Assessing the System Value of Optimal Load Shifting

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***Abstract*—We analyze a competitive electricity market, where consumers exhibit optimal load shifting behavior to maximize utility and producers/suppliers maximize their profit under supply capacity constraints. The approach stands in contrast to approaches where either the electricity price distribution or the electricity demand distribution is a fixed input in assessing optimal load shifting value. While the general form of this problem is non-convex, we show that our formulation can be represented as a convex program. The resulting computationally tractable formulation can be used to inform market design or policy analysis in the context of increasing availability of smart grid technologies. Through analytic and numeric assessment of the convex program, we extract basic principles about the equilibrium value of optimal electricity load shifting. For our illustrative numerical case, derived from the Current Trends scenario of the ERCOT Long Term System Assessment, the value of optimal load shifting technologies is estimated to be $3 per ERCOT customer per annum. We assess the sensitivity of this result to the flexibility of load, along with its relationship to the deployment of renewables.**

I. Introduction

Power systems are undergoing widespread change. On the demand side of the market, smart grid technologies provide the potential to automate the response of electricity demand to price signals. The electrification of various energy demands creates new types of load. Much work is ongoing in developing low cost storage technologies. On the supply side, variable renewable power sources are exhibiting exponential decreases in costs. Policies that support low carbon emission power sources are in place, or proposed, in many jurisdictions. Information technology enables non-traditional entities to participate in electricity markets.

This paper introduces a computationally tractable model of an electricity market that incorporates optimal load shifting capabilities, and subsequently uses the model to provide a high-level assessment of the value of such capabilities. By optimal load shifting capabilities, we refer to technologies that allow electricity demand to adjust automatically, by some programmed or learned rules, in response to a price signal, while maintaining the same quality of service for the electricity user. Examples of such technologies include behind-the-meter batteries, 'smart' dishwashers, automated pool pumps, the scheduling of electric vehicle charging, and systems that utilize the thermal inertia of a building when powering air conditioning systems. The unifying theme behind these technologies is the existence of an enhanced communication layer on top of the existing core power system.

Studies of optimum electricity dispatch and investment

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often assume the demand profile is exogenous and constant, while studies of smart grid value often assume the price profile is exogenous and constant. We here consider the equilibrium between these two modeling paradigms. Through the modeling of the electricity supply-demand system as an Arrow-Debreu competitive economy, this paper aims to inform market design and policy analysis, while also elucidating some basic principles on the equilibrium system value of shiftable electricity demand. In particular, we model electricity in each time period as the bundle of goods in the economy, for example 8760 electricity goods in an hourly model of yearly power system operation.

The existence of an equilibrium for an integrated model of production, exchange and consumption was proven in a non-constructive fashion by Arrow and Debreu [1]. While finding the general Arrow-Debreu equilibrium can be computationally hard [2], our application of the formulation to this problem can be shown to be equivalent to solving a convex programming problem, with associated favorable computational properties.

In addition to presenting the model, we find the following in our accompanying analysis. For an example scenario derived from the ERCOT Long Term System Assessment, if only 10-15% of each hour's demand is shiftable within a 12-24 hour window, the same benefits accrue as if all electricity demand could shift to any other hour of the year. Equivalently, 10-15% represents the point at which the arbitrage opportunities are saturated and no further installation of technologies that enable adjustable demand will be economical. For the example dataset the value of the system is of the order of \$3 per customer for the 2031 scenario year. Value in this context is defined as the difference in total costs between operating the power system with optimal automatic load shifting in place relative to a system where no automatic load shifting occurs. Whether this amount of money is enough to cover the cost of implementing such a system is not assessed here. Additionally, while the total benefits are non-negative under the structure of the model, it is possible for either the consumers or producers to lose relative to the world where demand is non-shiftable. Finally, a term for the marginal value of shiftable demand is derived to express how the value changes as prices adjust toward equilibrium.

Section II provides background, Section III introduces our core model, Section IV contains examples and introduces our numerical findings, while Section V discusses implications for the value of smart grid technologies, optimal generation capacity investment, market design, system operation, and policy analysis.

II. Background

To our current knowledge, this is the first framing of the electricity supply-demand system in this particular equilibrium form. Hobbs and Helman [3] discuss complementarity-based equilibrium modeling for electric power markets, mentioning conditions when the complementarity problem can be modeled as a linear program, but not considering shiftable electricity demand in their formulation. [4], [5], and [6] discuss complementarity problems more abstractly, showing conditions when such problems are equivalent to convex optimization problems. The formulation in this paper is one such example. Kiany and Annaswamy [7] show that the KKT conditions of the optimization problems of the various agents in an electricity market can be shown to form a linear complementarity problem, but do not enquire if the complementarity problem is equivalent to a social convex optimization formulation.[1] [8] and [9] both show how a system where the disaggregated agents solve their own

[1] The term 'a social formulation' or 'a social problem' is used throughout this paper to refer to the problem that a benevolent central agent with full control and full information would solve.

optimization problems can produce the social optimum, but consider disaggregation on the demand side and not the supply side also. De Jonghe et al. [10] present a range of methods to incorporate short-term demand response in power sector modeling, including both complementarity programming and quadratic programming, and outline conditions for one to be equivalent to the other. In contrast, the model we will present comprises a more general convex programming formulation, with fewer conditions on the structure of the problem.[2] Other formulations related to this work include [11] and [12]. In addition to the nature of the proof we will present, our paper differs from many of these references in how the resulting model is applied, focussing on the system value of load shifting capability.

