Some Evidence on the Sources of

Economic Fluctuations

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1. The Issues

Sharply different views about the reasons for booms and recessions

coexist among knowledgeable economists. The traditional Keynesian view

attributes fluctuations to shifts in the expenditure schedules of various

agents, especially in private investment. An even older view, still

vigorously promoted today, attributes fluctuations in real output to changes

in the stock of money. Lastly, there is now a growing body of opinion

holding that shifts in supply schedules are an important source of fluctua-

tions. The object of this paper is to measure the relative importance of

these three sources of fluctuations.

The raw material for this investigation is the observed co-variation

in the United States of three major variables over the postwar period—real

GNP, the price level, and the money supply. Each of the three sources of

fluctuations has a different pattern of effects on the three variables: An

upward shift in the aggregate demand schedule raises real output and the

price level; an upward shift in the aggregate supply schedule raises real

output and depresses the price level; and a positive surprise in money

growth raises output and prices and shows up directly in the money stock as

well.

The investigation is carried out within a simple model of the aggregate

economy. On the demand side, the reduced form of an IS-LM model is repre-

sented as a relation between real output, the money stock, and the price

level. The random disturbance in this relation is the model's characteriza-

tion of the source of fluctuations arising from demand. On the supply side,

aggregate supply is considered as a function of the price level relative to

a predetermined level based on expectations formed in earlier years. Again,

the random disturbance characterizes fluctuations arising from supply. The

price elasticity of aggregate supply determines the "Keynesness" of the model. If aggregate supply is price-elastic, prices are fairly rigid and quantity responses to demand shocks are strong. Horizontal shifts of the aggregate supply schedule have relatively little effect on output or prices in the Keynesian economy. On the other hand, if the price elasticity is low, the model is more classical--demand shifts influence prices instead of output, and supply shifts influence both output and prices.

The behavior of the money supply is viewed as the combination of a response of monetary policy to the demand and supply shocks and an independent monetary disturbance. If the primary motivation of monetary policy is to stabilize interest rates, then the money supply will respond positively to demand shocks, while if the motivation is to stabilize real GNP (and the economy is at least somewhat Keynesian), the response will be negative. Only the part of monetary fluctuations not associated with policy responses is treated here as an independent source of fluctuations.

The co-variation of the three variables that identifies the magnitude of the sources of fluctuations and the structure of the economy is across time periods as well as across variables. The response of this year's real output to a demand surprise from an earlier year gives information about the time pattern of price flexibility, for example. The model assumes classical behavior and perfect price flexibility in the very long run, but the amount of flexibility in the medium run is estimated from the data. The expectations that enter the process whereby prices are set in advance are assumed to be formed rationally--this assumption has important econometric identifying power.

The results of estimation suggest that both demand and supply shocks are important sources of macroeconomic fluctuations, whereas the independent

component of monetary policy is relatively unimportant. Some of the effects of the disturbances operate indirectly through their influence on the money supply rather than directly by shifting the aggregate supply and demand schedule, though. In this respect, money has a central intermediary role in fluctuations. With respect to the structural characteristics of the economy the study finds first, that the elasticity of aggregate demand with respect to the real money supply is approximately one half. Upward displacements of the money stock or downward displacements of the price level can have potent expansionary effects. Second, the estimated elasticity of aggregate supply with respect to prices in the short run is almost infinity—the economy is almost strictly Keynesian. Even after two years, prices are still relatively insensitive to demand, though the statistical reliability of this finding is not very high. Third, monetary policy tends to intensify demand shocks and weaken supply shocks in the year they occur, but both reactions are relatively weak.

In general, the findings of this study support some aspects of the views of all of the major schools of thought about the sources of fluctuations. Shifts in the aggregate demand schedule do seem to matter and are not offset by immediate price adjustments. Fluctuations in the money stock are also important and affect real output more than prices in the short run. Shifts in aggregate demand are the major source of fluctuations in real GNP, but shifts in supply are important as well. An eclectic view of economic fluctuations emerges from this study of the data.

2. The Basic Model and Estimation Technique

This section introduces the basic macroeconomic model and the econometric technique used to infer values of structural parameters and variances of disturbances. It abstracts from the fruitful but somewhat complex examination of the co-variation of the variables across time periods. It also ignores the monetary disturbance in order to focus on the demand and supply disturbances.

Consider first the simple IS-LM model. The price level is taken as fixed, for the moment. Then the position of the IS curve is determined by the expenditure process and the position of the LM curve by the money demand function and by the real money supply. The interest rate and the level of real GNP are determined by the intersection of the two curves. The model makes use of the derived relationship between real output, the real money supply, and a demand shift:

$$X_t = \phi(M - p_t) + d_t \; ;$$

here X_t is the log of real GNP, M is the log of the money supply, p_t is the log of the price level, ϕ is the elasticity of aggregate demand with respect to the real money supply, and d_t is the random variable characterizing the demand shift. Here and in subsequent equations, some constants are omitted for simplicity. For now, d_t is assumed serially uncorrelated and unpredictable given information available before year t. Note also that the money supply is assumed unchanging over time. The omission of the interest rate from the aggregate demand function indicates that it is a reduced form, not that the expenditure process is unaffected by interest rates. To keep the model simple, the companion reduced form equation for the interest rate is omitted and the information contained in the co-variation of interest rates, output, and prices is not used.

