Optics with Gain

How does the presence of laser gain modify ordinary optical phenomena?

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Adding gain to an optical medium

Optical wave propagation with real-valued refractive index n:

$$\mathcal{E}(z) = \exp\left[-j\,\frac{2\,\pi\,n}{\lambda}\,z\right]$$

Adding gain or loss makes n complex-valued:

$$\mathcal{E}(z) = \exp\left[-j\,\frac{2\,\pi\,n}{\lambda}\,z + \alpha\,z\right]$$

$$= \exp\left[-j\,\frac{2\,\pi}{\lambda}\left(n + j\,\frac{\lambda}{2\pi}\,\alpha\right)\,z\right]$$

$$= \exp\left[-j\,\frac{2\,\pi\,\tilde{n}}{\lambda}\,z\right]$$

The added index component is very small

The imaginary part of this complex refractive index

$$\tilde{n} = n_r + jn_i = n_r + j\,\frac{\lambda}{2\pi}\alpha$$

will be very small, except in very lossy cases (e.g., metals)

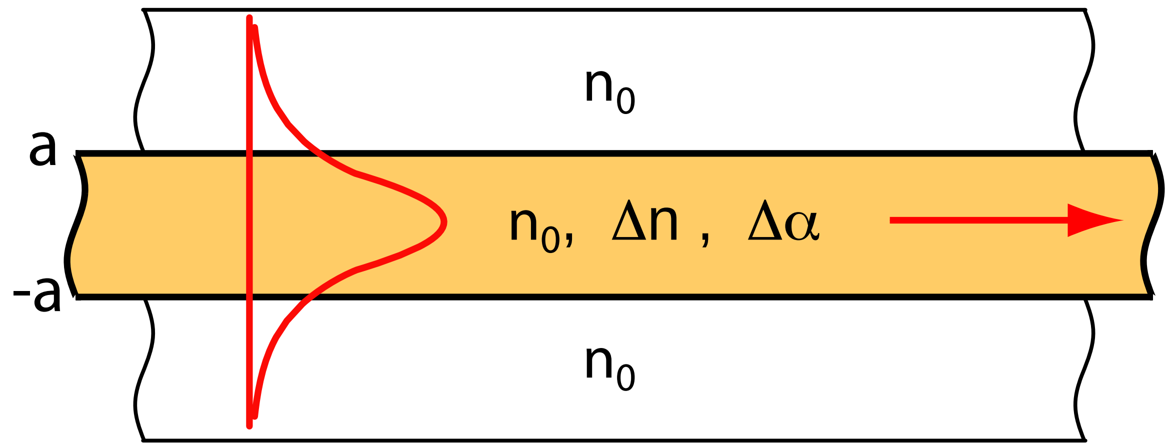
For example, the gain coefficient in a heavily doped, heavily pumped laser medium might be $\alpha \sim 1$ cm^{-1}, leading to

$$n_i \equiv \left(\frac{\lambda}{2\pi}\right)\alpha \sim 10^{-5} \quad \text{at } \lambda = 1 \text{ micron}$$

Consequences of gain in optical systems

- Laser amplifiers
  - Optical fiber amplifiers, EDFAs
- Laser oscillators
  - Regenerative gain in etalons and laser cavities
- Gain guiding of optical waves
  - Used in diode lasers, perhaps in optical fibers
- Modified optical wave propagation
  - Inhomogeneous plane waves
- Modified Fresnel reflection
  - Total internal reflection from a “gainy” medium?
  - Evanescent gain: does it exist?

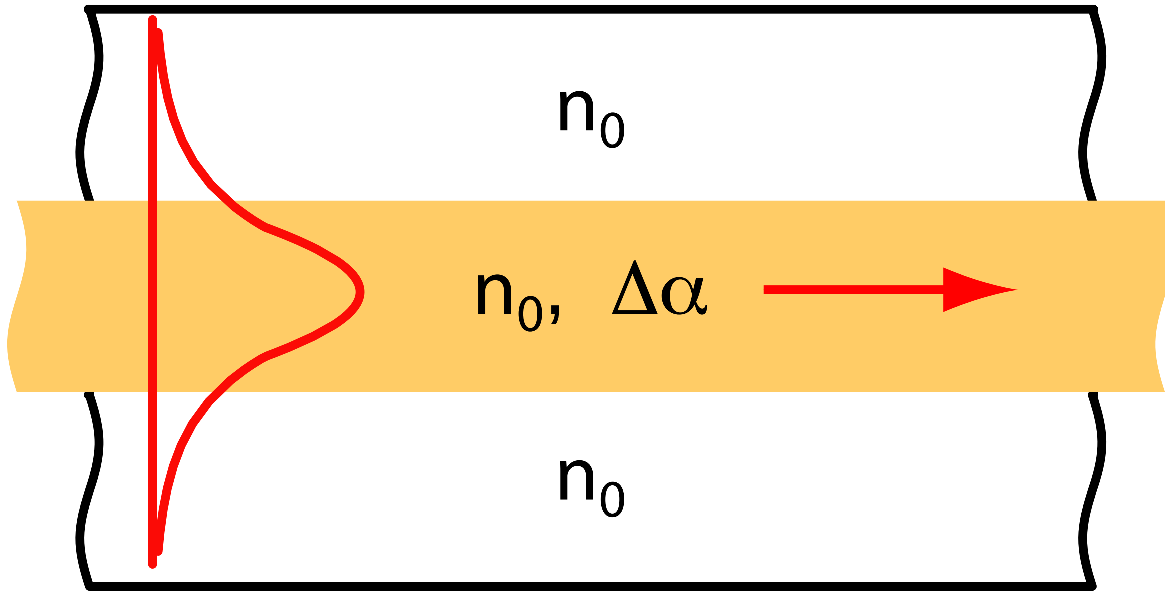
Guided optical modes (slab waveguides, fibers)



- Optical waves are usually guided by <u>index</u> variations:
  - Conventional dielectric waveguides and optical fibers
- They can also be guided by <u>gain</u> variations:
  - <u>Gain-guided</u> waveguides and optical fibers
  - <u>Mixtures</u> of gain and index guiding

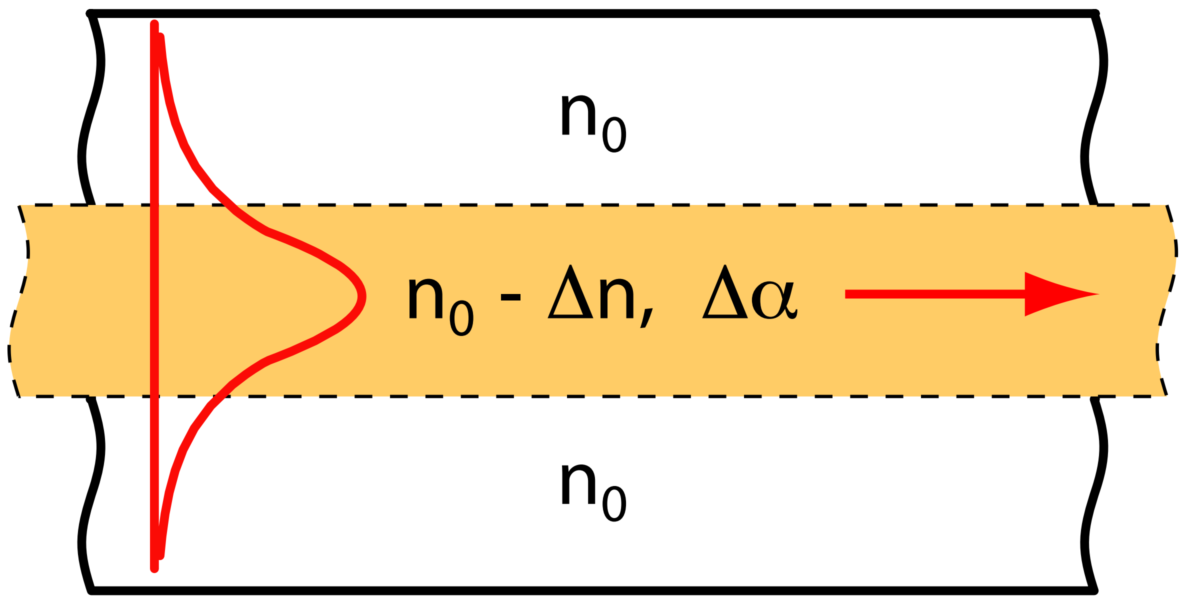
Purely gain guided optical modes

Pure gain guiding, with no index step

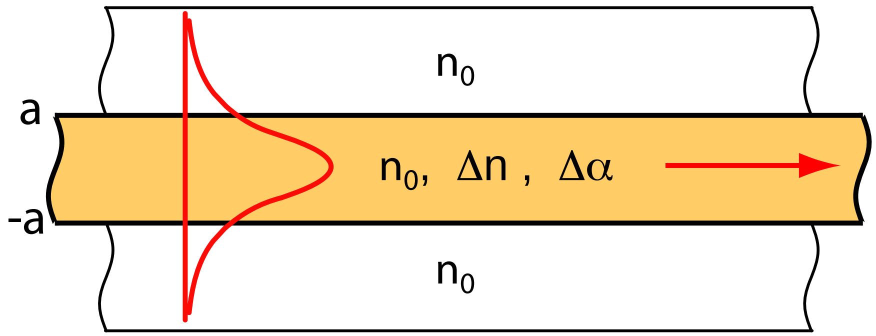


Gain guided, index anti-guided optical modes

Gain guiding plus index anti-guiding, with a negative index step: $-\Delta n$ in the core



Waveguide/fiber v and v-squared parameters



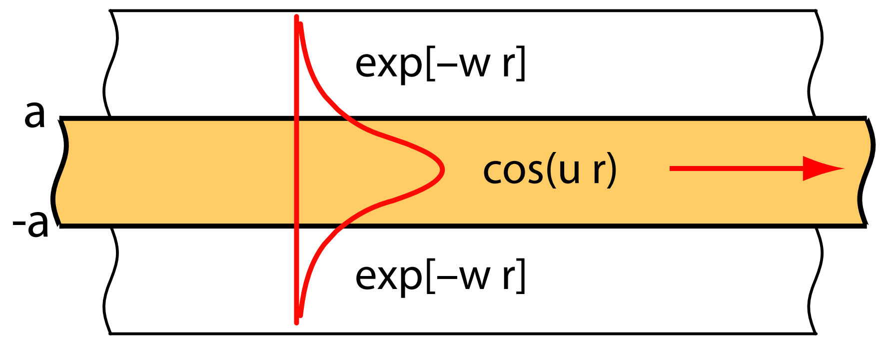
"v parameter" for purely index-guided optical waveguides

$$v \equiv \left(\frac{2\pi a}{\lambda}\right) \sqrt{2\, n_0\, \Delta n}$$

"ṽ-squared" parameter for gain plus index guiding

$$\tilde{v}^2 \equiv \left(\frac{2\pi a}{\lambda}\right)^2 2n_0 \left[\Delta n + j\,\frac{\lambda}{2\pi}\Delta\alpha\right]$$

Slab waveguide analysis



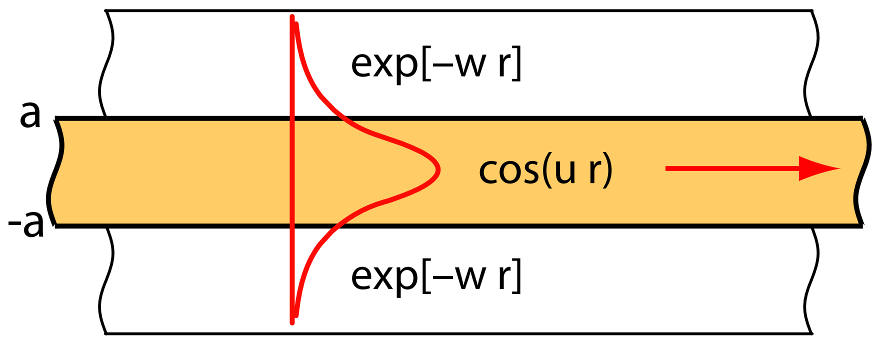
Core region: $\tilde{E}(x) = \cos(\tilde{u}\, x/a), \qquad |x| \leq a$

Cladding region: $\tilde{E}(x) = \exp[-\tilde{w}\,(|x| - a)/a], \quad |x| \geq a$

With gain guiding, the $\tilde{u}$ and $\tilde{w}$ parameters become complex-valued, and related to the $\tilde{v}^2$ by

$$\tilde{w}^2 + \tilde{u}^2 = \tilde{v}^2 \equiv \left(\frac{2\pi a}{\lambda}\right)^2 2n_0 \left[\Delta n + j\,\frac{\lambda}{2\pi}\Delta\alpha\right]$$

