Trigonometric identities

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1 Algebra

1.1 Definitions

$$\sin x = \frac{1}{\csc x} = \frac{e^{ix} - e^{-ix}}{2i} \qquad \sinh x = \frac{1}{\operatorname{csch} x} = \frac{e^{x} - e^{-x}}{2}$$

$$\cos x = \frac{1}{\sec x} = \frac{e^{ix} + e^{-ix}}{2} \qquad \cosh x = \frac{1}{\operatorname{sech} x} = \frac{e^{x} + e^{-x}}{2}$$

$$\tan x = \frac{1}{\cot x} = -i\frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} \qquad \tanh x = \frac{1}{\coth x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

1.2 Quotient identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

1.3 Hyperbolic relations

$$\sinh x = -i\sin(ix) \qquad \cosh x = \cos(ix) \qquad \tanh x = -i\tan(ix)$$

$$\operatorname{csch} x = i\csc(ix) \qquad \operatorname{sech} x = \sec(ix) \qquad \coth x = i\cot(ix)$$

1.4 Pythagorean identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sec^2 x - \tan^2 x = 1 \qquad \csc^2 x - \cot^2 x = 1$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad \operatorname{sech}^2 x + \tanh^2 x = 1 \qquad \coth^2 x - \operatorname{csch}^2 x = 1$$

1.5 Co-function identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

1.6 Parity relations

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \qquad \sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

$$\sinh(-x) = -\sinh x \qquad \cosh(-x) = \cosh x \qquad \tanh(-x) = -\tanh x$$

$$\operatorname{csch}(-x) = -\operatorname{csch} x \qquad \operatorname{sech}(-x) = \operatorname{sech} x \qquad \coth(-x) = -\coth x$$

1.7 Sum-difference formulas

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y \qquad \sinh(x \pm y) = \sinh x\cosh y \pm \cosh x\sinh y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y \qquad \cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y} \qquad \tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$$

1.8 Double angle formulas

$$\sin(2x) = 2\sin x\cos x \qquad \sinh(2x) = 2\sinh x\cosh x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\cosh(2x) = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$

$$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x} \qquad \tanh(2x) = \frac{2\tanh x}{1 + \tanh^2 x}$$

1.9 Power-reducing, half angle formulas

Using these as half angle formulas requires taking a square root whose sign must be chosen carefully.

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \qquad \cos^2 x = \frac{1 + \cos(2x)}{2} \qquad \tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

$$\sinh^2 x = \frac{\cosh(2x) - 1}{2} \qquad \cosh^2 x = \frac{\cosh(2x) + 1}{2} \qquad \tanh^2 x = \frac{\cosh(2x) - 1}{\cosh(2x) + 1}$$

1.10 Sum to product formulas

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

1.11 Product to sum formulas

$$\sin x\sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right] \qquad \cos x\cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\sin x\cos y = \frac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$$

1.12 Euler equations

$$e^x = \cosh x + \sinh x \qquad e^{-x} = \cosh x - \sinh x$$

$$e^{ix} = \cos x + i\sin x \qquad e^{-ix} = \cos x - i\sin x$$

1.13 Triangle laws

These are for any triangle with sides of length a, b, and c with angles opposite those sides A, B, C.

Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}$$

where R is the circumradius.

Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Law of tangents

$$\frac{a+b}{a-b} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$$

1.14 Tangent formulas

The first is a generalization of the sum formula. The second is derivable from the double and half angle formulas. The third follows directly from the tangent power reducing formula. The fourth follows from the second and third. The last follows directly from the tangent sum formula. In this way, hyperbolic versions can be similarly derived.

$$\begin{aligned}
\tan(nx) &= \frac{\tan[(n-1)x] + \tan x}{1 - \tan[(n-1)x]\tan x} \\
\tan\frac{x}{2} &= \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x} = \frac{\tan x \sin x}{\tan x + \sin x} \\
\cos x &= \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \\
\sin x &= \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \\
\tan\left(x + \frac{\pi}{4}\right) &= \frac{1+\tan x}{1-\tan x}
\end{aligned}$$

1.15 Inverse relations

These are derivable from the co-function identities.