The aggregate supply schedule makes supply depend on the current price level relative to a predetermined level, and on a random supply disturbance:

$$X_t = \bar{X}_t + \psi(p_t - z) + s_t$$

$\bar{X}_t$ is the trend part of the potential or natural level of output, ψ is the elasticity of aggregate supply with respect to the price level, z is the predetermined element of the price level, and s_t is a random variable characterizing supply shifts. Aggregate supply is not the outcome of profit-maximizing supply decisions of producers; such a supply function always depends on price ratios and not on the level of a single price as in this equation. Rather, aggregate supply is the Phillips curve of the simple model. It expresses the temporary rigidity of prices: The larger is the supply elasticity, ψ, the more Keynesian is the economy. In the simple model, the constancy of the money supply and the lack of serial correlation of the disturbances imply that any predetermined element is a constant. All that matters is the ability of sellers to adjust prices to current conditions. The lower is the supply elasticity, ψ, the more responsive are prices within the year.

The specification of the aggregate supply function used throughout this paper falls within the prevailing modern view that, in the short run, prices are somewhat unresponsive but sellers stand ready to meet all demand that is forthcoming at the prevailing price. Though practical economists find this characterization of the behavior of the economy compelling, it lacks a strong theoretical foundation. Recently it has been suggested that the existence of contracts that predetermine wages and prices might explain the phenomenon of high price elasticity of aggregate supply in the short run, but the short-comings of that explanation have been pointed out by Barro (1977b). The

older explanation that appeals to the slow diffusion of information is unpersuasive to many because information seems to be transmitted very quickly in most markets. There is a growing recent literature that rejects price-elastic aggregate supply and offers alternative explanations where the elasticity is zero in all but the very shortest run. See Lucas (1975) and Sargent (1977). In any case, the _possibility_ of price-elastic supply will be a feature of all of the models considered in this paper.

The reduced form of the simple model is

$$p_t - M = \frac{1 - \mu}{\phi} d_t - \frac{1 - \mu}{\phi} s_t$$

$$X_t - \bar{X}_t = \mu d_t + (1 - \mu)s_t$$

μ is the coefficient of Keynesness,

$$\mu = \frac{\psi}{\phi + \psi}$$

The price variable is expressed as the log of the price-money ratio for reasons that will be explained later in the paper. At this stage, M is just a constant, so subtracting it from p has no substantive content.

In the classical economy, where supply is totally inelastic ($\psi = 0$) and so $\mu = 0$,

$$p_t - M = \frac{1}{\phi} d_t - \frac{1}{\phi} s_t \; ;$$

$$X_t - \bar{X}_t = s_t \; ;$$

output is perturbed only by supply shifts. Demand shifts and supply shifts

are both accommodated by price responses. At the other extreme, in the Keynesian economy with complete price rigidity ($\mu = 1$),

$$p_t - M = 0 \qquad \text{(recall that the constant is omitted)}$$

$$X_t - \bar{X}_t = d_t$$

Supply shifts have no effect on either prices or output. Demand shifts affect output one-for-one and have no effect on prices.

Suppose that one observes prices and output in an economy whose Keynesness is unknown. What can be learned from the data about the structure of the economy and the relative importance of demand and supply shifts? In a linear statistical model of this kind, all of the relevant information from the data can be summarized in the variances and covariances of the data. With just two variables, the information is contained in three numbers:

$$M_{pp} = V(p_t - M)$$

$$M_{XX} = V(X_t - \bar{X}_t)$$

$$M_{pX} = \text{Cov}(p_t - M, X_t - \bar{X}_t)$$

The model implies values for these as functions of the underlying parameters:

$$M_{pp} = \left(\frac{1 - \mu}{\phi}\right)^2 \sigma_d^2 + \left(\frac{1 - \mu}{\phi}\right)^2 \sigma_s^2$$

$$M_{XX} = \mu^2 \sigma_d^2 + (1 - \mu)^2 \sigma_s^2$$

$$M_{pX} = \frac{\mu(1 - \mu)}{\phi} \sigma_d^2 - \frac{(1 - \mu)^2}{\phi} \sigma_s^2$$

Here it is assumed that the demand and supply shocks are statistically independent. Even with that strong assumption, it is apparent that estimation of the underlying parameters is hopeless. There are four parameters, ϕ, μ, σ_d^2, and σ_s^2, and only three distinct numbers from which to estimate them. Only with prior knowledge of one of the parameters is estimation possible. For example, under the hypothesis that aggregate demand is unit-elastic with respect to the real money supply ($\phi = 1$), it is easy to solve for the values of the parameters implied by the data:

$$\mu = \frac{M_{pX} + M_{XX}}{M_{pp} + 2M_{pX} + M_{XX}}$$

$$\sigma_d^2 = M_{XX} + 2M_{pX} + M_{pp}$$

$$\sigma_s^2 = \frac{M_{XX} - \mu^2 \sigma_d^2}{(1 - \mu)^2}$$

In the data used later in the paper, the variances and covariances are $M_{pp} = 14.6$, $M_{XX} = 10.5$, and $M_{pX} = -8.0$. Then, under the hypothesis that ϕ is unity, the implied estimates of the other parameters are: $\mu = 0.28$, $\sigma_d^2 = 9.0$, and $\sigma_s^2 = 18.9$. According to these estimates, the economy is not very Keynesian, and the variance of the supply shock is more than double that of the demand shock. The earlier formulas for the variances and covariances together with the statistical independence of the demand and supply shocks make it possible to decompose the variances and the covariance into demand and supply components:

$$M_{pp} = 14.6 = 4.8 \text{ demand} + 9.8 \text{ supply}$$

$$M_{XX} = 10.5 = 0.7 \text{ demand} + 9.8 \text{ supply}$$

$$M_{pX} = -8.0 = 1.8 \text{ demand} - 9.8 \text{ supply}$$

In this relatively un-Keynesian economy, demand shocks have an important effect on the variance of p-M because they influence the market-clearing price level. However, about two-thirds of the variance of p-M comes from the larger supply shock. Almost none of the variance of real output comes from the demand side, both because the economy is only slightly Keynesian and because the demand shocks are generally small compared to the supply shocks. Finally, the negative correlation of p-M and $X-\bar{X}$ is attributable entirely to the supply shock--demand shocks push the two variables in the same direction and so add an offsetting positive term to the covariance.