Slab waveguide dispersion equations



Requiring that mode function and slope be continuous at slab edges leads to dispersion relations:

$$\tilde{w} = \tilde{u} \tan \tilde{u}$$ for symmetric modes

$$\tilde{w} = \frac{-\tilde{u}}{\tan \tilde{u}}$$ for antisymmetric modes

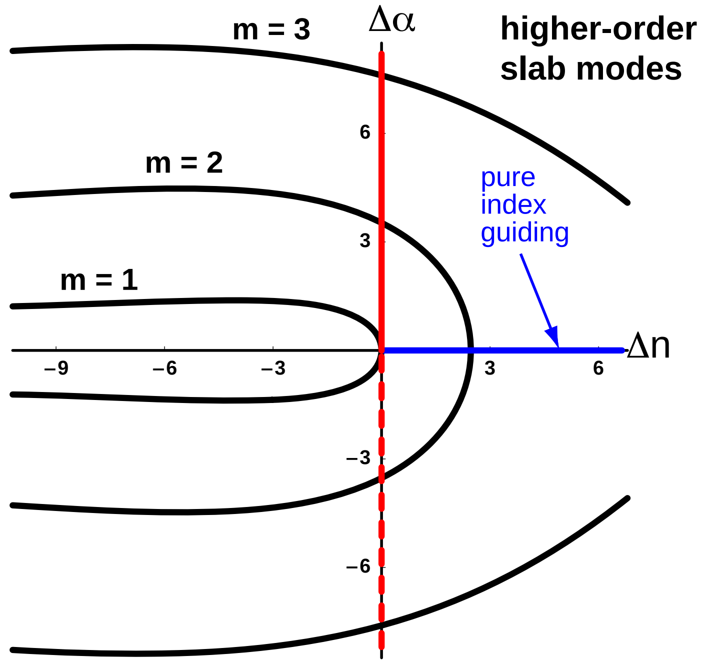
which must be satisfied in combination with

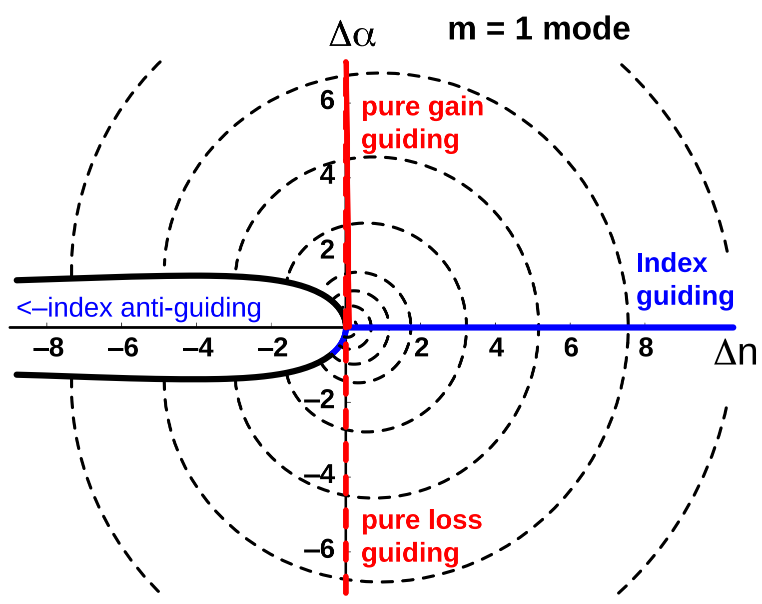
$$\tilde{w}^2 + \tilde{u}^2 = \tilde{v}^2$$

Allowed ṽ-squared values

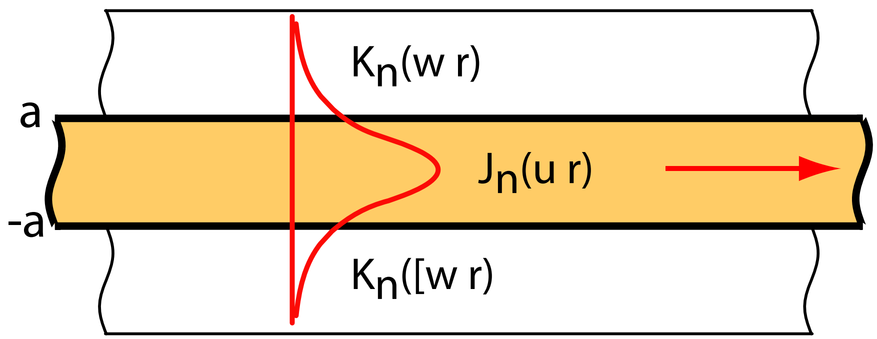
Allowed regions in complex $\tilde{v}^2$ plane for slab waveguide modes

(Horizontal axes are index guiding;
vertical axes are gain guiding)





Cylindrical fiber modes



LP01 mode of cylindrical fiber:

$$\tilde{E}_{01}(r) = \begin{cases} J_0(\tilde{u}\, r/a), & r \leq a \\ K_0(\tilde{w} r/a), & r \geq a \end{cases}$$

LP11 mode of cylindrical fiber:

$$\tilde{E}_{11}(r) = \begin{cases} J_1(\tilde{u}\, r/a), & r \leq a \\ K_1(\tilde{w} r/a), & r \geq a \end{cases}$$

Fiber dispersion equations

Matching functions and slopes at the core-cladding interface again leads to dispersion equations:

$$\frac{\tilde{u}J_1(\tilde{u})}{J_0(\tilde{u})} = \frac{\tilde{w}K_1(\tilde{w})}{K_0(\tilde{w})} \qquad \text{for LP01 modes}$$

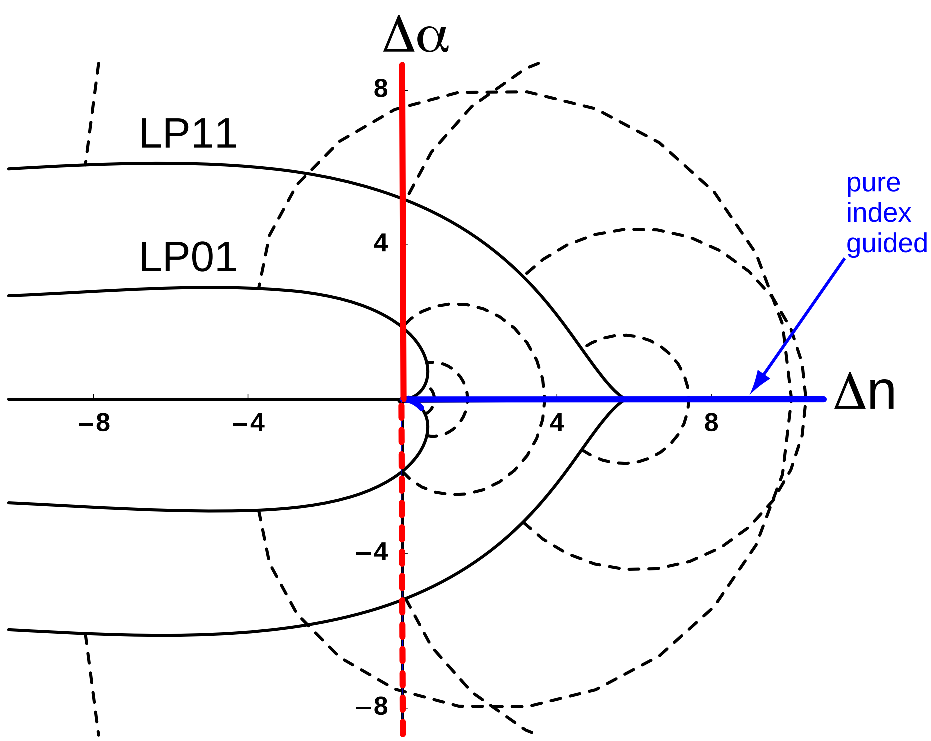
$$\frac{\tilde{u}J_1(\tilde{u})}{J_1(\tilde{u})} = -\frac{\tilde{w}K_0(\tilde{w})}{K_1(\tilde{w})} \qquad \text{for LP11 modes}$$

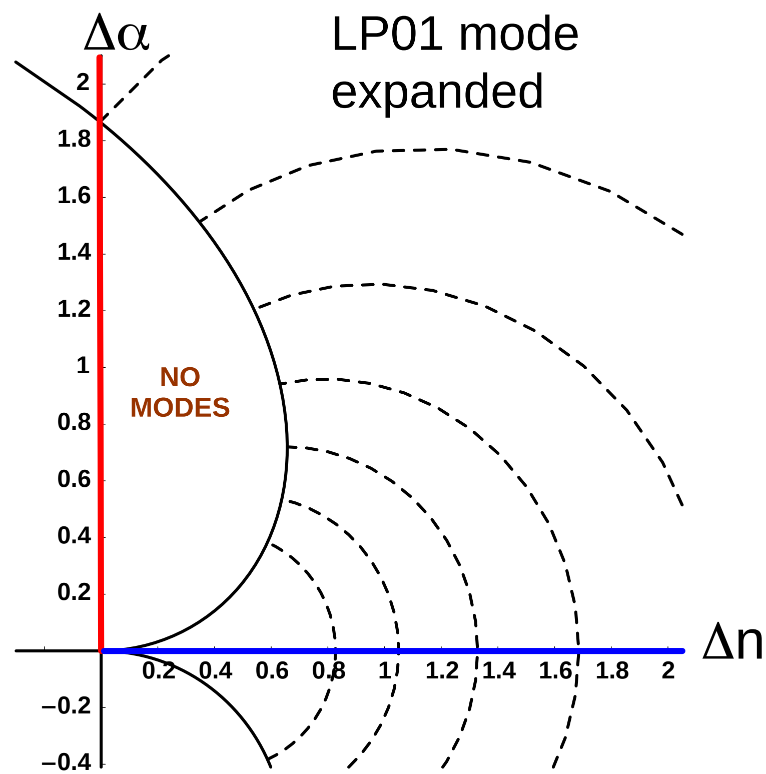
which must again be satisfied in combination with

$$\tilde{w}^2 + \tilde{u}^2 = \tilde{v}^2$$

Allowed ṽ-squared values

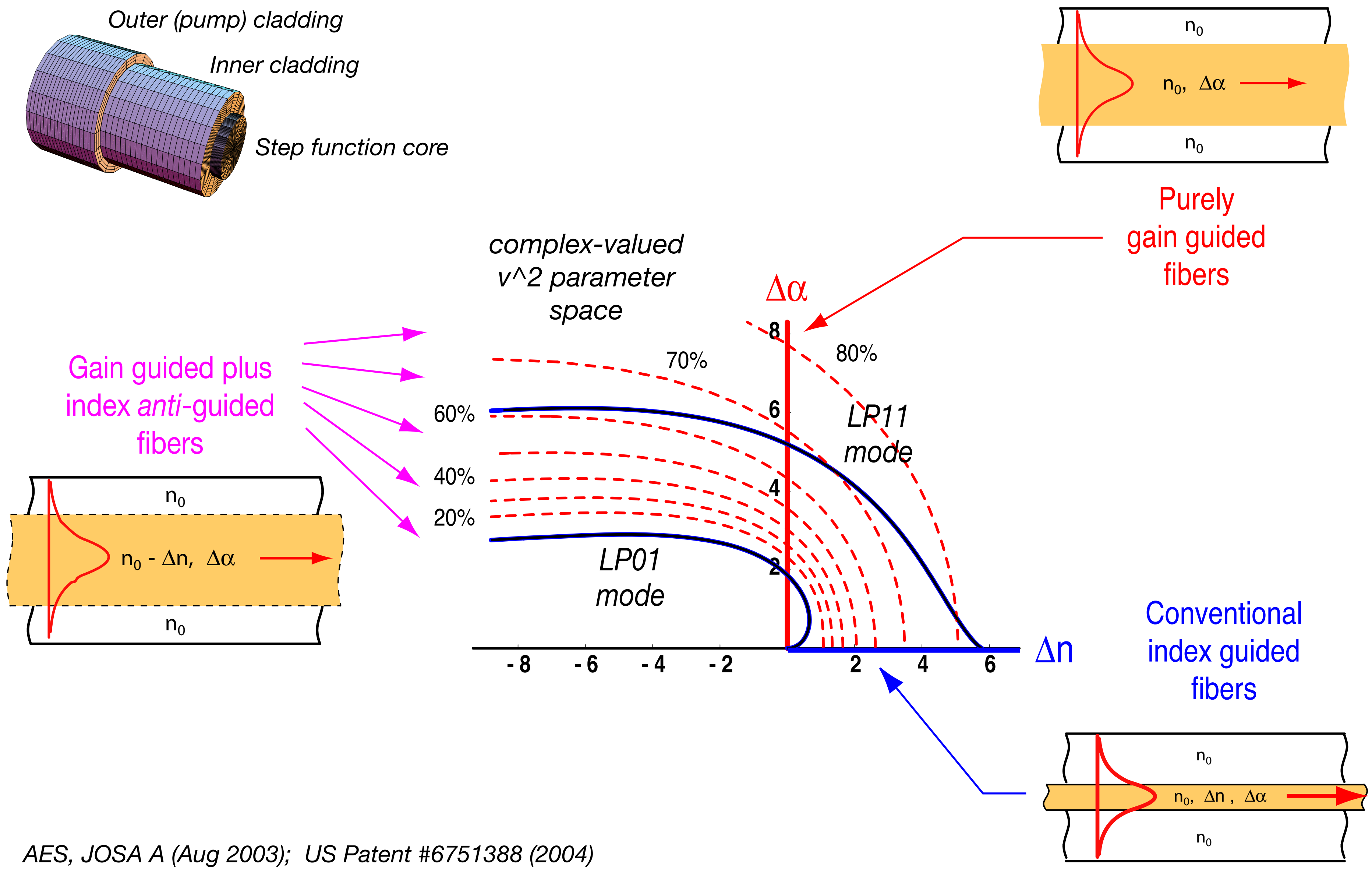
Allowed regions in complex $\tilde{v}^2$ plane for cylindrical fiber modes





