

# Sources and Mechanisms of Stagnation and Impaired Growth in Advanced Economies

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## Abstract

Advanced economies generally grew in real terms from 2000 until they stumbled badly after the financial crisis in late 2008. Their performance subsequent to the crisis has varied widely. This paper takes a close look at overall performance from 2000 onward of six advanced economies: France, Germany, Italy, Spain, the United Kingdom, and the United States. The paper's basic measure of performance is the level of real labour earnings per member of the population. This measure is appropriate because a main focus of concern about economic stagnation has been on low growth or even shrinkage in family incomes. Post-crisis performance was remarkably heterogeneous across the six countries. Although performance was general sub-par, no single theory of stagnation fits the data.

## 1

## Introduction

A central concern today is that the financial crisis of late 2008 sent many advanced economies into some form of stagnation. This paper takes real labour earnings per member of the population as a suitable measure of performance in judging stagnation. Labour earnings measure the well-being of the majority of the population. The primary alternative to this measure would be real output per person, the volume of resources available for all purposes. As the paper shows, downward shifts in the share of output accruing as income of workers are an important source of stagnation in several of the countries studied here.

Figure 1 shows the evolution of indexes of earnings using annual data stated in logs, over a period of seven years before the crisis and seven years following. The slope of each country's line shows its rate of growth. All six economies experienced some immediate decline immediately following the crisis. Germany and France had minor pauses in growth of earnings. The other four countries — Italy, Spain, the United Kingdom, and the United States — suffered large immediate declines in earnings. Among these, the UK and the US had weak recoveries with low but positive growth of earnings. Spain and Italy experienced declining earnings through the end of the data in 2014. Of the six countries, two — France and Germany — showed no

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obvious signs of stagnation, one — the US — suffered a substantial decline, followed by partial recovery, two — Spain and Italy — plunged almost continuously since the crisis, and the last — the UK — grew very rapidly from 2000 to the crisis and has been level since then.

Concerns about stagnation have been widespread as the disappointments about growth unfolded. At first, it appeared that the crisis had triggered unusually large cyclical contractions. Stagnation theories focused on deficient demand. The monetary policies of the three central banks responsible for the six countries were constrained by the effective lower bound on interest rates. But, as some of the economies — notably the UK and the US — returned to full employment, it became clear that other theories of stagnation were needed to explain poor growth.

This paper attacks the diagnosis of stagnation by breaking down the movements of real earnings per member of the population into seven components. Two of the components are related to traditional thinking about cyclical fluctuations: the employment rate (one minus the unemployment rate) and weekly hours, though a lingering low employment rate is a type of stagnation in some economies, notably Spain, and persistent declines in weekly hours have occurred in other economies. Multifactor productivity, the best available measure of technological advance, is a component with large contributions in five of the six economies. The capital/output ratio is the natural measure of capital intensity in the paper's framework; it is a component that helps understand the role of capital formation. Labour-force participation is a component with important movements in Spain and the US. And declines in the labour share of national income have an important role in declines in earnings in five of the six economies.

## 2 Real earnings per member of the population

This section presents the chain of relationships running from a set of variables, including multifactor productivity, the capital stock, and the population—to real earnings per person. The relationships are definitional. They reflect the way government agencies compile the data. I arrange the definitions in a way that is intended to be informative about the determinants of the variations in growth of real earnings. But it is important to keep in mind that the relationships are not causal. For example, it would be an overstatement to say that some of a decline in real earnings was caused by a decline in the income share of labour. Rather, what one can say is that forces that caused declines in the labour share also caused real earnings to grow more slowly than real income.

The first relationship is

$$\text{Total real earnings} = [\text{labour share}] \times [\text{real output}]$$

The labour share is one of the components of the ultimate decomposition. I further break down

$$\text{Real output} = [\text{output per unit of labour input}] \times [\text{volume of labour input}]$$

Output per unit of labour input = function of [multifactor productivity] and [capital/output ratio]

Volume of labour input = [hours per worker] × [workers per member of the labour force] × [members of the labour force per person of working age] × [people of working age as a fraction of the total population]

The result is a seven-way breakdown of real earnings per member of the population among the following:

4. Labour share;
5. Multifactor productivity;
6. Capital/output ratio;
7. Hours per worker;
8. Employment rate: 1– unemployment rate;
9. Ratio of labour force to working-age population;
10. Ratio of working-age population to the population of all ages.

I emphasize that the measure considered here is real earnings per member of the population, not per worker. This measure encompasses changes in the labour force and unemployment, as well as in the earnings of workers. The measure focuses on total resources created by workers per member of the population, before deduction of taxes, exclusive of additions from government transfers (public benefits), and inclusive of fringe benefits provided by employers. It includes contributions for public retirement and health programs.

