

**THE VOLATILITY OF EMPLOYMENT
WITH FIXED COSTS OF GOING TO WORK**

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Explanation of fluctuations in labor input is at the heart of any theory of aggregate fluctuations. A model of movement of aggregate output is of little interest if it implies that the level of employment is constant. The co-movement of employment and output with roughly unit elasticity is one of the central facts to be explained about fluctuations. Although considerable advances have been made recently in building models where fluctuations in employment have a firm theoretical foundation (Kydland and Prescott (1982), Rogerson (1984), Hansen (1985) and Prescott (1986), among others), serious questions have been raised about the realism of the assumptions of those models.

My purpose here is to develop a model of labor supply in which robust principles generate substantial movements in employment. Robustness has two dimensions. First, the economic mechanisms must be fully compatible with the basic economic principle that people do not permit unexploited opportunities for self-improvement to persist. Second, the assumptions must be realistic; they should not posit extreme responses to changes in relative prices.

I will be studying the fundamentals governing the allocation of labor, not the operation of particular institutions in the labor market. Because of the idiosyncratic nature of exchange in the labor market, it is unreasonable to speak as if all labor were allocated by a competitive spot market. Allocation of labor is guided by contracts and customs more than by markets. The important issue is how much labor is employed in each state of the world and time period. Questions of what institutions achieve the allocation have a subordinate role in this analysis.

The basic idea of this paper is to derive the allocation of labor that would be optimal within a certain broad class of preference orderings. The demand side appears simply in the form of the shadow value of labor services--the wage in a competitive model. The question is whether realistic assumptions about preferences can generate the right magnitude of fluctuations in labor input. Two features of preferences are prominent in the discussion. First is intertemporal substitution in work. If people are willing to concentrate their work in some years and their leisure in other years with little additional compensation, then fluctuations in labor input from one year to another are less of a mystery. The real question is whether the modest degree of intertemporal substitution compatible with microeconomic evidence is sufficient to generate the observed

volatility of employment.

The second issue is the tradeoff between days of work and hours of work per day. The tradeoff depends on pure preferences and on the household technology as well. In a simple case that I consider at some length, there is a constant marginal disamenity per day of work. Time spent getting ready for work at home, commuting time, setup time at work, the psychic cost of getting into work mode, and the indivisibility of some leisure activities are all elements of the disamenity. The disamenity explains why most people concentrate their work in a limited number of full work days and spend the remaining days away from work. Thus, a byproduct of the investigation of preferences about work is a theory of weekends, holidays, and vacations. But the motivation for the investigation is to consider a point raised by Rogerson (1984) and Hansen (1985)—the elasticity of the supply of annual hours of work with respect to the shadow wage can be much higher when workers choose between not working and working full time on a given day, in comparison to a model where they choose any level of work. I find that the situation described by Rogerson and Hansen—a completely horizontal labor supply schedule—is farfetched. However, under realistic assumptions, the labor supply schedule is significantly flattened in the presence of a fixed marginal disamenity per day of work. Part-time work does not pay in that case. Hence there is a certain critical wage where workers become indifferent between working somewhat below full-time hours and working zero hours. The labor supply schedule becomes perfectly elastic at that point.

1. Intertemporal substitution

Consider the worker whose preferences over consumption and work schedules are ordered by

$$(1.1) \quad A \log g^{-1} \left[(1 - \beta) \sum_{t=0}^{\infty} \beta^t g(c_t) \right] - \sum_{t=0}^{\infty} \beta^t u(L_t)$$

Here g and u are concave and convex functions respectively, β is a discount factor strictly less than one, c_t is consumption of goods in year t , and L_t is hours of work

in year t . The worker's budget constraint is

$$(1.2) \quad \sum_{t=0}^T R_t (w_t L_t - c_t) = 0$$

R_t discounts purchases in year t to year 0 and w_t is the real wage in year t .

The utility function 1.1 is additively separable in work, L_t , but not in consumption, c_t . The curvature of $g(\)$ controls intertemporal substitution in consumption, but that issue is not considered in this paper. The curvature of $u(\)$ controls intertemporal substitution in labor supply, a subject of central importance here. The log $g^{-1}(\)$ construction imposes the following constraint:

For the household with no initial wealth (as in 1.2), the long-run or stationary-state level of labor supply does not depend on the wage.

The stationary state is defined by $c_t = c$, $L_t = L$, $w_t = w$, and $R_t = \beta^t$. The worker's problem becomes

$$(1.3) \quad \text{Max } A \log c - \frac{1}{1-\beta} u(L)$$

subject to

$$(1.4) \quad wL - c = 0$$

This problem can also be expressed as

$$(1.5) \quad \text{Max } A (\log w + \log L) - \frac{1}{1-\beta} u(L)$$

where plainly the maximizing L does not depend on w .

The limitation of the investigation to preferences for which the long-run labor supply schedule is vertical is in keeping with the empirical evidence that cross-sectional uncompensated labor supply schedules are not very wage-elastic. The historical stability of hours of work in the face of large increases in real wages supports this restriction on preferences. Because of the restriction, absolutely none of the labor supply response described in this paper comes from the static or long-

run labor supply schedule.

