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Technical Appendix

EC.1. Proofs

LEMMA 1. *In equilibrium, there exists some $v^* \in [\underline{v}, \overline{v}]$ such that all strategic consumers with second period value less than v^* purchase in the first period, and all consumers with value greater than v^* wait for the sale period. A consumer with value v^* is indifferent between purchasing in the first or second periods.*

Proof. The surplus to a strategic consumer who purchases in the first period is $v_M - p$, which is constant and independent of a consumer's second period valuation. In the second period, a strategic consumer only purchases the product if (1) the sale price is less than or equal to their second period valuation, and (2) there is inventory available to purchase. Let $\int_0^x h(s, \widehat{v}, \widehat{q})\, ds$ be a strategic consumer's belief of the probability that the sale price is less than or equal to x and the consumer receives a unit. Then second period expected surplus of a strategic consumer with period 2 valuation equal to v is

$$\psi(v, \widehat{v}, \widehat{q}) = \int_0^v (v - s)\, h(s, \widehat{v}, \widehat{q})\, ds.$$

Since $h(\cdot)$ is independent of v due to the rational expectations hypothesis, this expression is increasing in v, and hence there is a unique v^* for which $v_M - p = \psi(v^*, \widehat{v}, \widehat{q})$. All consumers with greater valuations prefer to wait for the sale, while all consumers with lower valuations prefer to purchase at the full price. □

LEMMA 2. *Define the critical demand levels $D_l = q/\left(\xi + s_m \overline{G}(s_m)\alpha/s_l\right)$, $D_m = q/\left(\xi + \overline{G}(s_m)\alpha\right)$, and $D_h = q/\xi$, where l, m, and h stand for low, medium, and high, respectively. Then given a demand level D, there is a unique optimal sale price determined by*

$$s^*(D) = \begin{cases} s_h(D) & \text{if } D_m < D \le D_h \\ s_m & \text{if } D_l < D \le D_m \\ s_l & \text{if } D \le D_l \end{cases},$$

where $s_l = v_B$ is the low sale price, $s_m = \arg\max_{s \ge \widehat{v}} s(\overline{v} - s)$ is the medium sale price, and $s_h(D) = (\overline{v} - \underline{v})(D - q)/\alpha D + \widehat{v}$ is the high sale price, which is contingent on the demand realization and remaining inventory.

Proof. First, we note that in order for the retailer to have inventory to sell in the second period, we require $D \le D_h$. The retailer then has two choices:

(i) *Pricing to serve only strategic consumers ($s > v_B$).* Any price in the range $\widehat{v} > s > v_B$ is never optimal ($s = \widehat{v}$ always yields greater profit). The optimal price conditional on $s \ge \widehat{v}$ is the solution to

$$\arg\max_{s \ge \widehat{v}} \left(s \min\left(\overline{G}(s)\alpha D, I\right)\right).$$

If $D \le q$, then the retailer is demand constrained even if he serves all strategic consumers. That is, if $D \le q$, then $\min\left(\overline{G}(s)\alpha D, I\right) = \overline{G}(s)\alpha D$ for all $s \ge \widehat{v}$. The retailer's optimization problem then becomes

$$s_m = \arg\max_{s \ge \widehat{v}} \left(s \frac{\overline{v} - s}{\overline{v} - \underline{v}} \alpha D\right).$$

Since $s(\overline{v} - s)$ is concave, there may be an interior optimum determined by the solution to the first order condition, which yields $s^* = \overline{v}/2$, if $s^* \ge \widehat{v}$; otherwise, the optimal price is on the boundary. Note that the optimal price is independent of D and α, but does depend on $\widehat{v}$ and $\overline{v}$. The optimal profit in this region is $s_m \frac{\overline{v} - s_m}{\overline{v} - \underline{v}} \alpha D$.

Now consider the case in which $q < D \le D_h$. In this region, if the retailer sets a low sale price, he is inventory constrained, whereas if he sets a high sale price, he is demand constrained. For any demand level D, there exists some critical price $s_h(D)$, such that the retailer's revenue function is

$$R(s,I) = \begin{cases} sI & \text{if } s \le s_h(D) \\ s\overline{G}(s)\alpha D & \text{otherwise} \end{cases}.$$

In particular, $s_h(D)$ is determined by solving $I = \overline{G}(s)\alpha D$ for s, which yields

$$s_h(D) = (\overline{v} - \underline{v})\frac{D-q}{\alpha D} + \widehat{v}.$$

Recall that s_m is the maximizer of $s\overline{G}(s)\alpha D$. Because $s\overline{G}(s)$ is concave, if $s_h(D) \le s_m$, the optimal sale price is s_m, whereas if $s_h(D) > s_m$, the optimal sale price is $s_h(D)$. Thus, there exists some critical demand level D_m such that for $D < D_m$, it is optimal to price at s_m, and for $D > D_m$, it is optimal to price at $s_h(D)$. D_m is determined by solving $s_h(D) = s_m$ for D, which yields

$$D_m = \frac{q}{\xi + \overline{G}(s_m)\alpha}.$$

(ii) *Pricing to serve the bargain hunting segment ($s = v_B$).* If the retailer sets $s = s_l$, second period revenue is $s_l I$. This yields a greater profit than pricing at s_m if and only if

$$D \le \frac{s_l q}{s_m\overline{G}(s_m)\alpha + s_l\xi} \equiv D_l.$$

Since s_m maximizes $s(\overline{v} - s)$ in the interval $\overline{v} \ge s \ge \widehat{v} \ge s_l$,

$$s_l\overline{G}(\widehat{v}) \le \widehat{v}\overline{G}(\widehat{v}) \le s_m\overline{G}(s_m),$$

which implies $D_l \le q$. Thus, if demand is less than D_l, it is optimal to price low to clear all inventory $(s = s_l)$ and serve the bargain hunters. □