All of these conclusions are strongly dependent on the hypothesis that ϕ is one, and so are no more than illustrative. If ϕ is 0.75, for example, the implied estimate of the Keynesness parameter, μ, is 0.67, indicating much more price rigidity. As a general matter, the contemporaneous joint behavior of p-M and $X-\bar{X}$ does not contain enough information to answer the major questions about the origin of economic fluctuations. Information contained in the covariances with the money stock and the covariances across time periods is required as well.

3.   The Full Model

The simple model of the previous section is far wide of the mark in assuming that participants in the economy are incapable of forecasting any of the influences that determine prices and output. The simple model cannot generate sustained movements of either output or prices, yet the data for the United States show important movements lasting several years each. Further, the simple model makes the untenable assumption that the money stock never changes, when in fact, the stock drifts upward at a variable rate and is thought by some to be responsible for important fluctuations in

output. The full model in this section adds a monetary shock to the demand and supply shocks. It permits all three shocks to be serially correlated. The aggregate demand and supply functions remain the same, but the ability of participants to forecast future states of the economy requires a considerable elaboration of the process that predetermines prices and makes aggregate supply elastic with respect to the contemporaneous price level.

The aggregate demand function now lets the money stock vary over time:

$$X_t = \phi(M_t - p_t) + d_t$$

Similarly, the predetermined element of prices in the aggregate supply function also changes over time:

$$X_t = \bar{X}_t + \psi(p_t - z_t) + s_t$$

The most sensitive feature of the full model, and, indeed, the source of much controversy in modern macroeconomics, is the specification of the way that prices are predetermined in the aggregate supply equation, that is, the specification of the variable z_t. Consider first a specification introduced by Robert Lucas (1972): Suppliers set prices one period in advance. In doing so, they make use of all information about the future state of demand. They attempt to set the future price at what will turn out to be the market-clearing level. The motivation for this behavior is easiest to explain for perfectly competitive markets--if one seller sets a price above that of other sellers, he will sell nothing. If the price is below that of other sellers and also below cost, the seller will sustain a large loss. If it is below that of other sellers but above cost, he will make a large profit. The only point where the behavior of all sellers is consistent and

no excess profit is expected is where all sellers set the same price, and it is the expected market-clearing price.

Within the model, clearing of the market requires that actual output, X_t, equal current supply, $\bar{X}_t + s_t$. Thus expected market clearing involves

$$\underset{t-1}{E} X_t = \bar{X}_t + \underset{t-1}{E} s_t$$

The reduced form for X_t, given z_t, the money stock, M_t, and the disturbances d_t and s_t is

$$X_t = \bar{X}_t + s_t + \mu(d_t - s_t + \phi M_t - \phi z_t - \bar{X}_t)$$

Taking expectations and solving for the expected-market-clearing z_t gives

$$z_t = \frac{1}{\phi}(\underset{t-1}{E} d_t - \underset{t-1}{E} s_t + \phi \underset{t-1}{E} M_t - \bar{X}_t)$$

Now let q_t be the same combination of the actual random variables less their expected values:

$$q_t = d_t - \underset{t-1}{E} d_t - (s_t - \underset{t-1}{E} s_t) + \phi(M_t - \underset{t-1}{E} M_t)$$

This "disequilibrium" term measures all of the new information that becomes available in period t and condenses it to a single number. Note that $\underset{t-1}{E} q_t = 0$. It turns out that the reduced form for the model incorporating the hypothesis of expected market clearing can be written in the following way:

$$p_t - M_t = \frac{1}{\phi}(d_t - s_t - \bar{X}_t) - \frac{\mu}{\phi} q_t$$

$$X_t = \bar{X}_t + s_t + \mu q_t$$

Surprises from all three sources affect the price-money ratio and real output in a way that can be described by the same variable, q_t, because surprises relative to expectations formed last year matter this year only because of the predetermined element in the aggregate supply equation.

In the reduced form, the log of the price-money ratio is the sum of an equilibrium term, $(d_t - s_t - \bar{X}_t)/\phi$, and a disequilibrium or surprise term, $-\frac{\mu}{\phi} q_t$. Disequilibrium occurs only in an economy with some Keynesness (μ positive). Similarly, output is equal to its equilibrium level, $\bar{X}_t + s_t$, plus a disequilibrium term, μq_t. Again, disequilibrium occurs only with Keynesness.

The definition of the disequilibrium variable, q_t, makes it clear that disequilibrium can arise because of mistakes in predicting demand $(d_t \neq \underset{t-1}{E} d_t)$, mistakes in predicting supply $(s_t \neq \underset{t-1}{E} s_t)$, or mistakes in predicting the money stock $(M_t \neq \underset{t-1}{E} M_t)$. Any of these errors will cause the predetermined element of prices to depart from the market-clearing level, and thus result in disequilibrium in the output market. Because all prediction errors have their effects through the predetermined element of prices, it is possible to write a structural relation between aggregate supply and the departure of the actual price level from its expected level:

$$X_t = \bar{X}_t + \psi(p_t - \underset{t-1}{E} p_t) + s_t$$

This is the celebrated "Lucas supply function." It has been widely misunderstood as suggesting that errors in price expectations are a _cause_ of supply shifts, rather than a manifestation of surprises occurring in all parts of the economy. This misinterpretation seems to lie behind the critical remarks of Modigliani (1977) and Gordon (1977) on this subject, for example. This paper will avoid stating the aggregate supply function in Lucas' form

(though his form is perfectly valid), both because of its history of misinterpretation and because it does not generalize in the expected way for the case where some prices take more than one period to respond to new information. Throughout, the paper will instead focus on the reduced form relation between the random shifts in demand, supply, and money, and the observed levels of prices, output, and the money stock.