Horizontal axes = index guiding

vertical axes = gain or loss guiding



AES, JOSA A (Aug 2003); US Patent #6751388 (2004)

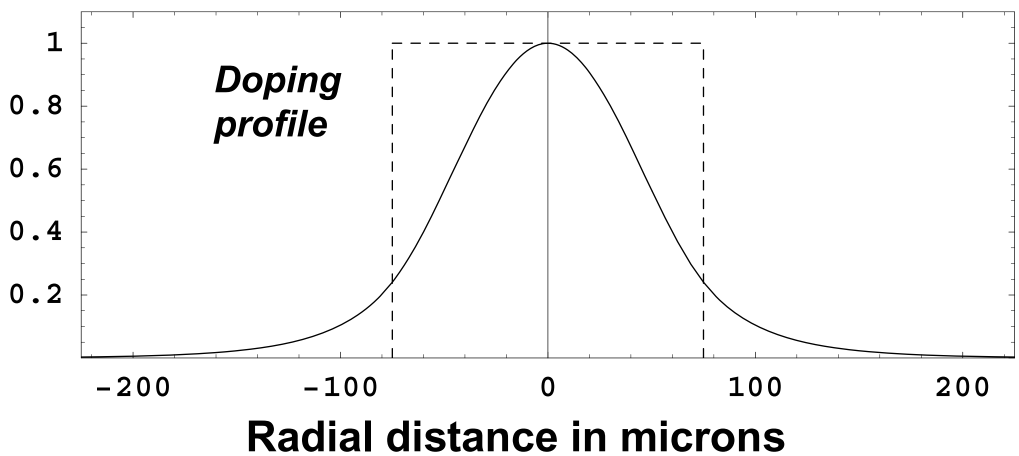
Step gain-guided example

Doping diameter = <u>150 microns</u>

On-axis gain = <u>1/cm</u>

TEM00 mode gain = 0.65/cm

Percent power in core = <u>65%</u>



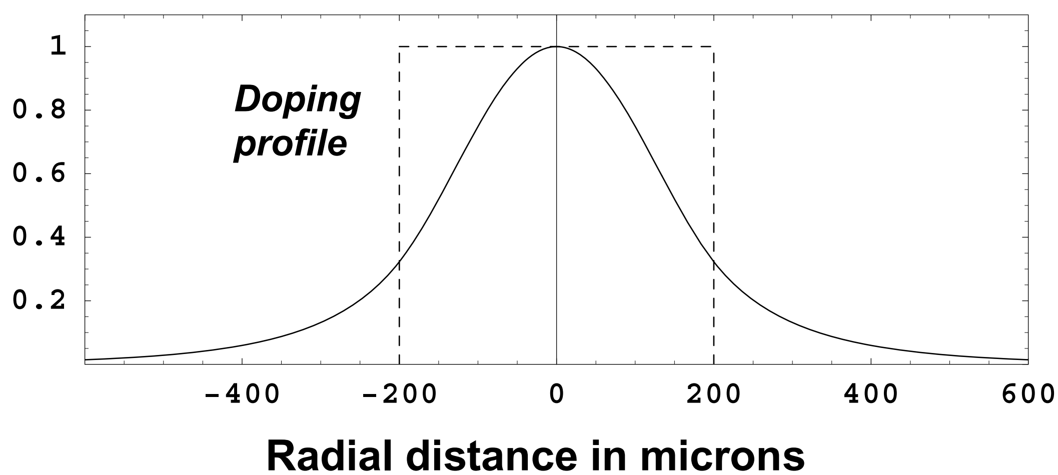
Step gain-guided example

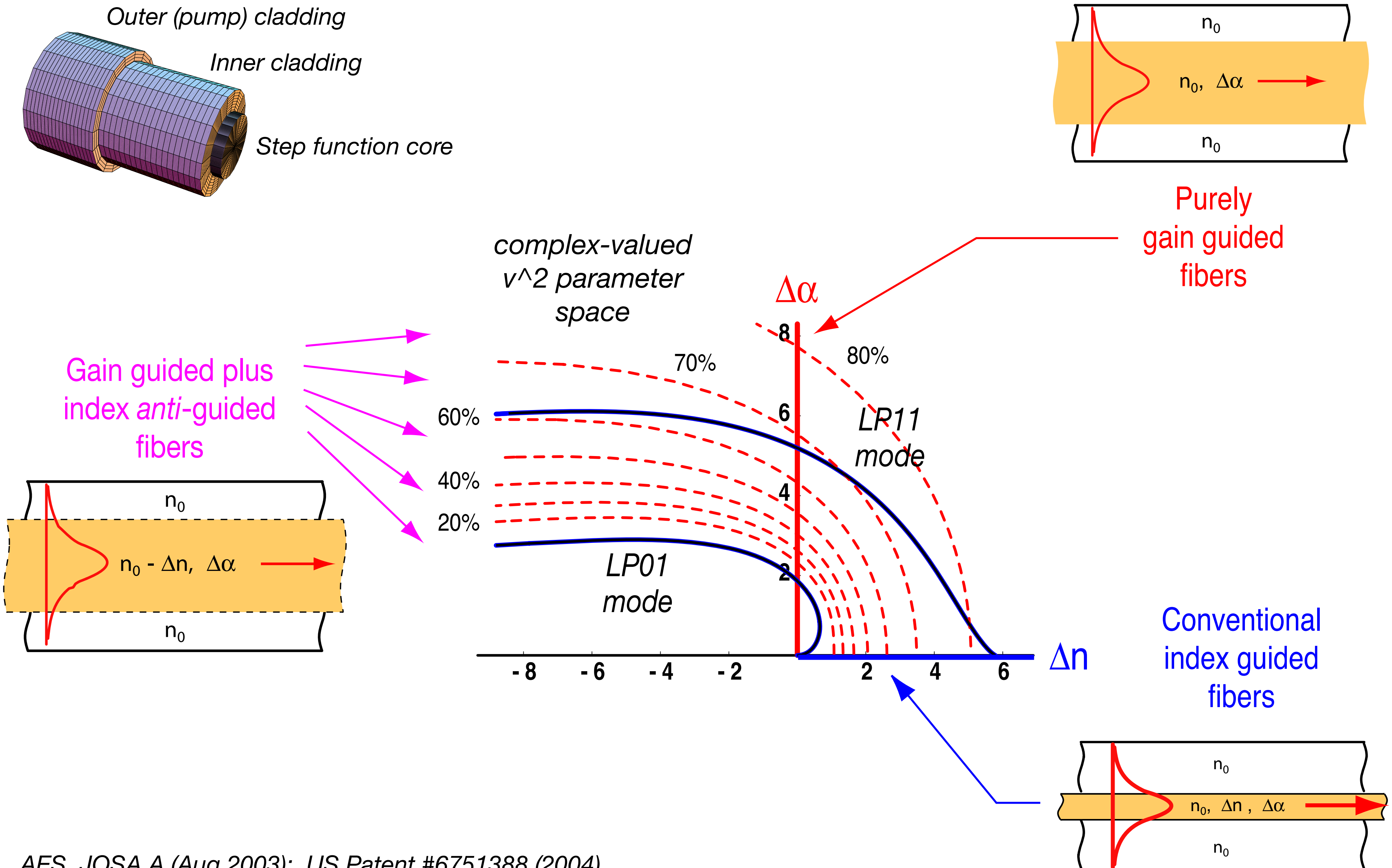
Doping diameter = <u>400 microns</u>

On-axis gain = <u>0.1/cm</u>

TEM00 mode gain = 0.48/cm

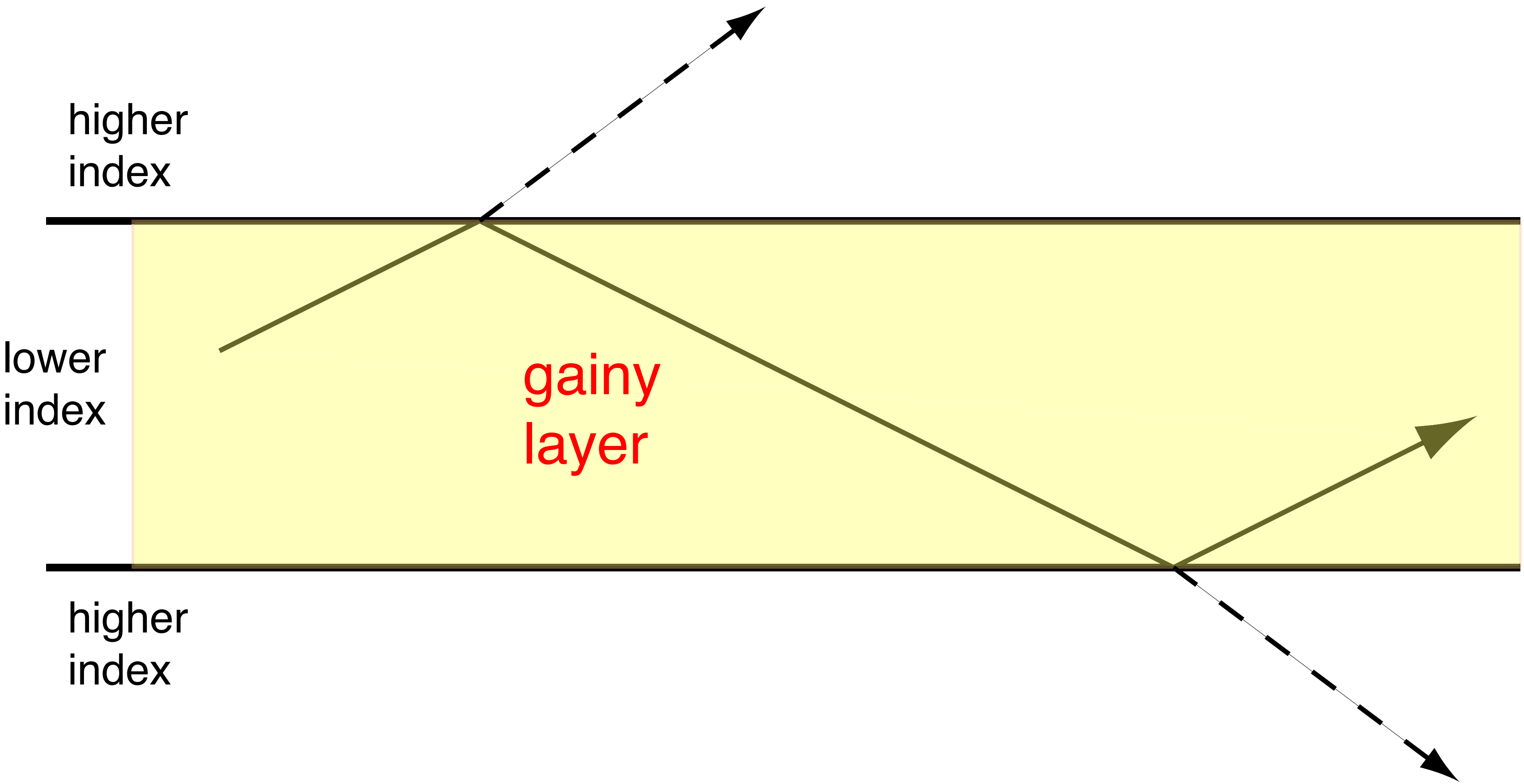
Percent power in core = <u>48%</u>





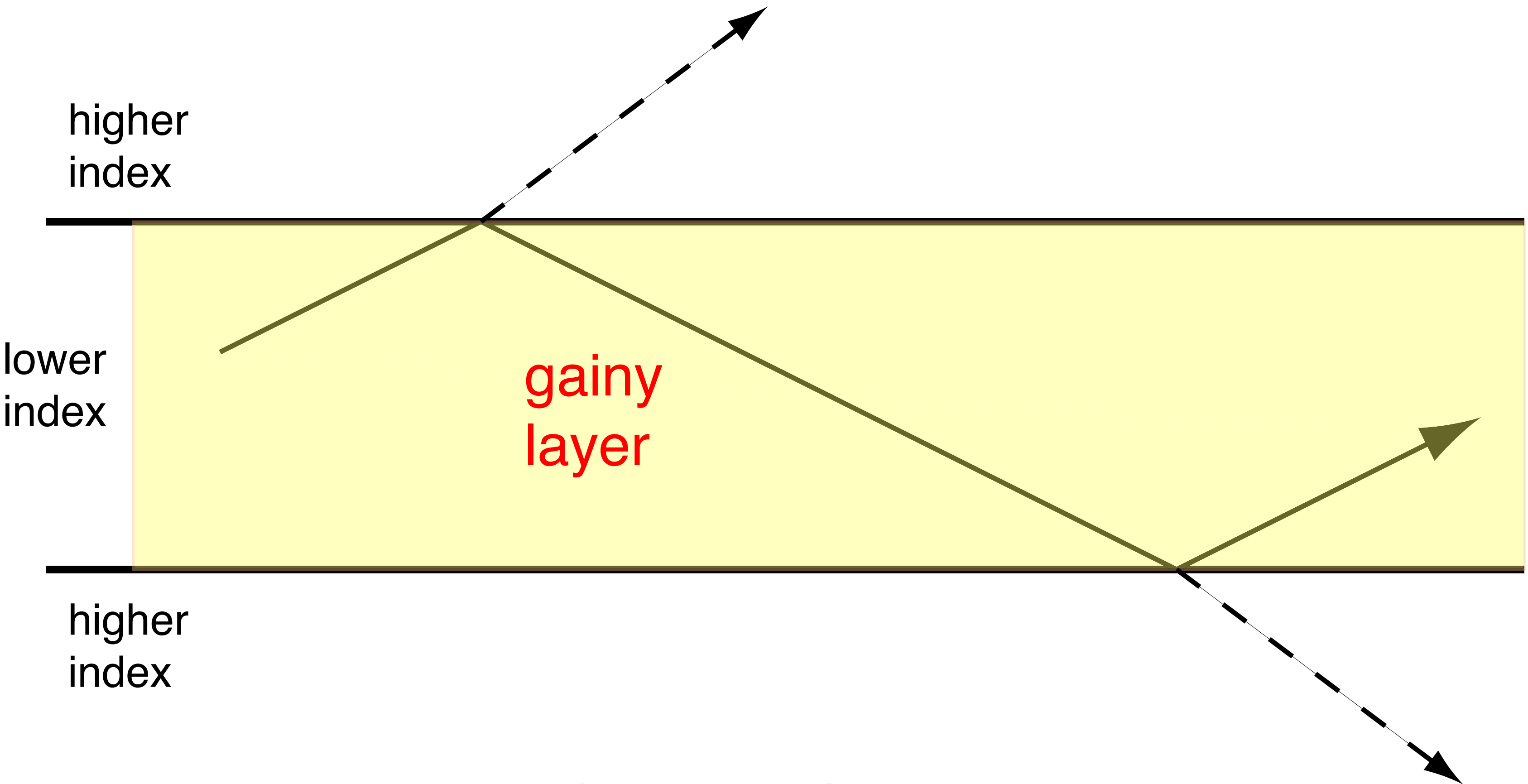
AES, JOSA A (Aug 2003); US Patent #6751388 (2004)

How does index anti-guiding plus gain guiding work?



Just make the layer thick

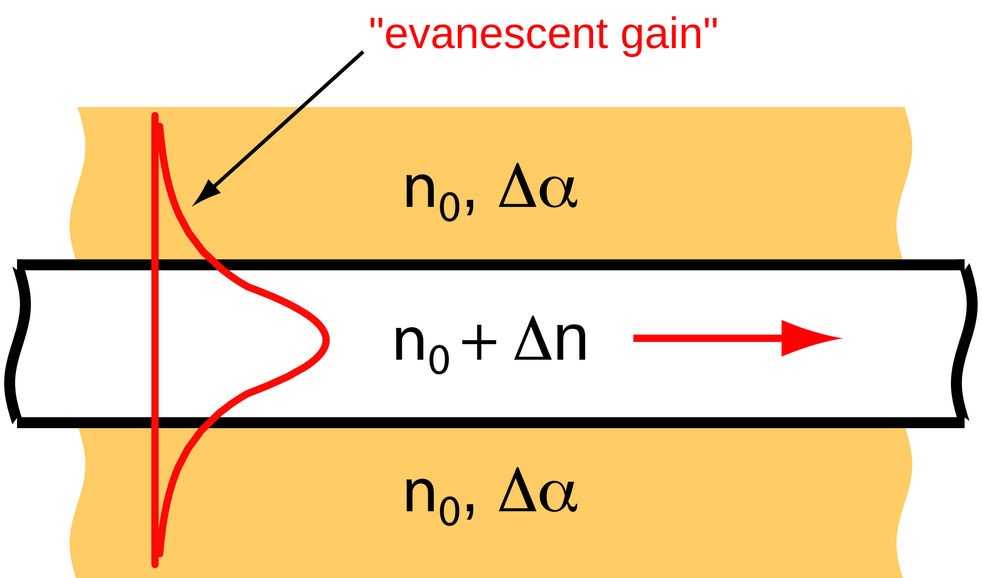
How does index anti-guiding plus gain guiding work?

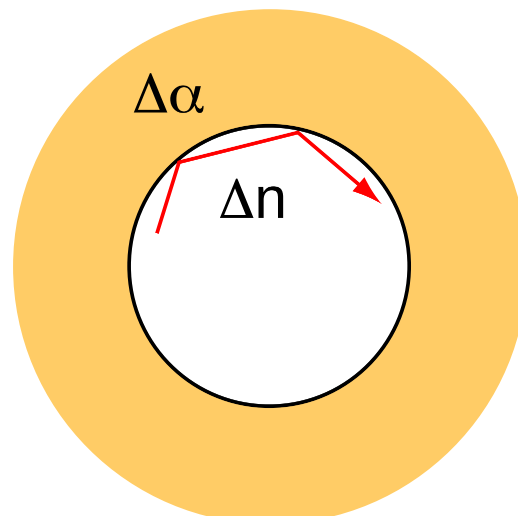


Surfaces always reflect . . . just make gain layer thick enough . . .

Evanescent gain?

Waveguides or fibers with gain only in the evanescent region (outside the core)?

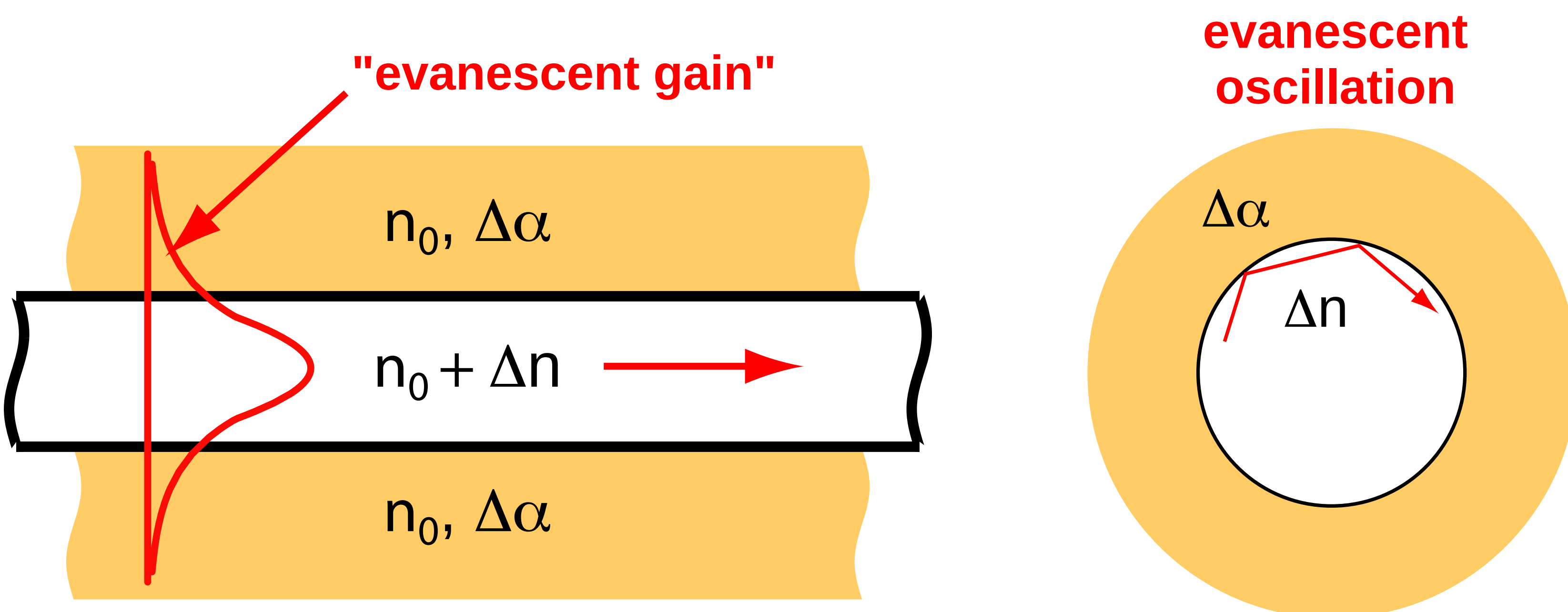




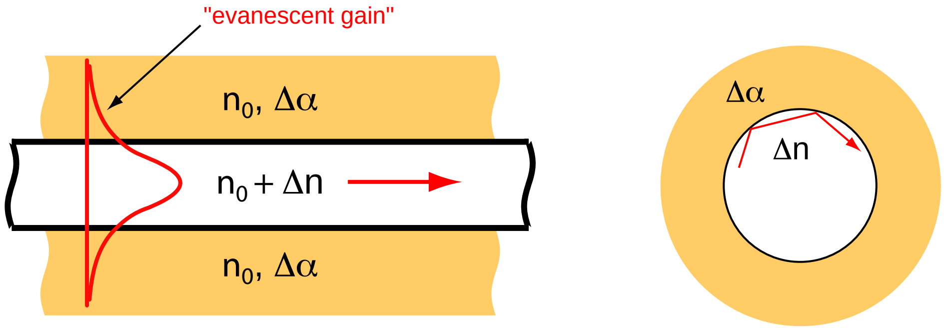
Dielectric spheres or similar dielectric resonators having whispering gallery modes, with gain only in the evanescent region outside the resonator?

