Procyclical Leverage

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January 21, 2015

Financial crises typically triggered by "small" shocks

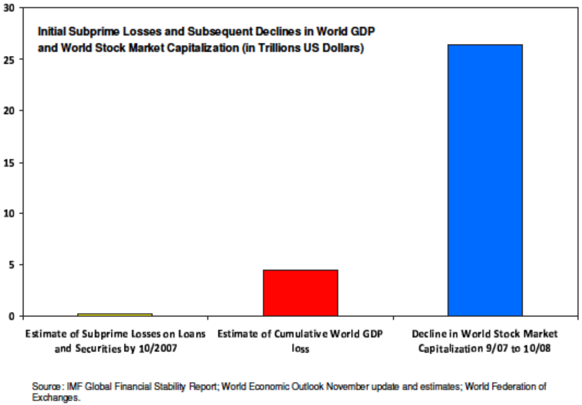


Figure: From Blanchard (2009), "The Crisis: Basic Mechanisms and Appropriate Policies."

How can a small shock have large effects?

Some amplification mechanisms:

1. **Non-contingent and procyclical leverage (today)**
2. Fire sales and asset market feedback
3. Uncertainty (exogenous and endogenous)
4. Coordination failures, e.g., bank runs
5. Macro amplification mechanisms, e.g., nominal rigidities...

Today: Procyclical leverage as a source of amplification

Roadmap:

- Accounting framework to illustrate how leverage can create damage.
- A static model of endogenous leverage:
  - Simsek (2013), "Belief Disagreements and Collateral Constraints."
- Dynamics and the leverage cycle:
  - Geanakoplos (2010), "The Leverage Cycle."
- Some empirical evidence on procyclical leverage and tail risk:
  - Adrian and Shin (2013), "Procyclical Leverage and Value-at-Risk."

Roadmap

1. Leverage and amplification channels
2. A static model of procyclical leverage
3. Dynamics and the leverage cycle
4. Empirics of procyclical leverage

A stylized model of financial institutions

- Suppose there are two periods, $\{0, 1\}$.
- Two types of agents, bank (B) and financiers (F).
- Everyone is risk-neutral with discount rate $r = 0$.
- B has investment opportunities but limited funds.
- Fs have funds but investment opportunities.
- Problem is how to transfer funds from Fs to B...

A stylized model of financial institutions

- Bank starts with net worth:

$$n_0 = \overbrace{s_0 k_0}^{\text{value of initial assets}} - \underbrace{b_0}_{\text{outstanding debt}}.$$

- Bank decides how much to invest k_1, to generate s_1 per asset.
- Can borrow b_1 with **non-contingent debt** (not contingent on s_1):

$$k_1 = n_0 + b_1.$$

- (For simplicity suppose debt is safe and the interest rate is $r = 0$.)

Bank faces a borrowing constraint

- Bank faces a **borrowing/collateral constraint:**

$$b_1 \leq \rho_1 k_1, \text{ where } \rho_1 < 1.$$

- Can microfound with frictions (e.g., asymmetric information/agency).
- Suppose s_1 sufficiently large so that the constraint binds. Then:

$$b_1 = \rho_1 k_1 \text{ and } k_1 = \frac{n_0}{1 - \rho_1}.$$

- Imagine the bank made a similar decision in the past so that:

$$b_0 = \rho_0 k_0 \text{ and } k_0 = \frac{n_{-1}}{1 - \rho_0}.$$

Key determinants of bank's investment

$$k = \overbrace{n}^{\text{net worth/internal funds}} \underbrace{\frac{1}{1-\rho}}_{\text{leverage ratio}}$$

Key implications:

- Net worth affects investment (Bernanke-Gertler, Kiyotaki-Moore....)
- But so do the borrowing conditions or leverage (corporate finance...)
- Some terminology:
  - $(1-\rho)\,k_1$ is **the downpayment**, $1-\rho$ is **the margin**,
  - $1/(1-\rho)$ is **the leverage ratio**,
  - ρ is the **loan to value ratio.**

Financial sector had high leverage before the crisis

- Greenlaw, Hatzius, Kashyap, Shin (2008), "Leveraged Losses: Lessons From the Mortgage Market Meltdown."

Exhibit 4.5 Leverage of Various Financial Institutions

	Assets ($bn)	Liabilities ($bn)	Capital ($bn)	Leverage
Commercial banks	11194	10050	1144	9.8
Savings Inst	1815	1607	208	8.7
Credit Unions	759	672	87	8.7
Finance Companies	1911	1720	191	10.0
Brokers/hedge funds	5597	5390	207	27.1
GSEs	1669	1598	71	23.5
Leveraged Sector	**22945**	**21037**	**1908**	**12.0**

Source: Authors' calculations based on Flow of Funds, FDIC Statistics on Banking, Adrian and Shin (2007), and balance sheet data for Fannie Mae, Freddie Mac, and broker-dealers under Goldman Sachs equity analysts' coverage

- Is high leverage ratio a problem? Why?

An example balance sheet

Assets	Liabilities
Assets k_0 (say 20)	debt, $b_0 = \rho_0 k_0$ (say 19)
	net worth, n_{-1} (say 1)

- Suppose the bank's current asset returns are realized to be s_0.
- Calculate n_0 when (i) $s_0 = 1$, (ii) $s_0 = 1.01$ and (iii) $s_0 = 0.99$.