My use of a utility function that is additively separable in work departs from Kydland and Prescott (1982). They introduce lagged work into the current utility function as a way of characterizing high intertemporal substitution. I would argue that essentially the same result can be achieved by choosing $u(L)$ to be close to linear. Although their work has come to be associated with the proposition that non-separable preferences help to explain the observed volatility of employment, what is really important is that intertemporal substitution in labor supply be greater than intertemporal substitution in consumption.

The first-order necessary condition for the maximization of utility with respect to L_t , in the general case, is

$$(1.6) \quad u'(L_t) = \lambda \beta^{-t} R_t w_t$$

λ is the Lagrange multiplier associated with the intertemporal budget constraint. Its value could be derived from the complete optimization problem, but this will not be necessary. Suppose that the amount of work in one year, say year zero, is known. Then λ can be calculated from

$$(1.7) \quad \lambda = \frac{u'(L_0)}{w_0}$$

Equation 1.6 can be considered the short-run labor supply equation in Marshallian form. Alternatively, it can be inverted to obtain the short-run labor supply function in Walrasian form:

$$(1.8) \quad L_t = u^{-1}(\lambda \beta^{-t} R_t w_t) = S(\lambda \beta^{-t} R_t w_t)$$

$S(\)$ is an increasing function of a single driving variable, the discounted wage. Browning, Deaton, and Irish (1985) calls this the Frisch supply function; the usefulness of stating the short-run labor supply function in a life cycle setting in this form has been stressed by MaCurdy (1981). Under the separability assumptions stated earlier, the real interest rate and the real wage do not have independent roles in labor supply (given λ). Rather, the single variable $R_t w_t$ controls labor supply. Years of high real wages will be years of unusually large

amounts of work. In addition, years when the discount function is high in relation to time preference ($\beta^{-t}R_t$ is high) will also be years of hard work.

The elasticity of labor supply

The constant-elasticity specification is a convenient family for appraising intertemporal substitution as a potential explanation of the volatility of employment:

$$(1.9) \quad u(L_t) = \frac{(\bar{L} - L_t)^{1-1/\sigma}}{1/\sigma - 1}$$

The parameter σ is the intertemporal elasticity of substitution in non-work, assumed to be a positive constant. The labor supply function is

$$(1.10) \quad L_t = \bar{L} - (\lambda\beta^{-t}R_t w_t)^{-\sigma}$$

The elasticity, ϵ , of labor supply with respect to the wage is given by

$$(1.11) \quad \epsilon = \frac{\bar{L} - L}{L} \sigma$$

that is, the elasticity of labor supply is the intertemporal elasticity of substitution multiplied by the ratio of non-work time, $\bar{L} - L$, to work time, L .

Estimation of the elasticity, ϵ , has been the subject of intensive research in labor economics in the past decade. Information is available for individual workers in random surveys of the population (where the variation in wages over time arises from random events and from life-cycle patterns) and from the participants in negative income tax experiments (where the variation comes from the experimental design). Pencavel (1986) has prepared a definitive survey of this body of research with respect to men. His conclusions can be summarized as follows: From survey data, ϵ appears to be in the range between 0 and 0.45 (Table 1.22, p. 85). From experimental data, the range is a little tighter, between

.06 and .19 (Table 1.21, p. 80). For women, research has concentrated more on the substitution of goods for labor, rather than intertemporal substitution, and even there the research has reached no consensus at all (Killingsworth and Heckman (1986)). For the purposes of this paper, I will use two values of ϵ , a low value of 0.1 and a high value of 0.4. If non-work time is double work time, the corresponding values of σ are 0.05 and 0.20.

The central problem of fluctuations theory can be seen starkly in the light of these estimates. Take as a benchmark that the annual hours of work of workers attached to cyclical sectors such as durables decline by 10 percent in a cyclical contraction. The corresponding reduction in the discounted wage needed to explain the contraction as a movement along the labor supply schedule is to $e^{-.1/.1} = 37$ percent of its normal level for the low elasticity of 0.1 and to $e^{-.1/.4} = 78$ percent for the high elasticity of 0.4. It does not take a careful review of data on the cyclical behavior of real wages and real interest rates to find that no such decline occurs in contractions

A central question, then, is whether the typical worker might not have an elasticity of labor supply close to 1, even though such a value is plainly rejected by empirical research. Could the research have such a large bias?

One of the pieces of evidence in favor of low elasticities is the stability of hours of work over the life cycle in the face of rising wages in the first 20 years or so of work. The worker with an elasticity of 1 would concentrate leisure in the early years when his time was less valuable. There are two reasons that this evidence is not definitive. First, the proper shadow value of labor is not the cash wage, but is the marginal product of labor plus the value of training and learning by doing. In view of the importance of training to younger workers, the life cycle pattern of the shadow value of labor is probably much flatter than is the pattern of cash earnings. Second, there may be a life-cycle pattern in preferences, such that people would work more when they were young, in the face of equal wages in all years. Then the pattern of rising wages would counteract the tendency for work to fall with age. A high degree of intertemporal substitution would be consistent with the observed patterns of wages and hours of work.