The Appendix explains the function relating output per unit of labour input to multifactor productivity and to the capital/output ratio.

Traditional macro theory separates the movements of output and employment into cyclical and trend components. Cyclical movements are transitory and the trend component moves smoothly and persistently over time. This view is encountering more and more scepticism as it has become clear that, at a minimum, there is a substantial residual component that is neither cyclical nor slow-moving.<sup>110</sup> In some countries, notably the US, the unemployment rate appears to be a reasonable indicator of a cyclical component, but in others, such as Italy and Spain, unemployment is not a mean-reverting variable and the identification of cyclical and non-cyclical components does not appear to be feasible. For that reason, this paper does not attempt such a decomposition. Its decomposition does assign roles to variables that differ in their traditional assignment as cyclical and non-cyclical — for example, the employment rate is a cyclical variable at least in the US — but the

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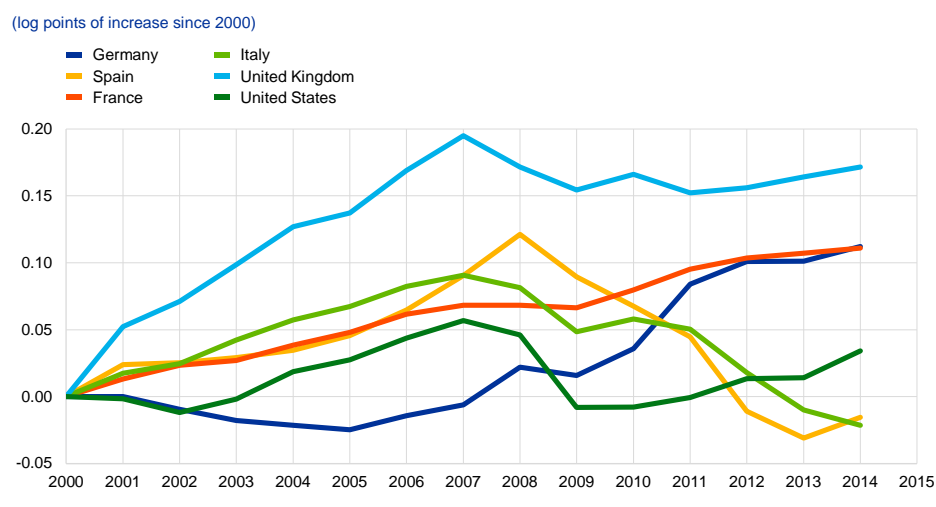
<sup>110</sup> See Fernald, John, Hall, Robert, Stock, James, and Watson, Mark (2017), “The Disappointing Recovery of U.S. Output since 2009,” *Brookings Papers on Economic Activity*, (1), forthcoming, for an extended discussion of this point in the US context.

basis for the decomposition is not a hypothesis of cyclicity, but rather is derived from ideas in growth theory.

### 3 Data

The data come from the website OECD.Stat.

**Figure 1**  
Real Compensation per Member of the Population



### 4 Results

The following series of tables and figures, all in the same format, show the movements of real compensation and its seven components. Each reports the log of an index, where the index itself starts at one, so the log starts at zero. The vertical axis is in log units, so the slopes are rates of growth. Each unit of increase of 0.1 is growth of a bit over 10 percent ( $100 \times (\exp(0.1) - 1)$  to be exact). In the figures, the vertical axis runs from  $-0.2$  percent per year to  $+0.2$  percent. Thus all the figures are comparable to one another. The log index for earnings over population is exactly the sum of the log indexes of the components, by construction.

Table 1 compares the annual percentage growth of real compensation per member of the population, in the crisis and later, to its growth in the calm years from 2000 through 2007. Growth rates were heterogeneous in the earlier period, ranging from just below zero to over three percent per year. Germany's negative rate could be called stagnationary. After the crisis, four countries had negative growth over the seven-year span. Two of those, Italy and Spain, were deeply negative, but the other two, the UK and the US, were also somewhat negative. France, though positive, was low enough to be called stagnated. Only Germany grew at a non-stagnation rate.

**Table 1**

Average Annual Percentage Growth Rates of Real Compensation per Member of the Population in the Pre- and Post-Crisis Years

(average annual percentage growth rates, in percentage points (see title))

Country	2000-2007	2007-2014
Germany	-0.10	1.97
Spain	1.51	-1.77
France	1.14	0.71
Italy	1.51	-1.87
UK	3.25	-0.39
US	0.94	-0.38

Recall that Figure 1 shows the annual evolution of real compensation by country.

The behaviour of earnings per member of the population in the period after 2007 could hardly be more different. Only Germany grew faster after the crisis than before. France enjoyed positive growth after 2007, but at a substandard rate. Italy and Spain had alarming shrinkage of earnings and the UK and the US dropped to slightly negative growth rates, in contrast to positive growth in the earlier period. The drop in the case of the UK was about 3.6 percentage points of growth.