Evanescent gain — does it really exist?

Do these systems really exhibit evanescent gain?



Many researchers say it does



And several claim to have observed it experimentally:

Koester, Laser action by enhanced total internal reflection, IEEE JQE 1966

Hill et al, Amplification of bound modes via evanescent-wave interactions, JOSA 1971

Hill et al, Evanescent wave amplification in asymmetric slab waveguides, JOSA 1974

Sasaki et al, Thin film waveguide evanescent dye laser, JAP 1980

Pendock, Mackenzie and Payne, Dye lasers using tapered optical fibers, Appl Opt 1993

Kozlov et al, Evanescent field thin-film optical amplifier, Electron Lett 1994

Fujiwara and Sasaki, Lasing of a microsphere in dye solution, Japan JAP 1999

Moon, Chough, and An, Microcavity laser based on evanescent gain, PRL 2000

An, Cylindrical and spherical lasers based on evanescent gain, J Chinese Chem Soc 2001

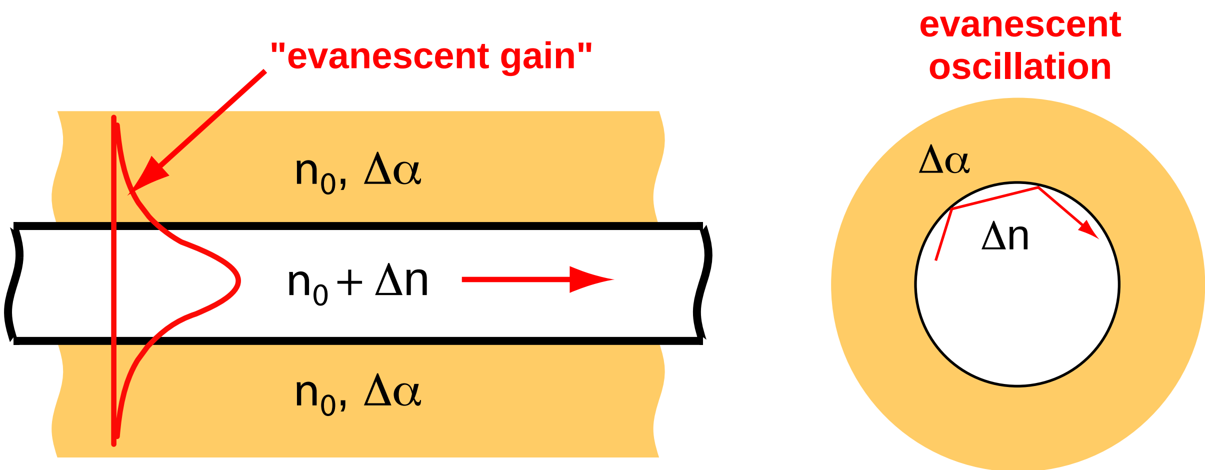
Choi et al, Microsphere laser using evanescent wave coupling, J Korean Phys Soc 2001

It's even been patented...

Hill, Watanabe and Chambers, U.S. Patent 3,950,707 (1976):

"Quantum amplifier having passive core and active cladding providing signal gain by interaction of evanescent-wave components of signal and pump beams propagating along the core."

