

Plenty of Room at the Hotel California

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Introduction

The purpose of this document is to establish a fair algorithm to determine monthly rent for the residents of 214 Oak Court, Menlo Park, CA 94025, i.e., the House. This document has contractual force and can only be modified in a manner consistent with the Liability Contract.

The Facts

Bernard Kirk Rihn has established the following survey of the dimensions of the bedrooms in the House. Slight modifications have been made to the names of the bedrooms, but the dimensions and other relevant details are presented in their original form.

The room sizes are as follows:

Hardwood large room (b_3): 12'x12'

Hardwood small room (b_4): 11 1/4' x 9 3/4'

Carpeted room (b_5): 11 1/2' x 9'

Pink room (b_2): 13 1/3' x 15 3/4'

2 1/2 foot offset on long side for half the wall

2 feet offset on other wall for hangar space

Also has private bathroom--just toilet

Master bedroom (b_1):

16' x 12 1/2'

Closets extend additional 2 feet on 12 1/2 foot side

Also has bathroom that isn't accessible to other parts
of the house without the bedroom door being open.

Doesn't include bathroom or 3 1/2' x 7' walkway space

The Criteria

We maintain that certain logical criteria should be categorically observed in the determination of rent. The most evident of these criteria is

The Master Bedroom Criterion. The single resident of the master bedroom should pay the highest rent.

Of equal obviousness is

The Carpeted Bedroom Criterion. The single resident of the carpeted bedroom should pay the lowest rent.

A stronger statement establishes a total ordering of the set of bedrooms:

The Bedroom Ordering Criterion. Consider the set $B = \{b_1, b_2, \dots, b_5\}$ of bedrooms where b_1 is the master bedroom, b_2 is the pink room, b_3 is the larger of the medium-sized singles (hardwood large), b_4 is the smaller of the medium-sized singles (hardwood small), and b_5 is the carpeted room. Consider the mapping $r : B \mapsto \mathbb{R}_+$ which is the rent function that maps the bedroom $b_i \in B$ to a monthly U.S. dollar rent $r_i := r(b_i) \in \mathbb{R}_+$ where \mathbb{R}_+ is the set of all positive reals. The following total ordering of the set $R = r(B)$ is intuitive: $r_i > r_j$ for all $i < j$, $i, j \in \{1, 2, \dots, 5\}$. That is, $r_1 > r_2 > r_3 > r_4 > r_5$.

We also maintain that closet space should be considered in rent calculations, but should be discounted relative to living space.

The Closet Space Discount Criterion. There exists some closet discount rate $\delta \in (0, 1)$ such that the utility U_l derived from some amount of living space σ is equivalent to the utility U_c derived from the same amount of closet space:

$$U_c(\sigma) = \delta U_l(\sigma)$$

If we approximate utility as a strictly linear (not affine) function of space, we can rewrite the above as

$$\tilde{\sigma} = \delta \sigma$$

where $\tilde{\sigma}$ is the closet-discounted equivalent living space of some quantity σ of closet space. That is, in intermediate calculations of room space, one unit of closet space is equivalent to some $\delta \in (0, 1)$ units of living space.

We establish, further, some metric for the personal full bathroom in b_1 and the personal half bathroom (toilet only) in b_2 . Consider

The Bathroom Criterion. There exist some s, t such that $s > t > 0$, where s is the equivalent living space premium of the full bathroom (shower) and t is the equivalent living space premium of the half bathroom (toilet).

Finally, we establish that the bedroom disparity should only determine a fraction of the rent, since common areas comprise the majority of the house and are equally accessible to all residents.

The Common Space Criterion. There exists some fraction $\beta \in (0, 1)$ of the total rent that should be determined based on bedroom disparity.

The Five-Person Rent Schedule

Let us consider the total equivalent living space λ_i in square-feet for each bedroom $i = \{1, 2, \dots, 5\}$. Due to Rihn in conjunction with the Closet Space Discount and Bathroom Criteria, we have

$$\begin{aligned} \lambda_1 &= (16.0 \times 12.5) + \delta(2.0 \times 12.5 + 3.5 \times 7.0) + s \\ \lambda_2 &= (13.3 \times 15.8 + 2.5 \times 7.9) + \delta(2.0 \times 13.3) + t \\ \lambda_3 &= (12.0 \times 12.0) + \delta(2.0 \times 8.0) \\ \lambda_4 &= (11.3 \times 9.8) + \delta(2.0 \times 8.0) \\ \lambda_5 &= (11.5 \times 9.0) \end{aligned}$$

where we have discounted the “walkway space” of b_1 at the closet rate. We have also taken the closet space of b_3 and b_4 to be two feet deep by eight feet wide as an upper bound, since these figures were not reported in Rihn.