LEMMA 3. *The retailer's profit $\pi(q,\widehat{v})$ is quasi-concave in q, and the optimal order quantity is determined by the unique solution to the first order condition,*

$$\frac{d\pi(q,\widehat{v})}{dq} = p - c - pF(D_h) + s_l F(D_l) + \int_{D_m}^{D_h} (2s_h(x) - \overline{v})\,dF(x) = 0. \tag{EC.1}$$

Proof. The retailer's expected profit under the optimal salvage pricing policy is

$$\pi(q,\widehat{v}) = p\int_0^{D_h} \xi x\,dF(x) + p\int_{D_h}^{\infty} q\,dF(x) - cq + s_l\int_0^{D_l}(q - \xi x)\,dF(x)$$
$$+ s_m\int_{D_l}^{D_m}\overline{G}(s_m)\alpha x\,dF(x) + \int_{D_m}^{D_h} s_h(x)(q - \xi x)\,dF(x).$$

Differentiation of this expression yields

$$\frac{d\pi(q,\widehat{v})}{dq} = p - c - pF(D_h) + s_l F(D_l) + \int_{D_m}^{D_h}\left(s_h(x) + \frac{ds_h(x)}{dq}(q - \xi x)\right)dF(x).$$

Taking the derivative of $s_h(x)$ with respect to q, we have $ds_h(x)/dq = -(\overline{v} - \underline{v})/\alpha x$. Then, the first derivative reduces to (EC.1). Let $\varphi(q) = d\pi(q,\widehat{v})/dq$. Noting $\varphi(0) = p - c > 0$ and $\lim_{q\to\infty}\varphi(q) = -c + s_l < 0$, it is apparent that $\pi(q,\widehat{v})$ possesses at least one local maximum. To demonstrate quasi-concavity of $\pi(q,\widehat{v})$, we must show that $\varphi(q)$ has a unique zero, i.e., that $\pi(q,\widehat{v})$ possesses a single local optimum. Given the asymptotic behavior of $\varphi(q)$, a sufficient condition for this to occur is that $\varphi(q)$ itself possesses at most one local optimum. If this is the case, then $\varphi(q)$ is either quasi-concave or quasi-convex, and $\varphi'(q)$ will possess at most one interior zero. Substituting for $s_h(x)$, $\varphi'(q)=d^2\pi(q,\widehat{v})/dq^2$ is given by

$$\varphi'(q)=(\overline{v}-p)\,f(D_h)\frac{dD_h}{dq}+s_l f(D_l)\frac{dD_l}{dq}-(2s_m-\overline{v})\,f(D_m)\frac{dD_m}{dq}$$
$$-2(\overline{v}-\underline{v})\int_{D_m}^{D_h}\frac{1}{\alpha x}dF(x),$$

A local optimum is achieved ($\varphi'(q)=0$) if and only if, for any q on the interior of the support of f,

$$0=(\overline{v}-p)\frac{f(D_h)}{f(D_m)}\frac{dD_h}{dq}+s_l\frac{f(D_l)}{f(D_m)}\frac{dD_l}{dq}-(2s_m-\overline{v})\frac{dD_m}{dq}$$
$$-2(\overline{v}-\underline{v})\frac{1}{f(D_m)}\int_{D_m}^{D_h}\frac{1}{\alpha x}dF(x). \tag{EC.2}$$

Recall that the MSLR assumption implies $f(\lambda x)/f(x)$ is monotonic in x for all $\lambda\le 1$. Assume that $f(\lambda x)/f(x)$ is weakly increasing in x. (The proof is identical if $f(\lambda x)/f(x)$ is weakly decreasing in x.) Since $\overline{v}\le p$, the first term is negative and increasing in q by MSLR assumption. Similarly, the second term is positive and increasing in q by the MSLR assumption. The third term is constant, while the fourth term is negative. We will now demonstrate that the fourth term is also increasing in q by performing a change of variable. Let $yq=x$, such that $dx=qdy$, and let $\lambda_h=dD_h/dq$ and $\lambda_m=dD_m/dq$. Then, the integral in the fourth term is equivalent to

$$\int_{D_m}^{D_h}\frac{f(x)}{xf(D_m)}dx=\int_{\lambda_m}^{\lambda_h}\frac{f(yq)}{yf(\lambda_m q)}dy.$$

