Small-Scale Self-Focusing Effects in Tapered Optical Beams

Approximate analysis of small-scale self focusing effects in tapered (i.e., strongly diverging or converging) optical beams

1) Usual collimated beam case:

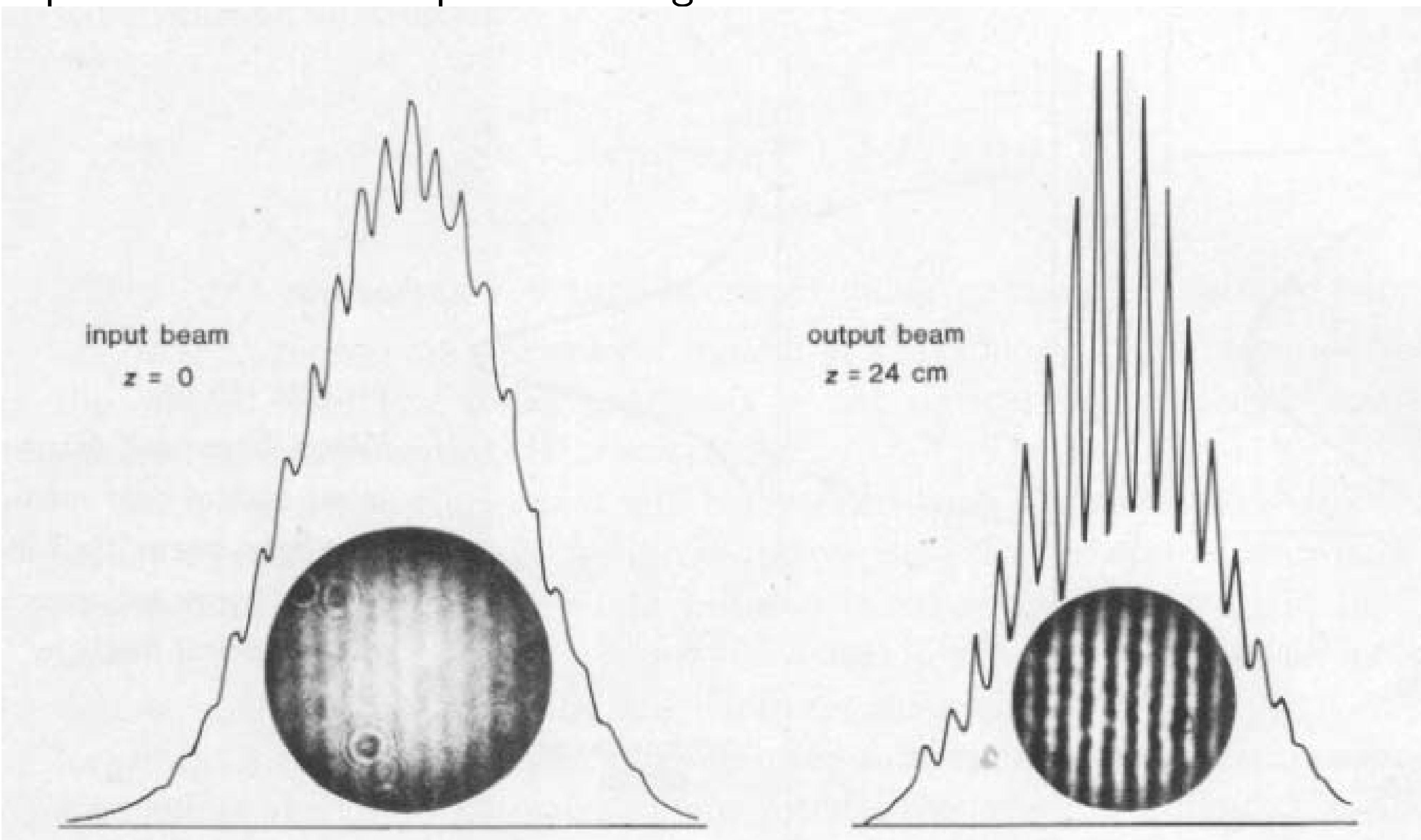
- Beam perturbations modelled as uniform sinusoidal ripples
- Ripple amplitudes grow exponentially with distance

2) Strongly tapered beam case:

- Beam perturbations modeled as nonlinear Newton's rings or Fresnel zone plates
- These patterns preserve their shape with distance but do not grow exponentially