$$\begin{array}{ccc}
\cos^{-1} = \frac{\pi}{2} - \sin^{-1} x & \csc^{-1} = \frac{\pi}{2} - \sec^{-1} x & \cot^{-1} = \frac{\pi}{2} - \tan^{-1} x \\
\sin^{-1}(-x) = -\sin^{-1} x & \cos^{-1}(-x) = \pi - \cos^{-1} x & \tan^{-1}(-x) = -\tan^{-1} x \\
\csc^{-1}(-x) = -\csc^{-1} x & \sec^{-1}(-x) = \pi - \sec^{-1} x & \cot^{-1}(-x) = \pi - \cot^{-1} x
\end{array}$$

$$\cos^{-1}\frac{1}{x} = \sec^{-1} x \qquad \sin^{-1}\frac{1}{x} = \csc^{-1} x \qquad \sec^{-1}\frac{1}{x} = \cos^{-1} x \qquad \csc^{-1}\frac{1}{x} = \sin^{-1} x$$

$$\tan^{-1}\frac{1}{x} = \frac{\pi}{2} - \tan^{-1} x = \cot^{-1} x \qquad x > 0$$
$$\tan^{-1}\frac{1}{x} = -\frac{\pi}{2} - \tan^{-1} x = -\pi + \cot^{-1} x \qquad x < 0$$
$$\cot^{-1}\frac{1}{x} = \frac{\pi}{2} - \cot^{-1} x = \tan^{-1} x \qquad x > 0$$
$$\cot^{-1}\frac{1}{x} = \frac{3\pi}{2} - \cot^{-1} x = \pi + \tan^{-1} x \qquad x < 0$$

These are derivable from the half-angle formulas:

$$\sin^{-1} x = 2\tan^{-1}\frac{x}{1+\sqrt{1-x^2}}$$
$$\cos^{-1} x = 2\tan^{-1}\frac{\sqrt{1-x^2}}{1+x} \qquad -1 < x \le 1$$
$$\tan^{-1} x = 2\tan^{-1}\frac{x}{1+\sqrt{1+x^2}}$$

Similarly, a few for hyperbolic functions are

$$\operatorname{sech}^{-1} x = \cosh^{-1}\frac{1}{x} \qquad \operatorname{csch}^{-1} x = \sinh^{-1}\frac{1}{x} \qquad \coth^{-1} x = \tanh^{-1}\frac{1}{x}$$

1.16 Inverse functions as logarithms

Note: give branch cut locations.

$$\sin^{-1} x = -i\ln\left(ix + \sqrt{1-x^2}\right) \qquad |x| < 1$$
$$\cos^{-1} x = -i\ln\left(x + \sqrt{x^2-1}\right) \qquad |x| < 1$$
$$\tan^{-1} x = \frac{i}{2}\ln\frac{1-ix}{1+ix}$$
$$\csc^{-1} x = -i\ln\left(\frac{i}{x} + \sqrt{\frac{x^2-1}{x^2}}\right) \qquad |x| \ge 1$$
$$\sec^{-1} x = -i\ln\left(\frac{1}{x} + \sqrt{\frac{1-x^2}{x^2}}\right) \qquad |x| \ge 1$$
$$\cot^{-1} x = \frac{i}{2}\ln\frac{x-i}{x+i}$$
$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$
$$\cosh^{-1} x = \ln\left(x + \sqrt{x-1}\sqrt{x+1}\right) \qquad x \ge 1$$
$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x} \qquad |x| < 1$$
$$\operatorname{sech}^{-1} x = \ln\left(x^{-1} + \sqrt{x^{-1}-1}\sqrt{x^{-1}+1}\right) \qquad 0 < x \le 1$$
$$\operatorname{csch}^{-1} x = \ln\left(x^{-1} + \sqrt{1+x^{-2}}\right)$$
$$\coth^{-1} x = \frac{1}{2}\ln\frac{x+1}{x-1} \qquad |x| < 1$$

Simplifications of root products are not made since we assume principal roots (this is irrelevant for real x). Note that $\operatorname{csch}^{-1} 2 = \ln \frac{1+\sqrt{5}}{2}$.