## 4.1 Labour's share of total income

Table 2 compares the annual growth rates of the log of the labour share in the two periods for the six countries. These and other calculations showing the components are in units that add up to the totals in Table 1. For example, the figure 0.25 for France's labour share in the earlier period means that the share rose, on average, by a multiple of  $\exp(0.0025) = 1.0025$  in a year, so a share that was 70 percent in one year would grow to  $70 \times 1.0025 = 70.18$  percent in the next year. The labour share was on a slightly upward trajectory in the earlier period — only Spain experienced a decline.

**Table 2**

Average Annual Growth of the Log of the Labour Share in the Pre- and Post-Crisis Years

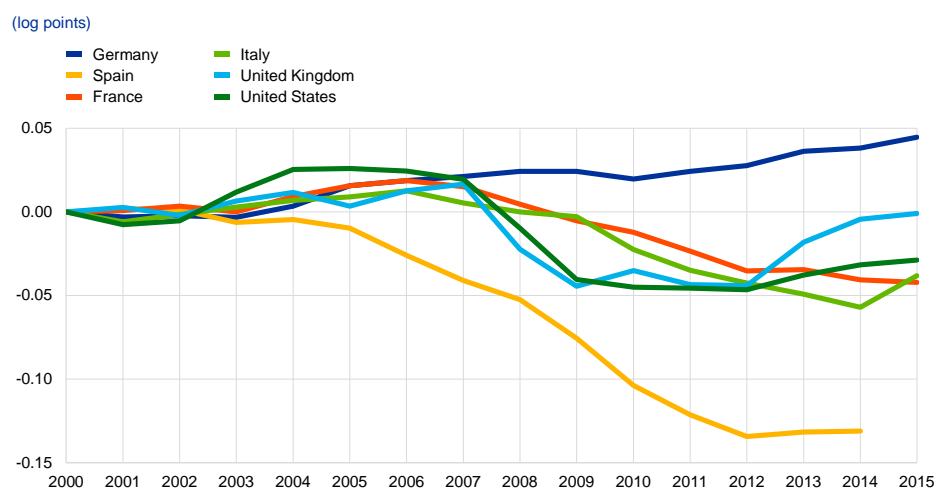
(log points of the share)

Country	2000-2007	2007-2014
Germany	0.35	0.28
Spain	-0.68	-1.50
France	0.25	-0.93
Italy	0.09	-1.04
UK	0.28	-0.35
US	0.32	-0.85

On the other hand, every country except Germany had a decline in the labour share after the crisis. For Italy and Spain, the share decline accounted for more than a percent per year of the decline in real earnings per person, and in France and the US, the decline was just under a percent per year. The declining labour share was an important contributor to the overall stagnation in labour earnings after the crisis. This aspect of the post-crisis stagnation has received relatively little attention.

Figure 2 shows the annual evolution of the log of the labour share.

**Figure 2**  
Annual Growth of the Log of the Labour Share



The role of the declining share of output, and thus of real income, in the overall decline in earnings growth is striking. The value of the income generated from the production of output has three major components — labour earnings (well over half), the return to plant and equipment (an important part of the remainder), and the return to intangibles (the rest). Research is approaching a consensus that the share of the return to plant and equipment has probably not grown enough to explain the decline in the labour share. Rather, growth in the intangible share accounts for the shift away from labour.

The intangible share has two distinct elements. One is intellectual property. Firms invest in technologies and earn returns reflecting the advantages over rivals that the technologies deliver. The value of newly created intellectual property is included in the national income and product accounts in the form of reported flows of investment, which the accounts cumulate to estimate the intellectual property component of the capital stock. The second element of the intangible share is the return to market power that cannot be attributed to new technology. Large businesses are growing relative to their smaller rivals, so product markets are becoming more concentrated. Oligopoly theory generally associates concentration with higher margins of price over marginal cost. Some economists believe that more vigorous policies to prevent concentration of markets might have avoided part of this shift.

Tech companies like Apple and Google sell their products for prices far above marginal cost, so their growth since 2000 would be a contributor to the rise in the overall markup ratio. Pharmaceuticals have also contributed to markup growth. The growing tendency for advanced-country firms to outsource production to other countries but to retain research, development, branding, and other costs domestically that are not part of marginal cost has further contributed to the change. The hypothesis of markup growth is fairly new to macroeconomics and it remains to undergo serious quantitative verification, however. The fact that the labour share only began to decline after the crisis is not easy to square with the intangible story. Other channels that raise markups of price above cost in times of stagnation call for further investigation. See Autor, et al., (2017) for a recent discussion of the US case.<sup>111</sup>

## 4.2 Multifactor productivity

Multifactor productivity is part of the growth of output not explained by growth in inputs. Over long spans of time, productivity growth is the most important component of overall earnings growth. It accounted for most of the growth in the earlier period, except in Italy and Spain. Table 3 compares the growth of multifactor productivity in the two periods. None of the six countries considered here had satisfactory average growth rates in the seven post-crisis years, and three of them suffered productivity shrinkage. Poor productivity growth is a major contributor to stagnation.