The co-ordination of electricity supply and demand, in particular the incorporation of deferrable demand into a stochastic unit commitment model, is discussed in [13]. We will show how the 'centralized load control case' in [13] is equivalent to a decentralized problem. On computation of equilibria, [14], [15], [16], [17] discuss the computation of equilibria in energy systems' models, while the computability of Arrow-Debreu equilibria is analyzed in [2], [18].

The value of smart grid technologies is considered in [19], with the price profile given as an exogenous input. How the price profile, and subsequently the value, change with increased deployment of smart grid technologies motivates the equilibrium price framework we consider here. Meanwhile, most models of the power sector from both an operation and capacity expansion point of view assume a fixed demand shape (for example see [20] or [21]). How responsive electricity demand affects the valuation of electricity supply technologies also motivates an equilibrium approach.

For the purposes of this paper, energy storage technologies are considered a subset of the technologies that can shift electricity demand. The arbitrage value of energy storage in a number of real markets has been assessed by [22]. A similar metric of valuation is derived directly from our equilibrium framework. The resulting identity for the marginal value of shiftable electricity demand illustrates how the value of the technologies can change with their increasing deployment. The price differences that incentivize the technologies can be arbitraged away by those same technologies, resulting in decreasing returns on investment.

The welfare impacts of energy storage are discussed in [23] and [24], showing that the societal benefits are always positive in a perfectly competitive market, but not necessarily so otherwise. We will also discuss the welfare impacts of shiftable electricity demand, and the associated distribution of benefits amongst producers and consumers.

Finally, underlying this paper is the concept that autonomous demand response technologies will be available. Examples of the enabling underlying algorithms include [25].

III. Model

In this study, we consider multiple independent electricity power consumers and multiple independent electricity suppliers / producers over a period of T discrete time points, where all of them are price-takers.

We will present a general case of our formulation, where both consumers and producers face convex optimization problems.

[2] A notable difference is that we consider the utility of total demand across the time window as opposed to the utility of demand in each hour.

A. Producer's Convex Optimization Problem

Let $\boldsymbol{p} \in \mathbf{R}_+^T$ be the given (equilibrium) price vector where its tth component $p_t,\ t = 1, 2, ..., T$, represents the electricity equilibrium price at time point t. Let $\boldsymbol{x}^i \in \mathbf{R}_+^T$ be the electricity production/output vector (its tth component x_t^i would be the power output at time point t) from producer i, $i = 1, ..., m$ (electricity producers such as thermal, hydro, renewable generators, or even discharging storage units). Similarly, let $z^i \in \mathbf{R}_+$ be the capacity investment variable from producer i. Then, the profit-maximization optimization problem of the producer i, $i = 1, ..., m$, is

$$\begin{array}{lll} \text{maximize} & \boldsymbol{p}^T\boldsymbol{x}^i - c^i(\boldsymbol{x}^i) - g^i z^i & \\ \text{subject to} & A^i\boldsymbol{x}^i + H^i z^i & \le \boldsymbol{b}^i, \\ & \boldsymbol{x}^i & \ge \boldsymbol{0}, \\ & z^i & \ge 0; \end{array} \tag{1}$$

where $c^i(\cdot) : \mathbf{R}_+^T \to R$ is the production cost function, and where g^i is the capital cost of new capacity. In addition to the set of hard physical limitations represented by vector $\boldsymbol{b}^i$, production $\boldsymbol{x}^i$ is constrained by the quantity of capacity investment z^i. A^i represents the constraint matrix of producer i, whereas the H^i matrix represents constraints associated with the availability of the new capacity. Note that some components of $\boldsymbol{x}^i$ could be negative if the supplier is a storage unit,[3] but, for simplicity, we assume here that $\boldsymbol{x}^i$ is a nonnegative vector. The producer problem possesses standard linear constraints, but all our results are applicable when $\boldsymbol{x}^i$ is constrained in a more general polyhedral set. We assume that $c^i(\cdot)$ is a continuous and differentiable convex cost function, that is, the marginal cost does not decrease as unit i increases production.

[3] Storage technologies can be represented in either, none, or both the producer's and consumer's problem. In the consumer's problem they are represented implicitly as a behind-the-meter device that can shift electricity demand from one period to another.

The optimality conditions of the problem facing producer i, $i = 1, ..., m$, are:

$$\begin{array}{rcl} \boldsymbol{p} - \nabla c^i(\boldsymbol{x}^i) & \le & (A^i)^T\boldsymbol{y}^i, \\ -g^i & \le & (H^i)^T\boldsymbol{y}^i, \\ A^i\boldsymbol{x}^i + H^i z^i & \le & \boldsymbol{b}^i, \\ \boldsymbol{p}^T\boldsymbol{x}^i - \nabla c^i(\boldsymbol{x}^i)^T\boldsymbol{x}^i - g^i z^i & = & (\boldsymbol{b}^i)^T\boldsymbol{y}^i, \\ (\boldsymbol{x}^i, \boldsymbol{y}^i) & \ge & \boldsymbol{0}, \\ z^i & \ge & 0 \end{array} \tag{2}$$

where $\boldsymbol{y}^i$ is the vector of dual variables associated with the physical constraints, and the equality is derived from the complementary slackness condition.