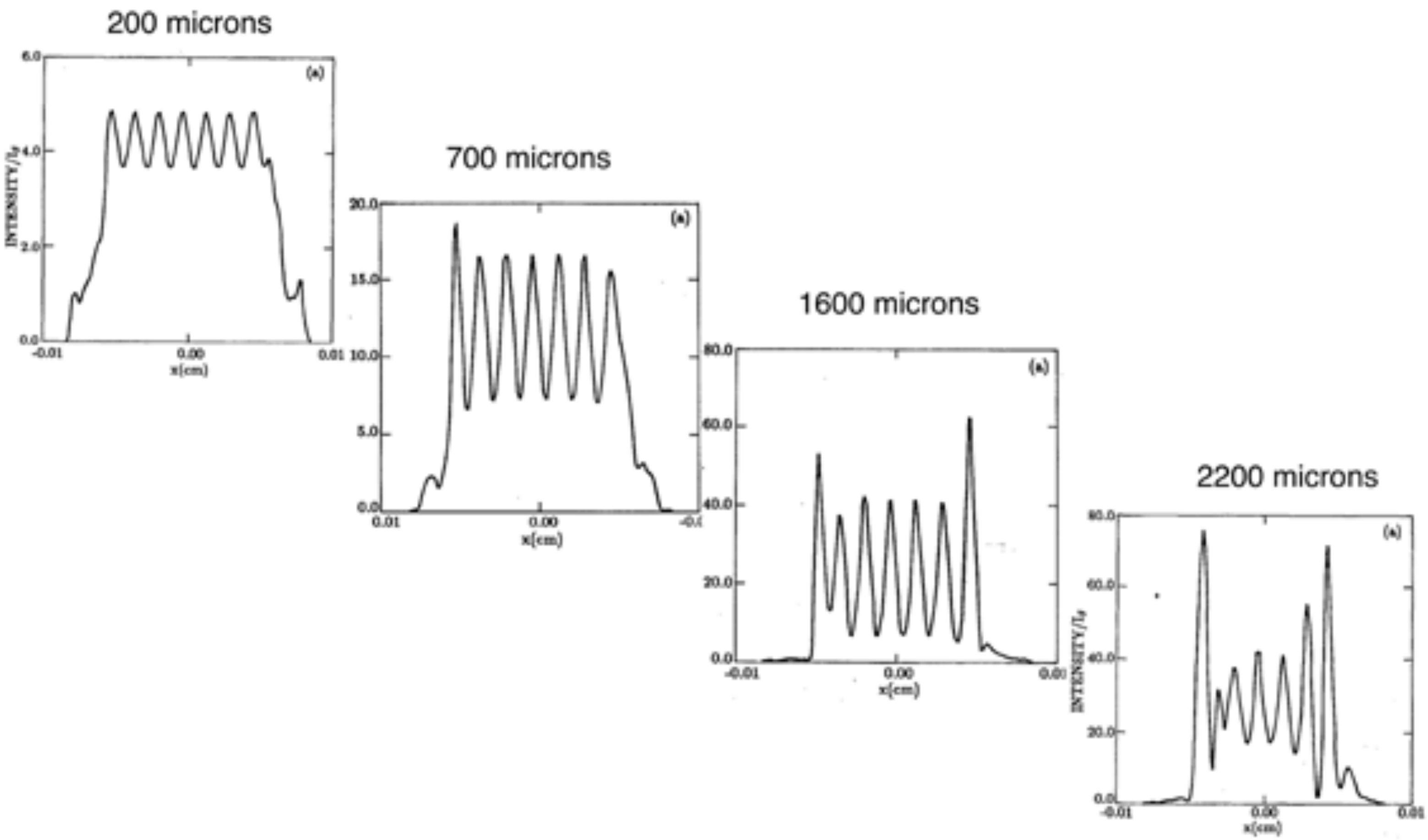
Collimated beam self focusing

Small initial ripples on a collimated gaussian beam grow in amplitude as the beam passes through a CS2 cell



Diode laser filamentation

Ripples on the beam in a wide-stripe diode laser grow with distance,
eventually breaking into tightly focused spots or filaments
that seriously degrade output beam quality



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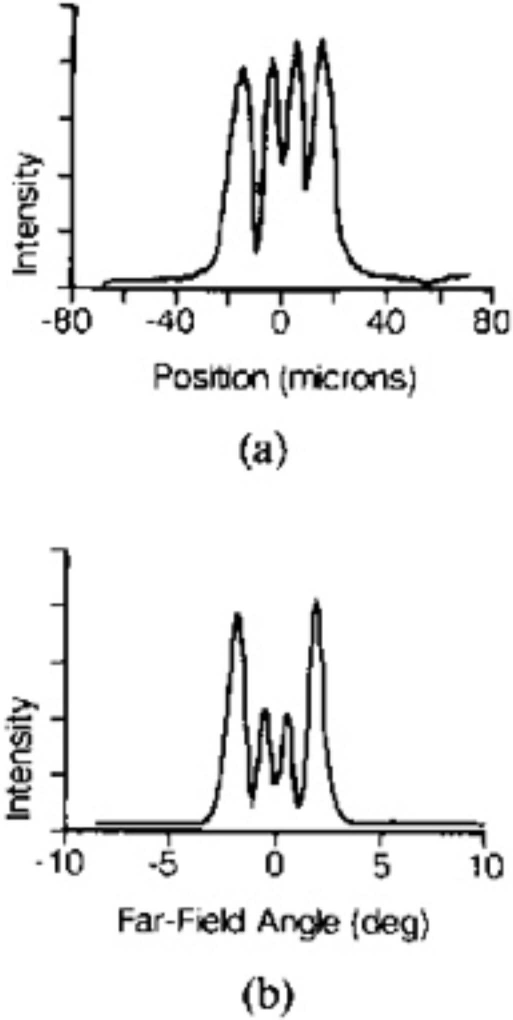


Fig. 6. Free-running emission of the 60 μm wide broad-area device used in our experiments without the application of external heating. (151 mA, $1.1 \times I_{\text{threshold}}$.)

Nonlinear beam propagation

Consider a nonlinear optical medium having either

- Nonlinear refractive index, with optical Kerr coefficient k_2
- Saturable gain or loss, with nonlinear coefficient g_2

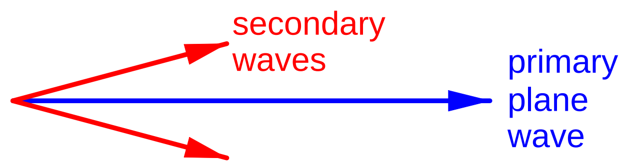
Nonlinear wave propagation coefficient $\tilde{k}$ in this medium is then

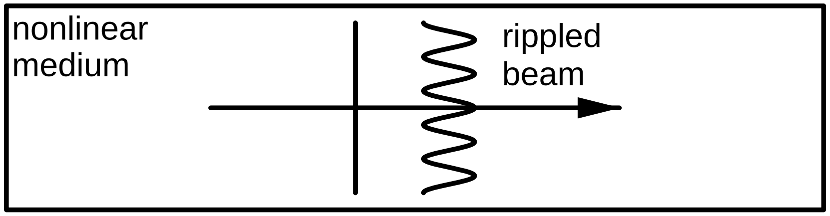
$$\tilde{k} = \left[k_0 + k_2 \, |\tilde{u}|^2\right] + j\left[g_0 - g_2 \, |\tilde{u}|^2\right]$$

and the paraxial wave equation for a beam propagating in this nonlinear medium is

$$\left[\nabla_{xy}^2 - 2jk_0 \frac{\partial}{\partial z} + 2jk_0 g_0 + 2k_0(k_2 - jg_2)\,|\tilde{u}|^2\right] \tilde{u} = 0$$

Collimated beam case





Expand the propagating beam as a strong primary plane wave plus two weaker secondary plane waves:

$$\tilde{u}(x,z) = u_0\, e^{g(z)-j\phi(z)} \times [1 + \tilde{a}_1(z)e^{j\kappa x} + \tilde{a}_2(z)e^{-j\kappa x}]$$

where

$u_0\, e^{g(z)-j\phi(z)}$ = growth and phase shift for overall beam

$\tilde{a}_1(z),\ \tilde{a}_2(z)$ = secondary waves traveling at small angles $\pm\theta$ to the main beam, with $|\tilde{a}_1, \tilde{a}_2| << 1$

$\kappa \equiv k_0 \sin\theta$ = transverse k vector

Coupled linear equations

This plane-wave formulation leads to two coupled linear equations for the secondary wave amplitudes

$$\frac{d\tilde{a}_1}{dz} = j\left[\frac{\kappa^2}{2k_0} - (k_2 - jg_2)\,u_0^2 e^{2g(z)}\right]\tilde{a}_1 - j\left[(k_2 - jg_2)\,u_0^2 e^{2g(z)}\right]\tilde{a}_2^\star$$

$$\frac{d\tilde{a}_2^*}{dz} = -j\left[\frac{\kappa^2}{2k_0} - (k_2 + jg_2)\,u_0^2 e^{2g(z)}\right]\tilde{a}_2^\star + j\left[(k_2 + jg_2)\,u_0^2 e^{2g(z)}\right]\tilde{a}_1$$

in which

$$\frac{\kappa^2}{2k_0} \text{ terms} = \text{linear Talbot phase shift terms}$$

$$k_2 - jg_2 \text{ terms} = \text{coupling terms produced by nonlinearly induced phase and amplitude gratings}$$