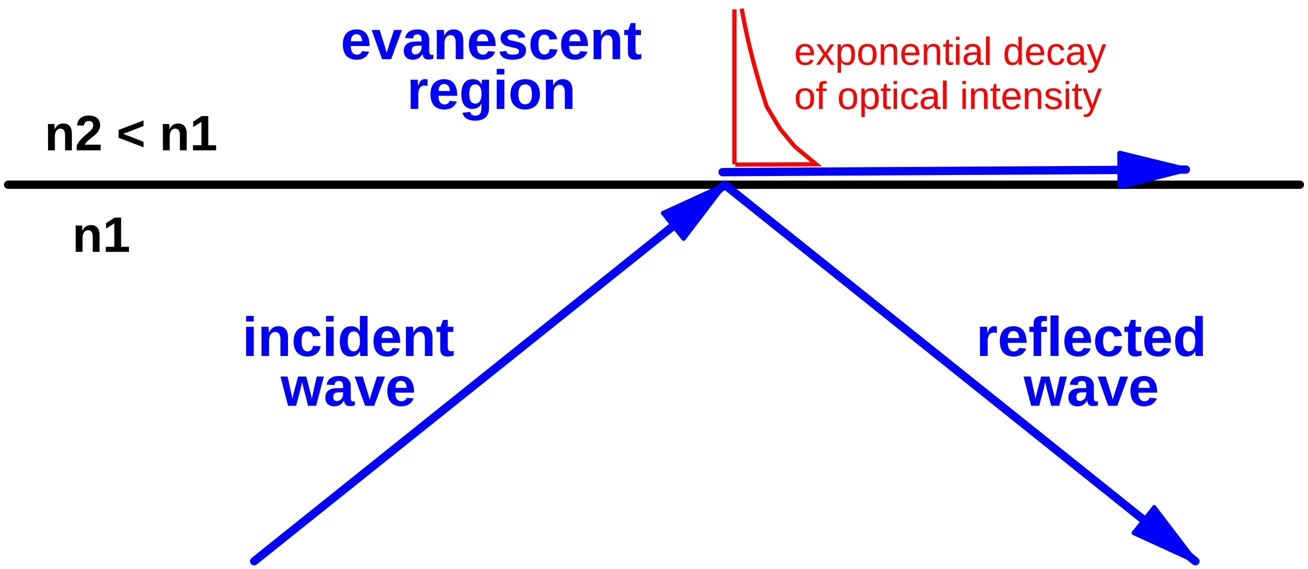
Evanescent gain implies TIR greater than unity



- Optical waveguiding implies total internal reflection (TIR)
- Evanescent gain implies TIR greater than unity
- Understanding TIR (with or without gain) requires an understanding of inhomogeneous plane waves

Total internal reflection (TIR)

Total internal reflection from a lossless dielectric interface

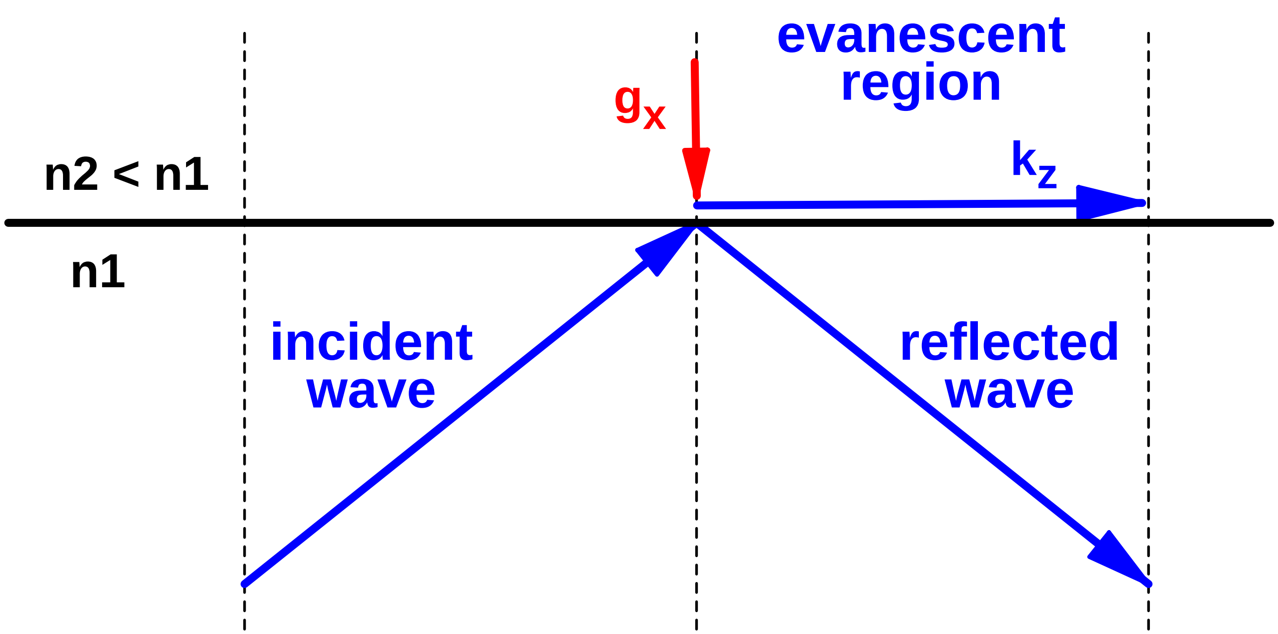


Intensity in evanescent region decays exponentially

Inhomogeneous plane wave concept

The optical field in the evanescent region is
an inhomogeneous plane wave

$$\tilde{u}(x, z) = \exp[-j\, k_z z + g_x x]$$



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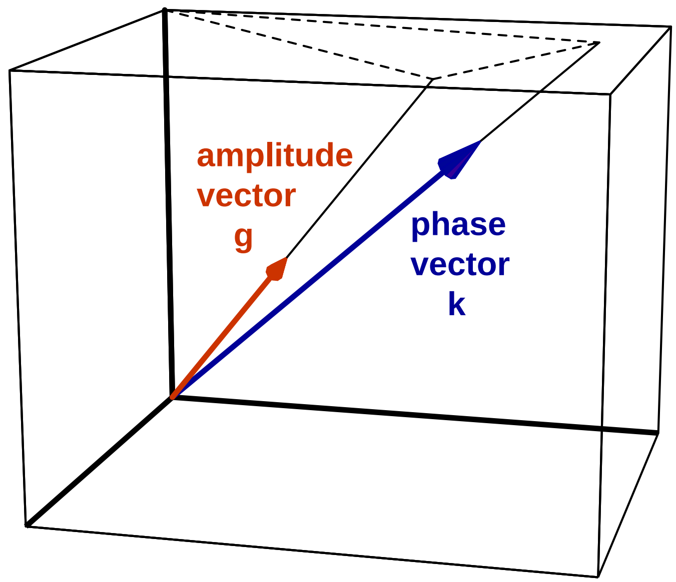
Inhomogeneous plane waves

An inhomogeneous plane wave is an infinite plane wave with nonparallel phase and amplitude propagation vectors



- Wavefronts (planes of constant phase) are perpendicular to **k**
- Wave amplitude increases exponentially in direction **g**

Inhomogeneous plane waves

Ordinary homogeneous plane wave propagates as

$$\tilde{u}(\mathbf{r}) = \exp[-j\mathbf{k} \cdot \mathbf{r}]$$

General inhomogeneous plane wave propagates as

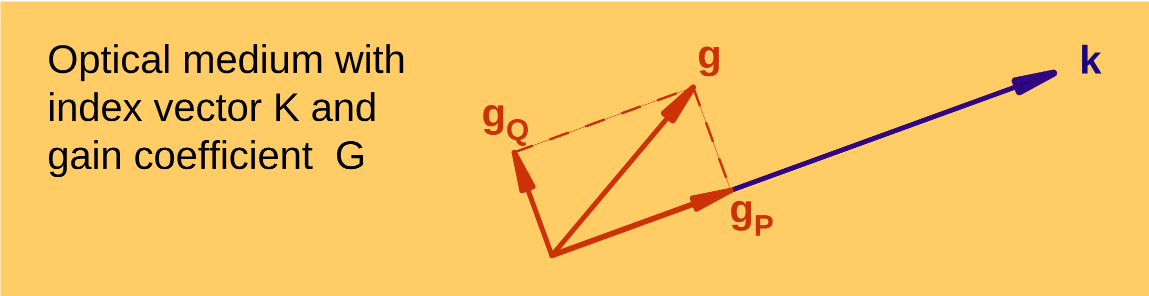
$$\tilde{u}(\mathbf{r}) = \exp[-j\,\mathbf{k} \cdot \mathbf{r} + \mathbf{g} \cdot \mathbf{r}]$$

where

$\mathbf{k} =$ "phase vector" or "wave vector"
$\mathbf{g} =$ "gain vector" or "amplitude vector"

and $\mathbf{k}$ and $\mathbf{g}$ are in general not parallel to each other

Inhomogeneous wave equation



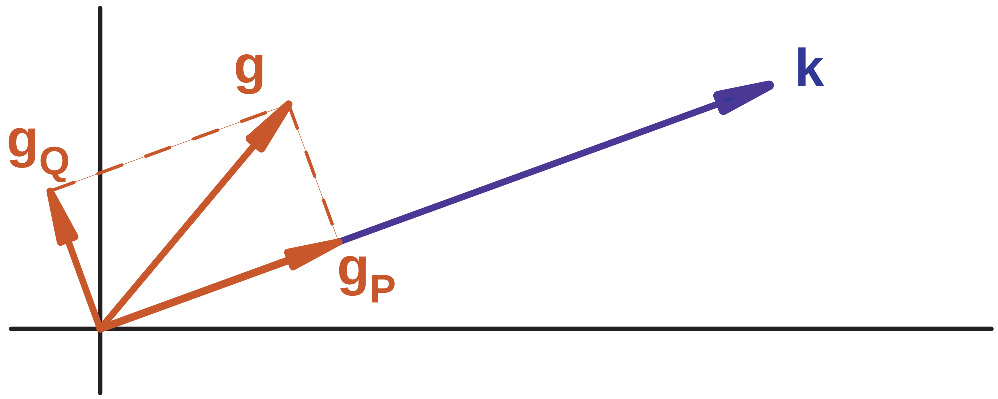
If $\mathbf{k}$ and $\mathbf{g}$ are wave quantities, while K and G are optical medium properties, wave equation becomes

$$\nabla^2 + (K + jG)^2 = (\mathbf{k} + j\mathbf{g}) \cdot (\mathbf{k} + j\mathbf{g}) + (K + jG)^2 = 0$$

which leads to

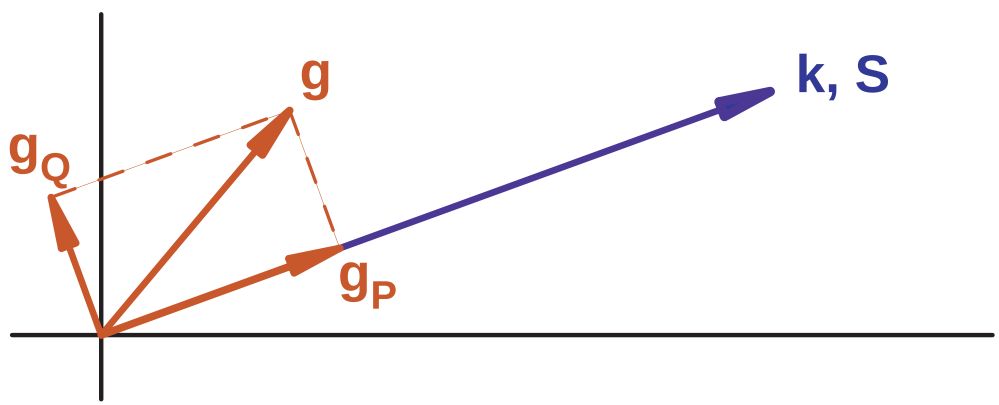
$$k^2 - g^2 = K^2 - G^2$$
$$k\, g_P = K\, G$$

Inhomogeneous plane wave characteristics



- Parallel component g_P must always have same sign as gain coefficient G, and magnitude $|g_P| \leq |G|$
- Quadrature component g_Q can have any arbitrary value (positive, negative or zero)
- Length k is always greater than K for $|g_Q| > 0$
- Only squares of G and g_Q appear in final results