Non-contingency of leverage creates amplification

Assets	Liabilities
Realized returns, $s_0 k_0$ (say $20 s_0$)	debt, $b_0 = \rho_0 k_0$ (say 19)
	realized net worth, n_0 (say $20 s_0 - 19$)

- **The bank's debt b_0 is fixed** regardless of its realized return s_0.
- Consequently, the profits are losses are entirely absorbed by n_0.
- This feature of debt creates amplification of losses (and gains):

$$\overbrace{n_0}^{20\% \text{ drop (or raise)}} = 20 \overbrace{s_0}^{1\% \text{ drop (or raise)}} - 19.$$

Non-contingency of leverage creates amplification

- Drop in net worth also affects banks' investment going forward:

$$\overbrace{k_1}^{20\%\text{ drop}} = \frac{1}{1-\rho_1} \overbrace{n_0}^{20\%\text{ drop}} .$$

- Reduction in banks' investment: **Credit crunch** for the real sector.
- (Can also trigger **fire sales** and further amplification–**Kiyotaki-Moore**).
- Making debt b_0 contingent on s_0 would greatly alleviate damage.
- But typically not observed in practice. Why not? Still open question.

In practice, leverage in some markets also seem to be procyclical...

Procyclical leverage in the housing market

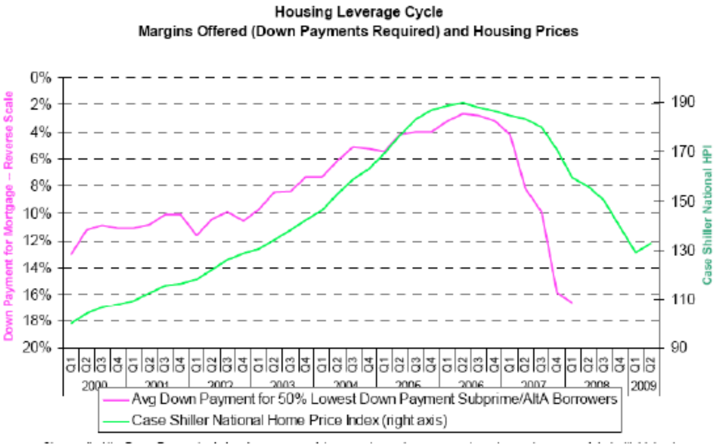
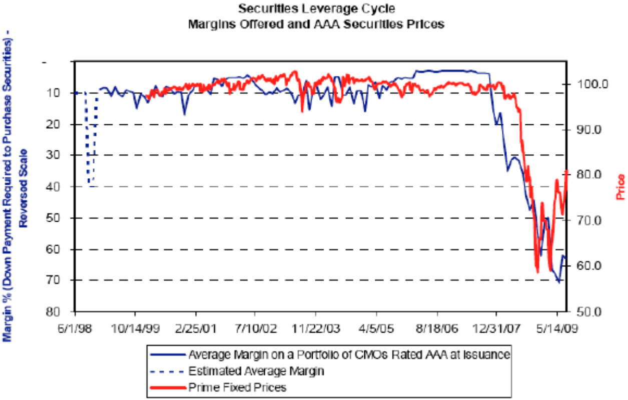


Figure: From Fostel and Geanakoplos (2010).

Procyclical leverage in the MBS market



Note: The chart represents the average margin required by dealers on a hypothetical portfolio of bonds subject to certain adjustments noted below. Observe that the Margin % axis has been reversed, since lower margins are correlated with higher prices.

Procyclical leverage can create further amplification. Why?

Procyclical leverage creates further amplification

$$\overbrace{k_1}^{\text{more than 90\% drop}} = \underbrace{\frac{1}{1-\rho_1}}_{\text{drops from 20 to 2 (say)}} \overbrace{n_0}^{\text{20\% drop}}, \text{ where } n_0 = 20 \overbrace{s_0}^{\text{1\% drop}} - 19.$$

- Geanakoplos (2010), "The Leverage Cycle" proposed a theory of procyclical leverage based on changes in uncertainty.
- I built on the Geanakoplos framework to obtain additional insights.

Basic features of Geanakoplos' models

- **Purely financial assets**: Pay dividends regardless of the owner.
- Nonetheless, **heterogeneous valuations** for several reasons:
  - Differences in prefs, beliefs, background risks...
- Heterogeneity generates **demand for borrowing/promises**.
- Promises are **collateralized by assets** and are **non-recourse.**
  - Borrower can walkaway. Loses collateral, no further punishment.
- **Contracts as commodities** in general competitive equilibrium.
  - GE forces "select" traded contracts.

Preview of Geanakoplos: Procyclical leverage

Geanakoplos (2003, 2010) baseline:

- Heterogeneity from belief disagreements, but viewed as a stand-in.
- Only non-contingent debt contracts.
- Leverage ratios/margins are endogenously determined.

Main results:

1. Leverage ratios/margins depend on **uncertainty**.
2. **Procyclical leverage** from **changes in uncertainty.**

High uncertainty $\Longrightarrow$ Lenders become nervous $\Longrightarrow$ Margins increase.

Preview of my paper: Belief disagreements

Simsek (2013) baseline:

- Focus on belief disagreements (speculation) as the reason for trade.
- **Result:** Leverage depends on **type of disagreements/uncertainty**.

- Start by Simsek's (2013) static model to facilitate exposition.
- Gradually build towards Geanakoplos (2010) and illustrate dynamics.

Roadmap

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My main insight: Asymmetric disciplining

Example: A single risky asset, three future states: G, N, B.

- Pessimists believe each state realized with equal probability.
- **Two types of optimism:**
  1. **Case (D):** Optimists believe probability of B is less than $1/3$.
     $\Longrightarrow$ Margin higher and price closer to pessimists' valuation.
  2. **Case (U):** Optimists believe probability of B is $1/3$. They believe probability of G is more than probability of N.
     $\Longrightarrow$ Margin lower and price closer to optimists' valuation.