Alternative cosine/sine formulation

An alternative expansion for the collimated beam case is

$$\begin{aligned}\tilde{u}(x,z) &= u_0\, e^{g(z)-j\phi(z)} \times [1 + \tilde{c}(z)\cos\kappa x] \\ &= u_0\, e^{g(z)-j\phi(z)} \times \big[1 + [c_r(z) + jc_i(z)]\, \cos\kappa x\big]\end{aligned}$$

The beam intensity profile is then

$$I(x,z) = |\tilde{u}(x,z)|^2 \simeq u_0^2\, e^{2g(z)}[1 + 2c_r(z)\cos\kappa x]$$

so that

- Real part $c_r(z) =$ periodic <u>amplitude</u> ripple
- Imag part $c_i(z) =$ periodic <u>phase</u> ripple

(to first order anyway)

Alternative coupled equations

The cosine formulation leads to an alternative set of linear coupled equations for $c_r(z)$ and $c_i(z)$:

$$\frac{dc_r(z)}{dz} = -2g_2u_0^2\, c_r(z) \;+\; \frac{\kappa^2}{2k_0}\, c_i(z)$$

$$\frac{dc_i(z)}{dz} = \left(\frac{\kappa^2}{2k_0} - 2k_2u^2\right)\, c_i(z)$$

where

$$\frac{\kappa^2}{2k_0} = \text{Talbot phase shift factor as before}$$

$$2k_2u_0^2 = \text{nonlinear index (optical Kerr) effect}$$

$$2g_2u_0^2 = \text{nonlinear (saturable) gain effect}$$

General result for ripple propagation

Assuming that the ripples propagate as

$$\tilde{a}_1(z), \tilde{a}_2(z) \quad \text{or} \quad c_r(z), c_i(z) \quad \sim \quad e^{\gamma z}$$

either set of coupled equations leads to the secular equation

$$\gamma^2 + 2g_2 u_0^2 \, \gamma \, + \, (\kappa^2/2k_0) \left[(\kappa^2/2k_0) - 2k_2 u_0^2 \right] = 0$$

which has roots or growth coefficients

$$\gamma = - \, g_2 u_0^2 \pm \sqrt{(g_2 u_0^2)^2 - (\kappa^2/2k_0)^2} \ ,$$

Pure Talbot case

For the purely linear case where $k_2 = 0$ and $g_2 = 0$, the ripple propagation is given by

$$\gamma = \pm j\,\kappa^2/2k_0$$

and hence

$$c_r(z), c_i(z) \quad \sim \quad \cos(\kappa^2/2k_0)\,z \;\; \text{or} \;\; \sin(\kappa^2/2k_0)\,z$$

This is the "pure Talbot" case: the ripple pattern oscillates periodically between pure amplitude profile and pure phase profile

Nonlinear index case

For a nonlinear index medium with $k_2 > 0$, $\kappa^2 < 4k_0 k_2 u_0^2$, and $g_2 = 0$, γ is given by

$$\gamma = \pm\sqrt{\left(\frac{\kappa^2}{2k_0}\right)\left[2k_2 u_0^2 - \left(\frac{\kappa^2}{2k_0}\right)\right]}$$

and the ripple amplitude thus has an exponentially growing component $c(z) \sim \exp[+\gamma\, z]$

This exponential growth rate has a maximum value for ripples with the spatial frequency $\kappa_{\max} = \sqrt{2k_0 k_2 u_0^2}$

Saturable gain case

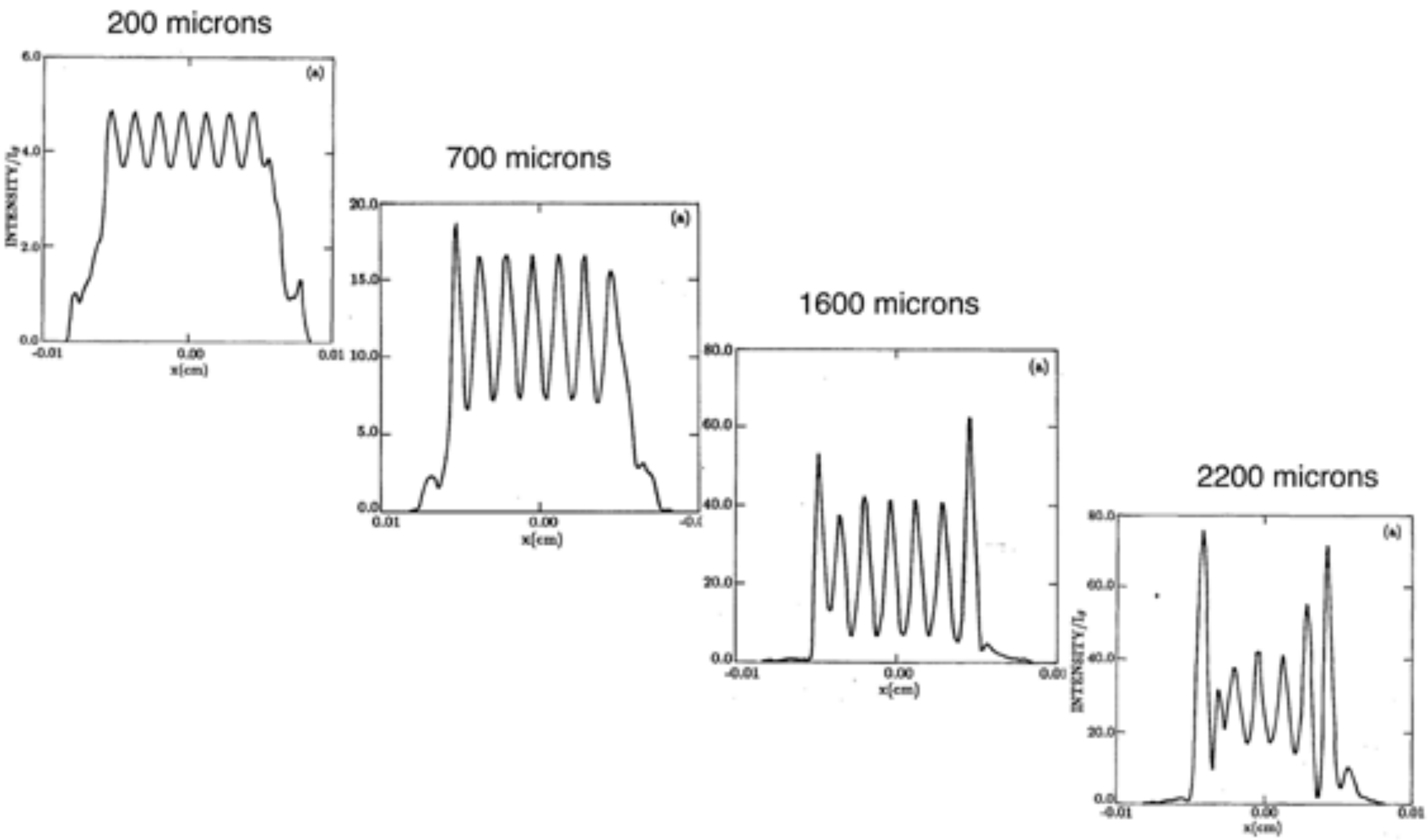
For the saturable gain case, with $k_2 = 0$ and a finite value of $g_2 > 0$, the roots for ripple growth become

$$\gamma = -\, g_2 u_0^2 \pm \sqrt{(g_2 u_0^2)^2 - (\kappa^2/2k_0)^2}\ ,$$

Both roots then have $\gamma_r < 0$ so that ripples always attenuate (and possibly oscillate periodically) with distance