B. Consumer's Convex Optimization Problem

Let $\boldsymbol{u}^j \in \mathbf{R}_+^T$ be the electricity consumption variables of consumer j (its tth component u_t^j would be the power usage at time point t) for consumer/device j, $j = 1, ..., n$, let $\boldsymbol{d}^j \in \mathbf{R}_+^T$ be the given fixed and non-shiftable power demand vector (its tth component d_t^j would be the fixed and non-shiftable demand at time point t for consumer j), and let variable $D^j \in \mathbf{R}_+$ be the aggregated shiftable electricity demand over the entire period for consumer j. Since D^j is a variable, consumer j is equipped with a continuous and differentiable concave utility function $u^j(\cdot) : \mathbf{R}_+ \to \mathbf{R}_+$ on D^j, that is, as more power is consumed, the marginal utility benefit decreases. Then, given the price vector $\boldsymbol{p}$, the jth consumer's cost-minimization problem can be represented as:

$$\begin{array}{lll} \text{minimize} & \boldsymbol{p}^T\boldsymbol{u}^j - u^j(D^j) & \\ \text{subject to} & \boldsymbol{u}^j & \ge \boldsymbol{d}^j, \\ & (\boldsymbol{e}^j)^T\boldsymbol{u}^j - D^j & \ge 0, \\ & \boldsymbol{u}^j & \le C\boldsymbol{e}, \\ & \boldsymbol{u}^j, D^j & \ge \boldsymbol{0}; \end{array} \tag{3}$$

where $\boldsymbol{e} \in \mathbf{R}_+^T$ is the vector of all ones, C is the circuit capacity, and $\boldsymbol{e}^j \in \mathbf{R}_+^T$ stands for the shiftable indicator vector. For example, $\boldsymbol{e}^j = (1; 1; 1; 0; ...; 0)$ implies that the electricity demand D^j could be met in any of time points 1, 2 or 3.

One may add multiple shiftable demand variables into the objective and constraints; but, for simplicity, we assume here that consumer j has one shiftable demand variable D^j.

The optimality conditions of the jth ($j = 1, ..., n$) consumer problem are

$$\begin{array}{rcl} \boldsymbol{p} & \geq & \boldsymbol{v}^j + \lambda^j \boldsymbol{e}^j - \boldsymbol{w}^j, \\ -u^j(D^j)' & \geq & -\lambda^j, \\ \boldsymbol{u}^j & \geq & \boldsymbol{d}^j, \\ (\boldsymbol{e}^j)^T \boldsymbol{u}^j - D^j & \geq & 0, \\ \boldsymbol{u}^j & \leq & C\boldsymbol{e}, \\ \boldsymbol{p}^T \boldsymbol{u}^j - u^j(D^j)' \cdot D^j & = & (\boldsymbol{d}^j)^T \boldsymbol{v}^j - C\boldsymbol{e}^T \boldsymbol{w}^j, \\ \boldsymbol{u}^j, \boldsymbol{v}^j, \lambda^j, \boldsymbol{w}^j & \geq & \boldsymbol{0}, \ \forall j. \end{array} \tag{4}$$

where $(\boldsymbol{v}^j, \lambda^j, \boldsymbol{w}^j)$ are the Lagrange multipliers for the three constraints, respectively, and the equality is derived from the complementary slackness conditions.

C. Competitive Market Equilibrium

In the Arrow-Debreu competitive market, the market clearing condition would have the total supply equalize the total consumption at every time point, that is,

$$\sum_{i=1}^{m} \boldsymbol{x}^i = \sum_{j=1}^{n} \boldsymbol{u}^j. \tag{5}$$

Then, the market equilibrium price $\boldsymbol{p}$, together with production $\boldsymbol{x}^i$ and consumption $\boldsymbol{u}^j$, are simultaneously optimal for every i and every j, under the conditions (2), (4) and (5).[4] Since both $\boldsymbol{p}$ and $(\boldsymbol{x}^i, \boldsymbol{u}^j)$ are equilibrium variables, these conditions possess undesirable nonlinear functions and non-convexities that make their solution computationally intractable. However, we prove the following theorem.

[4]Note that while the original equilibrium conditions in [1] included the maximization of utility given a budget constraint, our formulation entails maximization of utility net of costs.

Theorem 1: The market equilibrium price $\boldsymbol{p}$, together with optimal production $\boldsymbol{x}^i$ and optimal consumption $\boldsymbol{u}^j$, can be represented by a system of convex equality and inequality constraints on a convex set. More precisely, the set of conditions can be represented as:

$$\begin{array}{rcl} \boldsymbol{p} - \nabla c^i(\boldsymbol{x}^i) & \leq & (A^i)^T \boldsymbol{y}^i, \ \forall i, \\ -g^i & \leq & (H^i)^T \boldsymbol{y}^i, \ \forall i, \\ A^i \boldsymbol{x}^i + (H^i)^T z^i & \leq & \boldsymbol{b}^i, \ \forall i, \\ \boldsymbol{p} & \geq & \boldsymbol{v}^j + \lambda^j \boldsymbol{e}^j - \boldsymbol{w}^j, \ \forall j, \\ -u^j(D^j)' & \geq & -\lambda^j, \ \forall j, \\ \boldsymbol{u}^j & \geq & \boldsymbol{d}^j, \ \forall j, \\ (\boldsymbol{e}^j)^T \boldsymbol{u}^j - D^j & \geq & 0, \ \forall j, \\ \boldsymbol{u}^j & \leq & C\boldsymbol{e}, \ \forall j, \\ \sum_{i=1}^m \boldsymbol{x}^i & = & \sum_{j=1}^n \boldsymbol{u}^j, \ \forall j, \\ \sum_{i=1}^m (\nabla c^i(\boldsymbol{x}^i)^T (\boldsymbol{x}^i) + & g^i z^i & + (\boldsymbol{b}^i)^T \boldsymbol{y}^i) \\ = \sum_{j=1}^n ((\boldsymbol{d}^j)^T \boldsymbol{v}^j & + & u^j(D^j)' \cdot D^j - C\boldsymbol{e}^T \boldsymbol{w}^j), \\ \boldsymbol{x}^i, z^i, \boldsymbol{y}^i, \boldsymbol{u}^j, \boldsymbol{v}^j, D^j, \lambda^j, \boldsymbol{w}^j & \geq & \boldsymbol{0}, \ \forall i, j. \end{array} \tag{6}$$

Proof: As (6) contains the feasibility conditions associated with (1), any $\boldsymbol{p}$ and $((\boldsymbol{x}^i, z^i), \boldsymbol{y}^i)$ satisfying (6), $((\boldsymbol{x}^i, z^i), \boldsymbol{y}^i)$ is a feasible primal-dual pair of production maximization problem (1). Then, from the weak duality theorem, we have

$$\boldsymbol{p}^T \boldsymbol{x}^i - \nabla c^i(\boldsymbol{x}^i)^T \boldsymbol{x}^i - g^i z^i \leq (\boldsymbol{b}^i)^T \boldsymbol{y}^i, \ \forall i,$$

or

$$\boldsymbol{p}^T \boldsymbol{x}^i \leq \nabla c^i(\boldsymbol{x}^i)^T \boldsymbol{x}^i + g^i z^i + (\boldsymbol{b}^i)^T \boldsymbol{y}^i, \ \forall i.$$

Similarly, for any $\boldsymbol{p}$ and $(\boldsymbol{u}^j, D^j, \boldsymbol{v}^j, \lambda^j, \boldsymbol{w}^j)$ satisfying (6), $((\boldsymbol{u}^j, D^j), (\boldsymbol{v}^j, \lambda^j, \boldsymbol{w}^j))$ is a feasible primal-dual pair of consumer problem (3). Again from the weak duality theorem, we have

$$\boldsymbol{p}^T \boldsymbol{u}^j \geq (\boldsymbol{d}^j)^T \boldsymbol{v}^j + u^j(D^j) - C\boldsymbol{e}^T \boldsymbol{w}^j, \ \forall j.$$

Thus, summing up we have

$$\sum_{i=1}^{m} \left(\nabla c^i(\boldsymbol{x}^i)^T \boldsymbol{x}^i + g^i z^i + (\boldsymbol{b}^i)^T \boldsymbol{y}^i \right) \geq \boldsymbol{p}^T \left(\sum_{i=1}^{m} \boldsymbol{x}^i \right)$$

and

$$\boldsymbol{p}^T\left(\sum_{j=1}^n \boldsymbol{u}^j\right) \geq \sum_{j=1}^n \left((\boldsymbol{d}^j)^T\boldsymbol{v}^j + u^j(D^j) - C\boldsymbol{e}^T\boldsymbol{w}^j\right).$$

Then, from the market clearing condition

$$\sum_{i=1}^m \boldsymbol{x}^i = \sum_{j=1}^n \boldsymbol{u}^j,$$

we must have

$$\begin{aligned}\sum_{i=1}^m &\left(\nabla c^i(\boldsymbol{x}^i)^T\boldsymbol{x}^i + g^j z^j + (\boldsymbol{b}^i)^T\boldsymbol{y}^i\right)\\ &\geq \boldsymbol{p}^T\left(\sum_{i=1}^m \boldsymbol{x}^i\right) = \boldsymbol{p}^T\left(\sum_{j=1}^n \boldsymbol{u}^j\right)\\ &\geq \sum_{j=1}^n \left((\boldsymbol{d}^j)^T\boldsymbol{v}^j + u^j(D^j) - C\boldsymbol{e}^T\boldsymbol{w}^j\right).\end{aligned}$$

Furthermore, from (6), we have another equality

$$\begin{aligned}\sum_{i=1}^m &\left(\nabla c^i(\boldsymbol{x}^i)^T\boldsymbol{x}^i + g^i z^i + (\boldsymbol{b}^i)^T\boldsymbol{y}^i\right)\\ &= \sum_{j=1}^n \left((\boldsymbol{d}^j)^T\boldsymbol{v}^j + u^j(D^j) - C\boldsymbol{e}^T\boldsymbol{w}^j\right),\end{aligned}$$

taking this as given, we must then also have

$$\sum_{i=1}^m \left(\nabla c^i(\boldsymbol{x}^i)^T\boldsymbol{x}^i + g^i z^i + (\boldsymbol{b}^i)^T\boldsymbol{y}^i\right) = \boldsymbol{p}^T\left(\sum_{i=1}^m \boldsymbol{x}^i\right)$$

and

$$\boldsymbol{p}^T\left(\sum_{j=1}^n \boldsymbol{u}^j\right) = \sum_{j=1}^n \left((\boldsymbol{d}^j)^T\boldsymbol{v}^j + u^j(D^j) - C\boldsymbol{e}^T\boldsymbol{w}^j\right).$$