Poynting vector for inhomogeneous plane wave



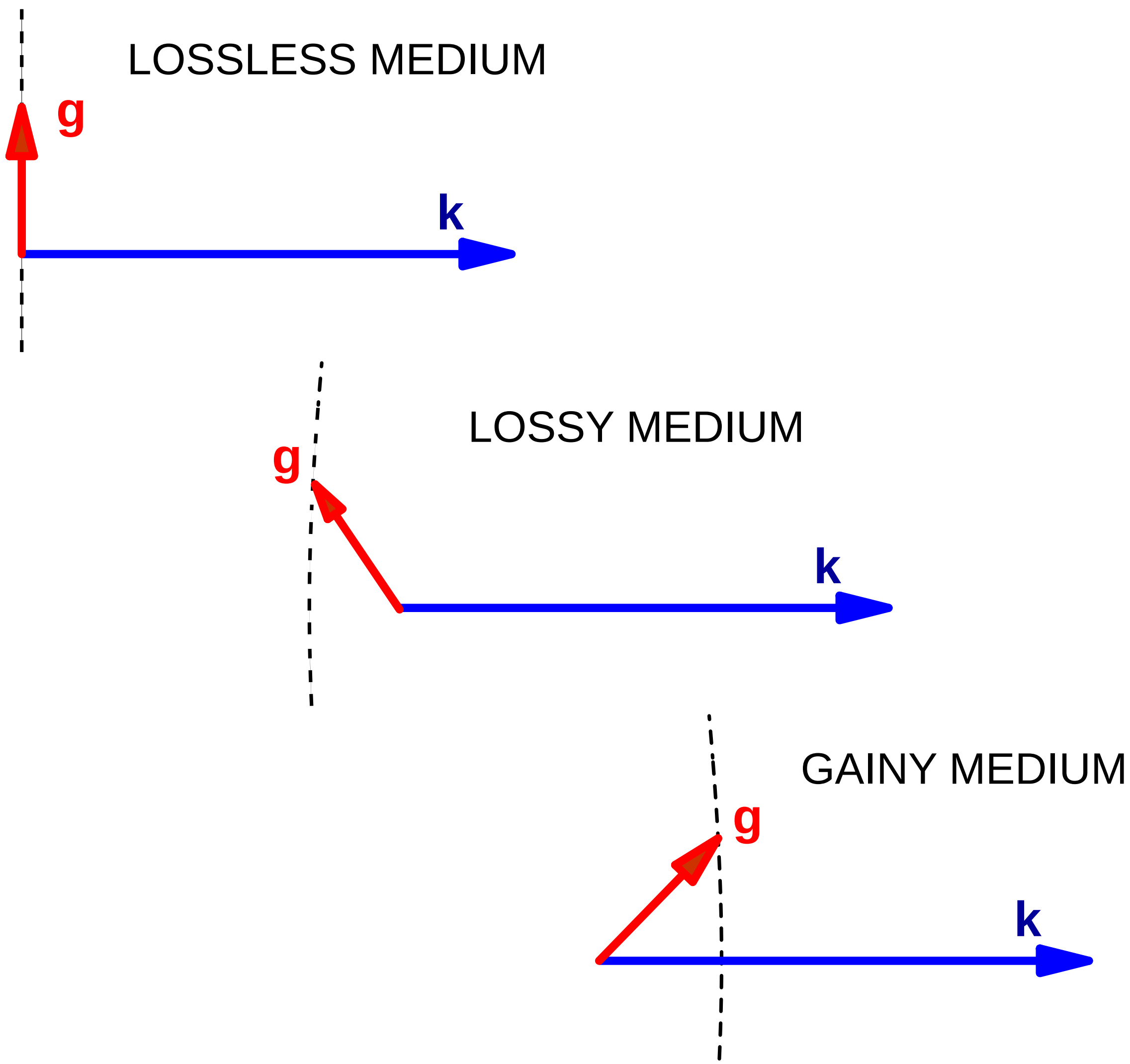
Energy flow or Poynting vector for an inhomogeneous plane wave is parallel to the $\mathbf{k}$ vector

$$\mathbf{S} = \mathrm{Re}\ \tilde{\mathbf{E}}^* \times \tilde{\mathbf{H}} = \left(\frac{|E_1|^2}{\omega\mu}\right) \exp[2\,\mathbf{g}\cdot\mathbf{r}] \times [k_x\hat{x} + k_z\hat{z}]$$

Vector g_P (parallel to $\mathbf{k}$) = "real growth" or attenuation

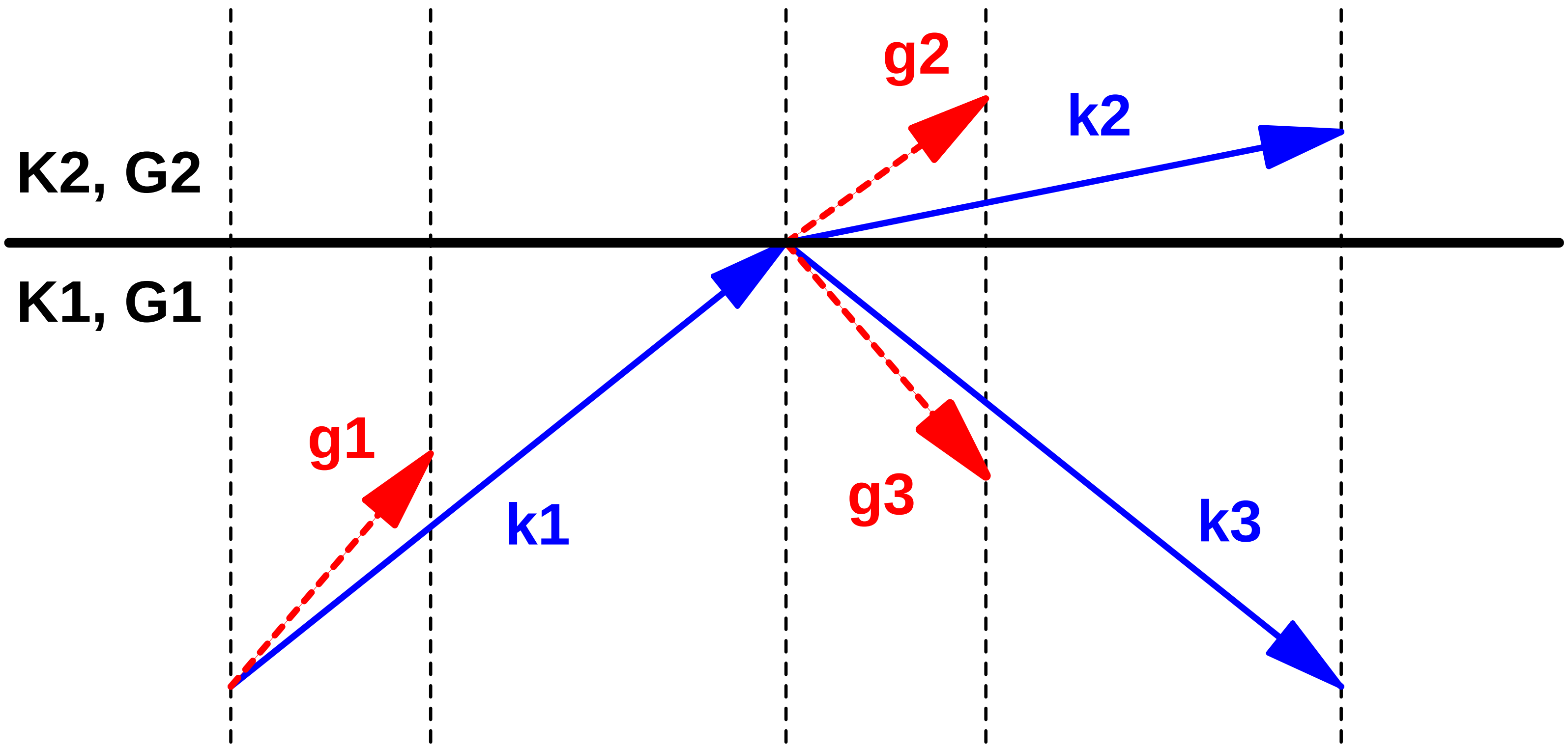
Vector g_Q (quadrature to $\mathbf{k}$) comes from initial conditions

Inhomogenous plane wave vectors



Fresnel reflection for inhomogeneous plane waves

Consider an inhomogeneous plane wave reflecting and refracting at a planar boundary between two gainy dielectric media

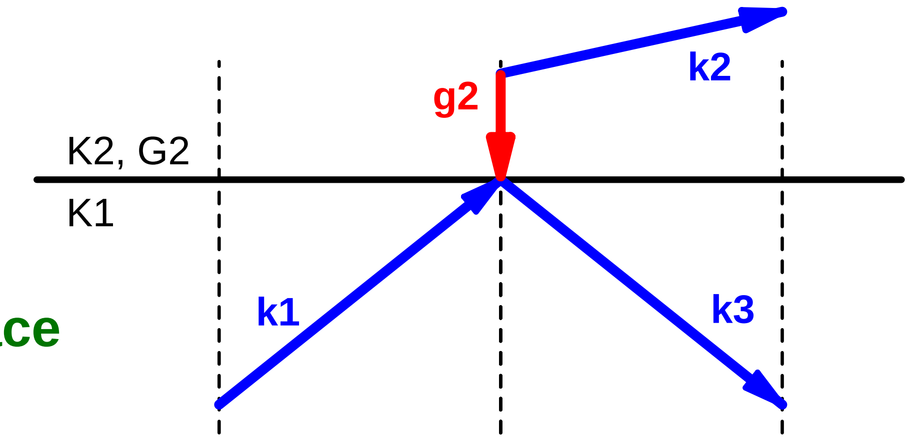


The **k** and **g** vectors must <u>both</u> be specularly reflected

Fresnel reflection: homog input wave, lossy upper half-space

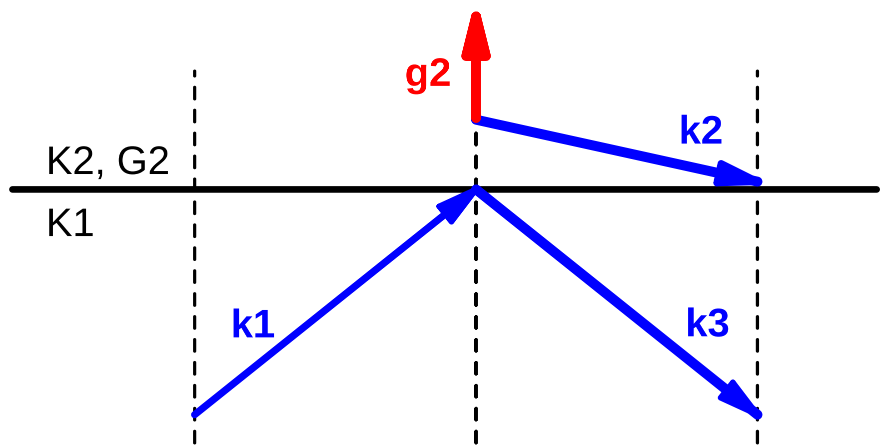
Physical solution

Power flowing (and attenuating) upward, away from the interface



Nonphysical solution

Power flowing downward, toward the interface (and growing upward)