Diode laser filamentation

Ripples on the beam in a wide-stripe diode laser grow with distance,
eventually breaking into tightly focused spots or filaments
that seriously degrade output beam quality



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Example: Diode laser filamentation

Spatial evolution of filaments in broad area diode laser amplifiers

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(Received 17 November 1992; accepted for publication 14 January 1993)

We report a numerical model that demonstrates the evolution of a uniform array of filaments from random fluctuations in the input of a single-pass semiconductor laser amplifier. We also report the first direct experimental observation of the spatial evolution of filaments in a broad area active grating semiconductor laser amplifier. The observed filamentation shows good agreement with the numerical model. This agreement suggests that such filaments may result from the unstable growth of microscopic fluctuations in the input and/or nonuniformities within the amplifier.

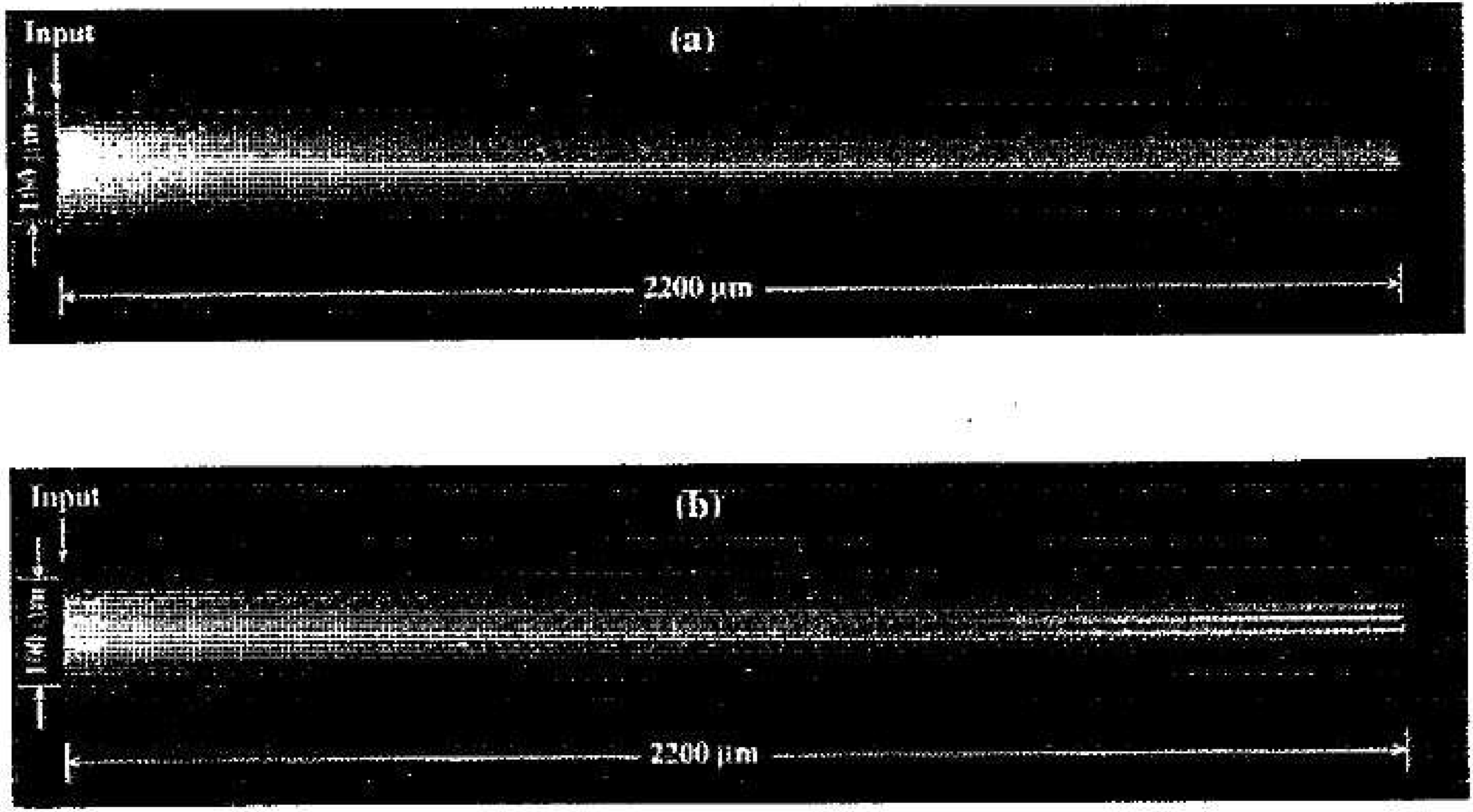


FIG. 1. Intensity vs position in a broad area amplifier. (a) With no random fluctuations. (b) With 2% random amplitude fluctuation. Both images are compressed by a factor of 8 longitudinally.