Intuition: Asymmetry of debt contract payoffs. Default in bad states.

- Disagreement about downside states $\Longrightarrow$ Tighter constraints.

Basic environment: Belief disagreements about an asset

- One consumption good (a dollar), two dates $\{0, 1\}$.
- Risk neutral traders have resources at date 0, consume at date 1.
- Invest in two ways:
  - Cash: One dollar invested yields one dollar at date 1.
  - **Asset** in fixed supply (of one unit). Trades at price p.
- Asset pays s dollars at date 1, where $s \in \mathcal{S} = \left[s^{\min}, s^{\max}\right]$.
- **Heterogeneous priors**: **Optimists** and **pessimists** with beliefs, F_1, F_0, with:

$$E_1[s] > E_0[s].$$

- **Endowments:** n_1, n_0 dollars at date 0 (asset endowed to outsiders).

Optimists (resp. pessimists) would like to borrow cash (resp. the asset).

Borrowing is subject to a collateral constraint

- A **borrowing contract** is

$$\beta \equiv \left(\underbrace{[\varphi(s)]_{s \in S}}_{\text{promise}}, \underbrace{\alpha}_{\text{asset-collateral}}, \underbrace{\gamma}_{\text{cash-collateral}} \right).$$

- **Collateralized and non-recourse**. Pays:

$$\min(\alpha s + \gamma, \varphi(s)).$$

- **GE treatment:** Traded in anonymous competitive markets at price $q(\beta)$.

Model can account for various borrowing arrangements

Examples of borrowing contracts:

1. **Simple debt contracts:** $\varphi(s) = \varphi$ for some $\varphi \in \mathbb{R}_+$.
2. **Simple short contracts:** $\varphi(s) = \varphi s$ for some $\varphi \in \mathbb{R}_+$.

Next: Baseline with only simple debt contracts:

$$\mathcal{B}^D \equiv \left\{ \left([\varphi(s) \equiv \varphi]_{s \in S}, \ \alpha = 1, \ \gamma = 0 \right) \mid \varphi \in \mathbb{R}_+ \right\}.$$

Denote by **outstanding debt per asset**, φ.

Definition of general equilibrium is standard

Type i traders choose $\left(\mu_i^+, \mu_i^-\right)$ and $\left(a_i, c_i\right)$ to maximize their **expected payoffs** subject to:

- **Budget constraint:**

$$pa_i + c_i + \underbrace{\int_{\mathcal{B}^D} q\left(\varphi\right) d\mu_i^+}_{\text{lending}} - \underbrace{\int_{\mathcal{B}^D} q\left(\varphi\right) d\mu_i^-}_{\text{borrowing}} \leq n_i.$$

- **Collateral constraint:** $\mu_i^-\left(\mathcal{B}^D\right) \leq a_i$.

A general equilibrium (GE) is $\left(\hat{p}, q\left(\cdot\right), \left(\hat{a}_i, \hat{c}_i, \hat{\mu}_i^+, \hat{\mu}_i^-\right)_{i\in\{1,0\}}\right)$ s.t. allocations are optimal and markets clear: $\sum_{i\in\{1,0\}} \hat{a}_i = 1$ and $\mu_1^+ + \mu_0^+ = \mu_1^- + \mu_0^-$.

Detour: Consider an alternative principle-agent equilibrium

Alternative to GE: Optimists choose contracts subject to collateral constraint and pessimists' participation constraint.

- When $p < E_1(s)$, optimists invest only in the asset, a_1.
- They choose, φ, which enables them to borrow $a_1 E_0 [\min(s, \varphi)]$.
- Given p, optimists solve:

$$\max_{(a_1,\varphi)\in\mathbb{R}^2_+} \quad a_1 E_1[s] - a_1 E_1[\min(s,\varphi)], \tag{1}$$

$$\text{s.t.} \quad a_1 p = n_1 + a_1 E_0[\min(s,\varphi)].$$

A **principal-agent equilibrium (PAE)** is $(p, (a_1^*, \varphi^*))$, such that optimists' allocation solves problem (1) and the asset market clears.

A regularity condition to capture the notion of optimism

Assumption (A2): The probability distributions F_1 and F_0 satisfy the hazard-rate order ($F_1 \prec_H F_0$), that is:

$$\frac{f_1(s)}{1 - F_1(s)} < \frac{f_0(s)}{1 - F_0(s)} \text{ for each } s \in \left(s^{\min}, s^{\max}\right). \tag{2}$$

- Optimism notion concerns upper-threshold events, $[s, s^{\max}]$.
- Ensures that problem (1) has a unique solution.

Existence, uniqueness, and equivalence of equilibria

Theorem: Under (A1) and (A2):

- There exists a unique PAE, $[p^*, (a_1^*, \varphi^*)]$.
- There exists an essentially unique GE,
  $\left[(\hat{p}, [q(\cdot)]), (\hat{a}_i, \hat{c}_i, \hat{\mu}_i^+, \hat{\mu}_i^-)_{i \in \{1,0\}}\right]$.
  - The allocations, the asset price, p, and the price of traded debt contracts uniquely determined.
- The PAE and the GE are equivalent, that is:

$$\hat{p} = p^*,\ \hat{a}_1 = a_1^* = 1,\ \hat{\varphi} = \varphi^*, \text{ and } q(\hat{\varphi}) = E_0[\min(s, \varphi^*)].$$

GE allocations are as if optimists have the bargaining power. Intuition?

