

The Rational Consumer: Theory and Evidence

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1 The Allocation of Wealth among the Generations of a Family That Lasts Forever: A Theory of Inheritance

Recent models of economic growth have been based on a variety of assumptions about consumption behavior. First, a large literature has grown out of the assumption that consumers make decisions by arbitrary rules, particularly the rule of consuming a fixed fraction of total income or that of consuming all wages and saving all profits. Second, in the past 2 or 3 years there has been a resurgence of interest in models of optimal accumulation in which consumption behavior is regulated by an authority that can see far beyond the lifetime of any individual and that maximizes a social welfare function defined over the consumption of present and future generations. Finally, an important series of papers by Samuelson (1958), Diamond (1965), and Cass and Yaari (1965) have investigated competitive models in which individuals determine their consumption for two or more periods by maximizing a utility function subject to a wealth constraint. The results of these investigations are somewhat disturbing—in particular, the competitive equilibrium interest rate may be permanently less than the rate of growth because of oversaving. This implies that the equilibrium is inefficient by a well-known theorem of Phelps and Koopmans (1965). Further, as Diamond has shown, some seemingly neutral fiscal activities of the government may have an important effect on the equilibrium—there may indeed be a “burden” of the public debt.

These models neglect an important aspect of the intertemporal decisions of the consumer—namely, that a person usually cares not only about his own consumption but also about the well-being of his children. A father has a considerable amount of control over his sons’ wealth because he can vary the size of the bequest that he makes to them. In this essay I derive some of the implications of hypothesis about the way in which a family makes decisions about inheritance. The hypothesis is that a father and his sons decide jointly how to allocate their wealth between the father’s and the sons’ consumption by maximizing a utility function in which each one’s consumption appears as an argument. Interestingly, while the spirit of this hypothesis is similar to that of the competitive utility maximizing models of Samuelson-Diamond-Cass-Yaari, the properties of the resulting model are very much like those of the centrally directed social-welfare-maximizing models of optimal growth. In fact, I will demonstrate that this competitive model has the catenary turnpike property common to almost all models of optimal growth.

This essay treats a highly stylized economy in which there is one kind of output, which may be either consumed or used as capital in production.

Each person lives two periods but consumes and earns wages only in the second period. At the beginning of the second period each person marries and has $1 + n$ sons and $1 + n$ daughters. A family consists of husband and wife, their children, and all their future descendants. There is a perfect market for loans between any two periods. Production is carried out by profit-maximizing entrepreneurs who borrow from the public to finance all their investments. Production is assumed to take place with constant returns to scale, and output is sold in a competitive market, so entrepreneurs earn no profit. Finally, all families are assumed to be identical.

1.1 Family Demand Functions with a Finite Horizon

The assumptions of this essay about the family allocation process can be stated in two main hypotheses. First, we have the following.

HYPOTHESIS ON THE ALLOCATION OF WEALTH BETWEEN FATHER AND SON: However much wealth a father and his sons devote to their consumption, they divide it among themselves so as to maximize a joint utility function,

$$U(c_f, c_s),$$

where c_f is the father's (and mother's) consumption and c_s is the consumption of each of their sons and daughters. We further assume that U is quasi-concave.

Then, as a consequence of this maximization process, for any given amount of wealth that they spend in total, there is a unique demand function giving the parents' share, where the parents are now identified as generation t :

$$c_t = d(x_t, r_t); \quad (1)$$

x_t is the present value of the consumption of parents and offspring, and r_t is the interest rate on loans between period t and period $t + 1$.

It is sensible to assume that the parent's consumption is not an inferior good:

$$\frac{\partial d}{\partial x_t} \geq 0. \quad (2)$$

Now if each pair of generations makes its decisions by this process and if the decisions are consistent in that each son's planned consumption is the

same as his actual consumption as a father, then the consumption of all future generations can be predicted exactly, given today's parents' consumption. This follows by induction after establishing the uniqueness of the sons' consumption given the parents' consumption, since each son subsequently becomes a father. For this purpose, consider the following diagram:

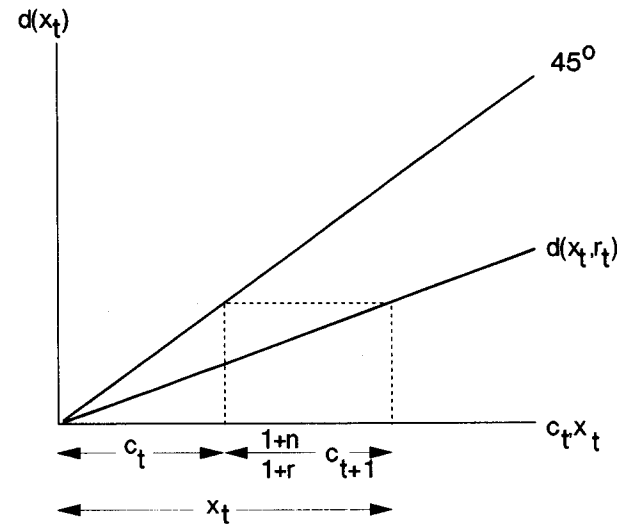


Figure 1.1

To determine the c_{t+1} corresponding to a particular c_t , draw a vertical line up from c_t to the 45° line and extend it horizontally to its intersection with the $d(x_t, r_t)$ curve (the intersection is unique by the assumption that the curve does not turn down). The horizontal distance at the intersection is the total expenditure x_t corresponding to c_t ; the consumption per son is given by

$$c_{t+1} = \frac{1 + r_t}{1 + n} (x_t - c_t). \quad (3)$$

Thus the parents can predict the consumption of every generation corresponding to a particular value of their own consumption. Analytically, this can be seen by substituting

$$x_t = c_t + \frac{1+n}{1+r_t} c_{t+1} \quad (4)$$

in equation 1 to get an implicit difference equation,

$$c_t = d\left(c_t + \frac{1+n}{1+r_t} c_{t+1}, r_t\right), \quad (5)$$

which can always be stated explicitly in the form

$$c_{t+1} = g(c_t, r_t). \quad (6)$$

Today's parents can use this relation to predict the consumption of all future generations.