The assumption that prices respond to new information after exactly one period is not satisfactory for empirical work, since the length of the period is left unspecified. A better starting point is a distribution of response lags. Perhaps the clearest assumption that would explain the existence of such a distribution is that the observed economy is the aggregation of a group of independent, non-interacting economies, each of which has a discrete price lag. The length of the lag is one year in economy 1, two years in economy 2, and so forth. In the process of aggregation, economy i is assumed to be given weight ω_i. The shocks perturbing the various economies are the same, and so the money stocks are the same. Trend output, $\bar{X}_t$, and all parameters are also the same in all of the economies. However, price levels and output differ among the economies because some react more quickly than others to new information. The reduced form for economy i is

$$P_{it} - M_t = \frac{1}{\phi}(d_t - s_t - \bar{X}_t) - \frac{\mu}{\phi} q_{it}$$

$$X_{it} = \bar{X}_t + s_t + \mu q_{it}$$

$$q_{it} = d_t - E_{t-i} d_t - (s_t - E_{t-i} s_t) + \phi(M_t - E_{t-i} M_t)$$

Taking weighted averages with weights ω_i gives

$$p_t - M_t = \frac{1}{\phi}(d_t - s_t - \bar{X}_t) - \frac{\mu}{\phi} q_t$$

$$X_t = \bar{X}_t + s_t + \mu q_t$$

$$q_t = d_t - \sum_{i=1}^{\infty} \omega_i \mathop{E}_{t-i} d_t - (s_t - \sum_{i=1}^{\infty} \omega_i \mathop{E}_{t-i} s_t) + \phi(M_t - \sum_{i=1}^{\infty} \mathop{E}_{t-i} M_t)$$

Again, the influence of prediction errors on $p_t - M_t$ and X_t operates through the single disequilibrium variable q_t, but now q_t involves a distributed lag of past predictions of the disturbances.

The Lucas supply functions for the individual economies aggregate in the following way:

$$X_t = \bar{X}_t + \psi(p_t - \sum_{i=1}^{\infty} \omega_i \mathop{E}_{t-i} p_{it}) + s_t$$

Aggregation does not eliminate all of the variables associated with the individual economies-- $\mathop{E}_{t-i} p_{it}$ remains in the aggregated supply function. It is certainly not the case that $\mathop{E}_{t-i} p_{it} = \mathop{E}_{t-i} p_t$. What seems at first the obvious generalization of the Lucas supply function,

$$X_t = \bar{X}_t + \psi(p_t - \sum_{i=1}^{\infty} \omega_i \mathop{E}_{t-i} p_t) + s_t$$

is not a correct implication of the aggregation of independent economies, nor is it a sensible specification of aggregate supply for any economy. There does not seem to be a useful distributed lag version of Lucas' structural equation for supply. The rest of this paper will use the reduced form, which aggregates in a straightforward way.

4. Stochastic Specification and Econometric Technique

The empirical results reported later in the paper are based on the following specification for the random processes:

$$d_t = \sum_{i=0}^{\infty} \delta^i \, \varepsilon_{t-i}$$

$$s_t = \sum_{i=0}^{\infty} \lambda^i \, \eta_{t-i}$$

$$\Delta M_t = \rho \Delta M_{t-1} + \alpha \varepsilon_t + \beta \eta_t + \nu_t$$

ε_t, η_t, and ν_t are distributed independently of all variables dated earlier than t and are independent of each other. Thus the demand and supply disturbances obey first-order autoregressive processes with parameters δ and λ, and the rate of growth of the money stock, ΔM_t, is serially correlated with parameter ρ. Note that the level of M_t is nonstationary under this specification—it does not have any tendency to return to a normal value. The parameters α and β control the response of monetary policy to the demand and supply shocks, respectively.

The distribution of reaction times of prices is assumed to be geometric: The weight given to economy i is

$$\omega_i = (1 - \omega)\omega^{i-1}$$

Then the weighted averages of expectation errors are

$$d_t - (1 - \omega) \sum_{i=1}^{\infty} \omega^{i-1} E_{t-i} d_t = \sum_{i=0}^{\infty} (\delta\omega)^i \varepsilon_{t-i}$$

$$s_t - (1 - \omega) \sum_{i=1}^{\infty} \omega^{i-1} E_{t-i} s_t = \sum_{i=0}^{\infty} (\lambda\omega)^i \eta_{t-i}$$

$$M_t - (1 - \omega) \sum_{i=1}^{\infty} \omega^{i-1} E_{t-i} M_t = \sum_{i=0}^{\infty} \frac{1-\rho^{i+1}}{1 - \rho} \omega^i (\alpha\varepsilon_{t-i} + \beta\eta_{t-i} + \nu_{t-i})$$

and the disequilibrium variable is

$$q_t = \sum_{i=0}^{\infty} \left(\delta^i + \alpha\phi \, \frac{1-\rho^{i+1}}{1-\rho} \right) \omega^i \varepsilon_{t-i}$$

$$- \sum_{i=0}^{\infty} \left(\lambda^i - \beta\phi \, \frac{1-\rho^{i+1}}{1-\rho} \right) \omega^i \eta_{t-i}$$

$$+ \phi \sum_{i=0}^{\infty} \frac{1-\rho^{i+1}}{1-\rho} \omega^i \nu_{t-i}$$

The disequilibrium effect of the new information about demand contained in an ε_t diminishes over time at a geometric rate which is the product of the serial correlation parameter δ and the lag parameter ω. A demand surprise can have a lasting effect only if serial correlation is strong and the response of prices is sluggish (ω is close to one). A supply surprise enters in a similar way. An independent monetary surprise, ν_t, has an effect that grows over time at first (if ω is not too small) and then declines to zero. Although the money stock has a nonstationary component which never returns to a zero after a shock, the disequilibrium variable does return to zero, because of the eventual incorporation of a monetary disturbance into an offsetting price change.