Differentiating with respect to q,

$$\frac{d}{dq}\left(\int_{\lambda_m}^{\lambda_h}\frac{f(yq)}{yf(\lambda_m q)}dy\right)=\int_{\lambda_m}^{\lambda_h}\frac{dy}{y}\left(\frac{d}{dq}\frac{f(yq)}{f(\lambda_m q)}\right)\le 0,$$

where the inequality follows from the MSLR assumption combined with the fact that $y\ge\lambda_m$. Thus it follows that the fourth term in (EC.2) is increasing in q. Each term on the right hand side of (EC.2) is increasing in q, and if a solution to the equation exists, it is unique. This implies $\varphi(q)$ has at most one interior optimum, and consequently $\pi(q,\widehat{v})$ is quasi-concave in q. □

LEMMA 4. *(i) Define $\theta_c=s_l/s_m$ and let $D_\theta=\theta q/(1-\xi+\theta\xi)$. The probability an indifferent strategic consumer purchases and receives a unit in the sale period is*

$$\begin{cases} F(D_l) & \text{if } \theta_c\le\theta, \\ F(D_\theta)+\int_{D_\theta}^{D_l}\frac{\theta I}{(1-\xi)x}dF(x) & \text{otherwise.}\end{cases}$$

(ii) The consumer best response $v^*(\widehat{q})$ satisfies $\lim_{\widehat{q}\to 0}v^*(\widehat{q})=\overline{v}$ and $\lim_{\widehat{q}\to\infty}v^*(\widehat{q})=\max\{\underline{v},v_M-p+v_B\}$.

Proof. (i) The probability that $D<D_l$ and a strategic consumer receives a unit is

$$\int_0^{D_l}\frac{\min((1-\xi)x,\theta I)}{(1-\xi)x}dF(x).$$

A new critical demand level D_θ is determined by the $\min((1-\xi)D, \theta I)$ term,

$$(1-\xi)D_\theta = \theta(q - \xi D_\theta).$$

If $D < D_\theta$, all strategic consumers are served in the sale period. In particular, if $D_l \le D_\theta$, then the sale price is only low when all strategic consumers are served. By comparing D_l and D_θ, we see that this occurs when $s_l/s_m \le \theta$.

(ii) Note that $\lim_{\widehat{q}\to 0} D_l = \lim_{\widehat{q}\to 0} D_\theta = 0$. Thus, the probability term in (3) goes to zero as $\widehat{q}$ approaches zero. Consequently, any strategic consumers purchasing in the second period will receive zero surplus (in expectation), while purchase in the first period will yield a strictly positive surplus. Hence, all consumers purchase in the first period, and $\lim_{\widehat{q}\to 0} v^*(\widehat{q}) = \overline{v}$. Similarly, $\lim_{\widehat{q}\to\infty} D_l = \lim_{\widehat{q}\to\infty} D_\theta = \infty$, which implies the probability term in (3) goes to one as $\widehat{q}$ approaches infinity. This implies all strategic consumers purchase the product at the lowest sale price in period 2, and hence, since $v_M - p < \underline{v} - v_B$ by assumption, there are no indifferent consumers and all strategic type consumers wait for the sale. □

LEMMA 5. *Assume the retailer has quick response capabilities. (i) Let* $s_r = \arg\max_{s\ge\widehat{v}}(s - c_2)\overline{G}(s)$ *and let* $D_r = q/\left(\xi + \overline{G}(s_r)\alpha\right)$. *Then, if* $c_2 \le \overline{v}$, *given a demand level* D, *there is a unique optimal sale price determined by*

$$s^* = \begin{cases} s_r & \text{if } D_r < D \\ s_h(D) & \text{if } D_m < D \le D_r \\ s_m & \text{if } D_l < D \le D_m \\ s_l & \text{if } D \le D_l \end{cases}.$$

where D_l, D_m, s_l, s_m, *and* $s_h(D)$ *are as in Lemma 2, and* $D_r \le D_h$ *from Lemma 2. If* $c_2 > \overline{v}$, *then reactive capacity is never used to satisfy sale period demand, and the optimal sale price is identical to that derived in Lemma 2.*

(ii) The retailer's profit with quick response, $\pi_r(q,\widehat{v})$, *is quasi-concave in* q.

Proof. (i) We first note that the retailer may effectively make the sale price and second procurement decisions simultaneously (although the sale price is not enacted until the start of the second period). The retailer will always procure at least enough inventory to fulfill all first period demand. Let q_2 be the additional inventory procured *above* the total first period demand. Note that if $c_2 > \overline{v}$, then $q_2 = 0$ and the retailer's sale price decision is identical to the model without quick response; hence we need only analyze the case where $c_2 \le \overline{v}$. Then the retailer's profit function is

$$\pi_r(q,\widehat{v}) = \mathbb{E}\left[p\xi D - c_2(\xi D - q)^+ - c_1 q + \max_{s\le p, q_2\ge 0} R(s, I, q_2)\right],$$

where $I = (q - \xi D)^+$. There are consequently two cases: if $I = 0$, then the initial inventory procurement is insufficient to fill any demand in the sale period. Hence, the retailer will likely wish to procure additional inventory specifically for sale in the salvage period. Alternatively, if $I > 0$, then some inventory from the initial order remains for the sale period. We will treat each case separately.