The second piece of evidence in favor of low substitution is the finding of MaCurdy and others that workers who experience periods of unusually high wages do not work much longer hours during those periods. This finding survives

strenuous attempts to filter out transitory errors in measuring wages. However, it is vulnerable to the following criticism: Most of the workers in the data are in the middle of long-term employment relationships with their employers. Their pay may be governed by practices and implicit contracts that create discrepancies between earnings and the shadow value of labor. In particular, whenever there is a substantial amount of job-specific human capital, it is likely that the shadow value exceeds the wage paid. When the rules call for a wage change, it may occur without any change in the shadow value. It is possible that a majority of the wage changes reported in the surveys occur independently of actual changes in the shadow value of labor. Under the further assumption that the rules and practices governing the setting of hours of work respond to the true shadow value, not the wage paid, then one would expect a weak association between wage movements and hours movements, even in the presence of a strong association between movements in the shadow value of labor and movements of hours.

The third piece of evidence on substitution comes from the negative income tax experiments. Workers facing a temporary drop in after-tax wage rates of 30, 50, or even 70 percent did not typically stop work. Only small declines in average hours occurred, and there is a presumption that part of these were measurement errors, because the negative income tax provides an incentive to under-report wage income. However, the NIT experiments are less than definitive for the following reason: The experiments were an unexpected and dramatic intervention in the employment relationship. Implicit contracts and practices were not designed to handle the contingency presented by the NIT experiment. Although in principle, employers should have granted time off for experimental subjects for the months they were under the NIT provisions, there was no opportunity for this type of institution to evolve. The hours of the subjects continued to be set by rules than considered the shadow value of labor and the presumptive value of the subjects' time, but the rules were not rewritten to take account of the large wedge that the NIT puts into the calculation of optimal hours. Hours were set without regard to the NIT and so it appears that hours are insensitive to the after-tax wage.

Rogerson (1984) and Hansen (1985) proposed a formal model in which employment in a contract setting is highly elastic, even though movements in employment are not associated with movements in wages. They suggested that

workers are constrained either not to work at all on a given day or to work a prescribed full-time schedule. They grant to their employers the choice as to which days they work. The optimal contract, which maximizes the joint welfare of workers and employers, makes the implicit labor supply of workers perfectly elastic in the short run. This proposition holds independently of the workers' underlying preferences about intertemporal substitution. The next section shows that the Rogerson-Hansen conclusion is highly special. Literal flatness of the entire labor supply schedule is only a remote possibility. However, a generalization of their work, based on more reasonable assumptions, reveals that the labor supply schedule may have a flat segment. The flat segment occurs at a critical wage that is below the normal wage. It can help explain the volatility of employment in a model where the shadow value of labor drops below the critical wage during slumps.

2. Implications of a cost or disamenity related to days of work

The starting point for this analysis is the characterization of preference orderings over annual work schedules. Let h_τ be the number of hours of work in day τ . The theory will deal with weekends, so τ runs over all 365 days of the year. The theory does not consider lunch breaks or other aspects of work schedules within the day. The problem is to find a simple representation of a suitable preference ordering in R_{365} . For reasons that will become apparent shortly, it is possible to restrict attention to orderings over schedules (h_1, \dots, h_{365}) whose elements are either 0 or a common value, h . Moreover, little is lost by making the assumption of symmetry: The worker is indifferent among all schedules that involve the same number of days of work, irrespective of their timing. With that assumption, the theory can explain the total number of weekend days and vacation days, but not their timing over the year.

Under these simplifications, preferences depend on two measures: daily hours, h , and the number of days spent working, d . In order to reach sharp conclusions, it will be appropriate to make assumptions about separability. These

are:

1. As in section 1, the utility function is separable in goods consumption and work in other years; the marginal value of earnings is a constant, λ , derived from the life-cycle optimization problem described in section 1.

2. The strictly convex function $u(h)$ measures the disamenity of working h hours in a day. The disamenity arising from d days of work is $du(h)$. The level of satisfaction from days off is $(365 - d)u(0)$.

3. The convex function $v(d)$ measures the disamenity of working d days per year.

Under these assumptions, the utility function is

$$(2.1) \quad \lambda w d h - d u(h) - (365 - d) u(0) - v(d)$$

The constraints are

$$(2.2) \quad 0 \leq h \leq 24$$

$$(2.3) \quad 0 \leq d \leq 365$$

I will assume that $u(24) = \infty$, so it will not be necessary to treat the upper constraint explicitly. The other constraints will have an important role in the discussion.

Interior maximum

The first-order necessary conditions for an interior maximum are

$$(2.4) \quad u'(h) = \lambda w$$

and

$$\begin{aligned}
 (2.5) \quad v'(d) &= \lambda w h - [u(h) - u(0)] \\
 &= \lambda w h - \Delta u(h)
 \end{aligned}$$

It is easy to show that the second-order conditions are satisfied for the interior maximum. The first-order conditions have a straightforward interpretation. The worker chooses h , hours of work on days when work occurs, by equating the marginal disamenity of work, $u'(h)$, to the marginal benefit of work, λw . The result can be inverted to obtain a Walrasian supply function for daily hours:

$$(2.6) \quad h = S_h(\lambda w)$$

The supply function is increasing in λw —it expresses a pure substitution effect when stated as a function of λw .