Estimation of the parameters of the full model will be carried out by a generalization of the technique of matching the observed and theoretical variances and covariances of the three variables. Covariances across time are informative in the full model, both about the serial correlation of the various disturbances and about the lags in responses to those disturbances. The technique can only be applied in cases where the model implies that the variances and covariances are finite. Both the log of the price-money ratio, $p_t - M_t$, and the departure of real output from trend, $X_t - \bar{X}_t$, have

finite variances, according to the model (as long as the serial correlation parameters, δ and λ, are both less than one). However, the money supply does not. This choice of stochastic specification is intentional--the money stock shows no tendency to a normal level, so the most natural specification is one with an "integrated" component where only the first difference tends to return to normal. But the first difference of the log of the money stock does have a finite variance, according to the model, so ΔM_t is a good choice for the third variable in the covariance matrix, along with $p_t - M_t$ and $X_t - \bar{X}_t$. The requirement of finite variances and covariances is achieved in three ways in the model:

1. Neither p_t nor M_t is required to have a finite variance, but eventual market clearing (i.e., monetary neutrality in the long run) means that $p_t - M_t$ tends to a normal level.

2. Market-clearing causes the permanent movements in the money supply to have temporary effects on the departure of real output from trend, $X_t - \bar{X}_t$.

3. The money stock appears in first difference form, which is hypothesized to have a finite variance.

For the empirical results presented later in this paper, the specification was altered to reduce the computational burden in estimation: All error processes and the distributed lag were truncated after four years. The price determination lag coefficients were specified as

$$\omega_i = \frac{1 - \omega}{1 - \omega^4} \omega^{i-1}$$

which sum to one and so constitute a true distribution. Then the complete model with this truncation is

$$P_t - M_t = \frac{1}{\phi}(d_t - s_t - \bar{X}_t) - \frac{\mu}{\phi} q_t \tag{1}$$

$$X_t - \bar{X}_t = s_t + \mu q_t \tag{2}$$

$$d_t = \sum_{i=0}^{4} \delta^i \varepsilon_{t-i} \tag{3}$$

$$s_t = \sum_{i=0}^{4} \lambda^i \eta_{t-i} \tag{4}$$

$$\Delta M_t = \sum_{i=0}^{4} \rho^i (\alpha \varepsilon_{t-i} + \beta \eta_{t-i} + \nu_{t-i}) \tag{5}$$

$$q_t = \sum_{i=0}^{4} \frac{\omega^i - \omega^4}{1 - \omega^4} \left[\left(\delta^i + \alpha\phi \frac{1-\rho^{i+1}}{1-\rho} \right) \varepsilon_{t-i} \right.$$

$$\left. - \left(\lambda^i - \beta\phi \frac{1-\rho^{i+1}}{1-\rho} \right) \eta_{t-i} + \phi \frac{1-\rho^{i+1}}{1-\rho} \nu_{t-i} \right] \tag{6}$$

Let y be a vector consisting of all the observations on the variables $P_t - M_t$, $X_t - \bar{X}_t$, and ΔM_t. Suppose that the underlying disturbances are normal random variables with variances σ_d^2, σ_s^2, and σ_m^2. Then y is a linear combination of normals and is multivariate normal itself, with mean zero and covariance matrix $\Omega(\theta)$; θ is a vector containing the 11 parameters of the model: ϕ, μ, α, β, ω, δ, λ, ρ, σ_d^2, σ_s^2, and σ_m^2. The elements of $\Omega(\theta)$ can be computed from the model in a straightforward but tedious way. Only the first three columns of $\Omega(\theta)$ need be formed; the other columns are just lagged versions of the first three. Then the log-likelihood of the sample is

$$L = -\frac{1}{2} \log \det \Omega(\theta) - \frac{1}{2} y' \Omega(\theta)^{-1} y$$

plus an inessential constant. Maximization of this likelihood gives an estimator with the usual desirable properties.

5. <u>The Data</u>

Data were defined as follows:

p_t: Log of the GNP implicit price deflator, 1972 = 100, from the U.S.

national income accounts

X_t: Log of real GNP in billions of 1972 dollars

M_t: Log of money stock (currency plus demand deposits), average of

weekly data for December of each year.

The variables used in the computations, $p_t - M_t$, $X_t - \bar{X}_t$, and ΔM_t were

formed from these data. $\bar{X}_t$ was taken as a linear trend (constant rate of

growth of GNP itself), and $X_t - \bar{X}_t$ was simply the residuals from a regression

of X_t against time. Similarly, a small trend was removed from $p_t - M_t$ (this

corresponds to the net influence of GNP growth and the offsetting trend in

the velocity of money) and a small constant was subtracted from ΔM_t.

