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Technical Appendix to "Long-Term Contracts under the Threat of Supplier Default"

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1. Proofs

Theorem 1 *(i) In the absence of failure, the optimal short-term contract is* $p_{11}^s = \mu_d + \mu_1$, $p_{12}^s = d_1 + \mu_2(c_1)$, *and* $p_{22}^s = \mu_d(d_1) + \mu_2(c_1)$, *where* d_1 *is the realized value of supplier 1's idiosyncratic costs. The buyer switches suppliers in the second period if* $k + \mu_d(d_1) < d_1$ *(i.e., if* $\alpha < d_1$*). The resulting expected profit for the buyer is*

$$\pi_b^s = 2s - \mu_d - \mu_1 - \mu_2 - \mathbb{E}\min\left(k + \mu_d(d_1), d_1\right). \tag{1}$$

(ii) In the absence of failure, the optimal long-term contract is any pair $\{p_{11}^l, p_{12}^l\}$ *such that* $p_{11}^l + p_{12}^l = 2\mu_d + \mu_1 + \mu_2$, *and the resulting expected profit for the buyer is*

$$\pi_b^l = 2s - 2\mu_d - \mu_1 - \mu_2. \tag{2}$$

(iii) In the absence of failure, the buyer always prefers a short-term contract to a long-term contract. Furthermore, among single-sourcing contracts, the optimal short-term contract achieves the centralized system optimal profit, which we denote by $\overline{\pi}_b$.

Proof. (i) In a short-term contract, prices must be subgame perfect. Thus, it is easy to see that in the second period the buyer will offer the lowest prices that satisfy the participation constraints of the suppliers, i.e., $p_{22}^s = \mu_2(c_1) + \mu_d(d_1)$ for supplier 2 and $p_{12}^s = d_1 + \mu_2(c_1)$ for supplier 1 (since d_1 is known at the end of the first period). Recalling that the buyer must also incur a per unit switching cost of k if he decides to switch to supplier 2, we see that the buyer will switch suppliers if $k + \mu_d(d_1) < d_1$. Thus, in choosing the optimal first period price, the buyer maximizes his total expected profit,

$$\begin{aligned} \pi_b &= \max_{p_{11}} 2s - p_{11} - \mu_2 - \mathbb{E}\min\left(d_1, k + \mu_d(d_1)\right), \\ & \quad \text{s.t. } \mathbb{E}\left(p_{11} - c_1 - d_1\right) \geq 0 \end{aligned}$$

Note that the supplier earns, in expectation, zero in the second period, as the buyer knows the realized value of costs. Hence, the supplier is effectively myopic, agreeing to any contract with $\mathbb{E}(p_{11} - c_1 - d_1) \geq 0$. Thus, the buyer's optimal first period price is $p_{11}^s = \mu_d + \mu_1$. This implies the expected profit is given by (1).

(ii) In a long-term contract, the buyer maximizes $\pi_b = 2s - p_{11} - p_{12}$ subject to $p_{11} + p_{12} - 2\mu_d - \mu_1 - \mu_2 \geq 0$. The optimal contract is found by solving for the binding participation constraint.

(iii) Follows from parts (i) and (ii). ■

Lemma 1 *Define p_{11}^* as the solution to*

$$-1 + \int_{-\infty}^{\alpha} (\mu_d(x) + k - x) f(p_{11}^* - x) g(x) dx = 0.$$

Then the buyer's optimal short-term contract consists of $p_{12}^s = d_1 + \mu_2(c_1)$, $p_{22}^s = \mu_d(d_1) + \mu_2(c_1)$, and

$$p_{11}^s = \begin{cases} p_{11}^* & \text{if } p_{11}^* \geq \mu_d + \mu_c \text{ and } \pi_b^s(p_{11}^*) \geq \pi_b^s(\mu_d + \mu_c) \\ \mu_d + \mu_1 & \text{otherwise} \end{cases},$$

where $\pi_b^s(p_{11})$ is given by,

$$\begin{aligned} \pi_b^s(p_{11}) &= 2s - p_{11} - \mu_2 - \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(\mu_d(x) + k \,|c_1 + d_1 > p_{11}) \\ &\quad - \Pr(c_1 + d_1 \leq p_{11}) \times \mathbb{E}(\min(d_1, \mu_d(x) + k) \,|c_1 + d_1 \leq p_{11}). \end{aligned} \tag{3}$$

The buyer switches suppliers in period 2 if $k + \mu_d(d_1) < d_1$.

Proof. As in the proof of Theorem 1, we have $p_{22}^s = \mu_d(d_1) + \mu_2(c)$ and $p_{12}^s = d_1 + \mu_2(c)$. As in the no failure case, the buyer switches suppliers voluntarily if $\mu_d(d_1) + k < d_1$. However, the buyer involuntarily switches suppliers if the first supplier fails, i.e., if $d_1 + c_1 > p_{11}$. Thus we have a partitioning of the buyer's second period actions according to the realized values of d_1 and c_1 in the following manner (Figure 5 in the main paper): in Region I, the second period price is $d_1 + \mu_2(c_1)$; in Regions II and III, it is $\mu_d(d_1) + \mu_2(c_1)$. Thus, the buyer's expected profit is given by (3).

We next use the following identity:

$$\begin{aligned} & \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(\mu_d(x) + k \,|c_1 + d_1 > p_{11}) \\ = \; & \mu_1 + \mu_d - \Pr(c_1 + d_1 \leq p_{11}) \times \mathbb{E}(\mu_d(x) + k \,|c_1 + d_1 \leq p_{11}) \end{aligned}$$

to rewrite the profit function as

$$\begin{aligned} \pi_b^s(p_{11}) &= 2s - p_{11} - \mu_1 - \mu_d - \mu_2 \\ &\quad - \Pr(c_1 + d_1 \leq p_{11}, d_1 < \alpha) \times \mathbb{E}(d_1 - \mu_d(x) - k \,|c_1 + d_1 \leq p_{11}, d_1 < \alpha). \end{aligned}$$

Differentiating this expression with respect to p_{11}, we have, since d_1 and c_1 are independent and the joint density function is simply the product of the independent densities,

$$\frac{d\pi_b^s(p_{11})}{dp_{11}} = -1 + \int_{-\infty}^{\alpha} (\mu_d(x) + k - x) f(p_{11} - x) g(x) dx.$$

Since all costs have finite positive means, it is true that $\lim_{x \to \pm\infty} f(x) = 0$ and $\lim_{x \to \pm\infty} g(x) =$