These, along with our inequalities developed from the weak duality theorem, imply,

$$\nabla c^i(\boldsymbol{x}^i)^T\boldsymbol{x}^i + g^i z^i + (\boldsymbol{b}^i)^T\boldsymbol{y}^i = \boldsymbol{p}^T\boldsymbol{x}^i,\ \forall i,$$

and

$$\boldsymbol{p}^T\boldsymbol{u}^j = (\boldsymbol{d}^j)^T\boldsymbol{v}^j + u^j(D^j) - C\boldsymbol{e}^T\boldsymbol{w}^j,\ \forall j.$$

Which in combination with the remaining conditions in (6), imply that the optimality conditions for each producer and consumer are represented by (6). That is, the fixed $\boldsymbol{p}$, $((\boldsymbol{x}^i, z_i), \boldsymbol{y}^i)$ that meets the conditions of (6) is an optimal primal-dual pair of production profit maximization problem (1), simultaneously for every i, and $((\boldsymbol{u}^j, D^j), (\boldsymbol{v}^j, \lambda^j, \boldsymbol{w}))$ is an optimal primal-dual pair of consumer problem (3), simultaneously for every j. ■

Moreover, there is an aggregated social convex program representing equilibrium conditions (6).

Theorem 2: The cost-minimization convex program

$$\begin{array}{lrl} \text{minimize} & \sum_{i=1}^m \left(c^i(\boldsymbol{x}^i) - g^i z^i\right) - & \sum_{j=1}^n u^j(D^j) \\ \text{subject to} & -A^i\boldsymbol{x}^i - H^i z^i & \geq -\boldsymbol{b}^i,\ \forall i, \\ & \boldsymbol{u}^j & \geq \boldsymbol{d}^j,\ \forall j, \\ & (\boldsymbol{e}^j)^T\boldsymbol{u}^j - D^j & \geq 0,\ \forall j, \\ & -\boldsymbol{u}^j & \geq -C\boldsymbol{e},\ \forall j, \\ & \sum_{i=1}^m \boldsymbol{x}^i - \sum_{j=1}^n \boldsymbol{u}^j & = \boldsymbol{0}, \\ & \boldsymbol{x}^i, \boldsymbol{u}^j, z^i, D^j & \geq \boldsymbol{0},\ \forall i, j \end{array} \tag{7}$$

produces (i) $(\boldsymbol{x}^i, \boldsymbol{u}^j, z^i, D^j)$ satisfying (6), (ii) the equilibrium price vector $\boldsymbol{p}$, being the optimal Lagrange multipliers associated with the market clearing equality constraints, and (iii) the other optimal multipliers $(\boldsymbol{y}^i, \boldsymbol{v}^j, \lambda^j, \boldsymbol{w}^j)$.

The proof of the theorem is straightforward, comparing the KKT conditions of the social optimization problem (7) with (6). One can see that the social welfare objective of the social problem consists of two parts: the first is the total electricity production cost and the second is the utility values of all consumers. Significantly, Theorem 2 shows that the market-clearing equilibrium price vector can be computed as a convex optimization problem, which makes the computation tractable.

From the complementarity condition, we also have:

Corollary 1: If the aggregated supply or demand is positive at every time period, that is, $\sum_{i=1}^m \boldsymbol{x}^i > \boldsymbol{0}$, then

$$\boldsymbol{p} = \min_i \left(\nabla c^i(\boldsymbol{x}^i) + (A^i)^T\boldsymbol{y}^i\right) = \max_j \left(\boldsymbol{v}^j + \lambda^j\boldsymbol{e}^j - \boldsymbol{w}^j\right).$$

IV. Illustrative Examples

For clarity of exposition, the set of examples will be a simpler version of our general model, namely with constant costs of production given fixed capacity constraints and a utility of total electricity demand such that total demand remains fixed. In such cases, our problem reduces to a linear programming problem.

A. Toy Example

Here we consider an interval with three time periods $t = 1, 2, 3$. There are two producers, a thermal resource with constant availability, denoted by superscript th, and a renewable resource with variable availability, denoted by superscript r. The production problem is as follows for the thermal generator:

$$\begin{array}{lll} \text{maximize} & \boldsymbol{p}^T \boldsymbol{x}^{th} - 7\boldsymbol{x}^{th} & \\ \text{subject to} & x_i^{th} & \leq 16, \ i = 1, 2, 3, \\ & x_1^{th}, x_2^{th}, x_3^{th} & \geq 0; \end{array}$$

And the variable renewable generator's production problem:

$$\begin{array}{lll} \text{maximize} & \boldsymbol{p}^T \boldsymbol{x}^{r} - 0\boldsymbol{x}^{r} & \\ \text{subject to} & x_1^{r} & \leq 2 \\ & x_2^{r} & \leq 7 \\ & x_3^{r} & \leq 9 \\ & x_1^{r}, x_2^{r}, x_3^{r} & \geq 0; \end{array}$$

that is, the thermal producer has a capacity of 16 available every period at a variable cost of 7, while the renewable producer has a varying available capacity at zero variable cost.