1.17 Inverse sum formulas

This is derivable from the tangent sum formula.

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

2 Calculus

2.1 Series

$$\begin{aligned}
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots &&= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \\
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots &&= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \\
\tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots &&= \sum_{n=0}^{\infty} \frac{U_{2n+1} x^{2n+1}}{(2n+1)!} \\
& &&= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^{2n} \left(2^{2n}-1\right) B_{2n} x^{2n-1}}{(2n)!} && |x| < \frac{\pi}{2} \\
\csc x &= \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \dots &&= \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2 \left(2^{2n-1}-1\right) B_{2n} x^{2n-1}}{(2n)!} && 0 < |x| < \pi \\
\sec x &= 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \dots &&= \sum_{n=0}^{\infty} \frac{U_{2n} x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n E_{2n} x^{2n}}{(2n)!} && |x| < \frac{\pi}{2} \\
\cot x &= \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \dots &&= \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} B_{2n} x^{2n-1}}{(2n)!} && 0 < |x| < \pi
\end{aligned}$$

where U_n is the nth up/down number, B_n is the nth Bernoulli number, and E_n is the nth Euler number.

2.2 Inverse series

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{x^{2n+1}}{2n+1} \qquad |x| \le 1$$

$$\cos^{-1} x = \frac{\pi}{2} - \left[x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \dots\right] = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{x^{2n+1}}{2n+1} \qquad |x| \le 1$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad |x| \le 1,\; x \ne i, -i$$

$$\csc^{-1} x = x^{-1} + \frac{1}{2}\frac{x^{-3}}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^{-5}}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^{-7}}{7} + \dots = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{x^{-(2n+1)}}{2n+1} \qquad |x| \ge 1$$

$$\sec^{-1} x = \frac{\pi}{2} - \left[x^{-1} + \frac{1}{2}\frac{x^{-3}}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^{-5}}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^{-7}}{7} + \dots\right] = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2}\frac{x^{-(2n+1)}}{2n+1} \qquad |x| \ge 1$$

$$\cot^{-1} x = \frac{\pi}{2} - \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right] = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad |x| \le 1,\; x \ne i, -i$$

A more efficient series for inverse tangent:

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{2^{2n}(n!)^2}{(2n+1)!}\frac{x^{2n+1}}{(1+x^2)^{n+1}}$$

2.3 Hyperbolic series

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \dots = \sum_{n=1}^{\infty} \frac{2^{2n}\left(2^{2n}-1\right) B_{2n} x^{2n-1}}{(2n)!} \qquad |x| < \frac{\pi}{2}$$

$$\operatorname{csch} x = \frac{1}{x} - \frac{x}{6} + \frac{7x^3}{360} - \frac{31x^5}{15120} + \dots = \frac{1}{x} + \sum_{n=1}^{\infty} \frac{2\left(1-2^{2n-1}\right) B_{2n} x^{2n-1}}{(2n)!} \qquad 0 < |x| < \pi$$

$$\operatorname{sech} x = 1 - \frac{x^2}{2} + \frac{5x^4}{24} - \frac{61x^6}{720} + \dots = \sum_{n=0}^{\infty} \frac{E_{2n} x^{2n}}{(2n)!} \qquad |x| < \frac{\pi}{2}$$

$$\coth x = \frac{1}{x} + \frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} + \dots = \frac{1}{x} + \sum_{n=1}^{\infty} \frac{2^{2n} B_{2n} x^{2n-1}}{(2n)!} \qquad 0 < |x| < \pi$$

where B_n is the nth Bernoulli number, and E_n is the nth Euler number.