The condition for optimal work days is less familiar but readily interpretable. The worker equates the marginal disamenity of extending the work year, $v'(d)$, to the net benefit, $\lambda w h - \Delta u$. The net benefit is the value of the earnings from the extra day of work, $\lambda w h$, less the disamenity, Δu , incurred by the discontinuous shift from 0 to h hours of work on that day. Again, the condition can be inverted to obtain a Walrasian supply function,

$$(2.7) \quad d = S_d(\lambda w)$$

This supply function is also an increasing function of λw ; its slope is

$$(2.8) \quad S'_d(\lambda w) = \frac{h}{v''(d)} > 0$$

The following property of the supply of days will play a central role in the ensuing discussion:

Proposition 1: If the marginal disamenity of work, $v'(d)$, is close to a constant, $v''(d)$ is small and the supply of days is highly elastic with respect to λw .

The annual supply of hours of work, L , from each worker is

$$(2.9) \quad L = S(\lambda w) = S_h(\lambda w) S_d(\lambda w)$$

and is increasing in λw . Note that an increase in the wage will bring forth an increase in annual hours that is the result of increases in both daily hours and days of work. The data on cyclical variations in work effort show that weekly hours and weeks per year are positively correlated (see Lilien and Hall (1986)), which is consistent with the model. The great bulk of the variation in annual hours is in weeks per year. Unfortunately, there do not seem to be any data on daily hours. It is possible, but not likely to my mind, that all of the variation in weekly hours is in days per week and none in hours per day.

Discussion of the separability assumptions

It is a fact of every urban society, I believe, that workers bunch their hours into roughly 220 to 300 days per year--weekends, holidays, and vacations are virtually universal in non-agricultural work. One could simply invoke a preference against work on Saturdays, Sundays, the months of July and August, and so forth, to explain days off. But it seems more interesting to seek a more general explanation founded on non-convex technology and preferences.

The simplest non-convexity is a fixed cost of appearing at work. Suppose, for example, that there is a cost, k (measured in the same units as earnings) for each day a worker spends on the job-- k includes commuting costs, setup costs, and the like. A simple way to include the cost is to set

$$(2.10) \quad v(d) = \lambda k d$$

in the utility function, 2.1. Then equation 2.8 gives a startling and important conclusion:

Proposition 2: If the only nonconvexity arises from a fixed cost, then labor supply is perfectly elastic through the interior maximum.

An interior maximum for d will occur only when the wage is a root of

$$(2.11) \quad \lambda k = \lambda w S_h(\lambda w) - [u(S_h(\lambda w)) - u(0)]$$

Call the root w^* . Then labor supply is 0 for $w < w^*$, $365 S_h(\lambda w)$ for $w > w^*$, and any value in between if $w = w^*$.

Perfectly elastic labor supply has formed the basis of a theory of employment fluctuations by Rogerson (1984) and Hansen (1985). They consider a simple version of this model in which workers choose between an exogenously set number of daily hours, h_0 , and zero hours. In the Rogerson-Hansen case, $v(d) = 0$, although they refer informally to fixed costs as the justification for the constraint on daily hours.. Cogan (1981) noted that the individual worker has a discontinuous static labor supply function in the presence of fixed costs.

Maximization of the utility function, 2.1, in this case without a constraint on daily hours results in spreading hours over all the days of the year. The condition for an interior maximum for d , expressed in equation 2.5, cannot be satisfied for a convex $u(h)$ when $v'(d) = 0$. Rather, when workers have a free choice of h , $\lambda w = u'(h)$ and

$$(2.12) \quad 0 < u'(h) h - \Delta u(h)$$

by virtue of the convexity of $u(h)$. Hence the maximizing d occurs at the corner, $d = 365$.

Rogerson and Hansen do not permit the free choice of h that underlies this conclusion. If h is held at an arbitrary value, h_0 , as they assume, then the condition for an interior maximum is

$$(2.13) \quad 0 = \lambda w h_0 - \Delta u(h_0)$$

Such an interior maximum will occur if h_0 exceeds the hours chosen freely to maximize $\lambda wh - u(h)$; that is,

$$(2.14) \quad u'(h_0) < \lambda w$$

Under Rogerson's and Hansen's assumption, labor supply is perfectly elastic at the point,

$$(2.15) \quad \lambda w^* = \frac{\Delta u(h_0)}{h_0}$$

As Hansen (1985) notes, the conclusion of perfectly elastic supply holds for all preferences about hours of work. It appears to be a universal explanation of highly elastic labor supply. Employment will be realistically volatile in a general equilibrium setting with fluctuations in labor demand, as in Prescott (1986). However, the Rogerson-Hansen story depends critically on the assumption that whenever daily hours are positive, they are at a level exceeding the level the worker would choose freely.

As Hansen suggests, the existence of a fixed cost of going to work, or a disamenity proportional to days of work, could rationalize their conclusion of highly elastic labor supply. An interior solution to equation 2.11 is certainly possible. However, the story of a flat labor supply schedule has the following grave defect. The supply of annual days of work is perfectly elastic over the range from 0 to 365. On the other hand, the allocation of labor rarely exceeds about 235 days in the U.S. (49 5-day weeks less 10 holidays). In a model based on the Rogerson-Hansen setup, a temporary increase in the wage, no matter how small, will cause workers to drop all weekends and vacations. It strains credulity that an extra nickel an hour in a particular year will make people work straight through without any days off.