The resulting data are shown in Table 1, and their correlations, across

variables and across time periods, are shown in Table 2. The next section

of the paper will attempt to extract the information contained in the corre-

lations by formal econometric procedures. First, though, it is useful to

mention the most noticeable features of the data and their relation to the

questions under investigation in this study. Some hypotheses about the

sources of fluctuations can be dismissed simply by looking at the data.

One of the most salient features of the data is the high amplitude of

the movements in the price-money ratio and their obvious relation to conven-

tional ideas of when booms and recessions have occurred. The standard

deviation of $p_t - M_t$, 0.038, actually exceeds that of $X_t - \bar{X}_t$, 0.032, by a

considerable margin. Recall that the model's equation for the price-money

ratio is

$$p_t - M_t = \frac{1}{\phi}(d_t - s_t) - \frac{\mu}{\phi} q_t$$

and the disequilibrium variable, q_t, depends positively on demand and monetary shocks and negatively on supply shocks. If the economy is highly Keynesian (μ close to 1), neither demand nor supply shocks have a strong effect on $p_t - M_t$, unless monetary policy is strongly responsive to the shocks. Thus a Keynesian interpretation of the observed behavior of the price-money ratio emphasizes the movements of its denominator, the money stock. The movements of $p_t - M_t$ do cast doubt on one hypothesis about fluctuations-- that demand shifts are the principal source of movements in the economy, prices are unresponsive to demand for Keynesian reasons, and the money stock is quiescent. However, modern Keynesian views where the money stock is given a more important role are perfectly capable of explaining the observed behavior of the price-money ratio.

There are large, sustained movements of real GNP around its trend as well. Ever since the great depression and the Keynesian revolution, explanation of these movements has been the central focus of macroeconomics. The model's equation for real output,

$$X_t - \bar{X}_t = s_t + \mu q_t$$

makes it clear that the Keynesian interpretation--that demand shifts are the major source of fluctuations in real output--is quite consistent with the model. In a Keynesian economy with μ close to one, the demand surprises in the disequilibrium variable q_t translate into comparable movements in $X_t - \bar{X}_t$. Of course, the same movements could be caused by independent monetary fluctuations in a Keynesian economy. On the other hand, supply shifts have large effects in a classical economy (μ close to zero). Supply

Table 1

Price-Money Ratio, Departure of Real GNP from Trend,
and Rate of Monetary Growth, 1948–1976

Year	Price-Money Ratio	Departure of Real GNP from Trend	Rate of Monetary Growth
1948	0.3	-1.7	-2.3
1949	0.1	-4.6	-1.4
1950	-1.8	0.2	3.1
1951	-0.2	4.4	4.0
1952	-2.2	4.6	2.0
1953	-1.2	4.9	-0.8
1954	-2.1	0.1	0.7
1955	-1.6	3.0	0.0
1956	0.9	1.5	-1.2
1957	5.3	-0.2	-3.3
1958	3.8	-4.0	0.9
1959	4.7	-1.6	-1.3
1960	6.4	-3.0	-2.6
1961	4.7	-4.0	-0.2
1962	5.6	-1.9	-2.0
1963	4.0	-1.6	-0.1
1964	1.5	0.1	0.6
1965	-0.3	2.3	0.5
1966	0.9	4.5	-1.6
1967	-2.1	3.6	2.0
1968	-4.7	4.3	3.0
1969	-2.5	3.4	-1.6
1970	-1.9	-0.6	0.1
1971	-2.6	-1.1	1.1
1972	-6.9	1.0	3.5
1973	-6.5	2.7	0.3
1974	-1.3	-2.2	-1.1
1975	4.4	-7.1	-1.9
1976	4.3	-4.8	-0.4

Note: Data are multiplied by 100 so they may be interpreted as percentage deviations.

Table 2

Correlations, 1948–1976

Current Variable	Lagged Variable	Length of Lag (Years)				
		0	1	2	3	4
p–M	p–M	1.00	0.76	0.44	0.25	0.17
	X–$\bar{X}$	-0.65	-0.56	-0.36	-0.24	-0.16
	ΔM	-0.76	-0.64	-0.32	-0.17	-0.21
X–$\bar{X}$	p–M	-0.65	-0.45	0.00	0.20	0.24
	X–$\bar{X}$	1.00	0.70	0.30	0.01	-0.05
	ΔM	0.33	0.61	0.22	-0.14	-0.14
ΔM	p–M	-0.76	-0.31	-0.19	-0.13	-0.07
	X–$\bar{X}$	0.33	0.26	0.11	0.02	-0.01
	ΔM	1.00	0.28	0.04	0.08	0.13

surprises have a negative effect on q_t which offsets their direct effects in a Keynesian economy.

The strong negative correlation of the price-money ratio and real output is another striking feature of the data. In a Keynesian economy where the numerator of the price-money ratio is fairly unresponsive, the explanation must rest on movements of the money stock. Either independent monetary shocks are a major source of fluctuations, or monetary policy strongly amplifies other shocks. Demand shocks by themselves cannot explain this finding. In a classical economy, neither demand nor monetary shocks have much effect on real GNP. However, supply shocks would exactly explain the observed negative correlation—a favorable shift in aggregate supply drives real GNP up and prices down, and vice versa.

Additional evidence about the structure of the economy is provided by correlations across years. Among these, the strongest serial correlation is in the price-money ratio. If the economy is Keynesian both in the short run (μ close to 1) and in the medium run (ω close to 1), this finding must be attributable to the money stock. Movements of the money stock are persistent, and prices adjust slowly to offset their movements. This requires not only that prices are relatively unresponsive within the year, but also that information about demand that is available several years in advance is not fully assimilated in prices. In a classical economy, where both demand and supply shifts affect the price-money ratio through its denominator, high serial correlation of the ratio reflects high serial correlation of one or both of these shifts.