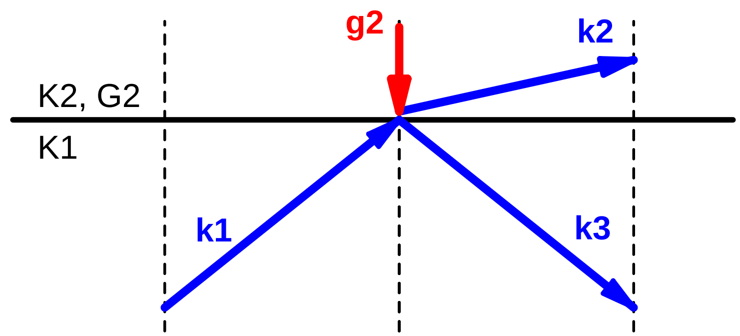


Fresnel reflection: gainy upper half-space

Lossy upper medium

G2 < 0

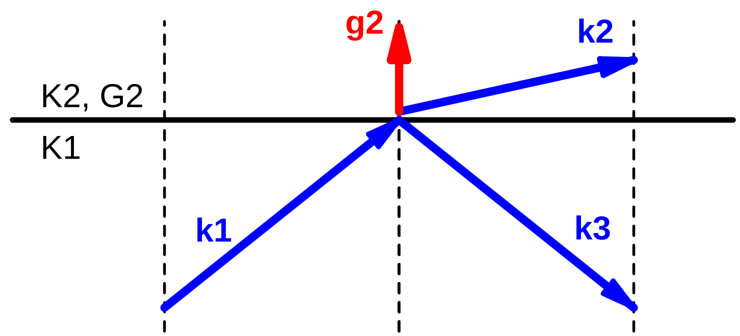
Power flowing (and attenuating) upward



Gainy upper medium

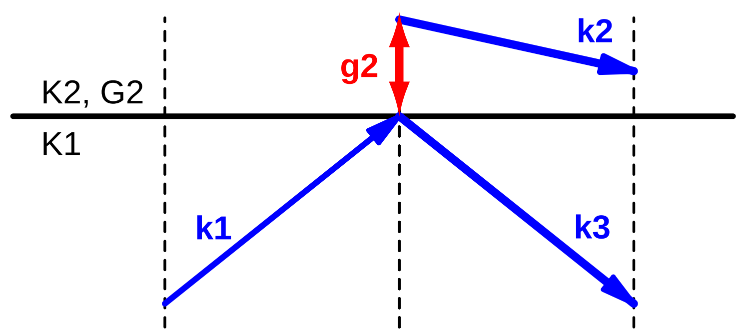
G2 > 0

Power flowing (and amplifying) upward



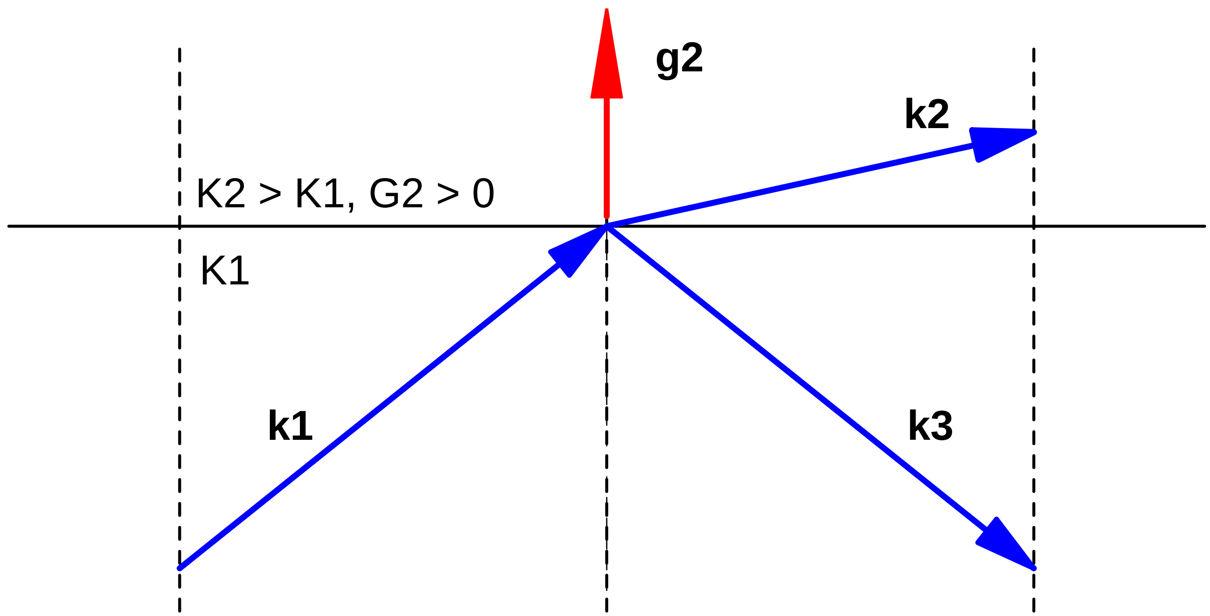
Nonphysical solution

Power flowing (and growing) downward, toward the interface



Fresnel reflection from a gainy medium

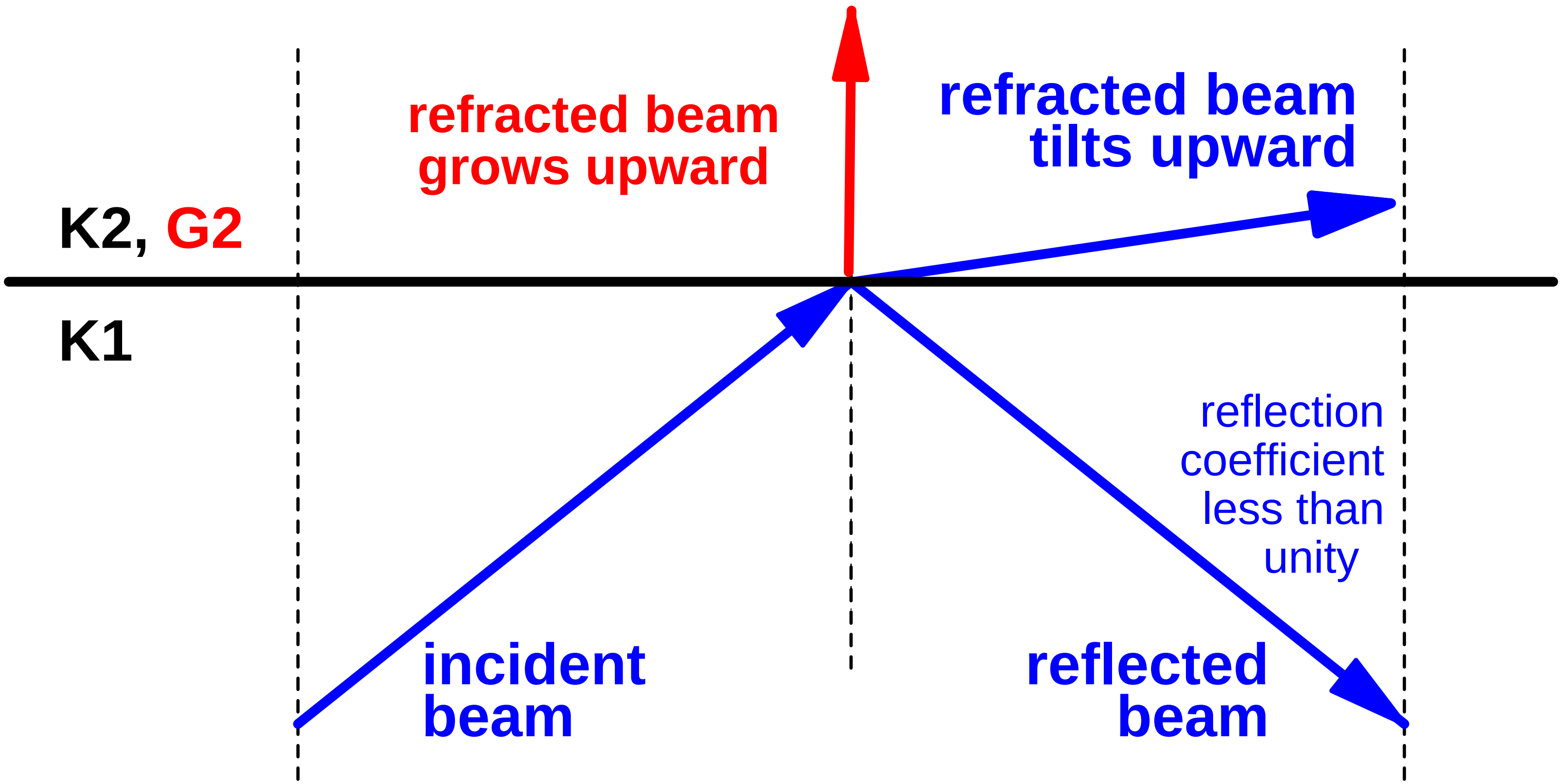
In a gainy upper medium the refracted $\mathbf{k_2}$ vector must tilt upward, which means the $\mathbf{g_2}$ (exponential growth) vector must also point upward



- Incident $\mathbf{k_1}$ beam launches shallow, amplifying refracted $\mathbf{k_2}$ beam
- $\mathbf{g_2}$ vector has mandatory positive projection on $\mathbf{k_2}$ vector

Summary: "amplified TIR" is not possible

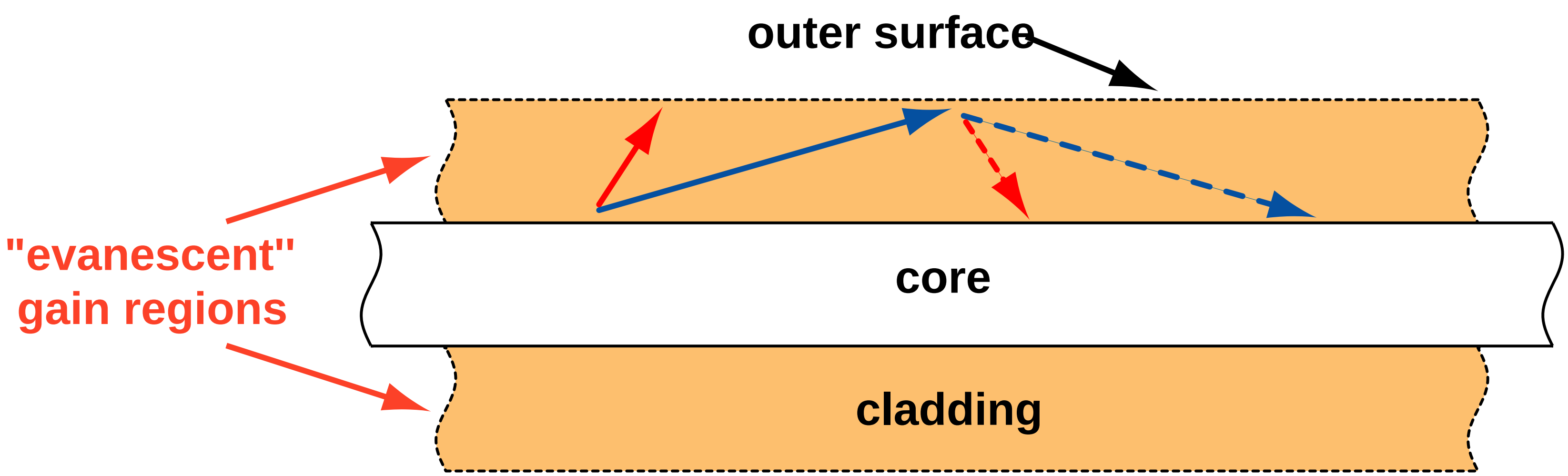
How "TIR" from a gainy medium really works



Interface functions as a *beam splitter . . .*

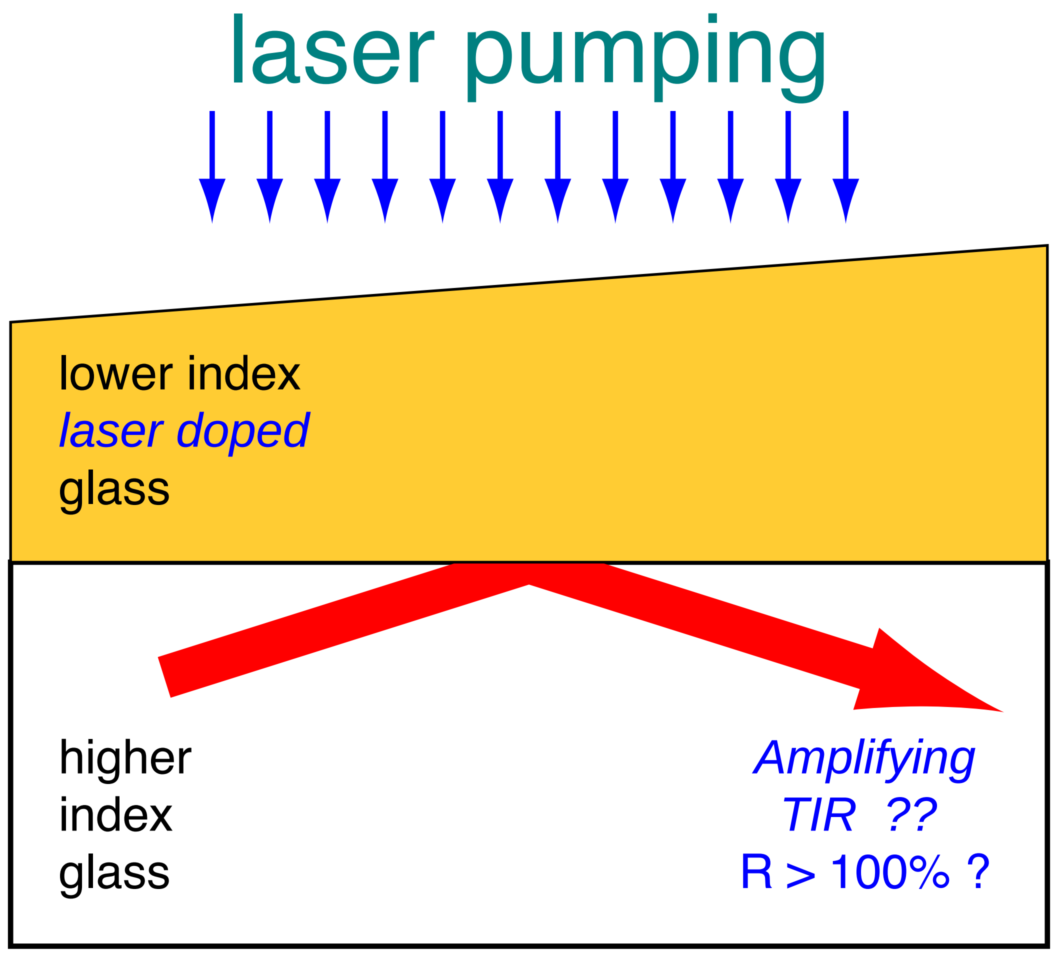
How to explain "evanescent gain" observations?