**Table 3**  
Average Annual Growth of the Log of the Labour Share in the Pre- and Post-Crisis Years

(percentages)

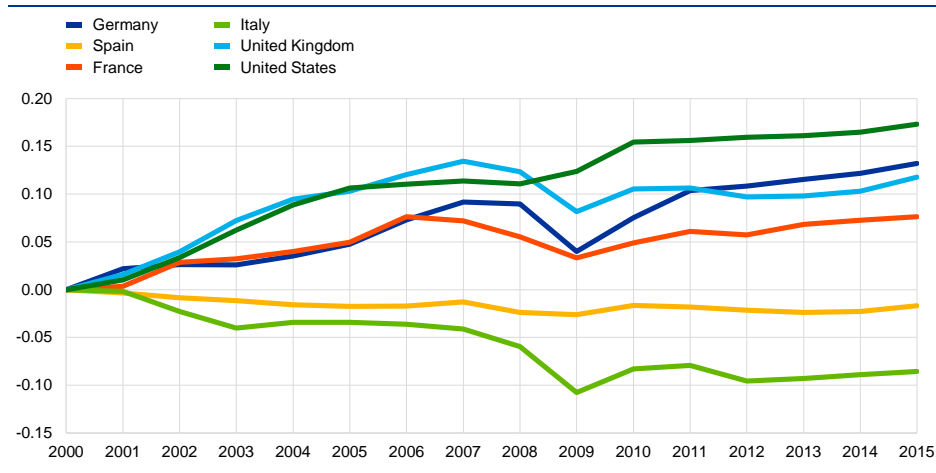
Country	2000-2007	2007-2014
Germany	1.53	0.50
Spain	-0.21	-0.17
France	1.20	0.01
Italy	-0.69	-0.80
UK	2.24	-0.52
US	1.90	0.85

Figure 3 shows the annual evolution of the log of multifactor productivity. In all six countries, productivity fell or its growth declined immediately after the crisis. Spain and the US had the smallest shortfalls at that time. Productivity growth in Germany, France, and the US has resumed in more recent years, but not at pre-crisis rates. In the other three countries, essentially no growth has occurred in productivity, except for recent growth in the UK.

<sup>111</sup> Autor, David, Dorn, David, Katz, Lawrence F., Patterson, Christina, and Van Reenen, John (2017), "The Fall of the Labor Share and the Rise of Superstar Firms", Cambridge, Mass., USA, *National Bureau of Economic Research Working Paper 23396*, May.

**Figure 3**

**Log of Index of Multifactor Productivity**



Fernald, Hall, Stock, and Watson carry out an analysis of the low rate of productivity growth in the US since the crisis. The rapid growth of productivity from 1995 to 2006 was likely the result of rapid adoption of information technology — such as the relational database — in many sectors, notably retail trade. Adoption may have slowed down. The heterogeneity in productivity growth among the six countries both before and after the crisis creates a challenge for technology-based explanations of productivity — advanced countries share access to new technologies widely. Other hypotheses about the slowdown in the US, such as a rising burden of regulation, remain plausible but are not supported by the limited data available.

### 4.3 Capital/Output ratio

Table 4 compares the growth of the capital/output ratio in the two periods. Capital is the stock of plant, equipment, and intellectual property. Though flows of capital formation have generally been weak in advanced countries, the table shows that the growth of capital relative to output increased in four of the six countries and declined only slightly in the other two, Germany and the US.



**Table 4**

Average Annual Growth of the Log of the Capital/Output Ratio in the Pre- and Post-Crisis Years

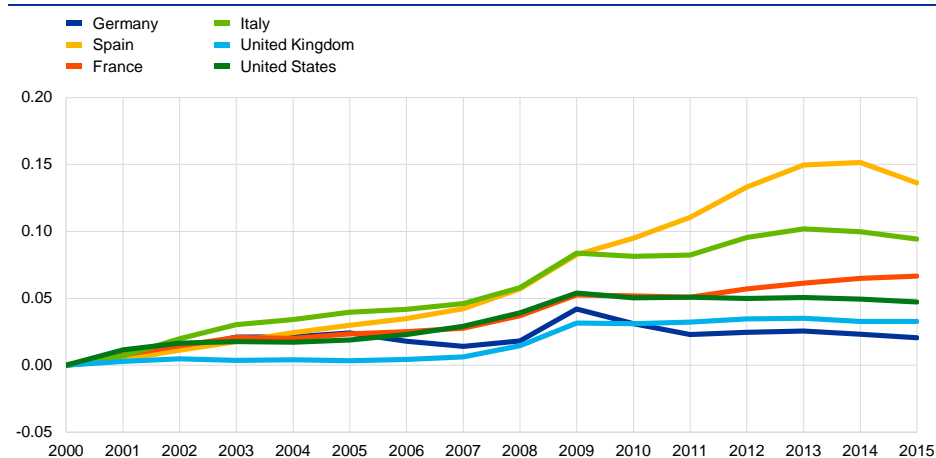
(percentages)