2.4 Inverse hyperbolic series

$$\sinh^{-1} x = x - \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^{2n}(n!)^2}\frac{x^{2n+1}}{2n+1} \qquad |x| < 1$$

$$\cosh^{-1} x = \ln(2x) - \left[\frac{1}{2}\frac{x^{-2}}{2} + \frac{1\cdot 3}{2\cdot 4}\frac{x^{-4}}{4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^{-6}}{6} + \dots\right] = \ln 2x - \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{2^{2n}(n!)^2}\frac{x^{-2n}}{2n} \qquad x > 1$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} \qquad |x| < 1$$

$$\operatorname{csch}^{-1} x = x^{-1} - \frac{1}{2}\frac{x^{-3}}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^{-5}}{5} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^{-7}}{7} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^{2n}(n!)^2}\frac{x^{-(2n+1)}}{2n+1} \qquad |x| < 1$$

$$\operatorname{sech}^{-1} x = \ln\frac{2}{x} - \left[\frac{1}{2}\frac{x^2}{2} + \frac{1\cdot 3}{2\cdot 4}\frac{x^4}{4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^6}{6} + \dots\right] = \ln\frac{2}{x} - \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{2^{2n}(n!)^2}\frac{x^{2n}}{2n} \qquad 0 < x \le 1$$

$$\coth^{-1} x = x^{-1} + \frac{x^{-3}}{3} + \frac{x^{-5}}{5} + \frac{x^{-7}}{7} + \dots = \sum_{n=0}^{\infty} \frac{x^{-(2n+1)}}{2n+1} \qquad |x| > 1$$

Expanding about ∞,

$$\sinh^{-1} x = \ln 2 - \ln x^{-1} + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2n-1)!!}{2n\,(2n)!!} x^{-2n}$$

$$\cosh^{-1} x = \ln 2 - \ln x^{-1} - \sum_{n=1}^{\infty} \frac{(2n-1)!!}{2n(2n)!!} x^{-2n}$$

$$\operatorname{csch}^{-1} x = \sum_{n=1}^{\infty} \frac{P_{n-1}(0)}{n} x^{-n}$$

$$\operatorname{sech}^{-1} x = \ln 2 - \ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n-1)!!}{2n(2n)!!} x^{2n}$$

where $n!! = n\cdot(n-2)\cdot(n-4)\dots$ down to 1 or 2 if n is odd or even (so $(2n+1)!! = (2n+1)!2^{-n}/n!$, $(2n-1)!! = (2n)!2^{-n}/n!$, and $(2n)!! = 2^n n!$). $P_n(x)$ is a Legendre polynomial.

2.5 Derivatives

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x \qquad \frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

2.6 Inverse derivatives

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\csc^{-1} x = -\frac{1}{x^2\sqrt{1-x^{-2}}} \qquad \frac{d}{dx}\sec^{-1} x = \frac{1}{x^2\sqrt{1-x^{-2}}} \qquad \frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$$

2.7 Hyperbolic derivatives

$$\frac{d}{dx}\sinh x = \cosh x \qquad \frac{d}{dx}\cosh x = \sinh x \qquad \frac{d}{dx}\tanh x = \operatorname{sech}^2 x$$

$$\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x \qquad \frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x \qquad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x$$

2.8 Inverse hyperbolic derivatives

$$\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x-1}\sqrt{x+1}} \qquad \frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2}$$

$$\frac{d}{dx}\operatorname{csch}^{-1} x = -\frac{1}{x^2\sqrt{1+x^{-2}}} \qquad \frac{d}{dx}\operatorname{sech}^{-1} x = -\frac{1}{x(x+1)\sqrt{\frac{1-x}{1+x}}} \qquad \frac{d}{dx}\coth^{-1} x = \frac{1}{1-x^2}$$

2.9 Integrals

$$\int \sin x = -\cos x \qquad \int \cos x = \sin x \qquad \int \tan x = -\ln|\cos x|$$

$$\int \csc x = -\ln|\csc x + \cot x| \qquad \int \sec x = \ln|\sec x + \tan x| \qquad \int \cot x = \ln|\sin x|$$

2.10 Inverse integrals

Compute by integration by parts using the derivative of the function.

2.11 Hyperbolic integrals

$$\int \sinh x = \cosh x \qquad \int \cosh x = \sinh x \qquad \int \tanh x = \ln|\cosh x|$$

$$\int \operatorname{csch} x = \ln\left|\tanh\frac{x}{2}\right| \qquad \int \operatorname{sech} x = 2\tan^{-1}\tanh\frac{x}{2} \qquad \int \coth x = \ln|\sinh x|$$

2.12 Inverse hyperbolic integrals

Compute by integration by parts using the derivative of the function.