Optimists' loan choice implies asymmetric disciplining

- Define: **loan riskiness**, $\bar{s} = \varphi$, and **loan size**, $E_0[\min(s, \bar{s})]$.

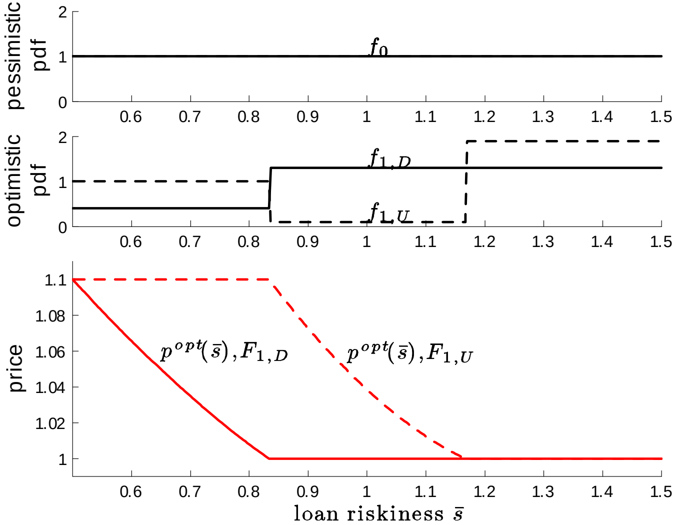
Theorem (Asymmetric Disciplining)

Suppose asset price is given by $p \in (E_0[s], E_1[s])$ and consider optimists' problem (1). The riskiness, $\bar{s}$, of the optimal loan is the unique solution to:

$$\begin{aligned} p &= p^{opt}(\bar{s}) \\ &\equiv F_0(\bar{s}) \int_{s^{\min}}^{\bar{s}} s \frac{dF_0}{F_0(\bar{s})} + (1 - F_0(\bar{s})) \int_{\bar{s}}^{s^{\max}} s \frac{dF_1}{1 - F_1(\bar{s})}. \end{aligned} \tag{3}$$

- $p^{opt}(\bar{s})$ is like an inverse demand function: Decreasing in $\bar{s}$.
- **Asymmetric disciplining:** Asset is priced with a mixture of beliefs.

Illustration of optimal loan and asymmetric disciplining



Optimists' trade-off: More leverage vs. borrowing costs

- Optimists choose $\bar{s}$ that maximizes the **leveraged return**:

$$\frac{E_1[s] - E_1[\min(s, \bar{s})]}{p - E_0[\min(s, \bar{s})]}.$$

- The condition $p = p^{opt}(\bar{s})$ is the first order condition for this problem.

Optimists' trade-off features two forces:

1. Greater $\bar{s}$ allows to leverage the unleveraged return:

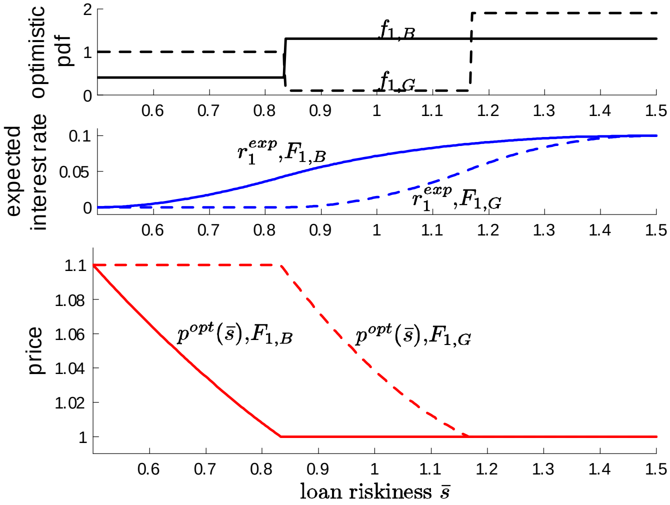
$$R^U \equiv \frac{E_1[s]}{p} > 1.$$

2. Greater $\bar{s}$ is also costlier. Optimists' **perceived interest rate**

$$1 + r_1^{per}(\bar{s}) \equiv \frac{E_1[\min(s, \bar{s})]}{E_0[\min(s, \bar{s})]}$$

is greater than benchmark and strictly increasing in $\bar{s}$.

Intuition for the asymmetric disciplining result



Equilibrium price is determined by asset market clearing

- Optimists' asset demand is:

$$a_1 = \frac{n_1}{p - E_0\left[\min\left(s, \bar{s}\right)\right]}.$$

- **Market clearing:** Set demand equal to supply (1 unit):

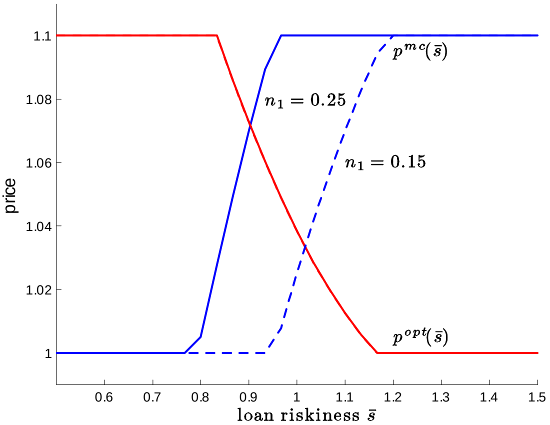
$$p = p^{mc}\left(\bar{s}\right) \equiv n_1 + E_0\left[\min\left(s, \bar{s}\right)\right].$$

  Increasing relation between p and $\bar{s}$.