The central defect of a model with a fixed cost or constant marginal disamenity of work is the following:

Proposition 3: If the reward to work, λwh , is enough to cover the cost or disamenity λk plus the value of the foregone hours, $\Delta u(h)$, on any one day of the year, it is enough to justify work on 365 days of the year.

The perfect elasticity of annual hours of work is not a fundamental property of all models with a fixed cost of work—it is the consequence of a special assumption about the disamenity of extra days of work.

Many jobs in the U.S. involve more than 8 hours of work a day. Professionals of most types, especially during the apprenticeship period, work 10 or 12 hours routinely and even more during peak periods. But 365-day work years are virtually unknown. Even MBAs in their early years at McKinsey and Company work just 5 of their 14-hour days each week. And among one of the groups with the highest annual hours—associates in major law firms—Sunday work is expected only in the most demanding New York firms. I conclude that a rising marginal disamenity of days of work is required for a realistic theory of the allocation of labor.

The separability assumptions expressed in the utility function 2.1 are not the only way to capture the reluctance of most people to put in 365 days of work. An alternative is to replace the disamenity of days of work, $v(d)$, with a disamenity of total annual hours, say $\hat{v}(\sum h_\tau)$. In addition, let there be a cost, λk , associated with each day of work (otherwise, convexity will give $d = 365$). The worker's problem is then

$$(2.16) \quad \text{Max } \lambda d (w h - k) - d \Delta u(h) - \hat{v}(d h)$$

Necessary conditions for an interior optimum are

$$(2.17) \quad \lambda w - u'(h) - \hat{v}'(d h) = 0$$

$$(2.18) \quad \lambda (w h - k) - \Delta u(h) - h \hat{v}'(d h) = 0$$

Putting the first into the second yields

$$(2.19) \quad u'(h) = \frac{\Delta u(h) + \lambda k}{h}$$

Note the absence of any role of the wage in determining daily hours, h , given λ . That is, the short-run supply schedule for hours is wage-inelastic. Hours are determined by equating the marginal disamenity, $u'(h)$, to the average disamenity and cost. This rule is optimal for any wage that gives an interior optimum. Then total annual hours, L , are determined from

$$(2.20) \quad \hat{v}'(L) = \lambda w - u'(h)$$

That is, the marginal disamenity of another hour of work during the year is equated to the marginal value of earnings, λw , less the marginal disamenity of that hour in the day that it occurs.

The alternative specification has the strong property that all fluctuations in annual hours take the form of fluctuations in annual days and none as fluctuations in daily hours. Hours change only if the setup cost, k , changes. Shifts of labor demand can change days of work only--people work on Saturday when there is more work to do, but they never stay late when there is more work to do. Again, although data on daily hours are scant, it is my impression that daily hours do rise when total annual hours rise, contrary to the prediction of the alternative model.

I conclude that the specification set forth at the beginning of this section seems most reasonable among the alternatives considered here. With a rising marginal disamenity of days of work, the central defect of the Rogerson-Hansen setup and its generalizations--perfectly elastic labor supply right up to 365 days of work a year--can be avoided. On the other hand, this specification lets daily hours respond to the marginal value of earnings, unlike the alternative where the only action is in days of work per year.

3. Labor supply schedules

This section examines the short-run labor supply function for a worker whose preferences about work are ordered by the life cycle utility function,

$$(3.1) \quad \frac{A}{1-\beta} \log g^{-1}[(1-\beta) \sum \beta^t g(c_t)] -$$

$$\sum \beta^t \left[d_t \frac{(24 - h_t)^{1-1/\sigma}}{1/\sigma-1} + (365 - d_t) \frac{24^{1-1/\sigma}}{1/\sigma-1} - v(d_t) \right]$$

where $v(d) = 0$ if $d \leq 235$ and $v(d) = \infty$ if $d > 235$. This utility function forces work schedules to exclude weekends and vacations by making the marginal disamenity of an extra day after 235 be infinite. The budget constraint includes a fixed setup cost, k , for each day of work:

$$(3.2) \quad \sum R_t (w_t d_t h_t - k d_t - c_t) = 0$$

Stationary state

The corresponding stationary state problem is

$$(3.3) \quad \text{Max } A \log [d (w h - k)] - d \frac{(24 - h)^{1-1/\sigma}}{1/\sigma - 1} - (365 - d) \frac{24^{1-1/\sigma}}{1/\sigma - 1}$$

The critical question is: In the stationary state, do workers choose to work a full year of 235 days, or do they specialize by working fewer days in order to limit setup costs? The higher the setup cost, k , the more likely that the worker will specialize. The issue of specialization is critical because the short-run labor supply schedule is perfectly elastic for workers who are chronically specialized.