By the Common Space Criterion, we maintain that an individual i 's fraction of rent $\rho_i \in (0, 1)$ consists of the fixed common portion, given by

$$\frac{1}{n}(1 - \beta)$$

(where $n = 5$ is the number of residents) and the variable bedroom portion given by

$$\beta \tilde{\lambda}_i$$

where $\tilde{\lambda}_i$ is linearly related to the equivalent living space λ_i by some factor such that

$$\sum_{i=1}^5 \tilde{\lambda}_i = 1$$

If we denote the vector of λ_i by λ and the vector of $\tilde{\lambda}_i$ by $\tilde{\lambda}$, the relation can be written as

$$\tilde{\lambda} = \frac{\lambda}{\sum_{i=1}^5 \lambda_i}$$

Thus, the final rent proportion vector ρ is given by

$$\rho = \frac{1}{n}(1 - \beta) + \beta \tilde{\lambda}$$

and the final rent vector r is given by

$$r = \rho Q$$

where $Q = 3000$ is the total monthly rent of the house in U.S. dollars.

We have yet to determine the bedroom proportion β , the closet discount rate δ , the shower premium s , and the toilet premium t . I maintain that the bedroom proportion should be $\beta = \frac{1}{4}$, as I believe this sufficiently encompasses the importance of a bedroom while giving due significance to the majority of the house, which is common space and appliances.

I also maintain that $\delta = \frac{3}{4}$, since I believe that closet space is almost as important as living space, particularly in the case of the carpeted bedroom, where an additional armoire providing closet space would in fact detract from living space; therefore, I have chosen a large δ to increase the rent of b_i for $i = \{1, 2, 3, 4\}$ relative to b_5 .

Further, I hold that s should be significant because the freedom it affords the user of the private full bath. I have chosen $s = 50$. That is, the full bath provides an additional equivalent living space of 50 square-feet over the shared hall bathroom. Similarly, I maintain that $t = \frac{s}{2} = 25$.

With these parameters, the numerical quantities for monthly rent in U.S. dollars are

$$\begin{aligned} r_1 &= 678.0689 \\ r_2 &= 668.3106 \\ r_3 &= 573.9138 \\ r_4 &= 547.4947 \\ r_5 &= 532.2120 \end{aligned}$$

We round the numbers to the nearest dollar (I round mine, r_4 , up to the nearest dollar to ensure $\sum_{i=1}^5 r_i = Q$) to obtain

$$\begin{aligned} r_1 &= 678 \\ r_2 &= 668 \\ r_3 &= 574 \\ r_4 &= 548 \\ r_5 &= 532 \end{aligned}$$

Indeed, the Bedroom Ordering Criterion is not violated and all of the rents sum to the total house rent of 3,000.

The Six-Person Rent Schedule

To ascertain the rent schedule in the presence of six people, where b_d for some $d \in \{1, 2, \dots, 5\}$ is considered a double, we simply modify the above calculation. The rent fraction vector is now $\rho' = [\rho'_1 \rho'_2 \cdots \rho'_{d-1} \rho'_d \rho'_d \rho'_{d+1} \cdots \rho'_5]^T$. An analogous modification is made to the rent and equivalent living space vectors.

In calculating the equivalent living space λ'_d for each of the double residents, however, we use

$$\lambda'_d = \frac{\lambda_d}{2} - p$$

where λ_d is the equivalent living space for b_d determined in the previous section and p is the privacy premium, which is subtracted to accommodate the inconvenience of a double. The equivalent living space of the double resident should not be one-half of the 5-person case, since this would imply that two singles of equal size provide the same equivalent living space. Indeed, the privacy premium $p > 0$ is an indispensable adjustment. For equivalent living spaces $i = \{1, 2, 3, 4, 5\} \setminus d$, we simply have $\lambda'_i = \lambda_i$.

The remainder of the rent calculation is as above. The normalized equivalent living space vector is

$$\tilde{\lambda}' = \frac{\lambda'}{2\lambda'_d + \sum_{i=\{1,2,3,4,5\} \setminus d} \lambda_i}$$

Thus, the final rent proportion vector ρ' is given by

$$\rho' = \frac{1}{n}(1 - \beta) + \beta \tilde{\lambda}'$$

where $n = 6$, in this case. The final rent vector r' is given by

$$r' = \rho' Q$$

We have yet to choose the value of the privacy premium p . I argue that the inconvenience afforded by double residence is significant. As such, I have chose p to be quite large at one and one-half times the full bathroom premium: $p = 75$. Using this value of p with $d = 1$, our final rent in U.S. dollars after rounding adjustments is

$$\begin{aligned} r'_1 &= 440 \\ r'_1 &= 440 \\ r'_2 &= 635 \\ r'_3 &= 522 \\ r'_4 &= 490 \\ r'_5 &= 473 \end{aligned}$$

If we take $d = 2$ (that is, the pink room as the double), the final rent in U.S. dollars after rounding adjustments is

$$\begin{aligned} r'_1 &= 646 \\ r'_2 &= 434 \\ r'_2 &= 434 \\ r'_3 &= 522 \\ r'_4 &= 491 \\ r'_5 &= 473 \end{aligned}$$

Conclusion

“Welcome to the Hotel California:
 Such a lovely place,
 Such a lovely face.
 Plenty of room at the Hotel California.
 Any time of year, you can find it here.”

