Optimal Financial Regulation and the Concentration of Aggregate Risk

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Balance Sheet Recessions

- Balance sheet recessions: Concentration of aggregate risk can create financial fragility and lead to financial crises
  - Kiyotaki and Moore [1997], Bernanke et al. [1999]
  - Cont. time: Brunnermeier and Sannikov [2012], He and Krishnamurthy [2011], Di Tella [2013]
- Why so much concentration of aggregate risk?
  - Incomplete contracts/ markets
- Di Tella [2013]: look at complete contracts
  - balance sheet channel disappears in standard setting
  - the type of aggregate shock plays a prominent role
  - e.g. uncertainty shocks create concentration of aggregate risk and balance sheet recessions

Optimal financial regulation

- Concentration of aggregate risk can create financial fragility and lead to financial crises
- Is this concentration of aggregate risk inefficient? Why?
- What is the optimal financial regulation policy?
- What are the right policy instruments?

Today and next class

- Use standard model with financial frictions derived from *moral hazard*
  - Di Tella [2013], Brunnermeier and Sannikov [2012], He and Krishnamurthy [2011]
- Competitive equilibrium where agents can write complete long-term contracts: Di Tella [2014]
- Compare to optimal allocation by planner facing same informational asymmetries
- Implement optimal allocation with simple policy instrument
- Two applications illustrate results

Setting I: preferences

- Experts and households, same EZ preferences: $U(c) = U_0^c$

$$U_t^c = \mathbb{E}\left[\int_0^\tau f(c_t, U_t^c)dt + U_\tau^c\right]$$

$$f(c, U) = \frac{1}{1 - 1/\psi}\left\{\frac{c^{1-1/\psi}}{((1-\gamma)U)^{\frac{\gamma-1/\psi}{1-\gamma}}} - \rho(1-\gamma)U\right\}$$

  where γ is RRA and ψ is EIS (with $\gamma = 1/\psi$ we get CRRA). We are interested in $\gamma > 1$ and $\psi > 1$ for models of stochastic volatility.

- The only difference is experts can use capital (next slide)

- Experts retire at time τ with Poisson arrival θ and become a household (for today take $\theta \to 0$)

Setting II: technology

- Experts trade capital at price p_t and produce consumption $ak_{i,t}$

$$\sigma_t k_{i,t} dZ_t + \nu_t k_{i,t} dW_{i,t}$$

  Z is aggregate BM, W_i is expert-idiosyncratic BM

- competitive investment sector with CRS technology:
  - rent capital k_t and invest $\iota_t(g_t)k_t$ consumption goods to produce flow of new capital $g_t k_t$
  - FOC: $\iota_t'(g_t) = p_t$
  - zero profits: just add $(p_t g_t - \iota(g_t))\, k_{i,t}$ to experts' profits

- Aggregate capital stock k_t follows:

$$dk_t = k_t g_t dt + k_t \sigma_t dZ_t$$

Setting III: aggregate shocks and markets

- Exogenous aggregate Markov state Y driven by aggregate shock Z
  - e.g. $\nu_t = \nu(Y_t)$ or $\iota_t(.) = \iota(Y_t, .)$

$$dY_t = \mu_Y(Y_t)dt + \sigma_Y(Y_t)dZ_t$$

- Complete markets: can trade consumption or capital contingent on any history of Z or $\{W_i\}$
  - Stochastic discount factor η

$$\frac{d\eta_t}{\eta_t} = -r_t dt - \pi_t dZ_t \qquad V_t = \mathbb{E}_t^P\left[\int_0^T \frac{\eta_s}{\eta_t}\delta_s ds\right]$$

  - or equivalent martingale measure Q

$$Z_t^Q = Z_t + \int_0^t \pi_s ds \qquad V_t = \mathbb{E}_t^Q\left[\int_0^T e^{-\int_t^s r_u du}\delta_s ds\right]$$

is a BM under Q.

Household's problem

- Choose a consumption stream c

$$\max_c U(c)$$

$$st: \quad \underbrace{\mathbb{E}^Q\left[\int_0^\infty e^{-\int_0^t r_s ds} c_t dt\right]}_{\text{PV of c}} \leq w_0$$

- Let $F_t(U)$ be the cost of delivering utility U to a household at time t.