The equilibrium, $(p, \bar{s}^*)$, is the unique solution to:

$$p = p^{mc}\left(\bar{s}\right) = p^{opt}\left(\bar{s}\right).$$

Illustration of equilibrium



Skewness is formalized by single crossing of hazard rates

- Obtain the comparative statics for $p, \bar{s}^*$ and the margin,

$$m \equiv \frac{p - E_0\left[\min\left(s, \bar{s}^*\right)\right]}{p}.$$

Definition (Upside Skew of Optimism)

Optimism of $\tilde{F}_1$ is *skewed more to upside* than F_1, i.e., $\tilde{F}_1 \succeq_U F_1$, iff:

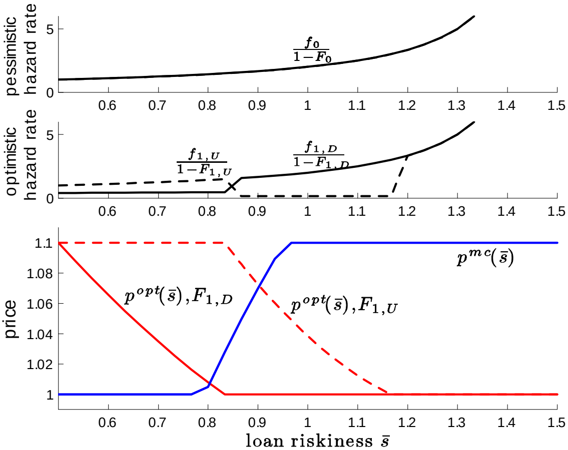
(a) $E\left[s\ ;\tilde{F}_1\right] = E\left[s;\ F_1\right]$.

(b) The hazard rates satisfy the (weak) single crossing condition:

$$\begin{cases} \frac{\tilde{f}_1(s)}{1-\tilde{F}_1(s)} \geq \frac{f_1(s)}{1-F_1(s)} \text{ if } s < s^U, \\ \frac{\tilde{f}_1(s)}{1-\tilde{F}_1(s)} \leq \frac{f_1(s)}{1-F_1(s)} \text{ if } s > s^U, \end{cases} \quad \text{for some } s^U \in S.$$

What investors disagree about matters

- **Theorem:** If optimists' prior is changed to $\tilde{F}_1 \succeq_U F_1$, then: the asset price p and the loan riskiness $\bar{s}^*$ weakly increase, and the margin m weakly decreases.



Extensions and other results

Other results from Simsek (2013):

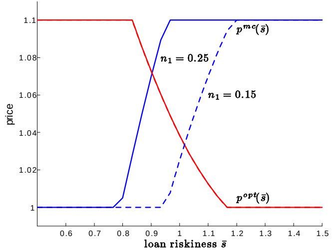
- Level of disagreement has ambiguous effects.
  - Type of disagreement more important.
- Results are robust to allowing for short selling.
  - Asymmetric disciplining of pessimism. Complementary.
- Richer contracts: Can replicate AD outcomes.
  - Bang-bang contracts as in Innes (1990).
  - Both asset and cash are split. Financial innovation?

The model offers a theory of procyclical leverage

- The main result suggests a theory of procyclical leverage.
- Driven by **changes in downside uncertainty/tail risk:**
    - Good times: Low downside uncertainty $\Longrightarrow$ High leverage.
    - Bad times: High downside uncertainty/disagreement $\Longrightarrow$ Low leverage.

- Do we really need to vary uncertainty? What if we vary net worth...

Net worth variation doesn't generate procyclical leverage



- Low net worth $\Longrightarrow$ Low prices/high expected returns $\Longrightarrow$ Optimists take greater leverage and speculate more!
- Result is general. Many models produce countercyclical leverage...

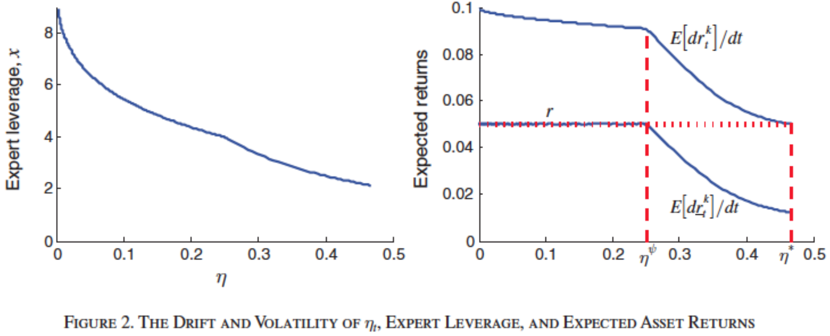


Figure: From Brunnermeier-Sannikov (AER, 2014): "A Macroeconomic Model with a Financial Sector."

Net worth variation doesn't generate procyclical leverage

- Similar feature in other prominent models, e.g., by He-Krishnamurthy.
- Here, leverage is essentially unrestricted (thanks to continuous time).
- Then, leverage pinned down by borrowers' desired portfolio size.
- This typically generates countercyclical leverage due to above logic.

- So procyclical leverage seems to require:
  Lenders' default concerns and **changes in uncertainty**.

The model also generates endogenous default

- Another interesting feature: **Default with positive probability.**
- (This is in contrast to the Geanakoplos model that we'll shortly see).
- Suppose $s \simeq s^{\min}$ is realized in this model.
- What happens to borrowers? Lenders? Do they make a loss?
- Why do lenders make these loans despite occasional default/losses?