0. Thus, $\lim_{p_{11}\to\pm\infty}\frac{d\pi_b^s(p_{11})}{dp_{11}}=-1$. Define p_{11}^* to be the solution to $\left.\frac{d\pi_b^s(p_{11})}{dp_{11}}\right|_{p_{11}=p_{11}^*}=0$ (if such a solution exists). As long as the density f is unimodal, the second derivative

$$\frac{d^2\pi_b^s(p_{11})}{dp_{11}^2}=\int_{-\infty}^{\alpha}(\mu_d(x)+k-x)\,f'(p_{11}-x)g(x)dx$$

is positive then negative, i.e., the profit function is convex-concave. Combined with the fact that the slope tends to -1 as p_{11} becomes large or small, the profit function is either always decreasing or has a local unconstrained maximum; see Figure 6 in the main paper. As in the no failure case, the supplier earns zero profit in expectation in the second period, thus the feasible region is any $p_{11}\geq\mu_d+\mu_1$, i.e., any p_{11} for which the supplier's participation constraint binds. Thus, there are three possibilities: (1) $p_{11}^*<\mu_d+\mu_1$, in which case the optimal price is $\mu_d+\mu_1$; (2) $p_{11}^*\geq\mu_d+\mu_1$ and $\pi_b^s(p_{11}^*)\geq\pi_b(\mu_d+\mu_1)$, in which case the optimal solution is p_{11}^*; and (3) $p_{11}^*\geq\mu_d+\mu_1$ and $\pi_b^s(p_{11}^*)<\pi_b^s(\mu_d+\mu_1)$, in which case the optimal price is $\mu_d+\mu_1$. ■

Lemma 2 *Let p_{11}^l be the solution to*

$$\int_{-\infty}^{\infty}(\mu_d(x)+k-x)\,f(p_{11}^l-x)g(x)dx=0. \tag{4}$$

Then, the optimal long-term contract under the threat of default is p_{11}^l, $p_{12}^l=p_{12}\left(p_{11}^l\right)$, and $p_{22}^l=\mu_d+\mu_2(c_1)$.

Proof. We first show that at optimality, supplier 1's participation constraint is binding. First, note that as in the short-term contract $p_{22}^l=\mu_d(d_1)+\mu_2(c_1)$. Then the buyer's profit is

$$\begin{aligned}\pi_b^l(p_{11},p_{12}) &= 2s-p_{11}-\Pr(c_1+d_1\leq p_{11})\times p_{12}\\ &\quad -\Pr(c_1+d_1>p_{11})\times\mathbb{E}\left(\mu_d(d_1)+\mu_2(c_1)+k|c_1+d_1>p_{11}\right),\end{aligned}$$

and supplier 1 accepts any contract with

$$\mathbb{E}(p_{11}-c_1-d_1)+\Pr(c_1+d_1\leq p_{11})\times\mathbb{E}(p_{12}-c_2-d_1|c_1+d_1\leq p_{11})\geq 0.$$

Constructing the Lagrangean $\mathcal{L}$ with multiplier λ and taking the derivative with respect to p_{12}, we see $\frac{d\mathcal{L}}{dp_{12}}=(\lambda-1)\Pr(c_1+d_1\leq p_{11})$. This implies that at optimality we must have $\lambda=1$, hence the constraint is binding. Then we may rewrite π_b^l, substituting the binding participation constraint, to yield

$$\begin{aligned}\pi_b^l(p_{11}) &= 2s-\mathbb{E}(c_1+d_1)-\Pr(c_1+d_1\leq p_{11})\times\mathbb{E}(c_2+d_1|c_1+d_1\leq p_{11})\\ &\quad -\Pr(c_1+d_1>p_{11})\times\mathbb{E}\left(\mu_d(d_1)+\mu_2(c_1)+k|c_1+d_1>p_{11}\right).\end{aligned}$$

Since

$$\begin{aligned} & \Pr\left(c_1 + d_1 \le p_{11}\right) \times \mathbb{E}\left(c_2 + d_1 | c_1 + d_1 \le p_{11}\right) \\ = \; & \mu_d + \mu_2 - \Pr\left(c_1 + d_1 > p_{11}\right) \times \mathbb{E}\left(c_2 + d_1 | c_1 + d_1 > p_{11}\right), \end{aligned}$$

we have

$$\pi_b^l\left(p_{11}\right) = 2s - \mu_1 - \mu_2 - 2\mu_d + \Pr\left(c_1 + d_1 > p_{11}\right) \times \mathbb{E}\left(d_1 - \mu_d\left(d_1\right) - k | c_1 + d_1 > p_{11}\right).$$

Taking derivatives with respect to p_{11}, we get

$$\begin{aligned} \frac{d\pi_b^l\left(p_{11}\right)}{dp_{11}} &= \int_{-\infty}^{\infty} \left(\mu_d\left(x\right) + k - x\right) f(p_{11} - x) g(x) dx, \\ \frac{d^2\pi_b^l\left(p_{11}\right)}{dp_{11}^2} &= \int_{-\infty}^{\infty} \left(\mu_d\left(x\right) + k - x\right) f'(p_{11} - x) g(x) dx \end{aligned}$$

Note that $\lim_{p_{11}\to\pm\infty} \frac{d\pi_b^l(p_{11})}{dp_{11}} = 0$ by the assumption that $\lim_{x\to\pm\infty} f(x), g(x) = 0$. Also note that, since f is unimodal, $\frac{d^2\pi_b^l(p_{11})}{dp_{11}^2}$ is positive, then negative, then positive again, i.e., π_b is convex-concave-convex. Since $\lim_{p_{11}\to\pm\infty} \frac{d\pi_b^l(p_{11})}{dp_{11}} = 0$, the sign of $\frac{d\pi_b^l(p_{11})}{dp_{11}}$ is $+/-$, i.e. π_b is quasi-concave. Thus, the optimal first period price is the solution to (4). ■

Theorem 2 *In the presence of failure risk, (i) $\pi_b^s, \pi_b^l \le \overline{\pi}_b$ and (ii) there exists some k^* such that, for all $k > k^*$, $\pi_b^s \le \pi_b^l$.*

Proof. (i) In the short-term static contract, $p_{11} \ge \mu_d + \mu_1$, so the expected profit is bounded above by

$$\begin{aligned} \pi_b^s\left(p_{11}\right) \le \; & 2s - \mu_1 - \mu_2 - \mu_d - \Pr\left(c_1 + d_1 > p_{11}\right) \times \mathbb{E}\left(\mu_d\left(d_1\right) + k | c_1 + d_1 > p_{11}\right) \\ & - \Pr\left(c_1 + d_1 \le p_{11}\right) \times \mathbb{E}\left(\min\left(d_1, \mu_d\left(d_1\right) + k\right) | c_1 + d_1 \le p_{11}\right). \end{aligned}$$