(1) $I = 0$. Any unit sold must be procured through quick response, thus $q_2 = \overline{G}(s)\alpha D$. Thus, the retailer's margin on each unit sold is $(s - c_2)$, and second period revenue as a function solely of s is $R(s) = (s - c_2)\overline{G}(s)\alpha D$, for $s \ge \widehat{v}$. The optimal sale price is thus $s_r = \arg\max_{s\in[\widehat{v},\overline{v}]}(s - c_2)\overline{G}(s)$, and is equal to $(\overline{v} + c_2)/2$ if this value is interior to the interval $[\widehat{v}, \overline{v}]$.

(2) $I > 0$. In this case, all first period demand was satisfied without the need to procure additional inventory, and the retailer will have positive on-hand inventory at the start of the second period even if no replenishment is made. The second period revenue is

$$R(s, I, q_2) = \begin{cases} s\min\left(\overline{G}(s)\alpha D, I + q_2\right) - c_2 q_2 & \text{if } s \ge \widehat{v} \\ sI & \text{if } s \le v_B \end{cases}.$$

If $D < q$, then the retailer is never inventory constrained in the second period, hence the revenue function is a identical to that derived in Theorem 1, and the optimal pricing scheme is also identical. On the other hand, if $D > q$, then demand exceeds the total supply, and the retailer may wish to procure additional units. Pricing to serve the bargain hunting segment is never optimal, and $q_2 = \left(\overline{G}(s)\alpha D - I\right)^+$, hence the second period revenue as a function of s is

$$R(s,I) = s\overline{G}(s)\alpha D - c_2\left(\overline{G}(s)\alpha D - I\right)^+. \tag{EC.3}$$

Note that additional inventory is required ($\left(\overline{G}(s)\alpha D - I\right)^+ > 0$) if

$$s \le (\overline{v} - \underline{v})\frac{D-q}{\alpha D} + \widehat{v} = s_h(D).$$

Thus, (EC.3) is equivalent to

$$R(s,I) = \begin{cases} s\overline{G}(s)\alpha D & \text{if } \overline{v} \ge s \ge s_h(D) \\ (s-c_2)\overline{G}(s)\alpha D + c_2 I & \text{if } s_h(D) > s \ge \widehat{v} \end{cases}.$$

This expression is a piecewise definition of two constrained concave functions. The unconstrained maximizers of these two functions are $s_m = \max(\overline{v}/2, \widehat{v})$ and s_r (defined above), respectively. This implies that if $s_m \ge s_h(D)$, the optimal sale price is s_m (just as in Theorem 1). If $s_m < s_h(D)$, the optimal sale price is $\min(s_r, s_h(D))$. Thus, by finding the demand value for which $s_r = s_h(D)$, we may find D_r, and the result follows.

(ii) The retailer's expected profit under the optimal salvage pricing policy is

$$\begin{aligned}\pi_r(q,\widehat{v}) = {} & p\xi\mu + c_2\int_{D_r}^{\infty}(q-\xi x)\,dF(x) - c_1 q + s_l\int_0^{D_l}(q-\xi x)\,dF(x) \\ & + s_m\int_{D_l}^{D_m}\overline{G}(s_m)\alpha x dF(x) + \int_{D_m}^{D_r} s_h(x)(q-\xi x)\,dF(x) + \int_{D_r}^{\infty}(s_r - c_2)\overline{G}(s_r)\alpha x dF(x).\end{aligned}$$

Differentiation of this expression yields

$$\frac{d\pi_r(q,\widehat{v})}{dq} = c_2 - c_1 - c_2 F(D_h^r) + s_l F(D_l) + \int_{D_m}^{D_r}(2s_h(x) - \overline{v})\,dF(x) = 0. \tag{EC.4}$$

Let $\varphi_r(q) = d\pi_r(q,\widehat{v})/dq$. Noting $\varphi_r(0) = c_2 - c_1 > 0$ and $\lim_{q\to\infty}\varphi_r(q) = -c_1 + s_l < 0$, it is apparent that $\pi_r(q,\widehat{v})$ possesses at least one local maximum. In the same manner as Theorem 2, by differentiating (EC.4) we may show that there is a unique solution to $d\pi_r(q,\widehat{v})/dq = 0$ and hence the retailer's profit function is quasi-concave in q. Noting that (EC.4) is independent of p yields the result. □

EC.2. Monotone Scaled Likelihood Ratio

Definition 2. A continuous, non-negative random variable X with density f satisfies the **monotone scaled likelihood ratio** (MSLR) property if, for all $\lambda \le 1$ and x in the support of X, $f(\lambda x)/f(x)$ is monotonic in x.

Note that the property implies the following: $f(bx)/f(ax)$ is monotonic in x, for all $a \geq b \geq 0$. The following table lists several non-negative distributions that satisfy this property. We use the notation and conventions of Bagnoli and Bergstrom (2005), Tables 1 and 2.