Experts' problem I

- Sign long-term contract $\mathcal{C}_i = (e_i, k_i)$ contingent on Z and W_i (and retirement) with full commitment
  - Di Tella [2013], Brunnermeier and Sannikov [2012], He and Krishnamurthy [2011]: short-term contracts

- Hidden action s_i "steal" capital: changes the probability measure over observed outcomes from P to P^s

$$\sigma_t k_{i,t} dZ_t + \nu_t k_{i,t} dW_{i,t}$$

$$\mathbb{E}^{s_i}\left[dW_{i,t}\right] = -\frac{s_{i,t}}{\nu_t} dt$$

  - Note 1: Z and W_i are both observable and contractible
  - Note 2: s_i is a stochastic process contingent on Z and W_i
  - Note 3: P and P^s are equivalent measures

Experts' problem II: IC

- Secretly sell it and consume (no hidden savings)

$$U^{s_i}\left(e_i + \phi p k_i s_i\right)$$

- where the utility is given by $U^{s_i}\left(e_i + \phi p k_i s_i\right) = U_0^{s_i}$

$$U_{i,t}^{s_i} = \mathbb{E}_t^{s_i}\left[\int_0^{\tau} f(e_{i,t} + \phi p_t k_{i,t} s_{i,t}, U_{i,t}^{s_i})dt + U_{i,\tau}^{s_i}\right]$$

- Notice how the expectation is taken under P^s, and note that $U_{i,\tau}^{s_i} = U_{i,\tau}^0$. Why?

- It is always optimal to implement $s_i = 0$. Why? (DeMarzo and Sannikov [2006])

$$IC: \qquad s_i = 0 \in \arg\max_{s_i} U^{s_i}\left(e_i + \phi p k_i s_i\right)$$

Experts' problem III

- Optimal contract

$$J_{i,0}(u_{i,0}) = \min_{(e_i, k_i)} \mathbb{E}^Q \left[\int_0^\infty e^{-\int_0^t r_s ds} \left(e_{i,t} dt - p_t k_{i,t} \left[dR_t - r_t dt \right] \right) \right]$$

$$\begin{aligned} st: \quad & U^0(e_i) = u_{i,0} \\ & (e_i, k_i) \in IC \end{aligned}$$

- where the return of capital is

$$dR_t = \left(\frac{a - \iota(g_t)}{p_t} + \mu_{p,t} + g_t + \sigma_t \sigma'_{p,t} \right) dt + (\sigma_t + \sigma_{p,t}) dZ_t + \nu_t dW_{i,t}$$

- Principals' free entry: set $u_{i,0}$ so that

$$J_{i,0}(u_{i,0}) = n_{i,0}$$

In general interpret $n_{i,t} = J_{i,t} > 0$ as the "net worth" of the expert.

Competitive Equilibrium

- Contracts, investment and HH consumption are optimal, and markets clear

$$\int_{\mathbb{I}} e_{i,t} di + c_t = (a - \iota(g_t))k_t$$

$$\int_{\mathbb{I}} k_{i,t} di = k_t$$

- with law of motion for capital

$$dk_t = k_t g_t dt + k_t \sigma_t dZ_t$$

First best without moral hazard

- Without moral hazard, perfect id. risk sharing
- No friction: so distribution of wealth doesn't matter (except for consumption)
- Assets are priced by arbitrage
- Experts and households share aggregate risk proportionally

Back to moral hazard: continuation utility

- Use continuation utility $U_{i,t}$ to provide incentives. For any contract the utility of not stealing U^0 follows

$$dU^0_{i,t} = \left(-f(e_{i,t}, U^0_{i,t}) + \theta\lambda_{i,t}\right) dt + \sigma_{U,i,t} dZ_t + \tilde{\sigma}_{U,i,t} dW_{i,t} - \lambda_{i,t} dN_{i,t}$$

- To see this, write

$$M_t = \int_0^t f(e_{i,s}, U^0_{i,s}) ds + \overbrace{\mathbb{E}_t\left[\int_t^\tau f(e_{i,s}, U^0_{i,s}) ds + U^0_{i,\tau}\right]}^{\boldsymbol{U^0_t}}$$

and notice it's a martingale under P, adapted to the filtration generated by Z, W_i, and N_i, so we can write $dM_t =$

$$f(e_t, U^0_{i,t}) dt + dU^0_{i,t} \quad = \quad \sigma_{U,i,t} dZ_t + \tilde{\sigma}_{U,i,t} dW_{i,t} - \lambda_{i,t} (dN_{i,t} - \theta dt)$$