Recall that the upper bound on profit $\overline{\pi}_b$ is given by (1), and since

$$\begin{aligned} \mathbb{E}\left(\min\left(d_1, \mu_d\left(d_1\right) + k\right)\right) \le \; & \Pr\left(c_1 + d_1 > p_{11}\right) \times \mathbb{E}\left(\mu_d\left(d_1\right) + k | c_1 + d_1 > p_{11}\right) \\ & + \Pr\left(c_1 + d_1 \le p_{11}\right) \times \mathbb{E}\left(\min\left(d_1, \mu_d\left(d_1\right) + k\right) | c_1 + d_1 \le p_{11}\right) \end{aligned}$$

we have $\pi_b^s\left(p_{11}\right) \le \overline{\pi}_b$ for all feasible p_{11}. The expected profit from the long-term static contract is, as a function of p_{11},

$$\begin{aligned} \pi_b^l\left(p_{11}\right) = \; & 2s - \mu_1 - \mu_2 - \mu_d - \Pr\left(d_1 + c_1 \le p_{11}\right) \times \mathbb{E}\left(d_1 | d_1 + c_1 \le p_{11}\right) \\ & - \Pr\left(d_1 + c_1 > p_{11}\right) \times \mathbb{E}\left(\mu_d\left(d_1\right) + k | d_1 + c_1 > p_{11}\right). \end{aligned}$$

Note that

$$\begin{aligned} \mathbb{E}\left(\min\left(d_1, \mu_d\left(d_1\right) + k\right)\right) \le \; & \Pr\left(c_1 + d_1 > p_{11}\right) \times \mathbb{E}\left(\mu_d\left(d_1\right) + k | d_1 + c_1 > p_{11}\right) \\ & + \Pr\left(c_1 + d_1 \le p_{11}\right) \times \mathbb{E}\left(d_1 | d_1 + c_1 \le p_{11}\right) \end{aligned}$$

so $\pi_b^l(p_{11}) \le \overline{\pi}_b$ for all p_{11}.

(ii) First, note that in the limit as $k \to \infty$, the buyer's expected profit in the optimal short-term contract satisfies

$$\begin{aligned}\lim_{k\to\infty} \pi_b^s &= \lim_{k\to\infty} 2s - p_{11}^s - \mu_2 - \Pr(c_1 + d_1 > p_{11}^s) \times \mathbb{E}(\mu_d(d_1) + k | c_1 + d_1 > p_{11}^s) \\ &\quad - \Pr(c_1 + d_1 \le p_{11}^s) \times \mathbb{E}(\min(d_1, \mu_d(d_1) + k) | c_1 + d_1 \le p_{11}^s).\end{aligned}$$

There are two possibilities: either (1) $\lim_{k\to\infty} p_{11}^s < \infty$, in which case $\lim_{k\to\infty} \pi_b^s = -\infty$ since the switching cost term dominates, or (2) $\lim_{k\to\infty} p_{11}^s = \infty$, in which case we also have $\lim_{k\to\infty} \pi_b^s(p_{11}) = -\infty$ since the p_{11}^s term dominates. In a long-term contract, on the other hand,

$$\begin{aligned}\lim_{k\to\infty} \pi_b^l &= \lim_{k\to\infty} 2s - \mu_1 - \mu_2 - \mu_d - \Pr\left(d_1 + c_1 \le p_{11}^l\right) \times \mathbb{E}\left(d_1 | d_1 + c_1 \le p_{11}^l\right) \\ &\quad - \Pr\left(d_1 + c_1 > p_{11}^l\right) \times \mathbb{E}\left(\mu_d(d_1) + k | d_1 + c_1 > p_{11}^l\right).\end{aligned}$$

Since p_{11}^l satisfies

$$\int_{-\infty}^{\infty} (\mu_d(x) + k - x) f(p_{11}^l - x) g(x) dx = 0,$$

it is easy to see that $\lim_{k\to\infty} p_{11}^l = \infty$, and hence $\lim_{k\to\infty} \pi_b^l = 2s - \mu_1 - \mu_2 - 2\mu_d$, i.e., the contract is equivalent to a long-term contract without failure (since the buyer pays such a high price that switching never occurs). In other words,

$$\lim_{k\to\infty} \pi_b^s = -\infty \text{ and } \lim_{k\to\infty} \pi_b^l = 2s - \mu_1 - \mu_2 - 2\mu_d > -\infty$$

hence for very large switching costs, the long-term contract is preferred. Thus, either the long-term contract is preferred for all k, or there must be some k^* for which $\pi_b^s < \pi_b^l$ for all $k > k^*$. ■

Lemma 3 *(i) The optimal short-term dynamic contract is* $p_{11}^s = \mu_d + c_1$, $p_{12}^s = d_1 + c_2$, *and* $p_{22}^s = \mu_d(d_1) + c_2$. *The buyer switches suppliers in the second period if* $k + \mu_d(d_1) < d_1$.

(ii) The optimal long-term dynamic contract is any pair $\{p_{11}^l, p_{12}^l\}$ *such that* $p_{11}^l + p_{12}^l = 2\mu_d + \mu_1 + \mu_2$.

(iii) The expected profit in each dynamic contract is equal to the expected profit in their static counterparts in Theorem 1.

Proof. Omitted; similar to Theorem 1. ■

Lemma 4 *(i) The optimal long-term dynamic contract is* $p_{11}^l = \alpha + c_1$, $p_{12}^l = p_{12}\left(p_{11}^l\right)$, *and* $p_{22}^l = \mu_d(d_1) + c_2$, *where* $p_{12}(p_{11}^l)$ *is the dynamic second period price for which the supplier's participation constraint binds.*

(ii) The optimal short-term dynamic contract is given by $p_{11}^s = c_1 + \max(x^*, \mu_d)$, $p_{12}^s = d_1 + c_2$, *and* $p_{22}^s = \mu_d(d_1) + c_2$, *where* x^* *is the solution to*

$$-1 + g(x^*)(-x^* + \mu_d(x^*) + k) = 0.$$

(iii) The long-term dynamic contract is preferred to the short-term dynamic contract, and yields expected profit equal to the system optimal expected profit without failure risk.

