j \geq (\mathbf{d}^j)^T \mathbf{v}^j + u^j(D^j) - C\mathbf{e}^T \mathbf{w}^j, \forall j.$$

Thus, summing up we have

$$\sum_{i=1}^m (\nabla c^i(\mathbf{x}^i)^T \mathbf{x}^i + g^i z^i + (\mathbf{b}^i)^T \mathbf{y}^i) \geq \mathbf{p}^T \left(\sum_{i=1}^m \mathbf{x}^i \right)$$

and

$$\mathbf{p}^T \left(\sum_{j=1}^n \mathbf{u}^j \right) \geq \sum_{j=1}^n ((\mathbf{d}^j)^T \mathbf{v}^j + u^j(D^j) - C\mathbf{e}^T \mathbf{w}^j).$$

Then, from the market clearing condition

$$\sum_{i=1}^m \mathbf{x}^i = \sum_{j=1}^n \mathbf{u}^j,$$

we must have

$$\begin{aligned} & \sum_{i=1}^m (\nabla c^i(\mathbf{x}^i)^T \mathbf{x}^i + g^i z^i + (\mathbf{b}^i)^T \mathbf{y}^i) \\ & \geq \mathbf{p}^T \left(\sum_{i=1}^m \mathbf{x}^i \right) = \mathbf{p}^T \left(\sum_{j=1}^n \mathbf{u}^j \right) \\ & \geq \sum_{j=1}^n ((\mathbf{d}^j)^T \mathbf{v}^j + u^j(D^j) - C\mathbf{e}^T \mathbf{w}^j). \end{aligned}$$

Furthermore, from (4), we have another equality

$$\begin{aligned} & \sum_{i=1}^m (\nabla c^i(\mathbf{x}^i)^T \mathbf{x}^i + g^i z^i + (\mathbf{b}^i)^T \mathbf{y}^i) \\ & = \sum_{j=1}^n ((\mathbf{d}^j)^T \mathbf{v}^j + u^j(D^j) - C\mathbf{e}^T \mathbf{w}^j), \end{aligned}$$

taking this as given, we must then also have

$$\sum_{i=1}^m (\nabla c^i(\mathbf{x}^i)^T \mathbf{x}^i + g^i z^i + (\mathbf{b}^i)^T \mathbf{y}^i) = \mathbf{p}^T \left(\sum_{i=1}^m \mathbf{x}^i \right)$$

and

$$\mathbf{p}^T \left(\sum_{j=1}^n \mathbf{u}^j \right) = \sum_{j=1}^n ((\mathbf{d}^j)^T \mathbf{v}^j + u^j(D^j) - C\mathbf{e}^T \mathbf{w}^j).$$

These, along with our inequalities developed from the weak duality theorem, imply,

$$\nabla c^i(\mathbf{x}^i)^T \mathbf{x}^i + g^i z^i + (\mathbf{b}^i)^T \mathbf{y}^i = \mathbf{p}^T \mathbf{x}^i, \forall i,$$

and

$$\mathbf{p}^T \mathbf{u}^j = (\mathbf{d}^j)^T \mathbf{v}^j + u^j(D^j) - C\mathbf{e}^T \mathbf{w}^j, \forall j.$$

Which in combination with the remaining conditions in (4), imply that the optimality conditions for each producer and consumer are represented by (4). That is, the fixed \mathbf{p} , $((\mathbf{x}^i, z_i), \mathbf{y}^i)$ that meets the conditions of (4) is an optimal primal-dual pair of production profit maximization problem (1), simultaneously

for every i , and $((\mathbf{u}^j, D^j), (\mathbf{v}^j, \lambda^j, \mathbf{w}^j))$ is an optimal primal-dual pair of consumer problem (2), simultaneously for every j . ■

APPENDIX B MARGINAL VALUE

To calculate the marginal value, we take our social linear program, (5), from Theorem 2, and explicitly add in a smart grid technology with capital cost s^j that enables a g^j quantity of shiftable demand in each hour for demand j .⁷ Assume that all of our load is non-shiftable \mathbf{d}^j in the absence of this technology. For brevity, but without loss of generality, we additionally do not include here the utility of total demand $u^j(D^j)$, the ability to invest in new supply capacity, z , or the power inflow constraint. (5) can then be written as follows.

$$\begin{aligned} & \text{minimize} && \sum_{i=1}^m \mathbf{c}^i \mathbf{x}^i + \sum_{j=1}^n s^j g^j \\ & \text{subject to} && -A^i \mathbf{x}^i \geq -\mathbf{b}^i, \forall i, && : \mathbf{y}^i \\ & && g^j \mathbf{e} \geq |\mathbf{u}^j - \mathbf{d}^j|, \forall j, && : \mathbf{v}^j \\ & && \mathbf{e}^T \mathbf{u}^j \geq D^j, \forall j, && : \lambda^j \\ & && \sum_{i=1}^m \mathbf{x}^i = \sum_{j=1}^n \mathbf{u}^j, && : \mathbf{p} \\ & && \mathbf{x}^i, \mathbf{u}^j \geq \mathbf{0}, \forall i, j \end{aligned} \quad (9)$$

