Hedonistic Calculus for the Individual
Corollary to Bentham's Hypothesis

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The nineteenth century British economist and philosopher Jeremy Bentham coined the term "hedonistic calculus" to refer to his intuitive statement on ethics, called Utilitarianism, by which every endeavor should be undertaken such that it maximizes the pleasure of the greatest number of people. He quantified his philosophy by a fundamental equation, which, by virtue of its naïve mathematical simplicity, insufficiently represents the nature of utility.

I have attempted here to develop a more precise correlation on an *individual* level, considering one's affinity for pleasure, one's participation in pleasurable behavior, and the economic consequences of decreasing marginal utility, all as a function of time, that when properly combined and integrated over the desired time period, will yield total utility.

We start first by attempting to understand Mr. Bentham's equation:

$$P = [N(C \times I \times D) + N_f(C_f \times I_f \times D_f)] \times F \times E$$

where P is the total pain or pleasure; N is the "propinquity," the remoteness or proximity in time of pain or pleasure, determined by an indifference curve as a function of utility; C is the certainty, expressed as a probability, that the pain or pleasure will occur; I is the intensity of the pain or pleasure; D is its duration; the subscript f is the future values of the respective factors; F is the "fecundity," or the tendency for the stimulation of further pain or pleasure; and E is the extent of the pain or pleasure, the number of people affected. Mr. Bentham's net utility is calculated by finding the difference between the total pleasure and total pain, both of which are attained by the above equation.

Mr. Bentham's equation, however, fails in practice, for his treatment of the nebulous concept of propinquity assumes no tangible, calculable form. It is more suitable, instead, to calculate a Riemann sum, expressed as an integral over a certain period of time, to find the precise utility. In doing so, we define two vector functions of time in n dimensions.

The first such vector function we call $\mathbf{A}(t)$, the affinity of an individual for a set of pleasurable techniques. Each of the n components of $\mathbf{A}$ represents a different technique (consuming chocolate, reading, sexual intercourse, etc.) where the magnitude of each component is the proclivity of the individual to receive pleasure from this activity and the unit vector of each component identifies the technique. The vector function $\mathbf{A}$ is,

$$\mathbf{A}(t) = \langle A_1(t), A_2(t), \ldots, A_{n-1}(t), A_n(t) \rangle$$

We can rewrite $\mathbf{A}$ by expressing its components as products of scalar quantities and unit vectors:

$$\mathbf{A}(t) = A_1(t)\mathbf{A}_1 + A_2(t)\mathbf{A}_2 + \ldots + A_{n-1}(t)\mathbf{A}_{n-1} + A_n(t)\mathbf{A}_n$$

The second function of time we call $\mathbf{p}(t)$, or the intensity with which an individual participates in each technique. The n components of $\mathbf{p}$, likewise, represent a different technique, where the magnitude of each component is a measure of said participation. We can express $\mathbf{p}$ similarly:

$$\mathbf{p}(t) = p_1(t)\mathbf{p}_1 + p_2(t)\mathbf{p}_2 + \ldots + p_{n-1}(t)\mathbf{p}_{n-1} + p_n(t)\mathbf{p}_n$$

Thus, the utility, U_t, achieved by an individual by one technique i at a time t is the scalar product of the affinity for this pleasure and the amount of participation. Since $\mathbf{A}_i = \mathbf{p}_i$,

$$U(t) = A_i(t)p_i(t)\mathbf{A}_i \cdot \mathbf{p}_i = A_i(t)p_i(t)$$

This instantaneous utility, however, is still mathematically impracticable without an understanding of what constitutes an individual's affinity. Most factors used in the calculation of this vector can only be assessed on an individual basis, but one factor is, by the laws of economics, universal: the law of decreasing marginal utility. How steep the drop in marginal utility must be assessed separately for every technique and every individual. We establish an index of depreciation k, which measures the steepness of the decline. We now express the affinity $\mathbf{A}$ in terms of the affinity due to decreasing marginal utility by implementing the index of depreciation for each technique and the affinity due to every other factor, which we call $\mathbf{a}$. The relationship is further complicated by the dependency of $\mathbf{A}$ on $\mathbf{p}$, for the decrease in marginal utility, by definition, must be expressed as a fraction of an individual's participation in a certain technique, and varies from technique to technique. Thus, we have the highly convoluted vector function:

$$\mathbf{A}(t) = a_1(t)\frac{\sqrt[k_1]{p_1(t)}}{p_1(t)}\mathbf{A}_1 + \ldots + a_{n-1}(t)\frac{\sqrt[k_{n-1}]{p_{n-1}(t)}}{p_{n-1}(t)}\mathbf{A}_{n-1} + a_n(t)\frac{\sqrt[k_n]{p_n(t)}}{p_n(t)}\mathbf{A}_n$$

which, for the sake of simplicity, we will abbreviate to,

$$\mathbf{A}(t) = \sum_{i=1}^{n} a_i(t) p_i^{\frac{1}{k_i}-1}(t)\mathbf{A}_i$$

The total utility within a given time period $\{t|[t_i, t_f]\}$, therefore, is the Riemann sum of all utilities from the initial time to the final time, better expressed as the integral,

$$U = \int_{t_i}^{t_f} \mathbf{A}(t) \cdot \mathbf{p}(t)\ dt = \int_{t_i}^{t_f} \sum_{i=1}^{n} a_i(t) p_i^{\frac{1}{k_i}-1}(t)\mathbf{A}_i \cdot \mathbf{p}_i\ dt$$

The dot product is unity. After simplifying, we are left with the convenient integral:

$$U = \int_{t_i}^{t_f} \mathbf{A}(t) \cdot \mathbf{p}(t)\ dt = \int_{t_i}^{t_f} \sum_{i=1}^{n} a_i(t) p_i^{\frac{1}{k_i}-1}(t)\ dt$$

This equation does for an individual case everything Mr. Bentham's equation does, however with a scientific precision uncharacteristic of the original parochial arithmetic jumble. The product of intensity and certainty, the expected value, is given by the components of $\mathbf{p}$, while the duration and indifference curve are substituted by the more precise integral. Fecundity may be expressed by dependency among components within the vector functions of both $\mathbf{A}$ and $\mathbf{p}$. Extent, however, is beyond the individual scope.