Proof. (i) With a dynamic long-term contract, the buyer offers $p_{11} = c_1 + \delta_1$, $p_{12} = c_2 + \delta_2$, and $p_{22} = c_2 + \delta_3$, optimizing over δ_i, $i = 1, 2, 3$. Failure occurs if $\delta_1 < d_1$. For the same reasons as in the previous lemmas, $p^l_{22} = c_2 + \mu_d(d_1)$. Thus, the buyer's profit is

$$\pi_b = 2s - \mu_1 - \mu_2 - \delta_1 - \Pr(\delta_1 \geq d_1) \times p_{12} - \Pr(\delta_1 < d_1) \times (\mu_d(d_1) + k),$$

maximized subject to

$$\delta_1 - \mu_d + \Pr(\delta_1 \geq d_1) \times \mathbb{E}(\delta_2 - d_1 | \delta_1 \geq d_1) \geq 0. \tag{5}$$

As in Lemma 4, one can show that the constraint is binding at any optimal solution by constructing the Lagrangean. Thus, by substituting a binding constraint into the objective function, we eliminate δ_2 and write the buyer's profit as

$$\pi_b = 2s - \mu_1 - \mu_2 - \mu_d - \int_{-\infty}^{\delta_1} xg(x)dx - \int_{\delta_1}^{\infty} (\mu_d(x) + k)\, g(x)dx.$$

Optimizing over δ_1, we have

$$\frac{d\pi_b}{d\delta_1} = g(\delta_1)\left(-\delta_1 + \mu_d(\delta_1) + k\right)$$

Since $g(x) > 0$ for all feasible x, π_b is quasiconcave, and is maximized by $\delta_1 = \mu_d(\delta_1) + k$, or $\delta_1 = \alpha$. Letting $\delta_2(\delta_1)$ be the value of δ_2 such that (5) holds, we have $p^l_{11} = c_1 + \alpha$, $p^l_{12} = c_2 + \delta_2(\delta_1)$, and $p^l_{22} = c_2 + \mu_d(d_1)$. Defining $p_{12}(p_{11}) = c_2 + \delta_2(\delta_1)$ yields the result.

(ii) In a dynamic short term contract, the buyer offers $p_{11} = c_1 + \delta_1$, $p_{12} = c_2 + \delta_2$, and $p_{22} = c_2 + \delta_3$, optimizing over the various δ. Failure occurs if $\delta_1 < d_1$. For the same reasons as in the previous lemmas, $\delta_3 = \mu_d(d_1)$ and $\delta_2 = d_1$, and the buyer switches suppliers if $d_1 > \mu_d(d_1) + k$. This implies

$$\begin{aligned}
\pi_b &= 2s - \mu_1 - \mu_2 - \delta_1 - \Pr(\delta_1 \geq d_1, \mu_d(d_1) + k \geq d_1) \times \mathbb{E}(d_1 | \delta_1 \geq d_1, \mu_d(d_1) + k \geq d_1) \\
&\quad - \Pr(\delta_1 < d_1) \times \mathbb{E}(\mu_d(d_1) + k | \delta_1 < d_1) \\
&\quad - \Pr(\mu_d(d_1) + k < d_1) \times \mathbb{E}(\mu_d(d_1) + k | \mu_d(d_1) + k < d_1).
\end{aligned}$$

Supplier 1 accepts any contract with $\delta_1 \geq \mu_d$. Clearly there are two cases. In the first, $\delta_1 > \alpha$, so

$$\begin{aligned}
\pi_b &= 2s - \mu_1 - \mu_2 - \delta_1 - \Pr(\alpha \geq d_1) \times \mathbb{E}(d_1 | \mu_d + k \geq d_1) \\
&\quad - \Pr(\alpha < d_1) \times \mathbb{E}(\mu_d(d_1) + k | \alpha < d_1).
\end{aligned}$$

The derivative is thus $\frac{d\pi_b}{d\delta_1} = -1$ and the optimal solution is $\delta_1 = \alpha$. In the second case,

$\delta_1 \le \alpha$, so

$$\pi_b = 2s - \mu_1 - \mu_2 - \delta_1 - \int_{-\infty}^{\delta_1} xg(x)dx - \int_{\delta_1}^{\infty} \left(\mu_d\left(x\right) + k\right) g(x)dx$$

and $\frac{d\pi_b}{d\delta_1} = -1 + g(\delta_1)\left(-\delta_1 + \mu_d\left(\delta_1\right) + k\right)$. Since $g(x) > 0$ for all x, the second term in this expression is negative for $\delta_1 \ge \alpha$, then positive for $\delta_1 < \alpha$, thus $\frac{d\pi_b}{d\delta_1}$ is either always negative (resulting in the optimal solution being boundary, $\delta_1 = \mu_d$) or has a unique zero in the interval $\mu_d \le \delta_1 < \alpha$.

(iii) The profit from the short-term contract is

$$\begin{aligned}\pi_b &= 2s - \mu_1 - \mu_2 - \delta_1 - \int_{-\infty}^{\delta_1} xg(x)dx - \int_{\delta_1}^{\infty} \left(\mu_d\left(x\right) + k\right) g(x)dx \\ &\le 2s - \mu_1 - \mu_2 - \mu_d - \int_{-\infty}^{\delta_1} xg(x)dx - \int_{\delta_1}^{\infty} \left(\mu_d\left(x\right) + k\right) g(x)dx,\end{aligned}$$

since $\mu_d \le \delta_1 < \alpha$. Comparing this to the long-term contract's profit of $\overline{\pi}_b$, we see that the difference between these two is

$$\overline{\pi}_b - \pi_b \ge \Pr\left(\delta_1 \le d_1 < \alpha\right) \times \mathbb{E}\left(\mu_d\left(d_1\right) + k - d_1 | \left(\delta_1 \le d_1 < \alpha\right)\right) > 0$$

hence the long-term dynamic contract is always preferred. ■

2. Contingent Transfer Payments and Loans

In what follows, we allow the buyer the option of making a transfer payment T to the supplier in the second period. The purpose of this transfer payment is to help support the supplier in the event that bankruptcy occurs at the end of the first period; that is, following a loss in the first period, if the buyer raises the total capital level of the supplier to zero or higher the supplier is spared from bankruptcy and may continue to do business with the buyer in the second period. See Babich (2006) for an analysis of why such threshold transfer payments may be optimal when suppliers face default risk.

We assume that some proportion $r \ge 0$ of the transfer payment T is repaid to the buyer at the end of the second period. If $r = 0$, then no amount is paid to the buyer; in this case, T is a direct operating subsidy. If $r > 0$, then T is a loan, some fraction of which is repaid to the buyer. If $r > 1$, then the buyer charges interest on the loan. For simplicity, we assume that r is exogenously determined (i.e., the firms do not bargain over the interest rate or repayment percentage of any transfer payment or loan). We also assume that the supplier only repays what he can based on profits in the second period.