Appendix: MATLAB Code for Rent Calculation

```
>> r = rent_schedule()
% r =
%   678 (master)
%   668 (pink)
%   574 (hardwood large)
%   548 (hardwood small)
%   532 (carpeted)

>> r = rent_schedule(1)
% r =
%   440 (master)
%   440 (master)
%   635 (pink)
%   522 (hardwood large)
%   490 (hardwood small)
%   473 (carpeted)

>> r = rent_schedule(2)
% r =
%   646 (master)
%   434 (pink)
%   434 (pink)
%   522 (hardwood large)
%   491 (hardwood small)
%   473 (carpeted)

function r = rent_schedule(varargin)
% This function determines the fair rent price based on the Two Fourteen
% Oak Court model detailed in Rabbani's "Plenty of Room at the Hotel
% California" and based on measurements made in Rihn's report on the
% dimensions and features of the bedrooms.
%
% PARAMETERS
%
% d      : Room that is used as the double, (0 for 5 people)
% beta  : Bedroom proportion
% delta : Closet discount rate
% s      : Full bathroom (shower) premium
% t      : Half bathroom (toilet) premium
% p      : Privacy premium
%
```

```

% Parameters should be input in the above order. A truncated parameter list
% or an omitted parameter using [] will cause the default parameter values
% to be used for the unspecified field.
%
% USAGE
%
% For a five-person house using the default parameters:
% >> r = rent_schedule();
%
% For a six-person house using the default parameters and b1 as the double:
% >> r = rent_schedule(1);
%
% For an arbitrary parameter list with "beta" and "p" as default:
% >> r = rent_schedule(d,[],delta,s,t);

% Default parameters
d = 0;
beta = 0.25;
delta = 0.75;
s = 50;
t = 25;
p = 75;

% Input parameters and error checking

% Parameter: d
if(nargin >= 1 && ~isempty(varargin{1}))
    d = varargin{1};
    if( d~=0 && d~=1 && d~=2 && d~=3 && d~=4 && d~=5 )
        error(sprintf(...
            'Invalid input parameter d = %g; must be in {0,1,2,3,4,5}.',d));
    end
end

% Parameter: beta
if(nargin >= 2 && ~isempty(varargin{2}))
    beta = varargin{2};
    if( beta<=0 || beta>=1 )
        error(sprintf(...
            'Invalid input parameter beta = %d; must be in (0,1).',beta));
    end
end

% Parameter: delta
if(nargin >= 3 && ~isempty(varargin{3}))
    delta = varargin{3};
    if( delta<=0 || delta>=1 )
        error(sprintf(...
            'Invalid input parameter delta = %d; must be in (0,1).',delta));
    end
end

```

```

% Parameter: s
if(nargin >= 4 && ~isempty(varargin{4}))
    s = varargin{4};
    if( s<=0 )
        error(sprintf(...
            'Invalid input parameter s = %d; must be in positive.',s));
    end
end

% Parameter: t
if(nargin >= 5 && ~isempty(varargin{5}))
    t = varargin{5};
    if( t<=0 )
        error(sprintf(...
            'Invalid input parameter t = %d; must be in positive.',t));
    end
end

if( s<=t )
    error(sprintf(...
        'Invalid input parameters s = %d, t = %d; must be s>t.',s,t));
end

% Parameter: p
if(nargin >= 6 && ~isempty(varargin{6}))
    p = varargin{6};
    if( p<=0 )
        error(sprintf(...
            'Invalid input parameter p = %d; must be positive.',p));
    end
end

% Equivalent living space quantities
lambda(1) = (16.0*12.5) + delta*(2.0*12.5 + 3.5*7) + s; % master
lambda(2) = (13.3*15.8+2.5*7.9) + delta*(2.0*13.3) + t; % pink
lambda(3) = (12.0*12.0) + delta*(2.0*8.0);           % hardwood lg.
lambda(4) = (11.3*9.8) + delta*(2.0*8.0);           % hardwood sm.
lambda(5) = (11.5*9.0);                             % carpeted

% Number of people
if( d == 0)
    n = 5;
else
    n = 6;
end

% Six-person case
if( d > 0 )
    lambda(d) = lambda(d)/2 - p;
    lambda = lambda([1:d d d+1:n-1]);
end

```

```
end

% Normalized living space quantities
lambda_norm = lambda/sum(lambda);

% Rent fraction and rent
Q = 3000; % Total rent
rho = (1-beta)/n + beta*lambda_norm;
r = round(Q*rho)';
r(end-1) = r(end-1) + (Q - sum(r))
```