The consumer problem is

$$\begin{array}{lll} \text{minimize} & \boldsymbol{p}^T \boldsymbol{u} & \\ \text{subject to} & u_1 & \geq 11, \\ & u_2 & \geq 16, \\ & u_3 & \geq 5, \\ & u_1 + u_2 + u_3 & \geq 37, \end{array}$$

where the constraints indicate that up to 5 units of electric energy can be shifted amongst the time periods.

The social linear program is:

$$\begin{array}{lll} \text{minimize} & 7\boldsymbol{x}^{th} & \\ \text{subject to} & -x_i^{th} & \geq -16, \ i = 1, 2, 3, \\ & -x_1^{r} & \geq -2, \\ & -x_2^{r} & \geq -7, \\ & -x_3^{r} & \geq -9, \\ & u_1 & \geq 11, \\ & u_2 & \geq 16, \\ & u_3 & \geq 5, \\ & x_i^{th} + x_i^{r} & = u_i, \ i = 1, 2, 3, \\ & u_1 + u_2 + u_3 & \geq 37, \end{array}$$

The equilibrium prices are $(7;\ 7;\ 7)$ and the $\boldsymbol{u}$ vector is $(11; 16; 10)$. The cost of production is 133, and the net profit of the producers is 0 and 126 respectively. We see that the 5 flexible units of demand were used in period 3, when there was highest availability of the zero variable cost renewable resource. If this demand was required at an earlier period and not flexible to move, the more expensive supply resource would have been used, with an associated decrease in system value. With fixed demand, this would have implied a lower electricity price in period 3. We will discuss these dynamics more systematically in later sections.

B. ERCOT 2031 Example

To illustrate the model further, we now implement the problem (7) with some aggregated data from the ERCOT power system in Texas. In so doing, we will undertake an assessment of the value of optimal load shifting enabled by smart grid technologies by comparing model outcomes when load shifting is available, and when it is not.

1) Data: The data used to conduct this exercise is from the 'Current Trends' scenario of the ERCOT Long Term System Assessment process [26]. The data includes capacity mix and hourly load, wind and solar data for the 2031 scenario year. The capacity mix used in the problem is displayed in Table I.

TABLE I
CAPACITY MIX IN EXAMPLE

Technology	Capacity (GW)	Marginal Cost ($/MWh)
Solar	21.7	0
Wind	21.5	0
Nuclear	5.2	11.4
Gas CC	37.3	48.7
Gas CT	12.1	72.6
Gas Steam	8.7	79.8
Coal	10.2	34.3

2) Abstract representation of optimal load shifting: Rather than an explicit representation of optimal load shifting technologies and associated capabilities, we consider a more abstract representation. In particular, we assess the impact of changing a) the percentage of benchmark load in every hour that is flexible, and b) the duration of the window in which load can shift. For example, a central scenario below is the case where 15% of load in each hour can shift to any other hour in the associated 24-hour window. The idea is that this high level approach can provide insight into what characteristics drive the value of technologies that enable optimal load shifting.

3) Results: Note that throughout this particular exercise we assume that the total consumer value of electricity consumption does not change with the introduction of the load shifting technologies. Thus, comparing consumer costs allows us to assess changes in consumer surplus. Table II compares the welfare losses and gains associated with two static cases, one where 15% of the reference demand in any hour is shiftable within each 24-hour window, and one where no demand is shiftable.

Figure 1 provides some graphical intuition behind these results. The consumers save due to the decline

TABLE II
DISTRIBUTION OF THE BENEFITS OF OPTIMAL LOAD SHIFTING BETWEEN CONSUMERS AND PRODUCERS IN NUMERIC EXAMPLE (MILLION $)

	No shiftable demand	Shiftable demand	Δ
Consumer Cost	22,434	21,780	-654
Producer Profit	10,575	9,992	-583
Welfare			+71

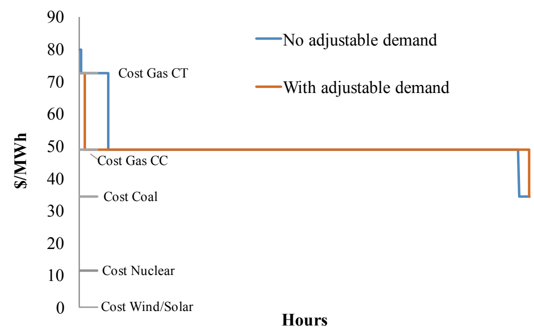

Fig. 1. Price duration curve for ERCOT 2031 Current Trends scenario. In adjustable demand case, 15% of demand is flexible within each 24-hour window.

in peak prices. Aggregate producer profit declines for the same reason. Profits that inframarginal generators such as nuclear and coal were making during peak hours decline as the number of peak hours declines.

This result is based on an assumption that at most 15% of load could shift forward or backward in time within each predefined 24-hour window. Figure 2 displays the sensitivity of the result to the duration of this window. If demand can shift from any hour of the year to any other, we do not get any additional benefits than if load can shift within a 24 hour period. Additionally, we can see the value is halved if the window is 12 hours.