It is easy to see that if the supplier does not enter bankruptcy, it is optimal to offer no transfer payment, $T = 0$. If the supplier does enter bankruptcy, then let $T(y) = -p_{11} + c_1 + d_1 + y$, where $y \ge 0$ is the excess cash that the buyer provides in the transfer payment that raises the supplier's capital level above zero. Let $R = \min\left(rT, \pi_{12}^+\right)$ be the repayment amount, where $\pi_{12}^+ = (p_{12} - c_2 - d_1 + y)^+$ is the positive part of the incumbent supplier's total profit at the end of the second period. Conditional on a transfer payment $T(y)$ having been made to a bankrupt supplier, the buyer's optimization problem in contracting with the incumbent supplier in the second period is

$$\max_{y,p_{12}} s - p_{12} + \mathbb{E}R$$
$$\text{s.t. } p_{12} - d_1 - \mathbb{E}(c_2 + R) + y \geq 0.$$

Differentiating the objective with respect to p_{12}, we see

$$\frac{d(s - p_{12} + \mathbb{E}R)}{dp_{12}} = -1 + \Pr(rT > p_{12} - c_2 - d_1 + y > 0) < 0,$$

i.e., profit is a decreasing function of price, so the participation constraint will be binding. By inserting the binding participation constraint into the objective function, we see that the buyer's expected profit from contracting with the incumbent supplier in the second period is, after subtracting the cost of the transfer payment,

$$s - d_1 - \mathbb{E}(c_2) + y - T.$$

The y term in this expression sums to zero with the y term in T, thus, in general, the optimal transfer payment satisfies $y = 0$, implying

$$T = (-p_{11} + c_1 + d_1)^+,$$

while the cost of contracting with a bankrupt supplier and providing an operating subsidy is $d_1 + \mu_2(c_1) + T$. Note that this expression is independent of r, i.e., it does not depend on whether or not the loan is repaid to the buyer. The reason for this is that the buyer extracts all of the supplier's surplus whether or not the loan is repaid, making the participation constraint binding; hence, if the supplier is to repay the buyer at the end of the second period, the buyer must compensate the supplier by paying a higher contract price. Since the buyer is already extracting all surplus from the supplier, he cannot extract further surplus, and his expected profit from contracting with the bankrupt supplier is thus independent of the magnitude of repayment. Thus, for the remainder of the analysis, we may ignore the precise value of r, and assume that the buyer pays a price of $p_{12} = d_1 + \mu_2(c_1) + T$ to subsidize and do business with a bankrupt supplier in period 2.

It may not be in the best interests of the buyer to support the bankrupt supplier. In particular, if the total expected procurement cost plus the necessary transfer payment exceed the total cost of switching to a new supplier, the buyer will choose the alternative supplier. In other words, if

$$d_1 + T \geq \mu_d(d_1) + k, \tag{6}$$

then it is optimal for the buyer to switch suppliers. We now define the following function:

$$\omega(p_{11}, c_1) \equiv \{x : 2x - \mu_d(x) = p_{11} - c_1 + k\},$$

which represents the value of the idiosyncratic cost d_1 such that (6) holds with equality for given values of p_{11}, c_1, and k. Since, by assumption, $2x - \mu_d(x)$ is monotonically increasing in x, there is a unique value of $\omega(p_{11}, c_1)$ for each p_{11} and c_1. Implicitly differentiating

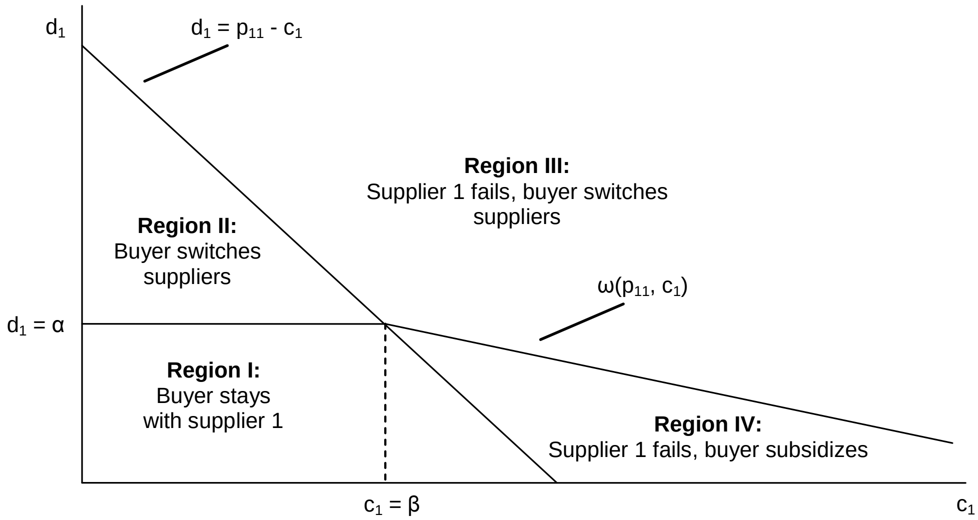

Figure 1. Optimal second period action of the buyer as a function of the realized values of c_1 and d_1 in the short-term contract with a contingent transfer payment in the second period.

$\omega(p_{11}, c_1)$ yields

$$\frac{d\omega(p_{11}, c_1)}{dc_1} = -\frac{d\omega(p_{11}, c_1)}{dp_{11}} = \frac{-1}{2 - \mu_d'(\omega(p_{11}, c_1))} \in [-1, -1/3] .$$

The curves dictated by $\omega(p_{11}, c_1)$ and $d_1 = p_{11} - c_1$ intersect when

$$d_1 - \mu_d(d_1) = k,$$

i.e., when $d_1 = \alpha$. Because the slope (as a function of c_1) of ω is always greater than the slope of $d_1 = p_{11} - c_1$, it must be true that there exists some β such that $\beta = p_{11} - \alpha$, and for all $c_1 > \beta$, $\omega(p_{11}, c_1) > p_{11} - c_1$, while for all $c_1 < \beta$, the opposite inequality holds. Figure 1 depicts this feature graphically in the case of short-term contracts.

Essentially, the addition of a transfer payment to the buyer's action space introduces Region IV to the diagram; there is now a region of the probability space for which it is optimal to subsidize a bankrupt supplier. In particular, the buyer chooses to support the distressed supplier if the idiosyncratic costs of the first supplier are small but the common cost component is high, as the figure shows. In this case, switching to the second supplier is more expensive than helping first supplier avoid bankruptcy.

The following proposition mirrors Lemma 1 in deriving the form of the optimal short-term contract.

Proposition 1 *With the option of contingent transfer payments, the buyer's optimal short-term contract consists of* $p_{11}^s = \mu_d + \mu_1$, $p_{12}^s = d_1 + \mu_2(c_1) + T$, *and* $p_{22}^s = \mu_d(d_1) + \mu_2(c_1)$. *The buyer switches suppliers in period 2 if* $d_1 \geq \min(\alpha, \omega(p_{11}, c_1))$.