Name	**Support**	**Density** $f(x)$	**Sign of** $\frac{d(f(\lambda x)/f(x))}{dx}$
Uniform	$[0,1]$	1	0
Exponential	$(0,\infty)$	$\beta e^{-\beta x}$	$+$
Power (all c)	$(0,1]$	cx^{c-1}	0
Weibull (all c)	$[0,\infty)$	$cx^{c-1}e^{-x^c}$	$+$
Gamma (all c)	$[0,\infty)$	$\frac{x^{c-1}e^{-x}}{\Gamma(c)}$	$+$
Chi-Squared (all c)	$[0,\infty)$	$\frac{x^{(c-2)/2}e^{-x/2}}{2^{c/2}\Gamma(c/2)}$	$+$
Chi (all c)	$[0,\infty)$	$\frac{x^{(c-1)/2}e^{-x^2/2}}{2^{(c-2)/2}\Gamma(c)}$	$+$
Beta ($\omega \geq 1$)	$[0,1]$	$\frac{x^{\nu-1}(1-x)^{\omega-1}}{B(\nu,\omega)}$	$+$
Beta ($\omega \leq 1$)	$[0,1]$	$\frac{x^{\nu-1}(1-x)^{\omega-1}}{B(\nu,\omega)}$	$-$

While many of the above distributions are log-concave, it is not true that the MSLR property is equivalent to log-concavity. For example, a normal distribution with a positive mean truncated to the non-negative half-space is log-concave, but does not exhibit the MSLR property over the entire support. In addition, the MSLR property is satisfied by many distributions without log-concave densities, such as the power, Weibull, gamma, chi, and chi-squared distributions for $c < 1$, and the beta distribution with $\omega < 1$. In general, if the distribution in question can be characterized by scale and location parameters and satisfies monotone likelihood ratio (MLR) property (see Karlin and Rubin 1956), then the distribution satisfies the MSLR property.

EC.3. Extension: Unknown Future Values

This section considers our model with one modification: now the strategics do not know their period 2 value for the product when they must make their buy/wait decision in period 1. The strategics do learn their value for the product at the start of period 2 but the firm remains unaware of their valuation when the period 2 pricing decision is made. Our objective is to demonstrate that quick response can be more valuable in this model when there are strategic consumers relative to the case in which there are only myopic consumers.

The notation with this model mimics our original notation, with some modification to account for the unknown period 2 valuation: D = period 1 demand; α = fraction of period 1 demand that is strategic; v_M = value of strategics and myopics in period 1; $v_2 \sim U[\underline{v}, \overline{v}]$ = value of strategics in period 2; v_B = value of bargain hunters in period 2; q = period 1 order quantity; γ = fraction of strategics that purchase in period 1; ξ = fraction of period 1 demand that purchases in period 1 = $(1-\alpha+\gamma\alpha)$; I = inventory in period 2. We assume

$$v_B \leq \underline{v} < \overline{v} \leq v_M$$
$$\underline{v} < \frac{1}{2}\overline{v} < p$$

Because $\underline{v} < \overline{v}/2$, it may not be profitable to try to sell to all of the strategics. The analysis with $\underline{v} < \overline{v}/2$ is similar, but omitted for expositional brevity. Because $\overline{v}/2 < p$, the firm never wants to markup inventory in period 2 .

EC.3.1. Period 2 pricing

Let s be the period 2 price. The firm can either take a deep discount to clear all inventory, $s = v_B$, or the firm can take a modest discount to sell to the strategics, $s \in [\underline{v}, \overline{v}]$. Define

$$R_2(s, D_2, I) = \begin{cases} s\left(\frac{\overline{v}-s}{\overline{v}-\underline{v}}\right)\min\{D_2, I\} & \underline{v} \leq s \leq \overline{v} \\ v_B I & s = v_B \end{cases}$$

where $D_2 = (1-\gamma)\alpha D$ and $I = q - D + (1-\gamma)\alpha D$ so

$$\min\{D_2, I\} = \begin{cases} (1-\gamma)\alpha D & D < q \\ (q - (1-\alpha+\gamma\alpha)D)^+ & \text{otherwise} \end{cases}$$

Define

$$\begin{aligned} s_H &= \arg\max s\left(\frac{\overline{v}-s}{\overline{v}-\underline{v}}\right) = \overline{v}/2 \\ s_L &= v_B \end{aligned}$$

So now we can write

$$R_2(D, I, \gamma) = \begin{cases} \frac{\overline{v}^2}{4(\overline{v}-\underline{v})}(q - D + (1-\gamma)\alpha D)^+ & q < D \leq \frac{q}{1-\alpha(1-\gamma)} \text{ and } s = s_H \\ \frac{\overline{v}^2}{4(\overline{v}-\underline{v})}(1-\gamma)\alpha D & D \leq q \text{ and } s = s_H \\ v_B(q - D + (1-\gamma)\alpha D)^+ & s = v_B \end{cases}$$

There are three situations describing R_2, with the first two being mutually exclusive. The first case is always preferred over the third because $\overline{v}/2 > \underline{v}$ implies

$$\frac{\overline{v}^2}{4(\overline{v}-\underline{v})} \geq v_B.$$

The second is preferred when there is a limited amount of inventory; in particular, the second is preferred over the third when $D > \hat{d}$ where

$$\hat{d} = \frac{q}{\left(\frac{\overline{v}^2}{4v_B(\overline{v}-\underline{v})} - 1\right)(1-\gamma)\alpha + 1} < q.$$