Specialization occurs when the worker is at an interior maximum with respect to days of work, d :

$$(3.4) \quad \frac{A}{d} - \frac{(24 - h)^{1-1/\sigma}}{1/\sigma - 1} + \frac{24^{1-1/\sigma}}{1/\sigma - 1} = 0$$

In the borderline case, the interior maximum will occur at the very boundary, $d = 235$. In addition, the model can be calibrated by requiring that the optimal h have the realistic value of 8. Then the first order condition for h can be used to eliminate the distribution parameter A :

$$(3.5) \quad \frac{A}{h - k/w} - d(24 - h)^{-1/\sigma} = 0$$

Inserting $d = 235$ and $h = 8$ into 3.4 and 3.5 and solving for k/w yields

$$(3.6) \quad \frac{k}{w} = 8 + 16^{1/\sigma} \left[\frac{24^{1-1/\sigma} - 16^{1-1/\sigma}}{1/\sigma - 1} \right]$$

When k/w exceeds this value, specialization occurs and the optimal work schedule covers fewer than 235 days in order to conserve setup costs. Specialization is much more likely when the elasticity of substitution, σ , is high, because the disamenity of crowding more work into fewer days is smaller when substitution is high. Critical values of k/w are:

σ	k/w
.05	7.2
.20	4.8
.40	3.1

The interpretation of k/w is the setup cost of a day's work expressed in hours of work. At the values of σ suggested by empirical research, setup costs have to be prohibitive to be high enough to induce specialization. On the other hand, with an elasticity of substitution of 0.4 (which implies an elasticity of labor supply of 0.8 in the short run), setup costs are equal in value to 3.1 hours of work, which may not be unrealistic for some workers.

The short-run labor supply function

To keep the notation simple, suppose that time preference and discounting offset each other ($\beta^{-t}R_t = 1$). The supply of daily hours for the utility function 3.1 is easily found to be

$$(3.7) \quad S_h(\lambda w) = 24 - (\lambda w)^{-\sigma}$$

Let w_0 be the wage in period 0 and assume that the worker chooses 8 hours of work in that period. Then λ is given by

$$(3.8) \quad \lambda = \frac{16^{-1/\sigma}}{w_0}$$

Thus

$$(3.9) \quad S_h(w) = 24 - 16 \left(\frac{w}{w_0} \right)^{-\sigma}$$

The critical wage, w^* , where labor supply becomes perfectly elastic is the root of equation 2.11 with these functions. The critical wage depends on the elasticity of substitution, σ , as shown in Figure 1. The upward-sloping curves in Figure 1 are two labor supply schedules, of the form of equation 3.9, with $\sigma = .1$ and $.2$. The downward-sloping curves measure the average "cost" of an hour of work,

$$(3.10) \quad f(h) = \frac{\Delta u/\lambda + k}{h}$$

$f(h)$ includes both the value of the foregone leisure and the fixed element, k . w^* is the wage at the intersection of the two schedules; this is just a restatement of the condition 2.11. Higher values of σ correspond to higher labor supply (for h less than 8 hours) and to higher cost schedules. Hence w^* is unambiguously higher for more wage-elastic labor supply:

Proposition 4: The critical wage, w^* , where labor supply becomes perfectly elastic, is low when intertemporal substitution in work hours is low and is high when substitution is high.