Proof. It is easy to see that $p_{12}^s = d_1 + \mu_2(c_1) + T$ and $p_{22}^s = \mu_d(d_1) + \mu_2(c_1)$. Substituting these expressions into the buyer's profit function, we see that profit in the optimal short term contract as a function of p_{11} is

$$\pi_b^s(p_{11}) = 2s - \mu_2 - p_{11} - \mathbb{E}\min(d_1 + T, \mu_d(d_1) + k),$$

where $T = (c_1 - d_1 - p_{11})^+$. Differentiating this expression with respect to p_{11}, we have

$$\frac{d\pi_b^s(p_{11})}{dp_{11}} = -1 + \Pr(\text{Region IV}) \leq 0,$$

where $\Pr(\text{Region IV})$ denotes the probability of Region IV in Figure 1, i.e. the probability that $c_1 > \beta$ and $\omega(p_{11}, c_1) > d_1 > p_{11} - c_1$. Since profit is decreasing in p_{11}, it is optimal to set p_{11} as small as possible such that the supplier's participation constraint is satisfied, i.e., $p_{11}^s = \mu_d + \mu_1$. ■

The following proposition describes the optimal two-period contract with contingent transfer payments. In the analysis below, we assume that long-term contracts are renegotiated if bankruptcy occurs and the buyer subsidizes the bankrupt supplier. This is reflective of the fact that often, when working with bankrupt suppliers, buyers must pay court-ordered price increases rather than continue with the terms of existing (likely unprofitable) contracts (see, for example, the case of Collins & Aikman discussed in Barkholz and Sherefkin 2007). We note that our results continue to hold even if prices are not renegotiated (see §3 of this appendix, which can be shown to apply to the case of contingent transfer payments as well) although the first order condition changes slightly (in particular, the expression for $\omega(p_{11}, t)$ changes).

Proposition 2 *Let p_{11}^l be the solution to*

$$\begin{aligned} &\int_{p_{11}-\alpha}^{\infty} f(t)\, g(\omega(p_{11}, t)) \frac{\partial \omega(p_{11}, t)}{\partial p_{11}} (\omega(p_{11}, t) + t - p_{11})\, dt \qquad (7)\\ + &\int_{\alpha}^{\infty} (k + \mu_d(x) - x)\, f(p_{11} - x)\, g(x)\, dx = 0. \end{aligned}$$

Then, the optimal long-term contract under the threat of default is p_{11}^l, $p_{12}^l = p_{12}(p_{11}^l)$, and $p_{22}^l = \mu_d + \mu_2(c_1)$.

Proof. We first show that at optimality, supplier 1's participation constraint is binding. Note that as in the short-term contract, $p_{22}^l = \mu_d(d_1) + \mu_2(c_1)$, and furthermore due to the renegotiation of the long-term contract if failure occurs, the cost of contracting with a bankrupt supplier is $d_1 + \mu_2(c_1) + T$ (effectively, regardless of the amount of loan repayment). Then the buyer's profit is

$$\begin{aligned} \pi_b^l(p_{11}, p_{12}) &= 2s - p_{11} - \Pr(\text{Region I,II})\,\mathbb{E}(p_{12}|\text{Region I,II}) \\ &\quad - \Pr(\text{Region IV})\,\mathbb{E}(d_1 + \mu_2(c_1) + T|\text{Region IV}) \\ &\quad - \Pr(\text{Region III})\,\mathbb{E}(\mu_d(d_1) + \mu_2(c_1) + k|\text{Region III}), \end{aligned}$$

where $T = (-p_{11} + c_1 + d_1)^+$, and supplier 1 accepts any contract with

$$\begin{aligned} & \mathbb{E}\left(p_{11} - c_1 - d_1\right) + \Pr\left(\text{Region I,II}\right)\mathbb{E}\left(p_{12} - c_2 - d_1 | \text{Region I,II}\right) \\ + \quad & \Pr\left(\text{Region IV}\right)\mathbb{E}\left(T | \text{Region IV}\right) \geq 0. \end{aligned}$$

Note that since the participation constraint of the supplier now considers two periods, the transfer payment T (and potential repayment R) are taken into account. Constructing the Lagrangean $\mathcal{L}$ with multiplier λ and taking the derivative with respect to p_{12}, we see

$$\frac{d\mathcal{L}}{dp_{12}} = (\lambda - 1)\Pr\left(\text{Region I,II}\right).$$

This implies that at optimality we must have $\lambda = 1$, hence the constraint is binding. Then we may rewrite π_b, substituting the binding participation constraint, which yields

$$\begin{aligned} \pi_b^l\left(p_{11}\right) &= 2s - \mu_1 - \mu_2 - \mu_d - \Pr\left(\text{Regions I,II,IV}\right)\mathbb{E}\left(d_1 | \text{Regions I,II,IV}\right) \\ & \quad - \Pr\left(\text{Region III}\right)\mathbb{E}\left(\mu_d\left(d_1\right) + k | \text{Region III}\right). \end{aligned}$$

Differentiating with respect to p_{11}, we have

$$\begin{aligned} \frac{d\pi_b^l\left(p_{11}\right)}{dp_{11}} &= \int_{p_{11}-\alpha}^{\infty} f\left(t\right) g\left(\omega\left(p_{11}, t\right)\right) \frac{\partial \omega\left(p_{11}, t\right)}{\partial p_{11}} \left(\omega\left(p_{11}, t\right) + t - p_{11}\right)\, dt \\ & \quad + \int_{\alpha}^{\infty} \left(k + \mu_d\left(x\right) - x\right) f\left(p_{11} - x\right) g\left(x\right)\, dx \end{aligned}$$

The first term is positive (since $\frac{\partial \omega(p_{11},t)}{\partial p_{11}}$ is positive) while the second term is negative. Furthermore, note that $\lim_{p_{11}\to\pm\infty} \frac{d\pi_b^l(p_{11})}{dp_{11}} = 0$ by the assumption that $\lim_{x\to\pm\infty} f(x), g(x) = 0$, together with the fact that

$$\lim_{p_{11}\to\pm\infty} \omega\left(p_{11}, c_1\right) = \lim_{p_{11}\to\pm\infty} \left\{x : 2x - \mu_d(x) = p_{11} - c_1 + k\right\} = \pm\infty.$$