- These insights could be relevant for last week's Franc turmoil. Swiss Central Bank (SCB) let the Franc float. Franc considerably appreciated and created losses for many (short) investors.
- What happened to their lenders/brokers? From last week's WSJ....

THE WALL STREET JOURNAL.

1098 866

MARKETS

Surge of Swiss Franc Triggers Hundreds of Millions in Losses

Brokerage FXCM Gets Rescue Package; Deutsche Bank and Citigroup Suffer Big Hits

At the center of this week's turmoil, analysts said, was the use by FXCM clients of borrowed money, or leverage. FXCM kept lower margin requirements, or the amount held as collateral for a loan, than its competitors, analysts said, a practice that enabled traders to boost returns by using borrowed money.

- Short selling is the mirror-image case. Losses when $s \simeq s^{\max}$.
- "Downside disagreement" (speculation on Franc's decline) combined with "upside agreement" (SCB will never let Franc appreciate) can generate low margin loans with default!

Roadmap

Towards the Geanakoplos (2010) model

Geanakoplos (2010): Same setting as before, with three differences:

1. Two continuation states, $s \in \{U, D\}$.
2. Continuum of beliefs. Type $h \in [0, 1]$ believes probability of U is h.
3. Dynamics, captured by three dates as opposed to two.

- We will build the model by gradually adding these ingredients.
- First consider only the first ingredient. This is the earlier model with $S = [D, U]$ and dF_0 and dF_1 that put all weight on states D and U.

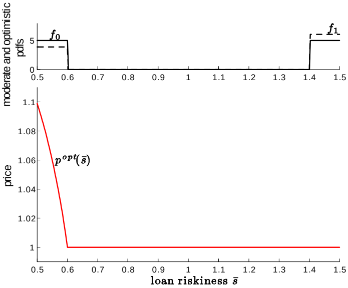
Geanakoplos as a special case of the earlier model

- Debt contract with promise $\varphi \in [D, U]$ priced by pessimists at $h_0\varphi + (1 - h_0) D$.
- Given price $p \in [D, U]$, optimists choose φ that maximizes:

$$\max_{\varphi \in [D,U]} \frac{E_1[s] - (h_1\varphi + (1 - h_1) D)}{p - (h_0\varphi + (1 - h_0) D)}.$$

How does $p^{opt}(\bar{s})$ (and thus, the optimal contract) look in this case?

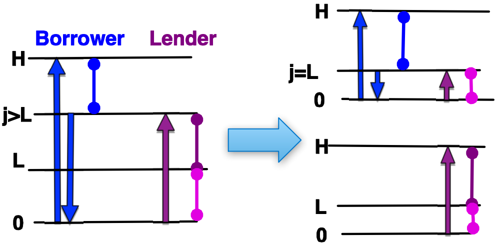
Geanakoplos as a special case of the earlier model



- For any $p \in (E_0[s], E_1[s])$, the optimal contract has riskiness $\bar{s} = D$.
- With two states, **no default**. Loans are **endogenously** fully secured.
- There is no default, but **fear of default** determines leverage.

Detour: A general no default theorem

- Fostel-Geanakoplos (2012): A very general **no default theorem**.
- Applies regardless of many details, e.g., reason for trade.
- Key step of the proof (similar to Modigliani-Miller):



- Can ignore default as long as **two states** and **purely financial asset**.
- LTV (and margins) are very simple: $\frac{L/p}{1+r}$. Above model: D/P.

Model with continuum of belief types

- Back to the model. Now introduce continuum of belief-types.
- Types denoted by h (beliefs for U), uniformly distributed over $[0, 1]$.
- Each type starts with (exogenous) net worth, $n > D$.
- No default theorem: Loans are fully secured. Downpayment D.
- Conjecture: There exists a cutoff $\hat{h}$ such that optimists (with $h > \hat{h}$) make a leveraged investment in the asset, and pessimists (with $h < \hat{h}$) hold the safe asset...

Equilibrium with continuum of belief types

- Optimists with $h > \hat{h}$ obtain a **leveraged return** of:

$$R(h) \equiv \frac{hU + (1-h)D - D}{p - D}.$$

- Pessimists with $h < \hat{h}$ obtain a return of 1.
- Indifference for the marginal type $\hat{h}$ implies **asset pricing condition**:

$$p = \hat{h}U + \left(1 - \hat{h}\right) D.$$

- We also have a **market clearing condition**:

$$\frac{n}{p - D}\left(1 - \hat{h}\right) = 1.$$

- We can solve these to obtain $p^*, \hat{h}$ in closed form.

Full fledged model with dynamics and continuum of beliefs

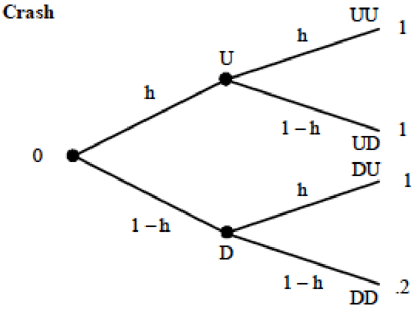
- Let us finally make the model dynamic, while keeping the continuum.
- Now suppose there is an additional date, 2. News arrive at date 1.

Asset pays only at date 2:

- If there is at least one good news (i.e., UU, UD or DU) asset pays 1.
- If there are two bad news (i.e., DD) asset pays 0.2....

Important ingredient: Bad news and uncertainty go in hand.

- Bad news **creates the possibility of a very bad event**.
- Shift from upside disagreement to downside disagreement.



Equilibrium: is a collection of asset prices, $(p_0, p_{1,U}, p_{1,D})$, and allocations for type h traders [at both dates 0 and 1] such that traders maximize and markets clear.