Country	2000-2007	2007-2014
Germany	0.24	0.15
Spain	0.70	1.82
France	0.46	0.62
Italy	0.77	0.89
UK	0.10	0.44
US	0.49	0.34

Figure 4 shows the annual evolution of the log of the capital/labour ratio. The ratio grew at similar moderate rates among the six countries through 2007, then jumped upward. The jump was the result of declining output, not a leap in capital formation. In countries with recoveries in output, the ratio fell back to its previous growth path. Spain and Italy, the countries most afflicted with output stagnation, saw further increases in the capital/output ratio.

**Figure 4**

Log of Index of Capital/Output Ratio



Weak capital formation is often cited as an aspect of stagnation. But it is a result of other forces of stagnation, in the sense that investment theory emphasizes that investment flows tend to stabilize the capital/output ratio, while raising the capital stock and raising the capital/labour ratio. The finding of stable and rising capital/output ratios shows that low productivity and declining labour-force participation, together with other adverse influences, lowered output growth and hence caused declining capital formation.

## 4.4 Weekly hours

Table 5 compares the growth of weekly hours per worker in the two periods. Figure 5 shows the annual evolution of the log of weekly hours per worker. It is close to proportional to the total hours of work in a year divided by the average number of people at work over the year. It is the number of hours the typical worker would put in if employed throughout the year.

**Table 5**

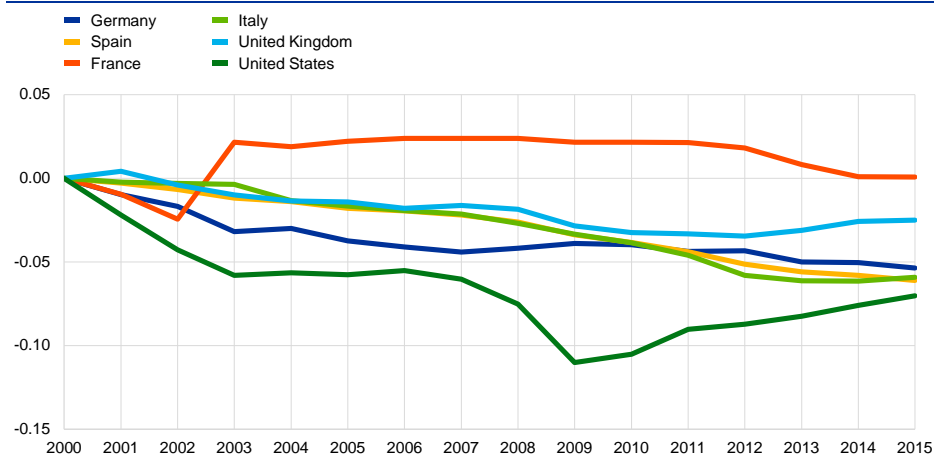
Average Annual Growth of the Log of Weekly Hours per Worker in the Pre- and Post-Crisis Years

(percentages)

Country	2000-2007	2007-2014
Germany	-0.73	-0.11
Spain	-0.37	-0.60
France	0.40	-0.38
Italy	-0.36	-0.67
UK	-0.27	-0.16
US	-1.01	-0.26

**Figure 5**

Log of Index of Weekly Hours per Worker



Hours trended downward in all six countries over the entire period from 2000 to 2014. The decline in hours after the crisis was a substantial contributor to the decline in those years in total real earnings per member of the population in Italy and Spain. These are the countries with the largest and most persistent increases in unemployment, so it would appear that increases in unemployment and decreases in hours respond to the same forces.

## 4.5 Employment rate of the labour force

Table 6 compares the growth of the log of the employment rate of the labour force in the two periods. Recall that this variable is  $\log(1-u)$ , where  $u$  is the unemployment rate stated as a decimal. In some economies, notably the US and the UK, the employment rate tracks the business cycle closely, rising in booms and falling in recessions. Because the employment rate returns fairly quickly to its normal level after a shock, its average over seven-year periods is close to normal, as shown in the table. In continental European countries, other factors are important, so the table has large positive figures for Italy and Spain during the earlier period and for Germany in the later period, and large negative figures for Italy and Spain in the later period. For these countries, those negative figures make important contributions to their declines in earnings. France had a moderate contraction in its employment rate in the later period.