Differentiating once more, we have

$$\begin{aligned} \frac{d^2\pi_b^l\left(p_{11}\right)}{dp_{11}^2} &= \int_{p_{11}-\alpha}^{\infty} f\left(t\right) g\left(\omega\left(p_{11}, t\right)\right) \frac{\partial \omega\left(p_{11}, t\right)}{\partial p_{11}} \left(\frac{\partial \omega\left(p_{11}, t\right)}{\partial p_{11}} - 1\right) dt \\ & \quad + \int_{p_{11}-\alpha}^{\infty} f\left(t\right) g\left(\omega\left(p_{11}, t\right)\right) \frac{\partial^2 \omega\left(p_{11}, t\right)}{\partial p_{11}^2} \left(t - p_{11} + \omega\left(p_{11}, t\right)\right)\, dt \\ & \quad + \int_{p_{11}-\alpha}^{\infty} f\left(t\right) g'\left(\omega\left(p_{11}, t\right)\right) \frac{\partial \omega\left(p_{11}, t\right)}{\partial p_{11}}^2 \left(t - p_{11} + \omega\left(p_{11}, t\right)\right)\, dt \\ & \quad + \int_{\alpha}^{\infty} \left(\mu_d\left(x\right) + k - x\right) f'\left(p_{11} - x\right) g\left(x\right) dx. \end{aligned}$$

Note that in the limit as $p_{11} \to \infty$, the first three integrals converge to zero, and hence

$$\lim_{p_{11}\to\infty} \frac{d^2\pi_b^l(p_{11})}{dp_{11}^2} = \lim_{p_{11}\to\infty} \int_\alpha^\infty (\mu_d(x) + k - x) f'(p_{11} - x) g(x) dx \geq 0$$

Thus, as p_{11} becomes large, $\frac{d\pi_b^l(p_{11})}{dp_{11}}$ tends to be increasing towards zero, i.e., it is negative, and hence the optimal first period price solves the first order condition, $\frac{d\pi_b^l(p_{11})}{dp_{11}} = 0$. ■

Finally, Proposition 3 demonstrates that the primary result of the paper holds: namely, long-term contracts are preferred if switching costs are high, and neither contract type coordinates the system in general. This proposition is referenced in Theorem 3 of the paper.

Proposition 3 *In the presence of failure risk with contingent transfer payments or loans, (i)* $\pi_b^s, \pi_b^l \leq \overline{\pi}_b$ *and (ii) there exists some* k^* *such that, for all* $k > k^*$, $\pi_b^s \leq \pi_b^l$.

Proof. (i) In the case of the short-term contract, by substituting the binding participation constraint of supplier 1 into the buyer's profit function, we see that the buyer's profit under the optimal single period contract is

$$\pi_b^s(p_{11}) = 2s - \mu_1 - \mu_2 - \mu_d - \mathbb{E}\min(d_1 + T, \mu_d(d_1) + k),$$

Since $T \geq 0$, it is easy to see that

$$\pi_b^s \leq 2s - \mu_1 - \mu_2 - \mu_d - \mathbb{E}(\min(d_1, \mu_d(d_1) + k)) = \overline{\pi}_b.$$

The expected profit from the long-term static contract is

$$\begin{aligned} \pi_b^l &= 2s - \mu_1 - \mu_2 - \mu_d - \Pr(\text{Regions I,II,IV})\,\mathbb{E}(d_1|\text{Regions I,II,IV}) \\ &\quad - \Pr(\text{Region III})\,\mathbb{E}(\mu_d(d_1) + k|\text{Region III}). \end{aligned}$$

Again, it follows by definition that

$$\pi_b^l \leq 2s - \mu_1 - \mu_2 - \mu_d - \mathbb{E}(\min(d_1, \mu_d(d_1) + k)) = \overline{\pi}_b.$$

(ii) In the case of the short-term contract, as $k \to \infty$, we have

$$\begin{aligned} \lim_{k\to\infty} \pi_b^s &= \lim_{k\to\infty} 2s - \mu_1 - \mu_2 - \mu_d - \mathbb{E}\min(d_1 + T, \mu_d(d_1) + k) \\ &= \lim_{k\to\infty} 2s - \mu_1 - \mu_2 - 2\mu_d - \mathbb{E}\left((d_1 + c_1 - \mu_1 - \mu_d)^+\right). \end{aligned}$$

In the long-term contract, on the other hand, from (7) we see that $\lim_{k\to\infty} p_{11}^l = \infty$, implying $\lim_{k\to\infty} \Pr(\text{Regions III,IV}) = 0$, and hence

$$\lim_{k\to\infty} \pi_b^l = 2s - \mu_1 - \mu_2 - 2\mu_d.$$

Thus, for large k, the long-term contract is preferred to the short-term contract, and the proposition holds. ■

As discussed in the main text of the paper, the assumption that any transfer payment is made at the start of the second period is key. However, if this assumption is relaxed, the results of the above theorem continue hold if one of several alternative conditions are met. We refer the reader to §7.2 of the main text for a discussion of this point.

3. Renegotiation

If the buyer is capable of renegotiating the long-term contract, then he will do so whenever the renegotiated price is lower than the contract price. Assume that during the renegotiation process, the supplier will accept any contract with non-negative profit in period 2. In that case, the renegotiated price is $d_1 + \mu_2(c_1)$. The buyer then pays the supplier $\min(p_{12}, d_1 + \mu_2(c_1))$ in the second period, where p_{12} is the second period contract price.

The profit to the buyer is thus

$$\begin{aligned}\pi_b &= 2s - p_{11} - \Pr(p_{11} \geq c_1 + d_1) \times \mathbb{E}(\min(p_{12}, d_1 + \mu_2(c_1)) | p_{11} \geq c_1 + d_1) \\ &\quad - \Pr(p_{11} < c_1 + d_1) \times \mathbb{E}(\mu_d(d_1) + \mu_2(c_1) + k | p_{11} < c_1 + d_1).\end{aligned}$$

There are subsequently two cases: either the supplier recognizes that the buyer will renegotiate the contract (and hence takes this into account in the participation constraint), or the supplier does not recognize that renegotiation will occur. In the former case, supplier 1 accepts any contract with

$$\mathbb{E}(p_{11} - c_1 - d_1) + \Pr(p_{11} \geq c_1 + d_1)\,\mathbb{E}(\min(p_{12}, d_1 + \mu_2(c_1)) - c_2 - d_1 | p_{11} \geq c_1 + d_1) \geq 0.$$

As in the proof of Lemma 2, we may construct the Lagrangean $\mathcal{L}$ with multiplier λ and take the derivative with respect to p_{12} to see that $\frac{d\mathcal{L}}{dp_{12}} = (\lambda - 1)\Pr(p_{11} \geq c_1 + d_1, p_{12} \leq d_1 + \mu_2(c_1))$. This implies that at optimality it must be true that $\lambda = 1$, and the constraint is binding. We may then rewrite π_b, substituting the binding participation constraint, to yield

$$\pi_b^l(p_{11}) = 2s - \mu_1 - \mu_2 - 2\mu_d + \int_{-\infty}^{\infty}\int_{p_{11}-x}^{\infty} (x - \mu_d(x) - k) f(t) g(x) dt dx,$$

which is precisely the same expression as the model without buyer renegotiation.