Differentiating the Lagrangian with respect to g^j yields the following:⁸

$$s^j = \mathbf{e}^T \mathbf{v}^j \quad (10)$$

Differentiating with respect to λ^j yields the following:

$$\mathbf{v}^j = |\mathbf{p} - \lambda^j \mathbf{e}| \quad (11)$$

Combining:

$$s^j = \mathbf{e}^T |\mathbf{p} - \lambda^j \mathbf{e}| \quad (12)$$

If \mathbf{e} is the vector of ones,⁹ λ^j will always equal $\text{median}(\mathbf{p})$,¹⁰ so we can progress as follows:

$$s^j = \mathbf{e}^T |\mathbf{p} - \text{median}(\mathbf{p}) \mathbf{e}| \quad (13)$$

$$\Rightarrow \text{marginal value} = \sum_t |p_t - \text{median}(\mathbf{p})| \quad (14)$$

APPENDIX C ADMM SCHEME FOR SOLVING MODEL

At each iteration k , the producers and consumers solve the Lagrangians of their respective optimization problems, \mathcal{L}_P and \mathcal{L}_C , with augmented terms as set out below. They then pass their respective supply and demand offers to the system operator and update their internal dual variables. With the gathered supply and demand values, the system operator updates the electricity price vector, \mathbf{p} and the aggregate supply-demand mismatch, Δ , communicating them to the agents

⁷In the case here, demand can increase only by g^j in any hour. Relaxing this limit changes the derivation, but not the associated qualitative points.

⁸The absolute value constraint can be transformed into equivalent linear constraints, with the resulting analysis translated into the results shown here.

⁹The derivation, and associated identity presented in footnote 4, extends naturally to when \mathbf{e} is not 1 for every time period.

¹⁰By (11) above, $\lambda^j = p_t$ when $v_t^j = 0$. This will occur when the constraint associated with \mathbf{v}^j is not binding, which will be at the median price. If at any other price, there is an arbitrage opportunity that the load shifting capability g^j can be used for, thus implying $v_t > 0$, and a contradiction is reached.

for the next iteration.¹¹ The penalty term, δ , is set by the modeler.¹²

A. Producer i

$$\mathbf{x}_k^i \leftarrow \arg \max_{\mathbf{x}^i} (\mathcal{L}_P^i(\mathbf{x}^i, \mathbf{p}_k, \mathbf{y}_k^i) + \frac{\delta}{2} \|\mathbf{A}^i \mathbf{x}^i - \mathbf{b}^i\|^2 + \frac{\delta}{2} \|\mathbf{x}^i - \mathbf{x}_{k-1}^i + \frac{\Delta_k}{|i|+|j|}\|^2) \quad (15)$$

$$\mathbf{y}_{k+1}^i \leftarrow \mathbf{y}_k^i + \delta(\mathbf{A}^i \mathbf{x}_k^i - \mathbf{b}^i) \quad (16)$$

B. Consumer j

$$\mathbf{u}_k^j \leftarrow \arg \max_{\mathbf{u}^j} (\mathcal{L}_C^j(\mathbf{u}^j, \mathbf{p}_k, \mathbf{v}^j, \lambda^j, \mathbf{w}^j) + \frac{\delta}{2} \|\mathbf{u}^j - \mathbf{d}^j\|^2 + \frac{\delta}{2} \|(e^j)^T \mathbf{u}^j - D^j\|^2 + \frac{\delta}{2} \|\mathbf{u}^j - \mathbf{C}e\|^2 + \frac{\delta}{2} (\mathbf{u}^j - \mathbf{u}_{k-1}^j + \frac{\Delta_k}{|i|+|j|})^2) \quad (17)$$

$$\mathbf{v}_{k+1}^j \leftarrow \mathbf{v}_k^j + \delta(\mathbf{d}^j - \mathbf{u}^j) \quad (18)$$

$$\lambda_{k+1}^j \leftarrow \lambda_k^j + \delta((D^j - e^j)^T \mathbf{u}^j) \quad (19)$$

$$\mathbf{w}_{k+1}^j \leftarrow \mathbf{w}_k^j + \delta(\mathbf{u}^j - \mathbf{C}e) \quad (20)$$

C. System Operator

$$\mathbf{p}_{k+1} \leftarrow \mathbf{p}_k - \frac{\delta}{|i|+|j|} (\sum_i \mathbf{x}_k^i - \sum_j \mathbf{u}_k^j) \quad (21)$$

$$\Delta_{k+1} \leftarrow \sum_i \mathbf{x}_k^i - \sum_j \mathbf{u}_k^j \quad (22)$$

ACKNOWLEDGMENT

We would like to thank Dr. Julia Matevosjana of ERCOT for assistance with data relating to the ERCOT Long Term System Assessment. We would also like to thank several anonymous reviewers whose feedback greatly improved this paper. James Merrick's primary funding for this project came from the Department of Energy, Office of Science PIAMDDI Grant (DE-SC0005171) to the Energy Modeling Forum at Stanford University. Yinyu Ye's primary funding for this project came from the Chinese Electric Power Research Institute and the Stanford Precourt Energy Efficiency Center.

¹¹The scheme assumes equality constraints, however the slack variables are not included below for clarity of exposition. Similarly, the capacity investment variable, z , is not included.

¹²For the implementation of the ERCOT numerical example, the model converged with a penalty value of 0.01.

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