**Table 6**

Average Annual Growth of the Employment Rate of the Labour Force in the Pre- and Post-Crisis Years

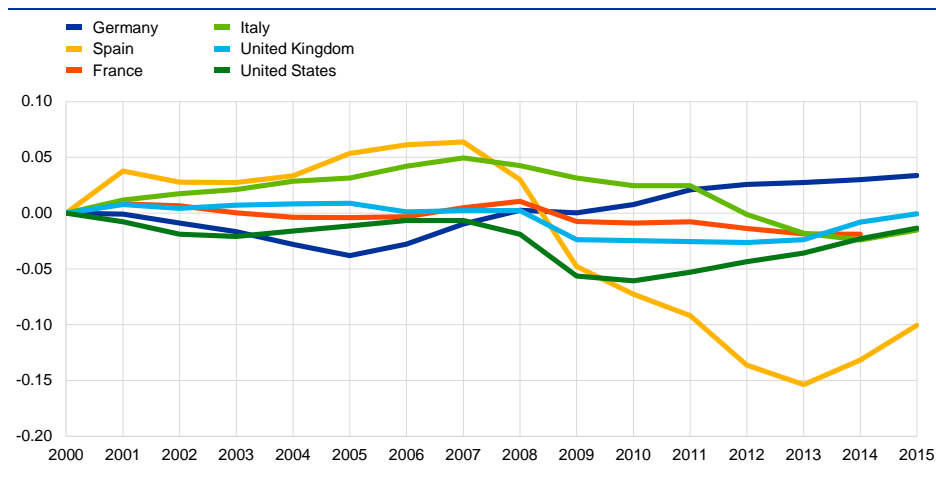
(percentages)

Country	2000-2007	2007-2014
Germany	-0.16	0.66
Spain	1.06	-3.26
France	0.08	-0.40
Italy	0.83	-1.23
UK	0.04	-0.17
US	-0.11	-0.27

Figure 6 shows the annual evolution of the log of the employment rate of the labour force. In all countries except Germany, a bulge of unemployment occurred immediately after the crisis in 2008. The bulge was largest for the US, where the employment rate fell by five percentage points, then recovered most of the decline by 2014 (and all by 2016, not shown). The decline and recovery in the UK employment rate was similar, but the amount of the decline was less than half that of the US employment rate. In France, no recovery occurred after the initial bulge — rather, the employment rate drifted downward. It would be hard to generalize about the responses of the employment rates of the four continental European countries.

**Figure 6**

Log of Index of the Employment Rate of the Labour Force



## 4.6

## Labour-force participation rate

The labour-force participation rate is the fraction of the working-age population who are looking for work or who are working. Table 7 compares the growth of the rate in the two periods. In the earlier period, participation grew in four countries, especially in Spain (but not much in Italy). It was close to steady in France and the US. In the post-crisis period, participation grew by small amounts in five countries but shrank significantly in the US.

**Table 7**

Average Annual Growth of the Labour-Force Participation Rate in the Pre- and Post-Crisis Years

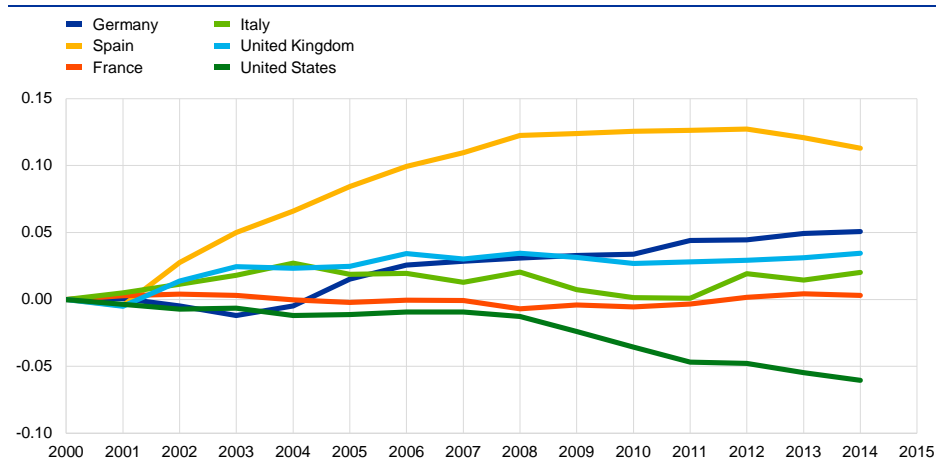
(percentage points)

Country	2000-2007	2007-2014
Germany	0.47	0.37
Spain	1.83	0.05
France	-0.02	0.07
Italy	0.21	0.12
UK	0.50	0.07
US	-0.16	-0.85

Figure 7 shows the annual evolution of the log of the labour-force participation rate. Spain and the US are conspicuous outliers. The post-crisis decline in US participation has received a great deal of attention. Though the decline began around the time of the crisis, most of it appears to be the result of noncyclical forces. One of these is demography — the large baby-boom cohort began to retire over this period. This factor accounts for about a third of the total decline. From 2010 through 2014, as unemployment returned to normal, participation continued to decline.