Incentive compatibility

- Use continuation utility $U_{i,t}$ to provide incentives

$$dU^0_{i,t} = \left(-f(e_{i,t}, U^0_{i,t}) + \theta\lambda_{i,t}\right) dt + \sigma_{U,i,t} dZ_t + \tilde{\sigma}_{U,i,t} dW_{i,t} - \lambda_{i,t} dN_{i,t}$$

- Incentive compatibility:

$$0 \in \arg\max_s f(e_t + \phi p_t k_{i,t} s, U^0_t) - \tilde{\sigma}_{U,i,t} \frac{s}{\nu_t} - f(e_t, U^0_t)$$

- FOC:

$$\tilde{\sigma}_{U,i,t} = \partial_e f(e_{i,t}, U^0_{i,t}) \nu_t \phi p_t k_{i,t} > 0$$

- the proof is a little complicated because of EZ preferences. Intuition?
  - See Di Tella [2014]

Optimal contract

- From homothetic preferences + linear technology, cost of delivering utility $U_{i,t}$ to an expert

$$J_{i,t} = \xi_t \underbrace{\left((1-\gamma)U^0_{i,t}\right)^{\frac{1}{1-\gamma}}}_{x_{i,t}}$$

- where $x_{i,t}$ is an increasing (convex) function of U: so it's also utility
- and ξ_t is a stochastic process that depends only on Z and N_i

$$\frac{d\xi_t}{\xi_t} = \mu_{\xi,t}dt + \sigma_{\xi,t}dZ_t + \left(\frac{\bar{\xi}_t}{\xi_t} - 1\right) dN_{i,t}$$

So $\xi_\tau = \bar{\xi}_\tau$ where $F_t(U) = \bar{\xi}_t \left((1-\gamma)U\right)^{\frac{1}{1-\gamma}}$

- ξ_t captures aggregate conditions (r, π, p, g and their future distributions)

HJB I

- The HJB equation associated with the minimization problem is

$$r_t J_t dt = \min_{e,k,\sigma_{\boldsymbol{U}},\lambda} edt - p_t k \mathbb{E}_t^Q \left[dR_t - r_t dt\right] + \mathbb{E}_t^Q \left[dJ_t\right]$$

- Notice all expectations are under Q. Why?

- Use $Z_t = Z_t^Q - \int_0^t \pi_s ds$ to write

$$\frac{d\xi_t}{\xi_t} = \left(\mu_{\xi,t} - \pi_t \sigma_{\xi,t}\right) dt + \sigma_{\xi,t} dZ_t^Q + \left(\frac{\bar{\xi}_t}{\xi_t} - 1\right) dN_t$$

$$dU_{i,t}^0 = \left(-f(e_{i,t}, U_{i,t}^0) + \theta\lambda_{i,t} - \pi_t \sigma_{U,i,t}\right) dt + \sigma_{U,i,t} dZ_t^Q + \tilde{\sigma}_{U,i,t} dW_{i,t} - \lambda_{i,t} dN_t$$

HJB II

- Now use Ito's lemma to compute

$$\mathbb{E}_t^Q\left[dJ_t\right] = \mathbb{E}_t^Q\left[d\left(\xi_t\left((1-\gamma)U_{i,t}\right)^{\frac{1}{1-\gamma}}\right)\right]$$

- and normalize controls:

$$e = \hat{e}x_{i,t}$$

$$k = \hat{k}x_{i,t}$$

$$\sigma_U = \hat{\sigma}_U(1-\gamma)U_{i,t}$$

$$\lambda = \hat{\lambda}(1-\gamma)U_{i,t}$$

HJB III

- We obtain the HJB:

$$r_t\xi_t = \min_{\hat{e},\hat{k},\hat{\sigma}_U,\hat{\lambda}} \hat{e} - p_t\hat{k}\left(\frac{a-\iota(g_t)}{p_t} + g_t + \mu_{p,t} + \sigma_t\sigma_{p,t}' - r_t - (\sigma_t + \sigma_{p,t})\pi_t\right)$$
$$+\xi_t\left\{\frac{1}{1-1/\psi}\left(\rho - \hat{e}^{1-1/\psi}\right) + \theta\hat{\lambda} - \hat{\sigma}_U\pi_t + \mu_{\xi,t} - \sigma_{\xi,t}\pi_t\right.$$
$$+\frac{\gamma}{2}\hat{\sigma}_U^2 + \frac{\gamma}{2}\left(\hat{e}^{-1/\psi}\phi p_t\hat{k}\nu_t\right)^2 + \sigma_{\xi,t}\hat{\sigma}_U$$
$$\left.+\theta\left(\left(1-\hat{\lambda}(1-\gamma)\right)^{\frac{1}{1-\gamma}}\frac{\bar{\xi}_t}{\xi_t} - 1\right)\right\}$$