On the other hand, if the supplier does not recognize that renegotiation will occur, then the buyer's profit is clearly higher than in the no-renegotiation case. Thus, the profitability of the long-term contracts is increased relative to the value of short-term contracts, and the results of the model continue to hold.

4. Normally Distributed Costs

Theorem 4. *(i) The optimal expected profit under all contract types is decreasing in ρ_d.*

(ii) The difference between the system optimal (long-term dynamic) profit and the profit under the long-term static contract is decreasing in ρ_d. In the limit as $\rho_d \to 1$, profits are equal.

(iii) The centralized system optimal expected profit is increasing in σ_d.

Proof. (i) Recall that, from the properties of the bivariate normal distribution, the expected value of d_2 conditional on $d_1 = x$ is

$$\mu_d(x) = (1-\rho_d)\,\mu_d + \rho_d x. \tag{8}$$

Using (8) and differentiating $\overline{\pi}_b$, we see that for the long-term dynamic contract,

$$\frac{d\overline{\pi}_b}{d\rho_d} = -\mu_d + \mathbb{E}\min\left(d_1, \mu_d + k/\left(1-\rho_d\right)\right) - \frac{k}{1-\rho_d}\overline{G}\left(\mu_d + k/\left(1-\rho_d\right)\right) < 0.$$

Now turning to the long-term static contract, by differentiating $\pi_b^l(p_{11})$ and using the Envelope Theorem,

$$\begin{aligned}\frac{d\pi_b^l}{d\rho_d} &= \frac{\partial \pi_b^l}{\partial \rho_d} = \int_{-\infty}^{\infty}\int_{p_{11}^l - x}^{\infty} \left(-x+\mu_d\right) f(t)g(x)dtdx \\ &= \mathbb{E}\left(-d_1 + \mu_d | d_1 > p_{11}^l - c_1\right)\Pr\left(d_1 > p_{11}^l - c_1\right) < 0.\end{aligned}$$

Finally, consider the short-term static contract. Applying the Envelope Theorem to $\pi_b^s\left(p_{11}\right)$,

$$\frac{d\pi_b^s}{d\rho_d} = \frac{\partial \pi_b^s}{\partial \rho_d} = \int_{\alpha}^{\infty}\left(\mu_d - x\right) g(x)dtdx + \int_{-\infty}^{\alpha}\int_{p_{11}^s - x}^{\infty}\left(\mu_d - x\right) f(t)g(x)dtdx \leq 0.$$

(ii) Consider the effect of taking the limit as $\rho_d \to 1$. Then, $\overline{\pi}_b = 2s - 2\mu_c - 2\mu_d$, and since $\lim_{\rho_d \to 1} p_{11}^* = \infty$, $\pi_b^l(p_{11}) = 2s - 2\mu_c - 2\mu_d$ as well, thus the contracts are equivalent. To prove the first part of the proposition, by examining $\frac{d\overline{\pi}_b}{d\rho_d}$ and $\frac{d\pi_b^l}{d\rho_d}$ and noting that in the long-term static contract $p_{11}^* = \mu_c + \mu_d + k/\left(1-\rho_d\right)$, it can be shown that $\frac{d^2\overline{\pi}_b}{d\rho_d^2} > 0$ and $\frac{d^2\pi_b^l}{d\rho_d^2} > 0$. Since $\overline{\pi}_b > \pi_b^l$ for $\rho_d < 1$, both functions are convex and decreasing, and they converge when $\rho_d = 1$, it must be true that $\frac{d\left(\overline{\pi}_b - \pi_b^l\right)}{d\rho_d} < 0$, i.e., the functions are smoothly converging to one another.

(iii) Using (8) and differentiating $\overline{\pi}_b$, we see that for the long-term dynamic contract,

$$\frac{d\overline{\pi}_b}{d\sigma_d} = -\frac{d}{d\sigma_d}\left(\mathbb{E}\min\left(d_1, \mu_d + k/\left(1-\rho_d\right)\right)\right).$$

This expression is merely the negative of a newsvendor expected sales function. It is well known that expected sales decrease as a function of standard deviation in a newsvendor with normally distributed demand, hence $\frac{d}{d\sigma_d}\left(\mathbb{E}\min\left(d_1, \mu_d + k/\left(1-\rho_d\right)\right)\right) < 0$ and thus $\frac{d\overline{\pi}_b}{d\sigma_d} > 0$.

5. Numerical Study

To study the magnitude of k^* (the switching cost above which long-term contracts are preferred), we calculated this critical switching cost both with and without transfer payments for every combination of the following parameters: $\mu_d = \{1,2,3\}$, $\mu_1 = \mu_2 = \{1,2,3\}$, $\sigma_d = \{1,3,6\}$, $\sigma_1 = \sigma_2 = \{1,3,6\}$, and $\rho_d = \{-0.5, 0, 0.5\}$, where costs are normally distributed and the common cost component is i.i.d. across time. The result is 243 distinct sets of parameters with coefficient of variation of idiosyncratic and common costs ranging from 0.333 to 6. The average total production cost in each period was 4. Our results are as follows:

Case	Average k^*	Average $k^*/(\mu_d + \mu_1)$	Median k^*
No Transfer Pmt	0.99	27.3%	0.81
Transfer Pmt	0.56	18.3%	0.00

In 42 of 243 cases, k^* increased as a result of allowing a transfer payment, while in the remainder, k^* decreased. Breaking down the results by coefficient of variation, we see:

CV of d_1	Avg $k^*/(\mu_d + \mu_1)$ with no Transfer	Avg $k^*/(\mu_d + \mu_1)$ with Transfer
0.3	13.8%	0.00%
0.5	17.5%	0.00%
1.0	23.0%	0.00%
1.5	27.8%	0.00%
2	25.6%	0.04%
3	35.4%	0.24%
6	44.8%	1.10%

In other words, k^* is smallest when there is low variability in the idiosyncratic cost term. This is intuitive, as the option to switch suppliers contains the most value (and hence short-term contracts contain the most value) when d_1 is highly variable. (Note that the behavior in the above table appears to be non-monotonic due to the fact that there are various parameter combinations–perhaps unequal in number–for each particular CV.)

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