Note, if the firm takes a deep discount, then inventory exceeds demand from the strategics. We assume that the strategics are the first customers served, so they are guaranteed to receive a unit in period 2. The period 2 revenue function can now be written as

$$R_2(D, Q) = \begin{cases} 0 & \frac{q}{1-\alpha(1-\gamma)} < D \\ \frac{\overline{v}^2}{4(\overline{v}-\underline{v})}(q - (1-(1-\gamma)\alpha)D) & q < D \leq \frac{q}{1-\alpha(1-\gamma)} \\ \frac{\overline{v}^2}{4(\overline{v}-\underline{v})}(1-\gamma)\alpha D & \hat{d} < D \leq q \\ v_B(q - (1-(1-\gamma)\alpha)D) & D \leq \hat{d} \end{cases}$$

EC.3.2. Strategic consumers' strategy

The strategics need to form expectations about the second period price and availability. To parallel our original model, the strategics assume they will be able to purchase a unit in period 2, i.e., availability is not a direct concern for the strategics. Therefore, the stategics form expectations about the second period price, which can be either s_H or s_L. Let ϕ be the probability of a deep discount, $\phi = \Pr(D \leq \hat{d})$. The strategics' expected surplus in period 2 is V:

$$V = \phi\left(\frac{\overline{v}+\underline{v}}{2} - v_B\right) + (1-\phi)\left(\frac{\overline{v}-\overline{v}/2}{\overline{v}-\underline{v}}\right)\left(\frac{\overline{v}+\overline{v}/2}{2} - \overline{v}/2\right):$$

the first term is their expect surplus if a deep discount occurs. The second term above is their surplus if $s = s_H$, and it is composed of three terms: (1) the probability $s = s_H$; (2) the probability the strategics are willing to purchase a unit at s_H; (3) their surplus conditional on being willing to purchase a unit. Algebraic simplification of V yields:

$$V = \frac{1}{2}\left(\frac{\overline{v}^2}{4(\overline{v}-\underline{v})} + \phi\left(\frac{3\overline{v}^2 - 4\underline{v}^2}{4(\overline{v}-\underline{v})} - 2v_B\right)\right).$$

It is straightforward to show that V is increasing in ϕ given that $v_B \le \underline{v} \le \overline{v}/2$.

Now consider the strategics' period 1 decision, which is either to purchase in period 1 at price p or to wait until period 2 to make a purchase. Recall, γ is the fraction of strategics that purchase in period 1. If $0 < \gamma < 1$, then the strategics must be indifferent between purchasing in the two periods:

$$v_M - p = V$$

It is straightforward to determine that the strategics will be indifferent between the two periods only if ϕ^* is the probability of a deep discount in period 2, where

$$\phi^* = \frac{2(v_M - p) - \frac{\overline{v}^2}{4(\overline{v}-\underline{v})}}{\left(\frac{3\overline{v}^2 - 4\underline{v}^2}{4(\overline{v}-\underline{v})} - 2v_B\right)}.$$

Note, this critical probability depends on the strategics' valuations and the period 1 price, but not on the strategics' period 1 decision.

Under what conditions is $0 < \phi^* < 1$? The first inequality holds when

$$\frac{\overline{v}^2}{8(\overline{v}-\underline{v})} < v_M - p$$

and the second when

$$\begin{aligned} v_M - p &< \frac{1}{2}\left(\frac{\overline{v}^2}{4(\overline{v}-\underline{v})} + \left(\frac{3\overline{v}^2 - 4\underline{v}^2}{4(\overline{v}-\underline{v})} - 2v_B\right)\right) \\ v_M - p &< \frac{1}{2}(\overline{v} + \underline{v} - 2v_B) \end{aligned}$$

Taking $v_B \le \underline{v}$ and $\underline{v} < \overline{v}/2$, the above reduces to the following sufficient condition:

$$v_M - p < \frac{\overline{v}}{4}.$$

Putting the two together, we have $\phi^* \in (0,1)$ when

$$\frac{\overline{v}^2}{8(\overline{v}-\underline{v})} < v_M - p < \frac{\overline{v}}{4}, \tag{EC.5}$$

which we will assume. (It is possible to show that this range is non-empty when $\underline{v} < \overline{v}/2$.) If the above condition is violated, then the strategics either always purchase in period 1 (even if a deep discount is guaranteed, the strategics prefer to purchase in period 1) or always purchase in period 2 (even if a deep discount is never offered, the strategics prefer to purchase in period 2): in the former the strategics act as if they are myopic and in the latter they act as if they are bargain hunters. Therefore, without (EC.5), we do not have a strategic segment and the model yields uninteresting dynamics.