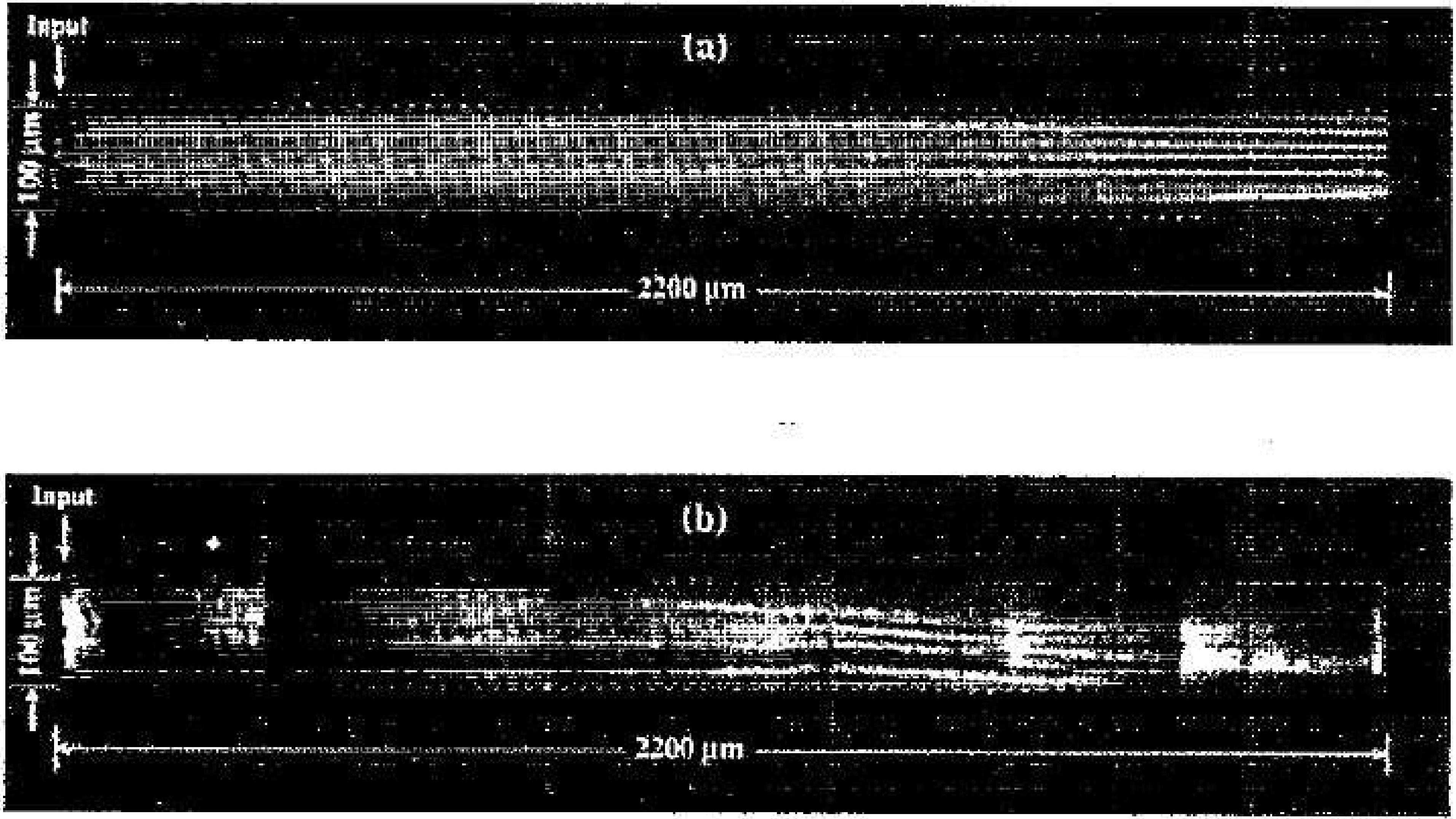
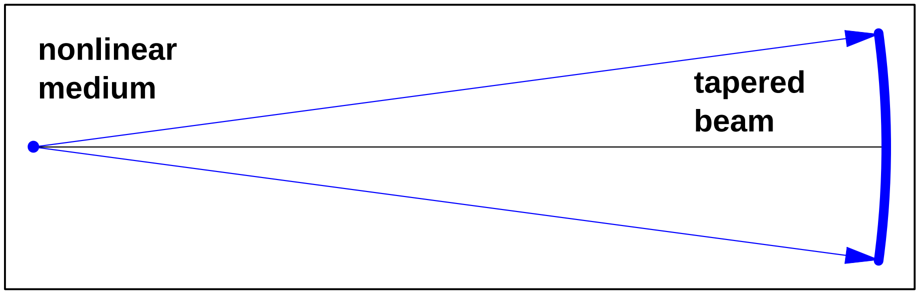


FIG. 3. Surface-emitted near field from a 100 μm active grating amplifier. (a) Theoretical plot with 10% peak-to-peak random fluctuations. (b) Experimentally measured near field. The vertical dashed lines indicate subfield boundaries in the mosaic image. Both images are compressed by a factor of 8 longitudinally.

Summary: collimated beam case

- The self-consistent small-amplitude perturbations for the collimated-beam case are periodic ripples
- In nonlinear-index or optical Kerr media these ripples grow exponentially with distance
- This leads, for example, to
  - Beam breakup and damage in high-power glass lasers
  - Severe filamentation in wide-stripe semiconductor lasers
- In saturable gain media the ripples decay with distance

Tapered beam case: converging or diverging waves



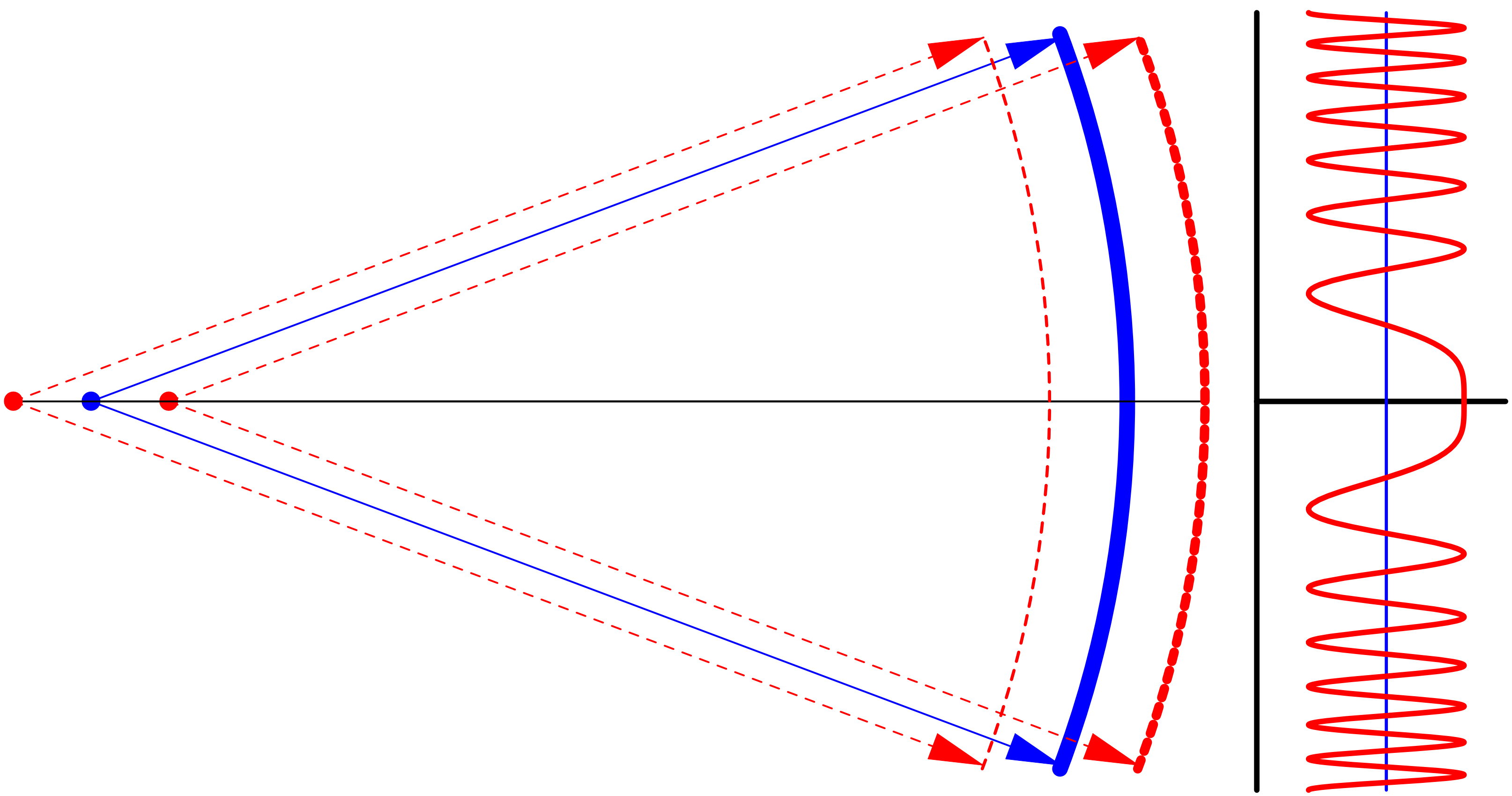
What are the analogous self-consistent patterns for tapered beams in similarly nonlinear media?