**Figure 7**

Log of Index of the Labour-Force Participation Rate



## 4.7

## Ratio of working-age population to total population

Table 8 compares the growth of the ratio of working-age population to total population in the two periods. This component accounts for children who are dependent on earners. Figure 8 shows the annual evolution of the log of the ratio. In the post-crisis period, only the US had a change in the ratio that was material, and it was fairly small but favourable.

**Table 8**

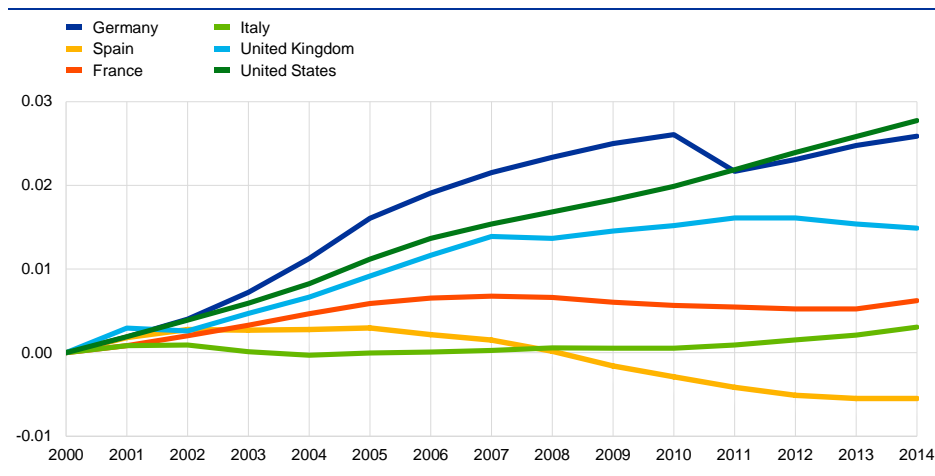
Average Annual Growth of the Ratio of Working-Age Population to Total Population in the Pre- and Post-Crisis Years

(percentages)

Country	2000-2007	2007-2014
Germany	0.36	0.07
Spain	0.02	-0.12
France	0.11	-0.01
Italy	0.00	0.05
UK	0.23	0.02
US	0.26	0.21

**Figure 8**

Log of Index of the Ratio of the Working-Age Population to Total Population



## 5

## Conclusions

The six countries have access to the same technologies, have economic advisors trained at the same universities, have professional elites who all can speak the same language, and have tightly integrated capital markets. They all elect governments with reasonably limited involvement in their economies — mainly in the form of generous redistributive programs. They rely on private economic actors to manage production, employment, and investment, subject to moderate regulation. They have no obvious sources of idiosyncratic shocks. They were all hit by a similar shock that resulted in steep drops in equity prices and in under-capitalized banks.

But even before the crisis, macroeconomic outcomes in these countries varied tremendously. Growth in real earnings per person was spectacular in the UK, mediocre in France, Italy, Spain and the US, and negative in Germany. Productivity growth was close to longer run historical levels in France, Germany, the UK, and the US, but negative in Italy and Spain. Unemployment fell dramatically in Italy and Spain but was stable in the other four countries. Labour-force participation grew a lot in Spain, moderately in Germany and the UK, and was essentially unchanged in the other three countries.

Heterogeneity in performance after the crisis was even greater. Real earnings per person grew from 2007 through 2014 by almost two percent per year in Germany and by 0.7 percent per year in France, but fell in the other four countries, especially Italy and Spain. The share of national income accruing to workers fell, in four cases dramatically, in all countries but Germany. Productivity growth was sub-par in all six countries, but positive in Germany and the US and negative in Italy and Spain.

Table 9 distills the figures for the major components, with shades of green indicating favourable positive values and red for unfavourable negative values. The figure

reinforces the conclusion that the outcomes are heterogeneous. Germany comes out as the least stagnated and Italy and Spain as the most stagnated.