Equilibrium conjecture

Conjecture:

- In period 0, optimists with $h \geq \hat{h}_0$ make a leveraged investment.
- In period $(1, U)$: asset is riskless and sells for $p_{1,U} = U$.
- In period $(1, D)$: optimists from period 0 are wiped out. New optimists, agents in $[\hat{h}_1, \hat{h}_0)$, step in and make a leveraged investment.

Characterization of date 1 equilibrium

- At date $(1, D)$, characterization is identical to before, with the only difference that beliefs are distributed over $\left[0, \hat{h}_0\right]$ instead of $[0, 1]$.
- Optimists with $h \in \left[\hat{h}_1, \hat{h}_0\right]$ make a leveraged investment and receive the leveraged return $R_1(h) = \frac{h(1-0.2)}{p_{1,D}-0.2}$.
- Date 1 equilibrium, $\left(p_{1,D}, \hat{h}_1\right)$, characterized by two equations:
  - **Asset pricing**:

$$p_{1,D} = \hat{h}_1 + \left(1 - \hat{h}_1\right) 0.2, \tag{4}$$

  - **Market clearing:**

$$\frac{n}{p_{1,D} - 0.2} \left(\hat{h}_0 - \hat{h}_1\right) = 1. \tag{5}$$

Date 0 equilibrium

Date 0 equilibrium characterization is similar with the following differences:

- Up and down payoffs, U and D, are **endogenous** and are given by $p_{U,1}$ and $p_{D,1}$.
- Marginal trader at date 0 has an option value of saving cash:

$$R\left(\hat{h}_0, \text{saving}\right) = \hat{h}_0 + \left(1 - \hat{h}_0\right) \max\left(1, \underbrace{R_1\left(\hat{h}_0\right)}_{\text{this is greater than 1. Why?}}\right).$$

- (This can be thought of as a precautionary savings motive. Future fire sales generate endogenous "risk aversion.")

Characterization of date 0 equilibrium

Date 0 equilibrium, $\left(p_0, \hat{h}_0\right)$, is also characterized by two equations:

- The indifference condition for date 0 marginal trader:

$$\frac{\hat{h}_0 \left(1 - p_{1,D}\right)}{p_0 - p_{1,D}} = \hat{h}_0 + \left(1 - \hat{h}_0\right) \frac{\hat{h}_0 \left(1 - 0.2\right)}{p_{1,D} - 0.2} \tag{6}$$

- Market clearing at date 0:

$$\frac{n}{p_0 - p_{1,D}} \left(1 - \hat{h}_0\right) = 1. \tag{7}$$

- **Equilibrium** $\left(\hat{h}_0, p_{0,D}, \hat{h}_1, p_{1,D}\right)$ is the solution to (4), (5), (6), (7).
- Solve equilibrium numerically. For $n = 0.68$, should give:

$$p_0 = 0.68, \; p_{1,D} = 0.43, \; \hat{h}_0 = 0.63, \; \hat{h}_1 = 0.29.$$

Main result: Countercyclical margins and leverage cycle

Three factors contribute to the price crash:

1. **Bad news** that lower expected value of asset for all agents.
2. **Net worth channel:** Loss of net worth for most optimistic investors. Asset sold to lower valuation investors.
3. **Procyclical leverage/countercyclical margins**
   - Margin at date 0: $\frac{p_0 - p_{1,D}}{p_0} = \frac{0.68 - 0.43}{0.68} \simeq 22\%$.
   - Margin at date 1: $\frac{p_{1,D} - 0.2}{p_{1,D}} = \frac{0.43 - 0.2}{0.43} \simeq 53\%$.

Leverage cycle: Leverage moves together with prices.

Key ingredient: Bad news and uncertainty go hand-in-hand.

Roadmap

Taking leverage theories to data

- These models suggest **leverage ratios** matter for investment/prices.
- They also emphasize **tail risk** as key determinant of leverage ratios.
- Adrian-Shin (2013) present empirical evidence consistent with these results, using data on **banks/broker-dealers** balance sheets.
- They (and coauthors) argue broker-dealer leverage affects asset prices.
- Let's take a brief look at their empirical evidence...

Measuring leverage ratio for banks/broker-dealers

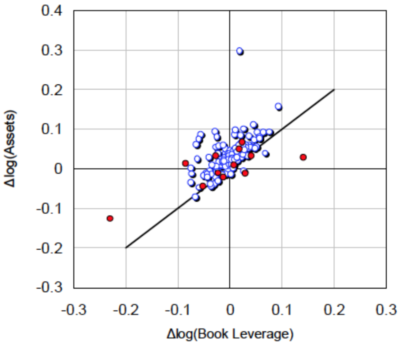
- Challenge: How to measure bank/broker-dealer leverage ratio?
- Two possibilities: **Book leverage** or **market-value leverage**.

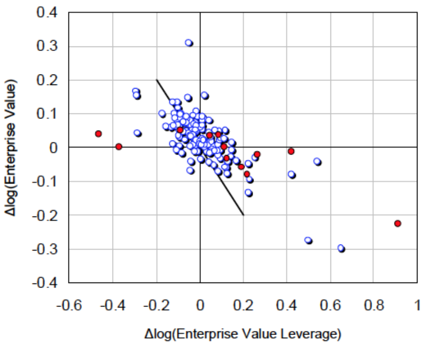
- Define "Book equity" as: Financial assets minus liabilities.
- **Book leverage** is financial assets divided by book equity.

- Define "net worth" as market capitalization.
- Define "enterprise value" as net worth plus debt.
- **Market/enterprise value leverage** is this divided by net worth.