Answers appear to be:

- The self-consistent patterns for tapered beams are nonlinear Newton's rings or Fresnel zone plates
- These patterns maintain their shape with distance, but do not grow exponentially

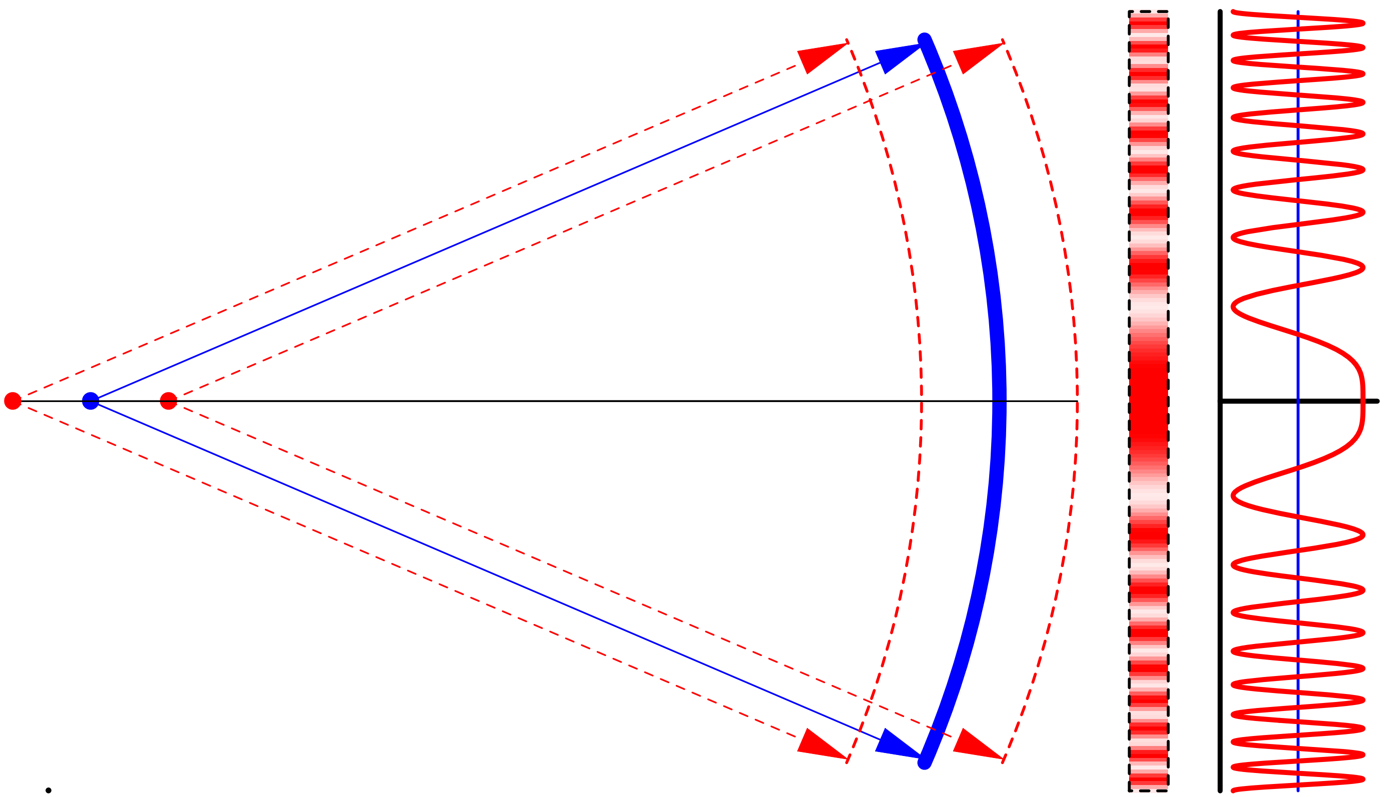
Tapered beam case: Newton's rings

Axially displaced spherical waves will interfere
to produce Newton's ring patterns:



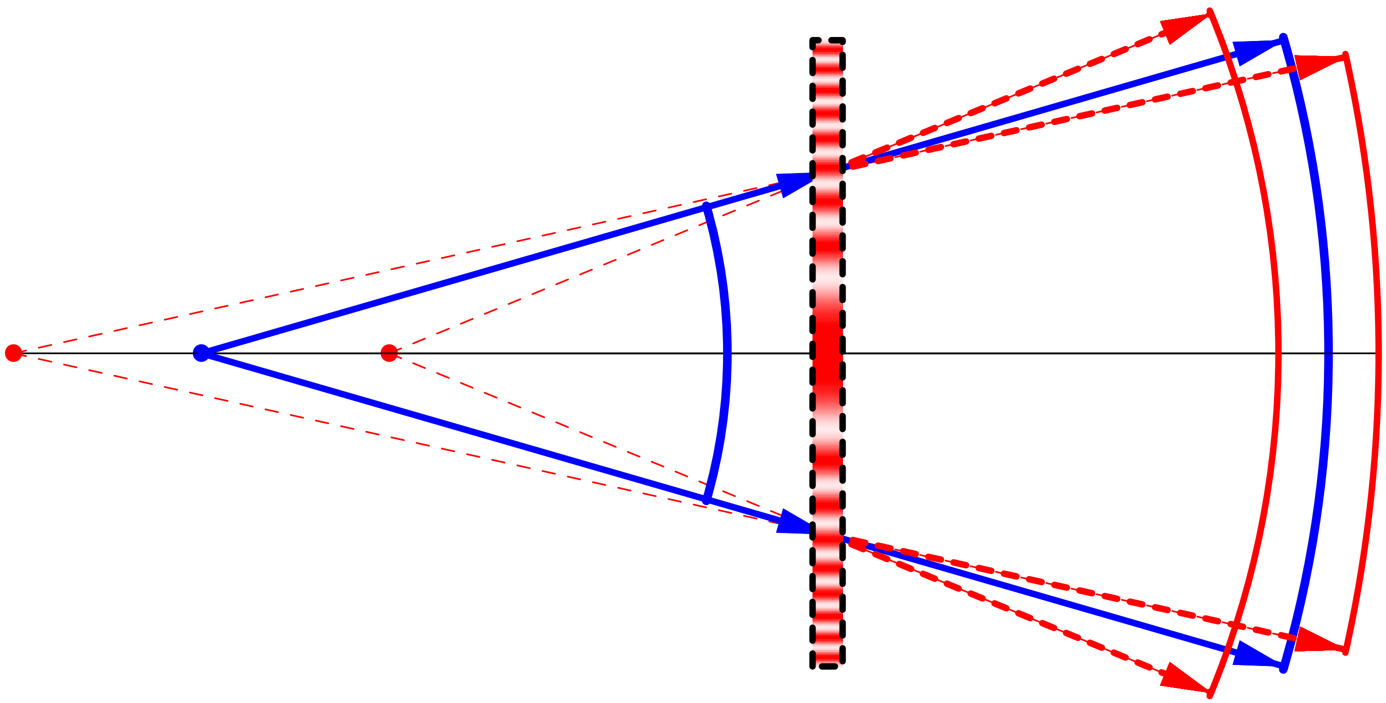
Tapered beam case: Fresnel zone plate

These Newton's rings will then produce in a nonlinear medium a nonlinear <u>Fresnel zone plate</u>:



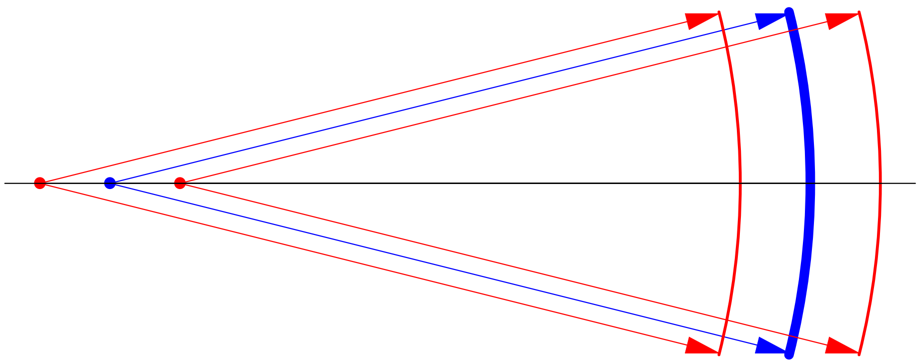
Tapered beam case: coupled spherical waves

This Fresnel zone plate will diffract energy from the primary spherical wave back into the same secondary waves that produced the zone plate



leading to linear coupling between the two secondary waves

Tapered beam expansion



Expand the tapered beam profile in the form

$$\tilde{u}(r,z) = u_0\, e^{g(z)-j\phi(z)} \left[\frac{\exp\left(-jkr^2/2z\right)}{z^{N/2}} + \tilde{c}_1(z) \frac{\exp\left(-jkr^2/2(z+a)\right)}{(z+a)^{N/2}} + \tilde{c}_2(z) \frac{\exp\left(-jkr^2/2(z-a)\right)}{(z-a)^{N/2}} \right]$$

Waves may be cylindrical $(N = 1)$ or spherical $(N = 2)$

Approximate formulation

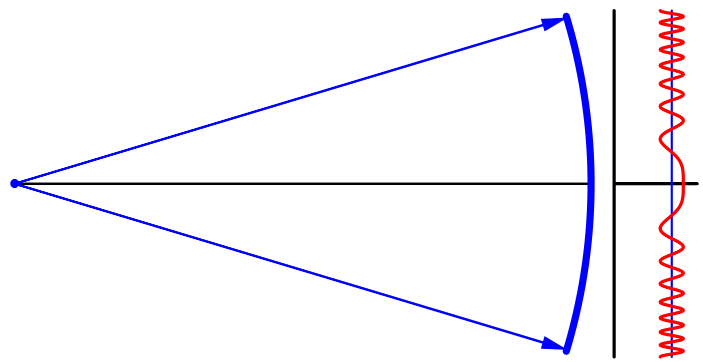
At large enough distances from the origin so that $|z| \gg a$, this formulation can be be expanded to first order in the form

$$\tilde{u}(x,z) \simeq \frac{u_0\, e^{g(z)-j\phi(z)-jkx^2/2z]}}{z^{N/2}} \left[1+\tilde{c}(z)\ \cos\frac{k\,ax^2}{2z^2}\right]$$

$$\simeq \frac{u_0\, e^{g(z)-j\phi(z)-jkx^2/2z]}}{z^{N/2}} \left[1+\left[c_r(z)+jc_i(z)\right]\ \cos\frac{k\,ax^2}{2z^2}\right]$$

in which the coefficients $\tilde{c}(z) \equiv c_r(z) + jc_i(z)$ describe the "Fresnel ripple pattern" on the tapered beam

Tapered beam intensity profile



The beam amplitude profile is then (approximately

$$I(x,z) \simeq \frac{u_0\, e^{g(z)-j\phi(z)-jkx^2/2z]}}{z^{N/2}} \left[1 + c_r(z)\, \cos\frac{k\,ax^2}{2z^2}\right]$$

which says that

$$c_r(z) \simeq \text{tapered beam amplitude ripple}$$

$$c_i(z) \simeq \text{tapered wavefront phase ripple}$$

Tapered beam coupled wave equations

The coupled linear equations for the cosinusoidal ripple coefficients $c_r(z)$ and $c_i(z)$ then become

$$\frac{dc_r(z)}{dz} \simeq -\frac{2g_2u_0^2}{z^N}\,c_i(z)$$

$$\frac{dc_i(z)}{dz} \simeq -\frac{2k_2u_0^2}{z^N}\,c_r(z)$$

No Talbot term $\kappa^2/2k_0$ in these equations
(at least for $z \gg a$)

(a) Cylindrical wave, nonlinear index only:

For cylindrical waves with finite k_2 and $g_2 = 0$,
coupled equations are

$$\frac{dc_r(z)}{dz} \simeq 0$$

$$\frac{dc_i(z)}{dz} \simeq -\frac{2k_2 u_0^2}{z}\, c_r(z)$$

with analytic solutions

$$c_r(z) = c_{r0}$$

$$c_i(z) = c_{i0} - \left[2k_2 u_0^2 \log(z/z_0)\right] c_{r0}$$

Intensity ripples remain constant with distance;
phase ripples decrease (slowly) with distance.