**Table 9**  
Summary of Post-Crisis Components

	Share	Productivity	Hours/week	Employment rate	Participation
Germany	0.28	0.50	-0.11	0.66	0.37
Spain	-1.50	-0.17	-0.60	-3.26	0.05
France	-0.93	0.01	-0.38	-0.40	0.07
Italy	-1.04	-0.80	-0.67	-1.23	0.12
UK	-0.35	-0.52	-0.16	-0.17	0.07
US	-0.85	0.85	-0.26	-0.27	-0.85

Notes: shaded green for positive and red for negative.

The column with the highest frequency of adverse scores is the labour share, which is negative for all countries but Germany. If market power obtained from inefficient concentration is responsible for the declining labour share, then that decline is a form of stagnation — a development that cuts real income persistently. On the other hand, if the decline comes from a shift in the product mix toward products with high ratios of price to marginal cost — tech products, entertainment, pharmaceuticals — then including their negative effects on the labour share as part of stagnation may be going too far. Economists have often proposed that the government should buy out the intellectual property rights to high-priced drugs and then set the price to the marginal cost of production. By the same logic, the government should buy out the rights to the iPhone and sell it for \$135.

Immediately after the crisis, when unemployment rose in many countries, including all six considered here, economists treated impending stagnation as an issue of deficient demand, arising from the inability of monetary policy to restore full employment by cutting interest rates far enough. Central banks' abilities to offset the effects of the crisis on demand were limited by the effective lower bound on interest rates. Huge fiscal deficits stood in the way of meaningful fiscal expansion. Two of the components of real earnings studied here — the employment rate and hours of work per week — are cyclical indicators in the US. Figure 5 shows a hint of a cyclical response of hours in the UK, but not in the four continental countries. Figure 3 shows cyclical declines with quick rebounds in multifactor productivity in all countries but the US. Figure 7 shows no sign of a cycle in labour-force participation triggered by the crisis in any of the six countries. In summary, the demand-related movements of the components seem to be quite limited. In the cases of Italy and Spain, where unemployment remained high through 2014 (and after), the possibility remains that lingering high unemployment could be cured by country-specific demand policies.

One idea about stagnation that links earlier shortfalls in demand to later levels of output is through investment. Weak investment cumulates to shortfalls in the capital stock in later years. It is true that, if the level of investment spending had remained at pre-2008 levels, the capital stocks in the six countries would have been higher in the post-crisis years, with the possible exception of Germany.

Stagnation shows through strongly in the data — real earnings per member of the population has grown much less or has even shrunk in the aftermath of the crisis. In thinking about the anatomy of stagnation, attention has shifted to stagnation in components that do not usually respond to fluctuations in demand — sometimes called supply components. In the US, for example, the two big components of stagnation are the declining labour share and declining labour-force participation. The first is not obviously related to supply, while the second is an aspect of labour supply. In recent years, productivity growth has been a third component of US stagnation. A better way to divide the components into two groups would be to ask which ones would respond to monetary or fiscal expansion — the demand components — and those that do not respond — the other components.

In some countries, the employment rate and weekly hours would be under the influence of demand, notably the US. The most seriously stagnated countries among the six considered here are Italy and Spain. It is an open question how much of their excess unemployment would yield to monetary or fiscal expansion.

I believe this paper makes it quite clear that unitary theories of stagnation are unhelpful in studying the behaviour of advanced economies since the crisis. Rather, each country has its own story, involving economic models of factor shares, productivity growth, unemployment, labour supply, and demographics. The great puzzle is how there can be so much heterogeneity in the stories, among countries that appear to be basically similar and were responding to similar shocks over the post-crisis period.

The primary lesson for economic policy in the findings of this paper is the importance of financial stability. All six countries studied here suffered from the paralysis of monetary policy from the effective lower bound, a direct result of the panic associated with the financial crisis. Advanced countries with stable financial systems and independent monetary policy, such as Canada, performed better during and after the financial crisis.

The basic principles of financial regulation to create shock-proof financial systems were well understood prior to the crisis, but supervision failed badly in many jurisdictions. Under the implicit protection of central banks and national governments, financial institutions took on far too much risk, making them vulnerable to shocks from declining real-estate prices and from other sources. Under the guidance of proper stress tests, bank regulators would have prevented instability, and under proper non-governmental resolution principles for insolvent institutions, investors would have disciplined excessive risk exposures and eliminated the temptation of the free government put.

## 6 Appendix

Let  $y$  be the log-change in output,  $A$  be the log-change in total factor productivity,  $n$  be the log-change in labour input,  $k$  be the log-change in capital, and  $\alpha$  be the



elasticity of the production function with respect to capital. Then the production function is, in log changes,

$$y = A + (1 - \alpha) n + \alpha k.$$

$$y - n = A + \alpha (k - n)$$

$$= A + \alpha (y - n) + \alpha (k - y).$$

So

$$y - n = \frac{A}{1 - \alpha} + \frac{\alpha}{1 - \alpha} (k - y)$$