The serial correlation of the departures of real output from its trend has been the subject of considerable interest in recent years (Lucas (1975), Sargent (1977), and Hall (1975)). Interestingly, the serial correlation of

$X_t - \bar{X}_t$ in Table 2 is weaker than is the serial correlation of $p_t - M_t$, though it is still pronounced. In a Keynesian economy with both short- and medium-run price rigidity, persistent demand or monetary shocks could explain the serial correlation of output. In a classical economy, only supply shifts can explain the serial correlation. As Sargent (1976) has pointed out, it is impossible to refute the hypothesis that a surprise from the demand side sets in motion a persistent supply response. The results of this study will rest on the hypothesis of statistical independence of demand and supply shifts. The results may invite interpretation along Sargent's lines, however.

Two major hypotheses survive this examination of the data. Either

1. The economy is Keynesian, both in the short and medium runs. Persistent movements in the money stock are the principal agent of fluctuations, though they may represent the response of monetary policy to demand and supply shocks, or

2. The economy is classical, prices respond quickly to new information, and shifts in the aggregate supply function are the major source of fluctuations.

The two hypotheses have dramatically different implications--in the first, an active countercyclical policy is effective and desirable, whereas in the second it is ineffective and probably undesirable. Separation of the two, if possible at all, requires a more concerted study of all the evidence simultaneously. Correlations across variables and across time yield additional information not so far considered, as do the correlations with the observed data on monetary growth.

6.   Results and Tests of Hypotheses

Maximization of the likelihood for the full model produced the following

parameter estimates and standard errors:

parameter	value	interpretation
ϕ	0.42 (0.22)	elasticity of aggregate demand with respect to the real money supply
μ	0.98 (0.02)	coefficient of Keynesness—the elasticity of real output with respect to a shift in demand
α	0.10 (0.17)	response of money stock to a shift in demand
β	-0.05 (0.04)	response of money stock to a shift in supply
ω	1.36 (0.43)	distributed lag parameter for response of prices to new information
δ	0.62 (0.07)	serial correlation parameter for demand shift
λ	0.40 (0.06)	serial correlation parameter for the supply shift
ρ	0.28 (0.15)	serial correlation parameter for the independent component of the rate of monetary expansion
σ_d^2	0.00049 (0.00013)	variance of the unexpected part of the shift in demand
σ_s^2	0.07693 (0.14433)	variance of the unexpected part of the shift in supply
σ_m^2	0.00017 (0.00005)	variance of the unexpected part of the independent component of the money stock

Briefly, these parameters tell the following story about the aggregate

economy: First, demand is quite responsive to the real money supply. The

point estimate of the elasticity is 0.42 but the sampling variation of this

estimate is fairly high. According to the estimate of μ, 0.98, the economy

is strongly Keynesian, in that virtually none of the effect of a surprise in

aggregate demand is offset by a movement of prices within the year. The

combined implication of the estimated values of ϕ and μ is that the money

stock has a powerful effect on real output. The estimated value of α implies

that monetary policy tends to amplify shifts in demand slightly, while the

negative value of β implies a slight contractionary response to supply shifts.

Neither of these effects is statistically unambiguous.

The estimated value of the lag parameter, ω, implies the following

distribution of response times for prices:

length of lag	fraction of prices responding with this lag	cumulative fraction of prices responding with this lag or a shorter lag
1 year	15 percent	15 percent
2 years	20 percent	35 percent
3 years	28 percent	63 percent
4 years	37 percent	100 percent

Table 3 shows the match between the correlations predicted by the model

and the correlations in the data. In many cases the fit is quite good. The

serial correlations of the three variables are reproduced quite accurately

in all cases. Both the contemporaneous and lagged correlations of the

price-money ratio and real GNP are matched reasonably well, though the

estimates understate the strength of the association between current p-M and

lagged X-$\bar{X}$. The match is good for the correlation of current X-$\bar{X}$ and p-M

lagged one year, but less satisfactory for longer lags.

The estimates do not do too badly for the correlations between the

current rate of monetary expansion and current and lagged values of the other

two variables. The contemporaneous correlation of ΔM and p-M is not as nega-

tive as it should be, but the weak correlations with lagged values of p-M are

reproduced. Apparently the problem here is the inability of the model to

match the strongly negative contemporaneous correlation without also making

Table 3

Fitted and Actual Correlations

Current Variable	Lagged Variable	Lag (Years)				
		0	1	2	3	4
p-M	p-M	1.00	0.71	0.43	0.19	-0.01
		1.00	0.76	0.44	0.25	0.17
	X-$\bar{\bar{X}}$	-0.56	-0.31	-0.11	0.03	0.11
		-0.65	-0.56	-0.36	-0.24	-0.16
	ΔM	-0.11	0.27	0.26	0.23	0.22
		-0.69	-0.69	-0.43	-0.22	-0.20
X-$\bar{\bar{X}}$	p-M	-0.56	-0.42	-0.26	-0.12	0.01
		-0.65	-0.45	0.00	0.20	0.24
	X-$\bar{\bar{X}}$	1.00	0.60	0.31	0.12	0.00
		1.00	0.70	0.30	0.01	-0.05
	ΔM	0.16	-0.12	-0.13	-0.13	-0.13
		0.33	0.78	0.40	-0.09	-0.16
ΔM	p-M	-0.11	-0.03	-0.01	0.00	0.00
		-0.69	-0.15	0.06	-0.09	-0.03
	X-$\bar{\bar{X}}$	0.16	0.04	0.01	0.00	0.00
		0.33	0.20	0.12	0.05	-0.04
	ΔM	1.00	0.28	0.08	0.02	0.01
		1.00	0.22	0.06	0.05	0.18

Note: Upper number is the fitted correlation and lower number is the
       actual correlation.

the lagged correlations too negative. The contemporaneous correlation of ΔM and $X-\bar{X}$ is not as large as it should be, and the lagged correlations are understated as well.

