Fiber Fourier optics

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The Fourier transform of a coherent optical image can be evaluated physically by use of a single lens plus free-space propagation, thereby providing the basis for the field of Fourier optics. I point out that one can similarly evaluate the discrete Fourier transform of a sampled or pixelated optical array physically by passing the discrete array amplitudes through a network of single-mode fibers or optical waveguides. A passive optical network that evaluates the fast Fourier transform of a coherent array can be fabricated by use of $(N/2)\log_2[N]$ optical 3-dB couplers plus small added phase shifts. Implementing such networks in fiber or integrated optical form could provide the basis for a possible technology of fiber Fourier optics. © 2001 Optical Society of America

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The Fourier transform of a continuous and coherent spatial distribution or image can be evaluated physically to a high degree of accuracy by use of one or more simple lenses plus free-space light propagation, leading to the well-established technology of Fourier optics as described in texts by Goodman,[1] Papoulis,[2] and Gaskill.[3] This Letter is to point out that the discrete Fourier transform (DFT), and specifically the fast Fourier transform (FFT), of a discrete or pixelated coherent image or array can similarly be evaluated physically by use of comparatively simple fiber-optic networks, thus leading to the prospective concept of fiber Fourier optics. In this approach a discrete set of coherently related optical input amplitudes a_n are fed into a lossless fiber or integrated optical network through a corresponding set of single-mode and single-polarization input fibers, and the DFT b_n of this sequence is taken out through N similar output fibers. Although there may be some practical difficulties in implementing this approach in real fiber-optic networks, this basic concept seems sufficiently powerful and promising that useful applications may emerge with further exploration.

In the standard approach to DFTs, the DFT of an N-term complex-valued input sequence a_n is given by

$$b_m = \sqrt{\frac{1}{N}} \sum_{n=0}^{N-1} a_n \exp(-jmn2\pi/N)\,,$$

$$0 \le (m,n) \le N-1\,. \qquad (1)$$

The inverse transform has the same form but with a plus sign in the exponential. If the summation over index n in Eq. (1) is separated into summations over the even and odd terms of a_n with n replaced with $2p$ and $2p+1$, respectively, the first $N/2$ terms of the transformed sequence b_m can be written as

$$b_{m=q} = \sqrt{\frac{1}{N}} \Bigg[\sum_{p=0}^{N/2-1} a_{2p} \exp(-jqp\pi/N) + \exp(-jq2\pi/N) \sum_{p=0}^{N/2-1} a_{2p+1} \exp(-jqp\pi/N) \Bigg], \qquad (2)$$

and the remaining $N/2$ terms can be written as

$$b_{m=q+N/2} = \sqrt{\frac{1}{N}} \Bigg[\sum_{p=0}^{N/2-1} a_{2p} \exp(-jqp\pi/N) - \exp(-jq2\pi/N) \sum_{p=0}^{N/2-1} a_{2p+1} \exp(-jqp2\pi/N) \Bigg], \qquad (3)$$

with the index q varying over $0 \le q \le (N/2) - 1$ in both cases. The summations on the right-hand sides of Eqs. (2) and (3) can be recognized as simply $(N/2)$th-order DFTs on the even and odd components of the Nth-order array, with some additional phase shifts of value $\exp(-jq2\pi/N)$ added to half of the transformed elements after the transformations.

Equations (2) and (3) illustrate the general principle that one can evaluate an Nth-order DFT by evaluating two $(N/2)$th-order transforms and combining the results with appropriate phase shifts. This provides the foundation for the Cooley–Tukey FFT algorithm,[4,5] which is universally employed for the numerical evaluation of DFTs. If this same procedure is applied again to the $(N/2)$th-order DFTs, they can each be separated into two $(N/4)$th-order transforms. If the original order N is a power of 2 so that $N = 2^M$, applying this procedure $M-1$ times reduces the initial Nth-order DFT to $M/2$ second-order DFTs. But a second-order DFT is given simply by

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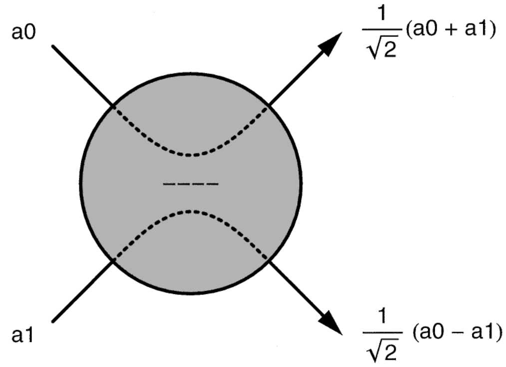

Fig. 1. A 2×2 Fourier transform is equivalent physically to an asymmetric 3-dB fiber-optic coupler or similar 50/50 beam splitter.

$$b_0 = \sqrt{1/2}\,(a_0 + a_1)\,, \qquad b_1 = \sqrt{1/2}\,(a_0 - a_1)\,, \qquad (4)$$

and this transform requires (amplitude scaling aside) no multiplications, only additions and subtractions. This second-order DFT is in fact physically identical to an antisymmetric 3-dB coupler or 50/50 optical beam splitter with a scattering matrix

$$\mathbf{S} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \qquad (5)$$

as shown in Fig. 1. The 2×2 DFT of a pair of coherently related optical values can therefore be evaluated by use of only a 3-dB fiber coupler or any equivalent 50/50 optical beam splitter.