Figure 3 illustrates the sensitivity of the benefits of optimal load shifting to what fraction of demand can shift in any given hour. Most of the value comes from

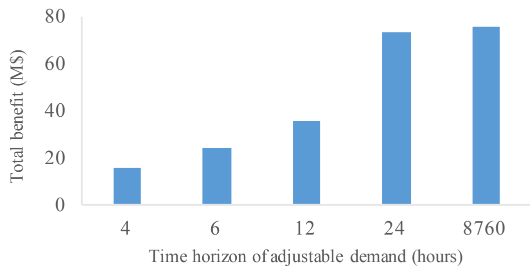

Fig. 2. Sensitivity of the value of load shifting to the duration of the window within which load can shift (assuming 15% of load flexible)

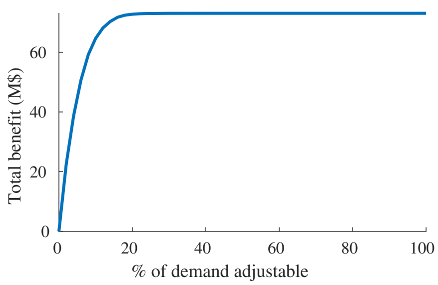

Fig. 3. Sensitivity of the value of load shifting to the fraction of demand that can shift within each hour (assuming 24 hour window).

being able to shift 10-15% of demand in any given hour. Equivalently, it is at this point that the benefits of technologies that enable load shifting saturate.

V. Implications

A. Economic Implications

These results indicate a value of system-wide optimal load shifting in this case of approximately \$3 per year per customer.[5] This figure assumes 15% of demand in each hour is available for shifting within each 24 hour period. If the window within which load can be shifted is 12 hours, the value is approximately \$1.50 per year. Whether this amount of money is enough to cover the cost of implementing such a system is not assessed here.

[5]\$76 million divided by an assumed 24 million customers served within the ERCOT area equals \$3.04 per customer

There are undoubtedly aspects of the benefits of the system not included in this study such as grid stability, meeting short term fluctuations, more transparency, and control for consumers. Additionally, the base case prices may not be as flat as presented here and more arbitrage opportunities may be available if the thermal units of the same class are not as homogenous in characteristic as modeled, and also if there are local pockets of more variable prices due to transmission constraints. On the other hand, this assessment did not consider rate structures, which typically exhibit less variability across an hourly basis that wholesale market prices. The purpose of this exercise however was to illustrate the use of the model. If this exercise showed either huge value or no value to optimal load shifting technologies, more detailed studies may not be necessary. The result from this example is somewhere in the middle, and this modeling approach could aid more detailed investigations.

Also for this case study, total welfare increases with the introduction of shiftable electricity demand, but producer surplus declines in this case while consumer surplus increases. As [24] shows, we expect the net welfare to be non-negative under the competitive market setup, however the distributional results have not been discussed extensively in the literature.

B. Optimal Capacity Mix

While included in the presentation of the model, our numerical case did not include endogenous capacity investments and retirements. Load shifting capabilities could be expected to change the optimal capacity mix, particularly in scenarios with tight carbon constraints and/or with significant ongoing declines in costs of wind and solar technologies. While we will not explore this numerically, the following section is relevant to this topic analytically.

C. Marginal Value of Shiftable Demand

The marginal value of the ability to shift electricity demand is as follows, where $\boldsymbol{p}$ is our equilibrium electricity price. Equivalently, the identity is the rational willingness to pay for the ability to shift electricity demand. See Appendix A for proof.[6]

$$\texttt{marginal value(\$/MW)} = \sum_t |p_t - \text{median}(\boldsymbol{p})| \tag{8}$$

The metric as derived is a measure of the dispersion of the electricity prices, being the sum of the absolute differences between the prices in each hour and the median price. While this is not a surprise, it is perhaps useful to derive an intuitive identity from the equilibrium model, an identity that uses price alone as a sufficient statistic, avoiding the need to include technical considerations such as the underlying capacity mix or flexibility of demand in the system that generated the prices. At the margin, this metric allows the value at the optimum to be calculated by simply assessing electricity prices, avoiding the need to run the full optimization model. A reduction in this metric indicates that fewer arbitrage opportunities are available, and implies a reduction in the value of load shifting technologies. We can expect this to happen as more such technologies are installed. For example in our numerical example earlier, we saw no additional value beyond the point where 10-15% of load was flexible.

Increased deployment of variable wind and solar resources can be expected to increase the dispersion of electricity prices and thus increase the value of load shifting technologies. In parallel, increased flexibility in load can alter the economics of investment in wind and solar generators. Using the marginal value of an investment in a wind or solar power generator, as first shown by [27], we can introduce the ratio between the marginal value of shiftable demand and investment in a wind or solar power generator.

$$\alpha = \frac{\sum_t |p - \text{median}(\boldsymbol{p})|}{|T|[E(\boldsymbol{p}).E(\boldsymbol{a}_g) + Cov(\boldsymbol{p}, \boldsymbol{a}_g)]} \tag{9}$$

Where $\boldsymbol{a}_g$ is the availability of generator g, and $E()$ is the time-weighted expectation. The essence of the formula is:

$$\alpha = \frac{\text{dispersion of prices}}{\text{fn(correlation of solar/wind \& price)}} \tag{10}$$

α, the relative marginal value metric, can frame how shiftable demand and renewable energy affect each other, with the former benefitting from the latter's dispersion of prices, and the latter benefitting from the potentially increased correlation between prices and renewables availability.