- this is a BSDE for ξ.
- If p, r, π, g, and $\bar{\xi}$ are functions of some state (X, Y), we can use Ito's lemma to look for $\xi(X, Y)$, and solve the HJB as a PDE.
- Boundary conditions?

FOC: $\hat{e}$

- The FOC for $\hat{e}$

$$\xi_t \hat{e}^{-1/\psi} + \underbrace{\xi_t \gamma \frac{1}{\psi} \left(\phi p_t \hat{k} \nu_t\right)^2 \hat{e}^{-2/\psi-1}}_{\text{front-loading}} = 1$$

- Standard: giving consumption is costly but reduces future continuation utility

- Front-loading: by giving the agent more consumption, reduce the private benefit of hidden action

- Tradeoff: intertemporal consumption vs idiosyncratic risk sharing

$$\tilde{\sigma}_{U,i,t} = (1-\gamma) U_{i,t} \hat{e}_t^{-1/\psi} \phi p_t \hat{k}_t \nu_t$$

recall $J = \xi \left((1-\gamma) U_{i,t}^0\right)^{\frac{1}{1-\gamma}}$ is convex in $U_{i,t}$

FOC: $\hat{k}$

- FOC for $\hat{k}_t = \frac{k_{i,t}}{x_{i,t}}$

$$\mathbb{E}[dR_t] - r_t = \underbrace{(\sigma_t + \sigma_{p,t})\pi_t}_{\text{agg. risk premium}} + \underbrace{\gamma \xi_t (\hat{e}_t^{-1/\psi} \phi \nu_t)^2 p_t \hat{k}_t}_{\text{id. risk premium}}$$

- excess return vs. costly incentives

FOC: $\hat{\sigma}_U$

- For $\hat{\sigma}_U$:

$$\hat{\sigma}_U = \frac{\pi_t - \sigma_{\xi,t}}{\gamma}$$

- Give more utility to the agent when:
  - a) the value of money is lower (captured by SPD η)

$$\frac{d\eta_t}{\eta_t} = -r_t dt - \pi_t dZ_t$$

  - b) when it is cheaper to provide utility to the agent (capture by ξ_t)

$$\frac{d\xi_t}{\xi_t} = \mu_{\xi,t} dt + \sigma_{\xi,t} dZ_t$$

- intertemporal hedging

FOC: $\hat{\lambda}$

- FOC for $\hat{\lambda}$:

$$1 = \left(1 - \hat{\lambda}(1-\gamma)\right)^{\frac{\gamma}{1-\gamma}} \frac{\bar{\xi}_t}{\xi_t}$$

$$\implies \quad \hat{\lambda} = \frac{1 - \left(\frac{\xi_t}{\bar{\xi}_t}\right)^{\frac{1-\gamma}{\gamma}}}{1-\gamma}$$

- For $\bar{\xi}_t = \xi_t$ we get $\hat{\lambda} = 0$
- For $\bar{\xi}_t \geq \xi_t$ we get $\hat{\lambda} \geq 0$
- Cont. utility drops after retirement: "give relatively more continuation utility when it is less costly"

Convexity

- Are the FOC sufficient?
- If $\psi \geq 2$ the HJB is jointly convex, so yes!
- But if $\psi < 2$? Principal is too powerful!
- In asset pricing: Bansal et al. [2012] $\psi = 2$, Gruber [2006] $\psi = 2$, Mulligan [2002] $\psi > 2$
- on the other hand: Hall [1988] and Vissing-Jorgensen [2002] $\psi < 1$
- average household vs fund managers and CEOs
- Hidden savings? Di Tella and Sannikov [2014]

Conclusions

- Optimal contracts can be characterized with simple HJB equation, taking arbitrary p, r, π, g, and $\bar{\xi}$ as given
- In a Markovian setting, solve as PDE
- Next class: general equilibrium and optimal regulation