(b) Cylindrical wave, gain saturation only:

For cylindrical wave with $k_2 = 0$ and finite g_2, equations are

$$\frac{dc_r(z)}{dz} \simeq -\frac{2g_2u_0^2}{z}\,c_i(z)$$

$$\frac{dc_i(z)}{dz} \simeq 0$$

with analytic solutions

$$c_r(z) = \left(\frac{z_0}{z}\right)^{2g_2u_0^2}\,c_{r0}$$

$$c_i(z) = c_{i0}$$

Intensity ripples decrease slowly with distance,
while phase ripples remain constant

(c) Spherical wave, nonlinear index only:

For cylindrical waves with finite k_2 and $g_2 = 0$,
coupled equations are

$$\frac{dc_r(z)}{dz} \simeq 0$$

$$\frac{dc_i(z)}{dz} \simeq -\frac{2k_2 u_0^2}{z^2}\, c_r(z)$$

with analytic solutions

$$c_r(z) = c_{r0}$$

$$c_i(z) = c_{i0} - 2k^2 u_0^2 \left(\frac{1}{z_0} - \frac{1}{z}\right) c_{r0}$$

Again intensity ripples remain constant, while
phase ripples increase (slowly) with distance

(d) Spherical wave, gain saturation only:

For spherical wave with $k_2 = 0$ and finite g_2, equations are

$$\frac{dc_r(z)}{dz} \simeq -\frac{2g_2u_0^2}{z^2}\,c_i(z)$$

$$\frac{dc_i(z)}{dz} \simeq 0$$

with analytic solutions

$$c_r(z) = c_{r0}\exp\left[-2k_2u_0^2\left(\frac{1}{z_0} - \frac{1}{z}\right)\right]$$

$$c_i(z) = c_{i0}$$

Again intensity ripples decrease with distance,
while phase ripples remain unchanged

Strongly Tapered Approximation

The strongly tapered approximation used in this analysis is $z \gg a$. The number of Fresnel rings N_R within a beam of half-angular spread $\pm\theta$ will be

$$N_R \simeq \frac{a x_1^2}{2z^2\lambda} \simeq \left(\frac{a}{2\lambda}\right)\theta_1^2$$

so that $N_R \gg 1$ also requires that $|a| \gg \lambda$

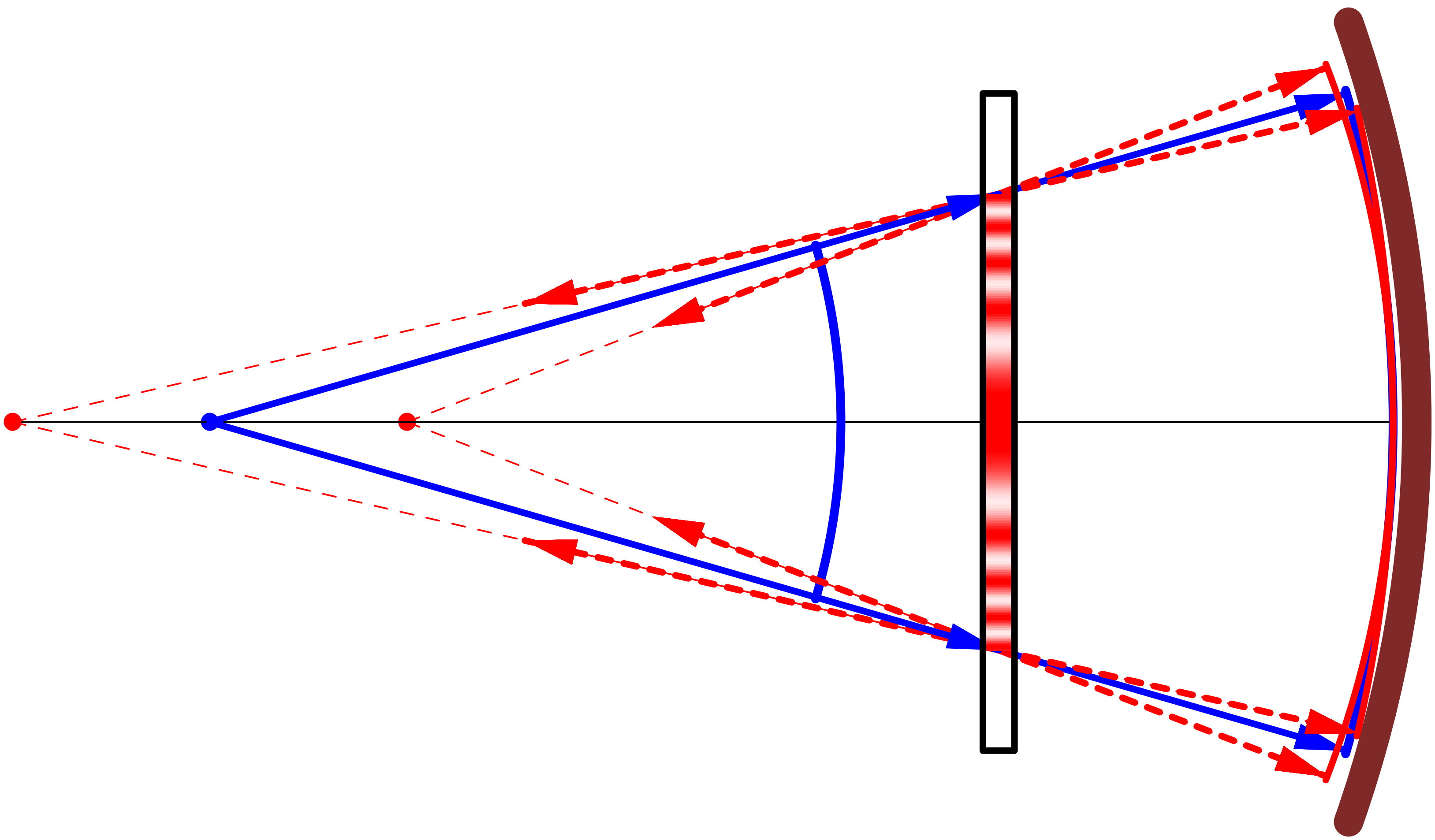
For a TEM$_{00}$ primary beam with Rayleigh range $z_R \equiv \pi w_0^2/\lambda$ one can further calculate that

$$N_R \simeq \frac{1}{8\pi}\,\frac{a}{z_R}$$

and hence $a \gg z_r$ along $|z| \gg a$

Tapered beam pattern formation?

Spontaneous Fresnel pattern formation in a nonlinear film?



Conclusions

1) Small-scale self focusing effects are very different for collimated and tapered beams
   - Ripples on tapered beams do not show exponential growth under any circumstances
   - Phase ripples do show weak and limited growth.

2) Difference is due to loss of Talbot phase shift term
   - In collimated beam case Talbot effect periodically converts phase ripples into amplitude ripples and vice versa
   - This makes possible exponential growth of phase-grating-induced ripples

3) Self-focusing is therefore (fortunately) a weak process for strongly tapered beams in optical Kerr or saturable gain media

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