Observations of "evanescent gain" probably result from regenerative feedback from outer surfaces or scattering elements



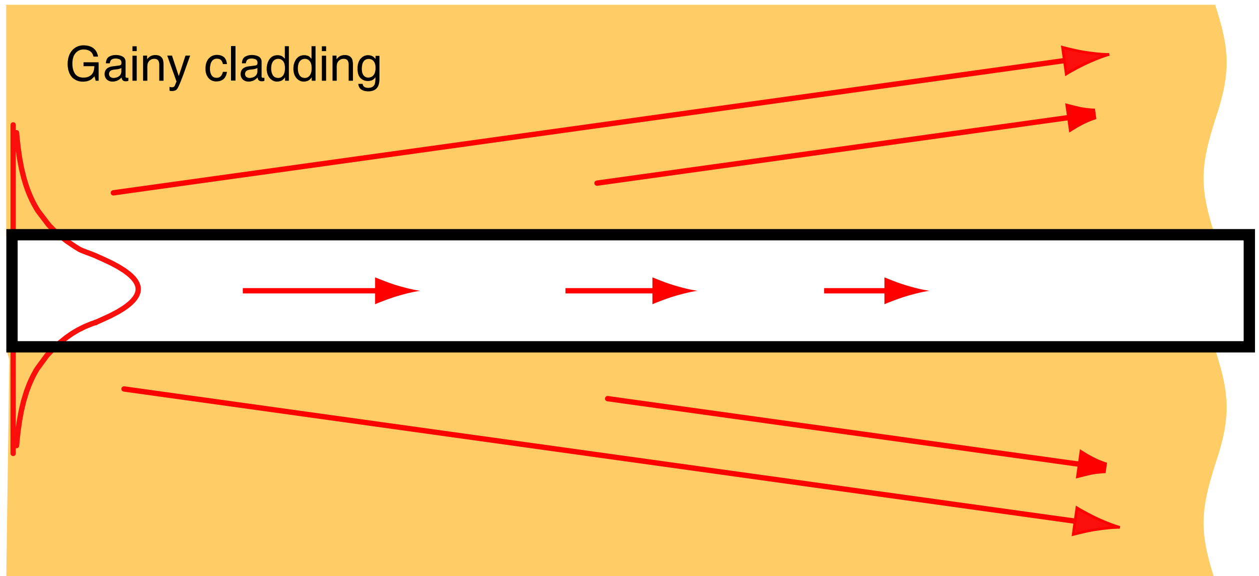
Structure really becomes a multi-layer waveguide; energy in cladding is amplified traveling outward, and in traveling back inward

Experimental (or numerical) tests?

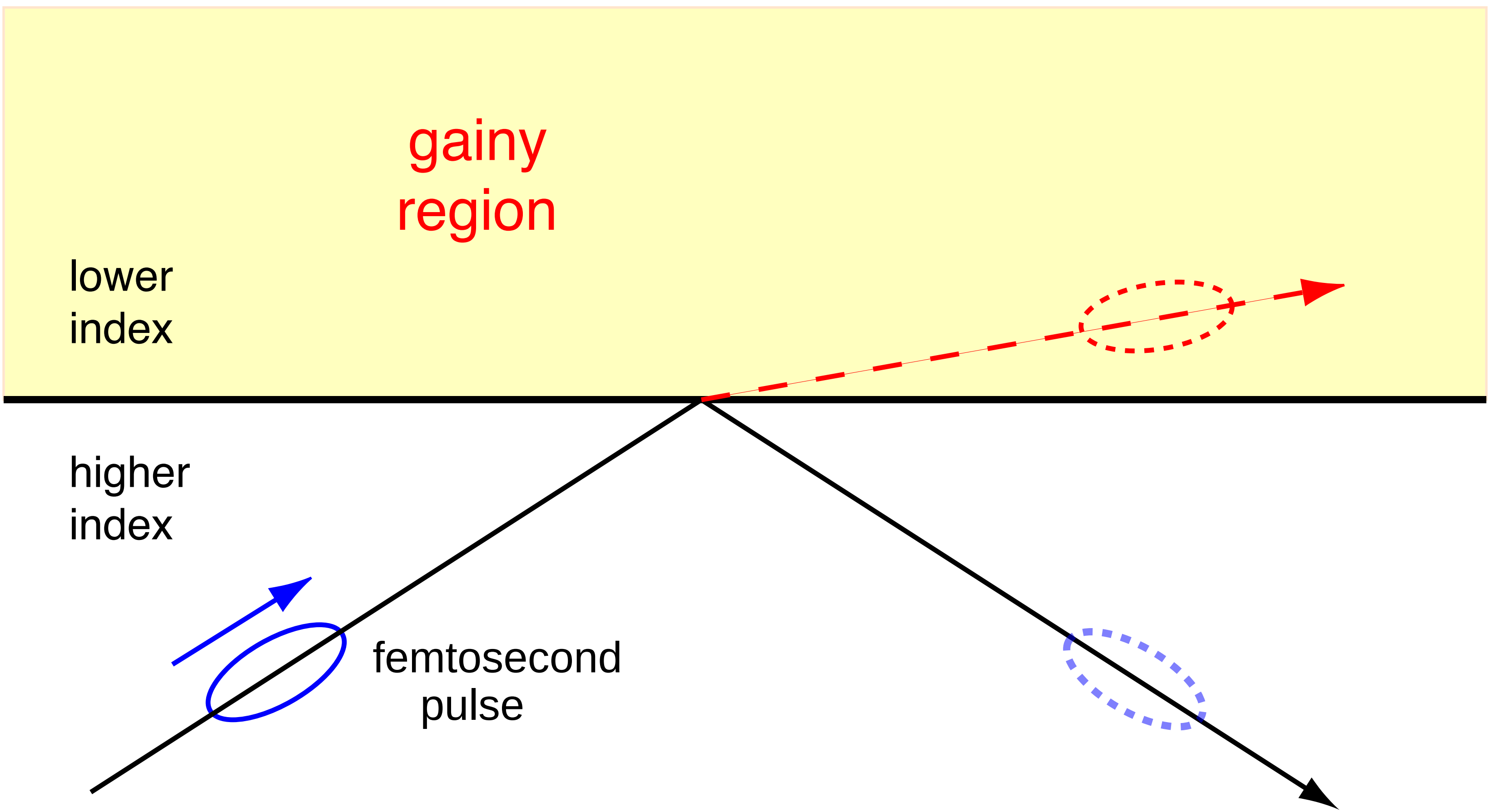


Evanescent waveguide BPM calculations

- Waveguide sheds energy outwards
- Waveguide mode attenuates

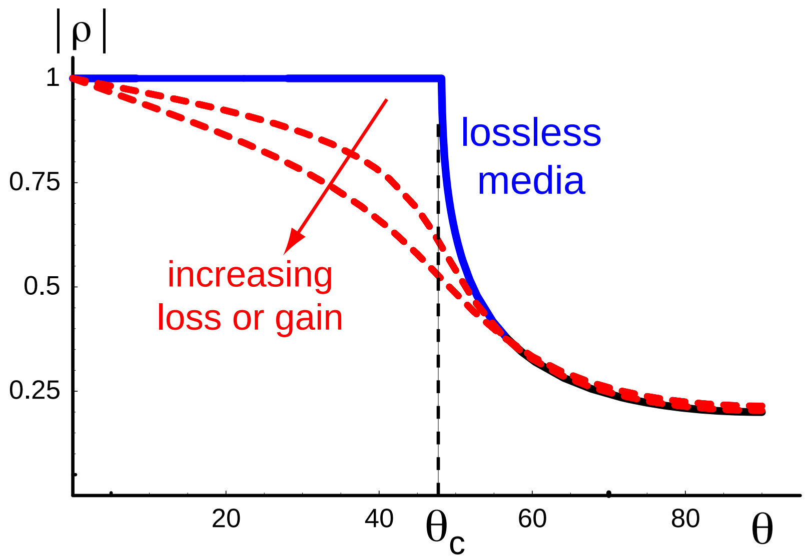


“Blob” model for TIR (numerical or experimental)

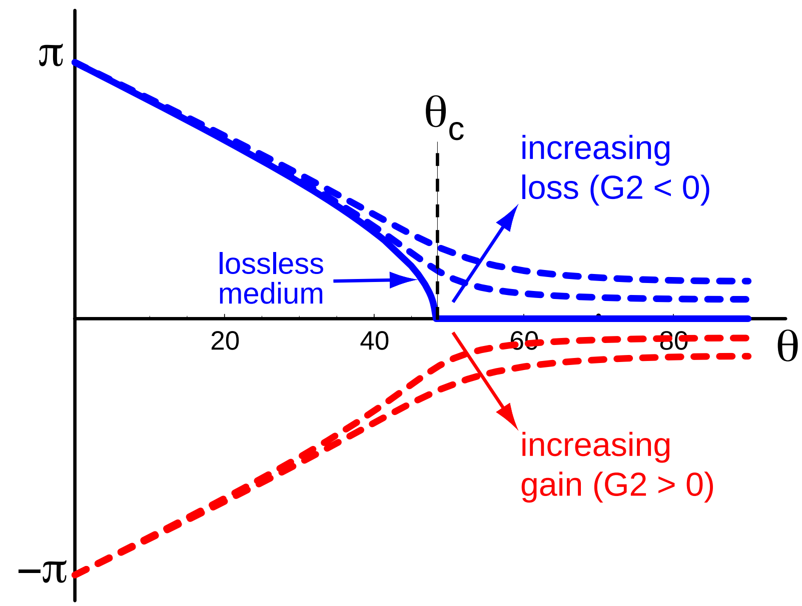


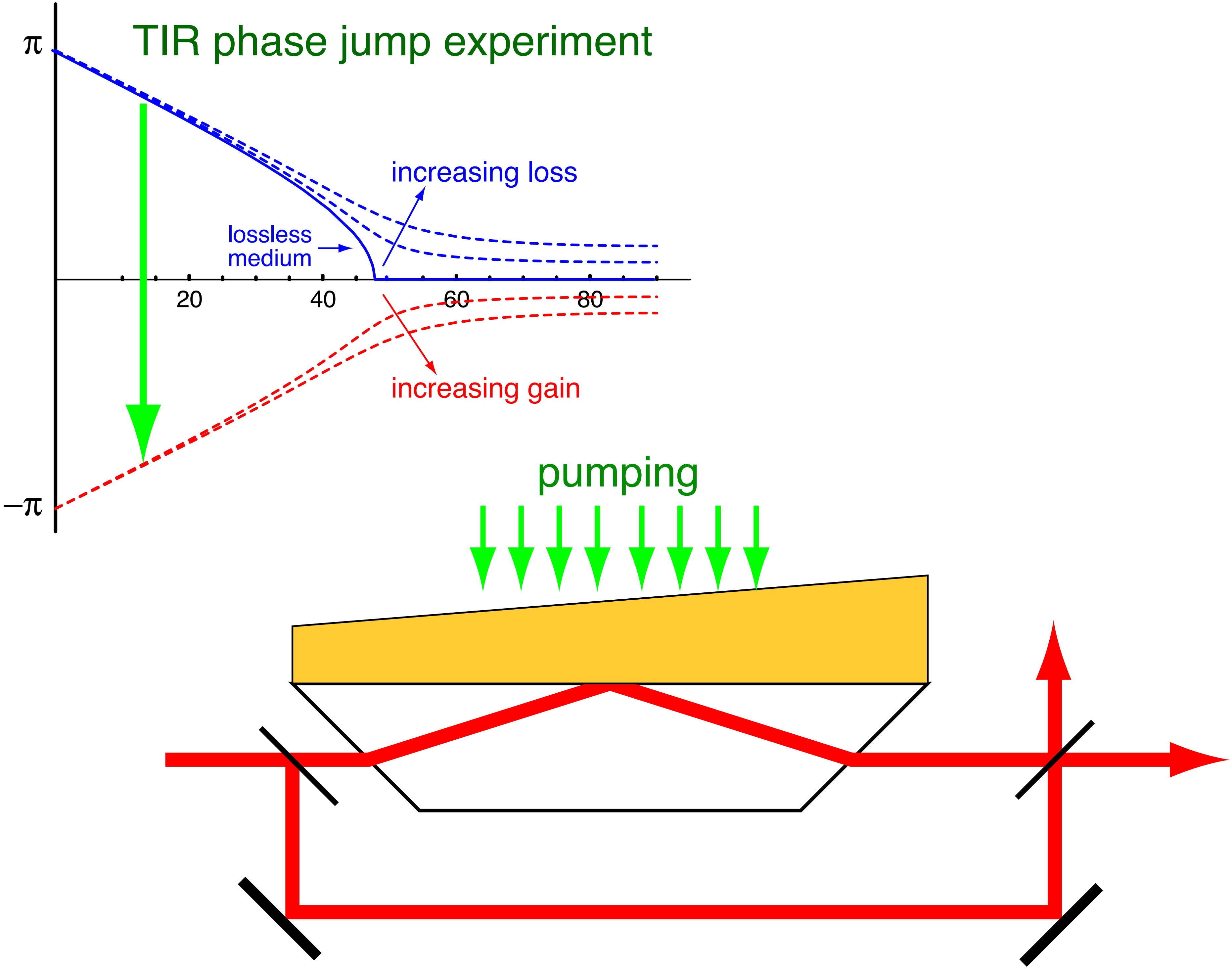
Reflection magnitude and phase versus angle

Reflection magnitude
versus incidence angle



Reflection phase shift
versus incidence angle




TIR phase jump experiment
π
−π
20
40
60
80
increasing loss
lossless
medium
increasing gain
pumping