D. Market Design / System Operation

The optimal load shifting model has a number of implications for electricity market design and system operations. We have presented a system where each supplier of electricity may submit their marginal cost of electricity production in each hour (the optimal bid in a competitive market setting), and where each consumer (or even each device) may submit their demand for electricity in each hour, along with their constraints on load shifting. The system operator then solves the tractable convex program to find the equilibrium solution. The associated price could then be released to the market participants. Each participant selfishly optimizing in a decentralized manner given this price signal will then lead to the socially optimum outcome, implying a zero 'price of anarchy' [28]. Note that centralized control

[6]For clarity, the formula as presented relates to fully flexible electricity demand across the model's full horizon. For those cases where the hours to which demand can move is restricted, the value is the cumulative application of the formula to prices within each window. Where the index j indicates each window: $\sum_j \sum_{t(j)} |p^j_{t(j)} - \text{median}(\boldsymbol{p}^j)|$

of load would not be required, nor would a demand aggregator be required to achieve the theoretical optimal market outcome, potentially mitigating market power concerns.

Alternatively, the convex program could be solved in a decomposed manner, with the system operator simply announcing an initial price, each agent solving their individual problems, returning quantities, then the system operator increasing or decreasing the price on the basis of aggregate excess demand. Such an approach is discussed in [8] and [9]. Such an approach allows the problem to be solved without any information requirements placed on the system operator.

Important issues for further research in the adoption of such an approach include non-competitive or strategic price settings, strategy-proof bidding systems, treatment of uncertainty, and latency in communication signals.

E. Policy Analysis

The work here can support the design of models for policy analysis, as the inclusion of load shifting capability can potentially change the policy-relevant insights produced by the model. In addition to the modelling tool, the identities in Section V-C can provide simple rules for understanding the policy implications of shiftable electricity demand.

VI. Conclusion

The electric power system is undergoing significant change on both the demand side and the supply side. This study has made two contributions to discussions relating to the evolving power system - a) a computationally tractable model that admits flexible functional forms is derived that can be used for policy analysis or market design/operation purposes, and b) using the model, a high level assessment of the value of optimal automated load shifting is carried out.

Numeric analysis for a 2031 ERCOT scenario indicates that to capture the majority of the value, an unrealistic 100% of demand is not required to be flexible, but a more achievable, yet far from trivial, 10-15% is. The majority of the value does not require adjustments across weeks or months, but again a more achievable yet far from trivial 12-24 hours. For this particular dataset and this particular methodology, the value unlocked by installing such a system is limited to a few dollars per year per customer. How this value would increase under greater deployment of variable renewables is shown analytically.

Finally, the equilibrium approach can offer qualitatively different insights than considering the impact of optimal load shifting from the supply or demand perspective alone.

Appendix A
Marginal Value

To calculate the marginal value, we take our social linear program, (7), from Theorem 2, and explicitly add in a smart grid technology with capital cost s^j that enables a g^j quantity of shiftable demand in each hour for demand j.[7] Assume that all of our load is non-shiftable $\boldsymbol{d}^j$ in the absence of this technology. (7) can then be written as follows.

$$\begin{array}{llll}
\text{minimize} & \sum_{i=1}^m \boldsymbol{c}^i \boldsymbol{x}^i & + \sum_{j=1}^n s^j g^j & \\
\text{subject to} & -A^i \boldsymbol{x}^i & \geq -\boldsymbol{b}^i,\ \forall i, & : \boldsymbol{y}^i \\
& g^j \boldsymbol{e} & \geq |\boldsymbol{u}^j - \boldsymbol{d}^j|,\ \forall j, & : \boldsymbol{v}^j \\
& \boldsymbol{e}^T \boldsymbol{u}^j & \geq D^j,\ \forall j, & : \lambda^j \\
& \sum_{i=1}^m \boldsymbol{x}^i & = \sum_{j=1}^n \boldsymbol{u}^j, & : \boldsymbol{p} \\
& \boldsymbol{x}^i, \boldsymbol{u}^j & \geq \boldsymbol{0},\ \forall i, j &
\end{array} \tag{11}$$

[7] In the case here, demand can increase only by g^j in any hour. Relaxing this limit changes the derivation, but not the associated qualitative points.

Differentiating the Lagrangian with respect to g^j yields the following:[8]

$$s^j = \boldsymbol{e}^T \boldsymbol{v}^j$$

Differentiating with respect to $\boldsymbol{u}^j$ yields the following:

$$\boldsymbol{v}^j = |\boldsymbol{p} - \lambda^j \boldsymbol{e}|$$

Combining:

$$s^j = \boldsymbol{e}^T |\boldsymbol{p} - \lambda^j \boldsymbol{e}|$$

If $\boldsymbol{e}$ is the vector of ones,[9] λ^j will always equal median($\boldsymbol{p}$),[10] so we can progress as follows:

$$s^j = \boldsymbol{e}^T |\boldsymbol{p} - \text{median}(\boldsymbol{p})|$$

$$\implies \texttt{marginal value} = \sum_t |p_t - \text{median}(\boldsymbol{p})| \tag{12}$$

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[8]The absolute value constraint can be transformed into equivalent linear constraints, with the resulting analysis translated into the results shown here.

[9]Proof and associated identity extends naturally to when $\boldsymbol{e}$ is not 1 for every time period. Not included here for brevity.

[10]By Corollary 1 above, $\lambda^j = p_t$ when $v_t^j = 0$. This will occur when the constraint associated with $\boldsymbol{v}^j$ is not binding, which will be at the median price. If at any other price, there is an arbitrage opportunity that the load shifting capability g^j can be used for, thus implying $v_t > 0$, and a contradiction is reached.

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