Now define η, which is the actual probability of a deep discount given γ:

$$\eta(q,\gamma) = \Pr\left(D \le \hat{d}\right) = \Pr\left(D \le \frac{q}{\left(\frac{\overline{v}^2}{4v_B(\overline{v}-\underline{v})}-1\right)(1-\gamma)\alpha+1}\right).$$

The strategics have three options, (1) $\gamma=0$, i.e., they all purchase in period 1; (2) $\gamma=1$, i.e., they all purchase in period 2 or (3) $0<\gamma<1$, i.e., they play a mixed strategy because they are indifferent between purchasing in either period given that all other strategics are adopting the same strategy. In the last case it must be that $\eta(q,\gamma)=\phi^*$, i.e., the stategics' choice, γ, must yield a probability of a deep discount that makes the strategics indifferent between the two periods. Otherwise, one of the two other strategies is optimal for the strategics.

Now define $\overline{q}$ and $\underline{q}$ such that

$$\Pr\left(D \le \underline{q}\right) = \Pr\left(D \le \frac{\overline{q}}{\left(\frac{\overline{v}^2}{4v_B(\overline{v}-\underline{v})}-1\right)\alpha+1}\right) = \phi^*.$$

We are now ready to define the strategics' reaction function:

$$\gamma(q) = \begin{cases} 1 & q \le \underline{q} \\ \eta^{-1}(q,\phi^*) & \underline{q} < q < \overline{q} \\ 0 & \overline{q} \le q \end{cases}$$

Furthermore, we note that $\gamma(q)$ is decreasing in q. To explain, if $q \le \underline{q}$, then quantities are sufficiently low that the strategics all purchase in period 1 because the actual probability of a deep discount is too low (even if they all purchase in period 1). Similarly, if $\overline{q} \le q$, then quantities are sufficiently high that the stategics all purchase in period 2 because the probability of a deep discount is sufficiently high (even if they all purchase in period 2). For intermediate quantities, the strategics choose a mixed strategy. Note, $\eta(q,\gamma)$ is increasing in q and γ, so the inverse, $\eta^{-1}(q,\phi^*)$, is unique (and exists for the range $\underline{q} < q < \overline{q}$).

EC.3.3. The firm's strategy

Now consider the firm's optimal strategy. The firm's profit function is

$$\pi(q) = -cq + pE\left[\min\{(1-(1-\gamma)\alpha)D, q\}\right] + E[R_2(D,q)]$$

.where

$$\begin{aligned} E[R_2(D,q)] = & \int_0^{\hat{d}} v_B\left(q-(1-(1-\gamma)\alpha)x\right)f(x)dx + \int_{\hat{d}}^{q} \frac{\overline{v}^2}{4\left(\overline{v}-\underline{v}\right)}(1-\gamma)\alpha x f(x)dx \\ & + \int_q^{\frac{q}{1-\alpha(1-\gamma)}} \frac{\overline{v}^2}{4\left(\overline{v}-\underline{v}\right)}\left(q-(1-(1-\gamma)\alpha)x\right)f(x)dx \end{aligned}$$

It follows that

$$\begin{aligned} \frac{dE[R_2(D,q)]}{dq} &= \int_0^{\hat{d}} v_B f(x)dx + \int_q^{\frac{q}{1-\alpha(1-\gamma)}} \frac{\overline{v}^2}{4\left(\overline{v}-\underline{v}\right)} f(x)dx \\ &= v_B F\left(\hat{d}\right) + \frac{\overline{v}^2}{4\left(\overline{v}-\underline{v}\right)}\left(F\left(\frac{q}{1-\alpha(1-\gamma)}\right) - F(q)\right) \end{aligned}$$

Furthermore,

$$E\left[\min\{(1-(1-\gamma)\alpha)D,q\}\right] = \int_0^{\frac{q}{1-\alpha(1-\gamma)}} (1-(1-\gamma)\alpha)xf(x)dx + \left(1-F\left(\frac{q}{1-\alpha(1-\gamma)}\right)\right)q$$

and

$$\frac{dE\left[\min\{(1-(1-\gamma)\alpha)D,q\}\right]}{dq} = \left(1-F\left(\frac{q}{1-\alpha(1-\gamma)}\right)\right)$$

So

$$\pi'(q) = (p-c) - \left(p - \frac{\overline{v}^2}{4(\overline{v}-\underline{v})}\right)F\left(\frac{q}{1-\alpha(1-\gamma)}\right) + v_B F\left(\hat{d}\right) - \frac{\overline{v}^2}{4(\overline{v}-\underline{v})}F(q)$$

We now establish that there is a unique q that satisfies $\pi'(q)=0$, i.e., the firm's profit is quasi-concave in q. Note,

$$\pi'(0) = (p-c) > 0$$

(assuming $F(0)=0$) and

$$\pi'(\infty) = -(c-v_B) < 0.$$

Furthermore

$$\frac{\pi''}{f(Q)} = \frac{1}{1-\alpha(1-\gamma)}\left(p - \frac{\overline{v}^2}{4(\overline{v}-\underline{v})}\right)\frac{f\left(\frac{Q}{1-\alpha(1-\gamma)}\right)}{f(Q)} + \frac{v_B}{\left(\frac{\overline{v}^2}{4v_B(\overline{v}-\underline{v})}-1\right)(1-\gamma)\alpha+1}\frac{f\left(\hat{d}\right)}{f(Q)} - \frac{\overline{v}^2}{4(\overline{v}-\underline{v})}$$

From the MSLR property, π'' is increasing or decreasing, which implies that π' is either quasi-concave or quasi-convex, which implies that π is quasi-concave.