The most important failure of the estimates is their inability to match the correlations between current values of the real GNP and current and lagged values of the rate of money expansion. The surprising strength of this correlation, especially with the rate of monetary expansion lagged one year, has been emphasized by Barro (1977a). The estimates presented here do not reflect that important aspect of the data. The model as it stands is not completely successful in incorporating the role of monetary fluctuations. The problem seems to arise from excessive simplicity in the specification of the stochastic process for the money stock. The results presented here suggest that there is enough information in the data to fit a more elaborate model. In the present simple model, the maximum likelihood "weighting" of the matches between the various correlations seems to put more emphasis on the serial correlation of ΔM rather than on the relation between money and other variables. There is another region in the parameter space where the opposite occurs--the strong positive correlations between money and GNP and negative correlations between money and the price-money ratio are reproduced, but the predicted serial correlation of ΔM is much too high. However, the points in this region all have significantly lower likelihood than the estimates presented here. This conclusion was verified by starting the iterative maximization procedure at a number of sets of parameter values in the region. In all cases, the procedure moved back to the results given here.

Because the demand, supply, and monetary disturbances are hypothesized to be statistically independent, the model implies an unambiguous decomposition of the variances of the three variables into components associated with

the three disturbances. In terms of percentages of the total variances,

this decomposition is:

Variable	Percent of variance attributable to		
	demand disturbance	supply disturbance	money disturbance
Price-money ratio, p-M	6	74	20
Real GNP less trend, X-X̄	63	29	8
Monetary growth,	3	51	46

About three-quarters of the variance in the price-money ratio comes from the

supply disturbance, either directly via the supply perturbation of the price

level, or indirectly via the response of the money stock. Demand fluctuations

account for only a little of the fluctuations in p-M, and the independent

money disturbance accounts for only a fifth of the variance. On the other

hand, almost two-thirds of the variance of real GNP around its trend is

attributable to demand disturbances. This includes the reinforcing effect

of the positive response of monetary policy to demand shocks. Supply

disturbances are an important factor in fluctuations in real GNP, accounting

for 29 percent of its variance. Part of the potential impact of supply

disturbances is offset by the negative response of monetary policy to those

disturbances. Finally, the variance of the rate of monetary growth is about

evenly split between the independent monetary surprise (46 percent) and the

response of the money stock to supply shocks (51 percent).

The most interesting hypotheses about the sources of fluctuations and

the nature of the economy relate to single parameters and so can be tested

directly from the results already presented. The test that reaches the

sharpest conclusion relates to the coefficient of Keynesness. The hypothesis

of full contemporaneous clearing of the market ($\mu = 0$) is overwhelmingly rejected—the "t-statistic" for it is 59. The result is so strong that any value of μ below 0.94 will be rejected at the 5 percent level. The model finds that almost no market-clearing response of prices to demand, supply, or monetary shocks occurs within the first year, and is extremely confident about this finding.[1]

Since the supply disturbance plays such a large role in the model's explanation of the fluctuations in the three observed variables, it is interesting to ask if the statistical evidence in favor of this conclusion is very strong. The "t-statistic" obtained from the estimate of 0.08 and its reported standard error of 0.14 suggests that the sampling variability of the estimate is so large as to prohibit any strong conclusion. However, that standard error apparently overstates the likelihood that the true value of the supply variance is close to zero. The likelihood ratio for the hypothesis that the supply variance equals 0.04 is 2.13, which rejects the hypothesis at the 5 percent level. The results thus contain quite strong evidence that supply fluctuations are an important source of overall economic fluctuations.

7. Concluding Remarks

The joint behavior of prices, real output, and the money stock provides a good deal of information about the nature of the economy and the sources of fluctuations. In common with most recent empirical work on prices and aggregate supply, this study finds evidence of important price rigidity in the short run. In spite of the repeated frustrations of theorists in putting

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[1]This conclusion was verified by computing the alternative likelihood ratio statistic for the test.

price rigidity on a firm theoretical ground, it remains a hypothesis that seems well supported by the data. In this study, the opposing view that the appearance of price rigidity is just an illusion caused by mistaking supply disturbances for demand disturbances is given a fair chance. The evidence does assign the supply disturbance an important role, but still favors price rigidity quite emphatically. In this respect, the study does not support a classical interpretation of fluctuations. The case against the classical model is far from closed, however. The model used here makes stringent assumptions about the statistical independence of demand and supply shocks from each other's current and lagged values. The most sophisticated recent statements of the classical model have not been tested here and it is known that estimation of the fully general model is impossible. Still, it is worth knowing that the simpler classical model does not square with the facts.

Much additional research could be done to improve the results of this paper. First, a more elaborate specification of the process determining the money supply would probably enable the model to fit both the observed serial correlation of monetary growth itself and its high correlation with real GNP. Second, the model could be expanded to take advantage of the information contained in other macroeconomic variables. One obvious candidate is a measure of fiscal policy, either real government expenditures or some kind of full employment deficit. In a model extended in this way, monetary and fiscal policy would be treated symmetrically. Another candidate is the interest rate. The apparatus of the model would make it possible to separate a nominal interest rate into a real interest rate and an expectation of inflation. The potential role of inflationary expectations in shifting the aggregate demand schedule, which has been ignored in this work to date, ought to be examined as well.

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