Figure 1. The critical wage, w^*

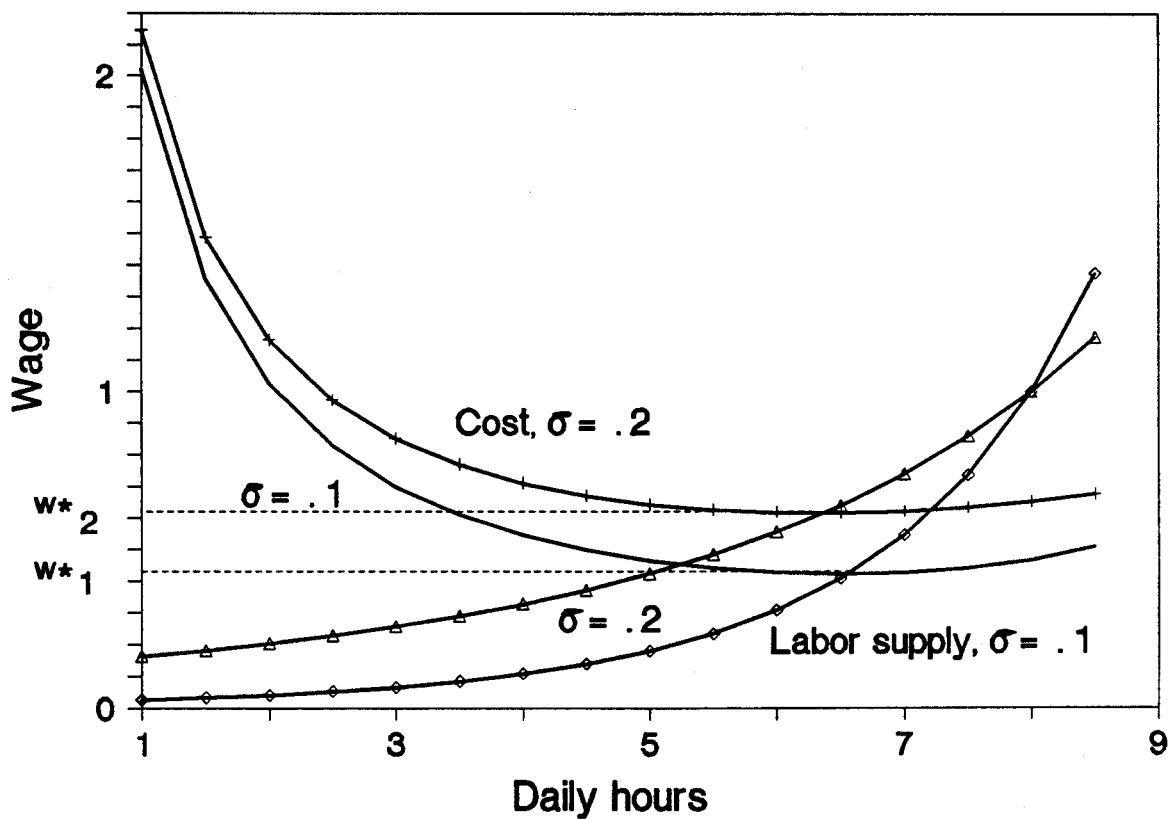


Figure 2 shows two short-run labor supply schedules, one for the low value, $\sigma = .05$, and one for the high value, $\sigma = .20$. The vertical axis is the wage, expressed as the ratio to the normal wage, w_0 . The fixed cost, k , is taken as $2w_0$ --the fixed cost is two hours of pay at normal wages.. Both labor supply schedules are far from the perfectly elastic ones proposed by Rogerson and Hansen. The worker with low σ is particularly reluctant to enter the region where work is not spread over all available days. Only when the wage has fallen to 32 percent of its normal level does the worker begin to specialize and enter the perfectly elastic part of his labor supply schedule. Even with the higher level of σ , the worker works a full-year schedule until the wage drops to 62 percent of normal.

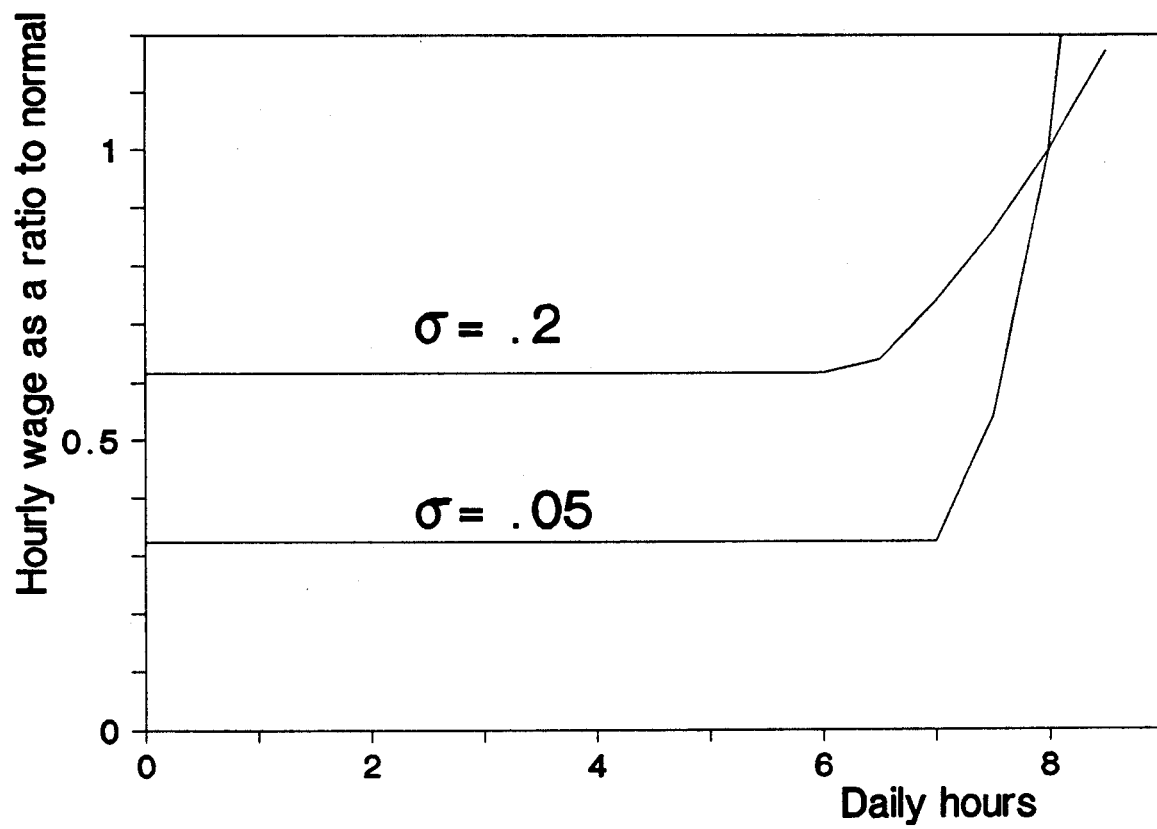
The upshot of this investigation is that modest variations in the wage or shadow value of work will not coax the worker onto the flat part of the labor supply schedule. The existence of fixed costs will have no impact at all on the allocation of labor unless the shadow value drops to perhaps 60 percent of its normal level. From that point on, however, perfect elasticity takes over. Fixed costs will contribute to the volatility of employment if there is a powerful driving force capable of sharply depressing the shadow value of labor. Fluctuations of the order of magnitude of observed fluctuations in real wages are incapable of moving employment into the region of perfectly elastic labor supply.