Let $q(\gamma)$ be the firm's optimal quantity given γ. From the implicit function theorem, $q(\gamma)$ is increasing in γ:

$$\frac{\partial q(\gamma)}{\partial \gamma} = -\frac{\frac{\partial \pi(q)}{\partial q d\gamma}}{\pi''(q)} > 0.$$

Therefore, there exists a unique equilibrium (γ^*, q^*) such that $q^* = q(\gamma^*)$ and $\gamma^* = \gamma(q^*)$.

EC.3.4. Quick Response

This section details the impact of quick response (QR) on equilibrium behavior and profits. As in the original model, assume that the second order is placed after observing D but before period 1 demand and that order is received in time to satisfy period 1 demand. Units in the second order cost c_2 per unit. Additional units can be procured even if they will be sold in period 2. However, we assume

$$\frac{\overline{v}^2}{4(\overline{v}-\underline{v})} < c_2 < p,$$

and will shortly explain that assumption.

Consider the impact of QR on the firm's optimal decisions. If $D < \hat{d}$, the firm prefers to sell all remaining inventory at v_B than to sell a portion of its inventory at s_H. The marginal value of additional units is then v_B, so no additional units are ordered at price c_2 and the optimal decision remains to discount at v_B. If $\hat{d} < D < q_r$, the firm prefers to sell a portion of its inventory at s_H rather than to take the deep discount. Given that some inventory will not be sold, the marginal value of additional inventory is zero and no additional product is procured. If $q_r < D < q_r\left(1-\alpha(1-\gamma)\right)$, the firm's inventory in period 2 is less than its potential demand at the price s_H. In this situation the firm may be able to sell additional units. However, the optimal selling price remains, s_H. It is not worthwhile to procure additional units if

$$-c_2 + \left(\frac{\overline{v}}{2\left(\overline{v}-\underline{v}\right)}\right)\left(\frac{\overline{v}}{2}\right) < 0, \tag{EC.6}$$

which simplies to

$$\frac{\overline{v}^2}{4\left(\overline{v}-\underline{v}\right)} < c_2:$$

the first term in (EC.6) is the marginal cost of an additional unit; the second term has two components, the first of which is the probability of selling an additional unit and the second is the revenue if the unit is sold. If (EC.6) does not hold, then it may be in the interest of the firm to procure additional units for sale in period 2, which would make the firm even more conservative with its initial purchase quantity.

As we have established, the firm's second period pricing remains unchanged. Therefore, the strategics' reaction function, $\gamma(q)$, remains unchanged as well. However, now the firm's profit function is

$$\begin{aligned}\pi_r(q_r) = &-c_1 q_r + (p-c_2)(1-(1-\gamma)\alpha)E[D]\\ &+(c_2-c_1)E\left[\min\{(1-(1-\gamma)\alpha)D, q_r\}\right] + E[R_2(D,q_r)].\end{aligned}$$

As $\pi(q)$ is quasi-concave in q, it is straightforward to show that $\pi_r(q_r)$ is quasi-concave in q_r as well. Thus, the firm's reaction function, $q_r(\gamma)$ is well behaved and increasing in γ. Furthermore,

$$\pi'(q) > \pi_r'(q_r),$$

which implies $q_r(\gamma) < q(\gamma)$. When the firm operates QR there exists a unique equilibrium, (γ_r^*, q_r^*), and the firm's initial purchase is smaller than without QR, $q_r^* < q^*$.

As in the original model, QR is not necessarily more valuable with strategic consumers than with just myopic consumers. If all strategic consumers are converted into myopic consumers, the guaranteed demand in period 1 (at the high selling price of p) is increased. Nevertheless, as in the original model, it is possible to construct a condition in which QR is always more valuable with strategic consumers than without them. As before, we want to know when $\gamma_r^* = 1$ occurs, i.e., q_r is sufficiently low that all of the strategics purchase in period 1. If $\gamma_r = 1$, we have

$$\begin{aligned}\pi_r(q_r, \gamma_r = 1) &= -c_1 q_r + (p-c_2)E[D] + c_2 E\left[\min\{D, q_r\}\right] + E[R_2(D,q_r)].\\ &= -(c_1 - v_B)q_r + (p-c_2)E[D] + (c_2 - v_B)E\left[\min\{D, q_r\}\right]\end{aligned}$$

and the optimal order quantity is

$$F(q_r^*) = \frac{c_2 - c_1}{c_2 - v_B}.$$

Thus, we have $q_r^* \leq \underline{q}$, which implies $\gamma_r^* = 1$, if

$$\frac{c_2 - c_1}{c_2 - v_B} \leq \frac{2\left(v_M - p\right) - \frac{\overline{v}^2}{4\left(\overline{v}-\underline{v}\right)}}{\left(\frac{3\overline{v}^2 - 4\underline{v}^2}{4\left(\overline{v}-\underline{v}\right)} - 2v_B\right)}, \tag{EC.7}$$

which is analogous to the condition in the original model. Furthermore, if (EC.7) holds, as in the original model, we know that QR is more valuable with strategic consumers than without them.

References

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