Search-and-Matching Analysis of High Unemployment Caused by the Zero Lower Bound

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Phelps and Winter (1970), p. 337

A landing on the non-Walrasian continent has been made. Whatever further exploration may reveal, it has been a mind-expanding trip: We need never go back to

$$\dot{p} = \alpha(D - S)$$

and

$$q = \min(D, S)$$

$$.$$

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(4) A *central bank* that accepts deposits (reserves) from endowed households that pay interest, in the form of the primary input, at a per-period rate of r, the *reserve rate*.

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Frictionless equilibrium

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The wage is $w_t = p_t$ and the supply of consumption by firms, integrated with the market for workers, is

$$\begin{aligned} c_t &= 0 \text{ if } p_t < z \\ &\in [0, (1-z)\lambda] \text{ if } p_t = z \\ &= (1-p_t)\lambda \text{ if } p_t > z \end{aligned} \tag{1}$$

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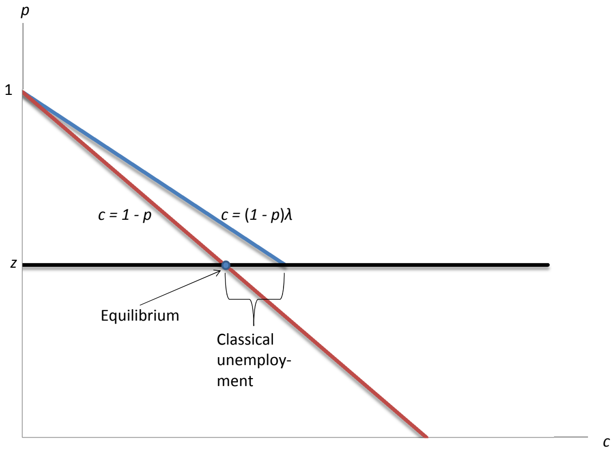
Equilibrium without frictions



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Let $\underline{a} = \min_t a_t$. The household will choose $c_t = 0$ for all t with $a_t > \underline{a}$

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The quick and dirty explanation is that adding a central bank that sets an interest rate different from the equilibrium rate of a model, without removing an equation, results in an over-determined system of equations that has no solution

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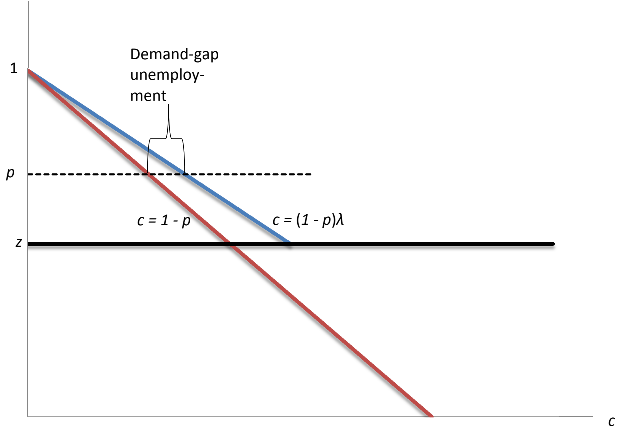
Demand Gap Resulting from a Price and Wage above the Equilibrium Level



Demand-gap unemployment

A feasible path of the economy exists with prices satisfying the intertemporal equality condition (the consumption Euler equation) of the endowed households and with demand-gap unemployment in every period. The price trajectory is

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Demand-gap unemployment is

$$u_t = \lambda - 1,$$

the excess of the labor force over maximum feasible employment

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The demand-gap model and the DMP model clash

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The number of postings is $V = \frac{\phi(q)}{q}U$, where U is the number of searchers. A reasonable specification for $\phi(q)$, based on the matching function $\alpha\sqrt{UV}$, is

$$\phi(q) = \frac{\alpha^2}{q}$$

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The surplus from a match is $p - z$; the worker receives a fraction β of the surplus and the firm retains the rest.

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The unemployment rate is

$$u = 1 - \frac{\alpha^2}{q}$$

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Unemployment, continued

The wage is

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The labor market imposes a functional relation between unemployment and the price:

$$u(p) = 1 - \frac{(1 - \beta)\alpha^2(p - z)}{k}$$

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A matched household and firm make a Nash bargain for the price of consumption goods, p

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Nash bargain in the product market

The firm's outside option is to sell to another household at the prevailing price, $\bar{p}$, but the firm faces a cost γ of breaking off bargaining with one household and starting up with another, so the outside option is worth $\bar{p} - \gamma$

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The surplus from the potential trade is $1 - (\bar{p}_T - \gamma)$

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Nash bargain in period T, continued

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In the symmetric equilibrium, where $\bar{p} = p$, the price is

$$p_T = 1 - \frac{b}{1-b}\gamma$$

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Earlier periods

The endowed household has the option to invest its endowment at the central bank at rate r for τ periods, and pay

$$\frac{p_{t+\tau}}{1 - u_{t+\tau}}$$

for conversion in period $t + \tau$ The effective price is boosted by division by $1 - u_{t+\tau}$ to account for the possibility that the household will not be matched to a firm

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Earlier periods, continued

The present value in period $t-1$ of output purchased by saving in period $t-1$ and purchasing in period $t+\tau$ is

$$X_{t,\tau} = \frac{p_{t+\tau}}{(1+r)^{\tau}(1-u(p_{t+\tau}))}$$

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The most advantageous outside option is

$$x_t = \min_{\tau} X_{t,\tau}$$

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This outside option for the household in period t is worth $1 - x_t$. If $x_t > 1$, it has no influence and the bargain becomes the same as in period T, in which case I redefine $x_t = 1$. The firm has the same option as in period T. The surplus is

$$S_{t-1} = 1 - (1 - x_t) - (\bar{p}_{t-1} - \gamma)$$

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The household's payoff is

$$\begin{aligned} 1 - p_{t-1} &= bS + 1 - x_t \\ &= b[1 - (1 - x_t) - (\bar{p}_{t-1} - \gamma)] + 1 - x_t. \end{aligned} \qquad (2)$$

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Symmetric equilibrium

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Given p_T, one can compute the equilibrium price path by backward recursion.

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Illustrative parameter values

Efficiency of matching: $\alpha = 0.28$
Bargaining weight of jobseekers: $\beta = 0.5$
Bargaining weight of endowment households: $b = 0.5$
Firm's cost of maintaining a posting of a vacancy: $k = 0.02$
Flow value of not working: $z = 0.5$
Number of years: $T = 10$
Central bank's real interest rate: $r = 0.01$

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Elasticity of the unemployment rate with respect to the product price is around 25, a value known to equip the model to turn small observed fluctuations in productivity into meaningful fluctuations in unemployment. The model's reliance on Nash bargaining with equal bargaining weights—shown in Shimer (2005) to generate pathetically small fluctuations in unemployment—is offset by the model's different specification of the matching process

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Paths of Unemployment and Consumption Price Induced by a Central-Bank Interest Rate of 0.01