The entire sequence of operations involved in successively subdividing an input array of N pixels and applying the added phase shifts to calculate its Nth-order FFT can therefore be implemented in a lossless optical fiber (or other) network with nothing more than $(N/2)\log_2[N]$ couplers or 50/50 beam splitters plus a smaller number of in-line optical phase shifts. In fact, one can directly convert any of the signal graphs or butterfly diagrams representing the signal flows for different variants of the FFT algorithm as shown in texts such as those by Brigham[5] and Rabiner and Gold[6] into fiber-optic or optical waveguide systems simply by converting the junction points in each of these diagrams into asymmetric 2×2 directional couplers and adding in the additional phase shifts as small optical delays. Figures 2 and 3 show, for example, the standard decimation-in-time and decimation-in-frequency FFT algorithms, respectively, of order $N = 8$, implemented as fiber-optic systems. The circles in the figures correspond to asymmetric 3-dB couplers as shown in Fig. 1, and the boxes correspond to small added optical lengths or phase shifts $\exp(-jq2\pi/N)$, where q is the integer in the rectangle. Extension of this approach to higher orders is straightforward.

These networks have been laid out in the form shown in Fig. 2 partly to emphasize the direct connection with standard textbook discussions of the FFT and partly to emphasize that, except for the small added phase shifts, exactly equal (but otherwise arbitrary) fiber lengths are required between the coupling elements in each successive pair of vertical columns. The sharp 90° corners in Fig. 2 would, for example, presumably be replaced by more-gentle curves in a real implementation. Note that, as is the usual case with the FFT procedure, the input elements are arranged in natural order and the output elements are arranged in a scrambled sequence for the decimation-in-time algorithm, with reverse holding true for the decimation-in-frequency algorithm. One could, of course, correct this simply by physically rearranging the fibers carrying the scrambled values.

It is not entirely clear at this point whether fiber Fourier optic networks of this sort can be of practical utility or will be limited to an interesting but academic concept. Obviously this general approach can be implemented in principle by use of any kind of lossless 50/50 beam splitter, including, for example, fiber 3-dB

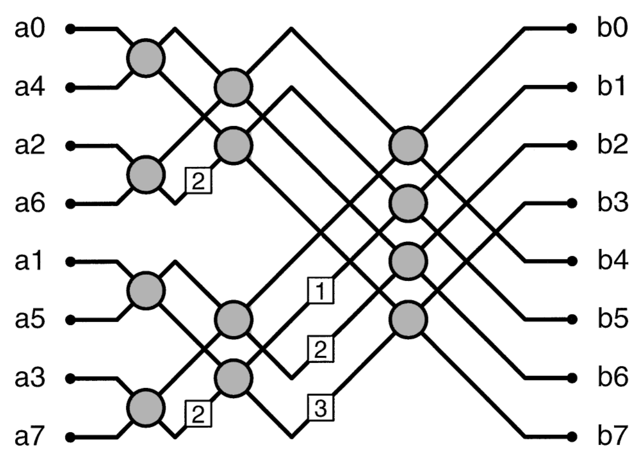

Fig. 2. Fiber-optic network that performs an eight-order DFT, using the decimation-in-time algorithm. The circles represent asymmetrical 3-dB couplers; the boxes represent added phase shifts of value $-q2\pi/N$, where q is the integer shown in the box.

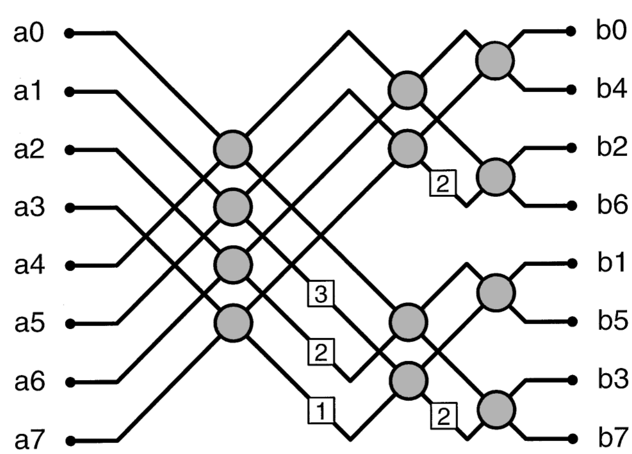

Fig. 3. Fiber-optic network that evaluates the decimation-in-frequency FFT algorithm.

couplers, waveguide or integrated optical couplers, optical beam splitters, or polarizing prisms or other kinds of polarization beam splitter. The major practical difficulty in implementing this concept is that the total phase shift along each of the fibers that spans any one individual vertical column or rank of the structure as one marches across the array must be exactly equal to within a small fraction of an optical wavelength. This would obviously be a difficult task in an assemblage of fiber couplers but might be much more feasible in a miniature integrated optical structure. Given the very broad bandwidth of optical fibers, one can envisage potential applications of this approach to very rapid simultaneous spatial and temporal optical image processing or multiwavelength image processing as well as optical scanning or spectral analysis simultaneously in both space and time. Applications to quantum optical signal processing and quantum communications may also be of interest. It might also be noted that an electro-optic approach for performing discrete Fourier transformations by use of incoherent light was proposed by Goodman *et al.*[7]

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