4. Labor mobility

Fluctuations in employment are concentrated in certain cyclical sectors: manufacturing and construction, in particular. A key question for fluctuations theory is why workers who are unemployed because of a contraction in a cyclical sector do not take work in a non-cyclical sector. For theories in which the driving force is shifts in the composition of demand, it takes an explanation of immobility to arrive at a theory of fluctuations in employment and unemployment.

Consider a worker in a sector in a temporary slump. The worker's long-term job sets the value of λ . There is another job available paying a wage w , expressed as a ratio to the worker's regular wage. To take the temporary job, the

Figure 2. Labor supply schedules



worker has to incur a moving cost m . The fixed cost per day of work at the new job is k . What is the minimum number of days of employment at the new job, d , such that it pays to take the job?

The point of indifference occurs when the net return from temporary employment equals the value of the time foregone:

$$(4.1) \quad (w h - k)d - m = \frac{d \Delta u(h)}{\lambda}$$

Solve for d :

$$(4.2) \quad d = \frac{m}{w h - k - \Delta u(h)/\lambda}$$

That is, the number of days is the ratio of the one-time mobility cost to the daily net benefit of working. There is an immediate connection with the previous discussion of the perfectly elastic part of the labor supply schedule. If the alternative job pays only the equivalent of w^* , the alternative job will have to last forever to justify the moving cost.

Suppose the moving cost is 500 hours of wages at the normal rate, the fixed cost is 2, and the alternative job pays 70 percent of the normal wage. Then the critical number of days to amortize the moving cost is

σ	d
.05	179
.20	901

In the low-elasticity case, $\sigma = .05$, it would pay for the worker to take a temporary job with a 30 percent pay cut over the year or two that a typical slump lasts. If σ is .20, it would take over 4 years of work (with 235 working days in a year) to amortize moving costs equal to 500 hours of work. Workers with low values of σ put little value on the extra leisure available during a period off the job. Hence they would choose a fill-in job during a slump even in the face of

substantial mobility costs. Workers with values of σ of .4 (corresponding to a short-run elasticity of labor supply of .8) put sufficient value on the extra leisure that they would choose to sit out a slump.

5. Conclusions

It makes a big difference whether the intertemporal elasticity of substitution is .05 or .40, that is, whether the short-run elasticity of labor supply is .1 or .8. The lower value is consistent with empirical evidence. The volatility of employment is hard to explain when the elasticity is so low--it requires huge movements in the shadow value of work to generate fluctuations in the 10-percent range typical of the business cycle. Fixed costs have little effect on the short-run labor supply schedule. The perfectly elastic segment of labor supply, as in Figure 2, occurs at just over 30 percent of the normal wage, when the fixed cost is equal in value to 2 hours of work. Moreover, even if the shadow value of work fell below the critical level in the worker's own sector, he would be willing to work temporarily in another sector in the face of a 30-percent lower wage than normal and a moving cost equal in value to 500 hours of work at the normal wage. Thus, an economy populated by workers with values of σ around .05 will have very stable employment in response to large fluctuations in the demand for labor.

Even at the upper end of the range of values of σ consistent with the empirical research--say $\sigma = .20$ --the same conclusion follows. The critical wage where labor supply becomes perfectly elastic is 60 percent of the normal wage. If a sector experiences a slump lasting a little less than a year, it pays for some of the workers to take alternative jobs. Aggregate employment is highly stable unless extreme fluctuations in demand occur.

On the other hand, if σ is 0.40, employment volatility becomes more readily explicable. At that level of substitution, the worker puts substantial value on a temporary period of extra leisure. The critical shadow value of work, below which the worker chooses not to work, is 85 percent of the normal wage. And such a worker would choose to remain out of work rather than take a temporary job at a pay cut of 15 percent or more. As I noted in section 1, elasticities of labor

supply of close to 1 are inconsistent with the empirical estimates and require a rationalization of an important downward bias in those estimates.

Robust economic explanations of the volatility of employment have two ways to turn at this point. One is to take the stand that the true short-run elasticity of labor supply is close to one, even though empirical studies find much lower values. Then relatively modest fluctuations in the shadow value of labor are consistent with the observed volatility of employment. Two enhancements of that view emerged from the discussion in the body of this paper: First, labor supply becomes perfectly elastic once the shadow value of labor has dropped about 15 percent below normal. Second, workers from a slumping sector will not take temporary jobs in another sector unless the wage is sufficiently higher than the critical cutoff wage to amortize moving costs over the period the slump is expected to last.

The alternative is to accept the finding of low intertemporal elasticity of substitution and to build a model in which very large movements in the shadow value of labor can occur. Such a model is almost certainly non-competitive. An example is Hall (1987). In comparison to a firm in a competitive industry, a firm with monopoly power is much more likely to maintain a stable relative price when demand falls, and to cut its output and level of employment by more. Then the perfectly elastic segment of the labor supply schedule discussed in this paper becomes relevant even when the critical shadow value is only 40 percent of the normal shadow value. The reduction in employment that occurs in a non-competitive cyclical industry can easily be large enough to put the workers on this segment of their labor supply.

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