It turns out the two measures behave very differently...

Measuring leverage ratio for banks/broker-dealers





Measuring leverage ratio for banks/broker-dealers

Which definition is **conceptually** more relevant for us?

- Recall we have a theory of asset-based leverage/margins.
- For banks, book equity reflects mostly margins on financial assets.
- In contrast, net worth contains claims to future profits/fees etc.
- Book equity appears more appropriate in our context.

Adrian, Moench and Shin (AMS, 2013): Book leverage also seems empirically more relevant for predicting asset prices/returns (later).

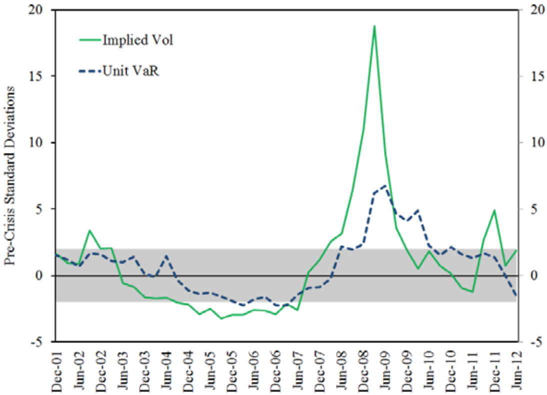
Measuring tail risk for banks/broker-dealers

- Another challenge: How to measure tail risk?
- In practice, banks/regulators use **Value-at-Risk** to assess health:

$$\text{Prob}\left(A < A_0 - V\right) \leq 1 - c.$$

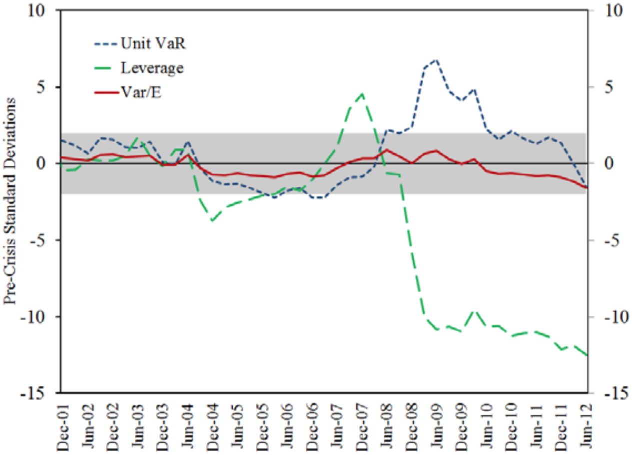
  - Here, A_0 is initial or some benchmark value of assets.
  - A is the end-of-period random value of assets.
  - c is the confidence level. Typically 99% or 95%.
  - V is the **Value-at-Risk** at c confidence level.
- Define also **unit VaR** as $v = V/A_0$, VaR per dollar invested.

Banks' VaRs and their implied volatility



- Banks' self-reported VaRs are highly correlated with implied vols.
- Dramatic increase in VaR (extreme losses) during the crisis.

Banks' leverage ratios are correlated with their VaRs



- Consistent with (a broad interpretation of) Geanakoplos (2010).
- Pictorial results supported by regression analysis (see the paper).

Leverage also has implications for asset prices

- AMS (2013): Do variations in banks' leverage affect asset prices?
- They run predictive return regressions along these lines:

	MKT	SPX	BAA
		1975Q1 - 2012Q4	
coeff	-0.070***	-0.065**	-0.025**
	[-2.975]	[-2.542]	[-2.122]
coeff-Stambaugh	-0.070***	-0.064**	-0.025**
	[-2.960]	[-2.527]	[-2.117]
R^2	0.056	0.089	0.029
N obs	151.000	151.000	151.000

- Coef: OLS coefficient on lagged broker-dealer leverage growth.

Leverage also has implications for asset prices

- Negative coefficient broadly consistent with Geanakoplos-Simsek:
  - Uncertainty$\Longrightarrow$ low leverage$\Longrightarrow$ low price$\Longrightarrow$ high expected return.
- AMS (2013) run similar regressions using **market value leverage.**
- They also run regressions using **banks' net worth** as opposed to leverage (measured either using market valuation or book valuation).
- Net worth is not significant, counter to much financial frictions theory!

	$yBDblevg$	$yBDmlevg$	$yBDbeg$	$yBDmeg$	$yCBblevg$	$yCBmlevg$
			Predictive Time Series Regressions			
MKT	-0.070***	0.012	0.044*	-0.009	-0.276	0.027
	[-2.857]	[0.342]	[1.761]	[-0.268]	[-1.377]	[0.658]
BAA	-0.026**	0.014	0.014	-0.021	-0.159	0.020
	[-2.122]	[0.819]	[1.086]	[-1.216]	[-1.595]	[0.992]
CMT10	0.008	0.006	-0.006	-0.006	-0.137	0.001
	[0.593]	[0.312]	[-0.475]	[-0.329]	[-1.277]	[0.049]

- Broker-dealers' financial distress seems best summarized by leverage.
- Net worth might matter for other institutions, e.g., commercial banks.

Taking stock: Procyclical leverage

- **Leverage**, especially if **procyclical**, can generate **amplification.**
- Geanakoplos: **Theory of procyclical leverage** driven by **uncertainty**.
- Simsek: Qualification in the context of trade driven by disagreements.
  - What matters is **type of uncertainty: downside vs. upside.**
  - Procyclical leverage from changes in **downside uncertainty/tail risk.**
- Adrian-Shin and coauthors: Broadly consistent empirical evidence.