It remains to introduce an additional hypothesis to specify in what way today's parents and offspring care about future generations of the family or, in other words, how they choose the part of the total family wealth that they will appropriate for themselves, x_t . This leads to the following hypotheses.

HYPOTHESIS OF FUTURE GENERATIONS: Today's parents and offspring spend the largest amount of wealth, x_t , that is consistent with the long-run family budget constraint that terminal wealth not be negative.

Then the decision-making process of the family may be visualized in the following way: today's parents and offspring examine the various family consumption trajectories that correspond to alternative values of x_t and pick the value of x_t whose consumption trajectory will exhaust family wealth at time T . The easiest way to state the exhaustion of wealth is in terms of family assets (nonhuman wealth); if A_t denotes family assets per person measured at the beginning of period t , then

$$A_{t+1} = \frac{1+r_t}{1+n} A_t + w_t - c_t, \quad (7)$$

where w_t is noninterest income (wages) per person in period t . Then the budget constraint is

$$A_{T+1} = 0. \quad (8)$$

Because of the continuity of the functions involved, there is always a value of the parents' consumption c_t corresponding to a trajectory that exactly meets this budget constraint.

The discussion so far has considered only the behavior and motivation of one pair of generations of a family and has carefully avoided the notion that any individual made plans that are binding on future generations. It is interesting at this point to investigate the consequences of this kind of behavior for the family as a whole. In particular, we inquire whether or not this behavior is reasonable in the sense that it resembles the behavior that might be prescribed if the family in fact had a planner.

The discussion will draw upon the results of Hall (1967), which proposes a hypothesis that is equivalent to the present one in the special case where the intergenerational utility function has the special additive form

$$U(c_t, c_{t+1}) = u(c_t) + (1+n)v(c_{t+1}). \quad (9)$$

This form is assumed in the following discussion.

The first important property of family consumption under the inheritance hypothesis is *efficiency*. A consumption trajectory is efficient if there is no generation whose consumption could be increased without violating the family budget constraint. Clearly with a finite horizon any trajectory that meets the budget constraint exactly is efficient; the real significance of this property is apparent only when the family is assumed to last forever. However, family consumption behavior based on arbitrary rules (such as a constant savings ratio) may fail to meet even this simple criterion.

The second important property is what Samuelson (1950) calls *reversibility*: for any consumption trajectory there is a total family wealth and an interest rate trajectory that yields the consumption trajectory as the family demand. In other words, there are no parts of the consumption space that are permanently in the dark in the sense that they would never be the demand of a family in a competitive economy. If a surrogate family utility function exists, this property is equivalent to quasi-concavity of the function. In the present model this property always holds if the horizon is finite. The conditions for family equilibrium are

$$s(c_t, c_{t+1}) \equiv \frac{v'(c_{t+1})}{u'(c_t)} = \frac{1+r_t}{1+n} \quad (10)$$

and

$$W = \sum_{t=1}^T \prod_{\tau=1}^{t-1} \left(\frac{1+n}{1+r_\tau} \right) c_\tau. \quad (11)$$

Thus the reversibility conditions are satisfied with

$$r_t = (1 + n)s(c_t, c_{t+1}) - 1, \quad (12)$$

and the value of W given in equation 11.

A third property, equivalent to the second if a surrogate family utility function exists, is *diminishing marginal rate of substitution*. This is important because it indicates a tendency for the family consumption plan to involve approximately equal consumption for all generations rather than concentrating on only a few generations. Hall (1967) showed that diminishing marginal rate of substitution will hold in this model if the rate of change of the rate of impatience with respect to the consumption level is small in absolute value. The rate of impatience, $\rho(c)$, is defined by

$$\rho(c) = (1 + n)s(c, c); \quad (13)$$

it is the interest rate at which the level of consumption c will remain constant. Diminishing marginal rate of substitution is guaranteed over any horizon T if

$$\frac{|\rho'(c)|}{1 + \rho(c)} < \varepsilon, \quad (14)$$

for a positive constant ε , independent of T .

A fourth property of possible interest is the existence of a surrogate family utility function $U^*(c_1, \dots, c_T)$ with the property that all family consumption decisions could be portrayed as if they were made by maximizing this function subject to the family budget constraint. We find from Hall (1967) that in general there is no such surrogate family utility, and hence no meaning can be given to the notion of family preferences among alternative consumption trajectories. This is neither destructive nor, in retrospect, a surprising conclusion. After all, the only connection that today's generations have with the future in this model is a concern for the financial integrity of the family; it would be surprising indeed if this were equivalent to having preferences between any pair of consumption trajectories even when the only difference between the trajectories was in the consumption of a generation far in the future.

There is one significant expectation to this conclusion. If the rate of impatience is constant over all consumption levels, then there is, in fact, a surrogate utility with the familiar form

$$U^*(c_1, \dots, c_T) = \sum_{t=1}^T \left(\frac{1+n}{1+\rho} \right)^t u(c_t). \quad (15)$$

Then the process of allocating wealth between succeeding generations is exactly the same as would be implied by the Euler equation for maximizing (15); the budget constraint is exactly the transversality condition for this maximization. In this case the inheritance hypothesis amounts to assuming that the family has adopted as its behavioral rule, not the notion of maximizing a utility function, but an operationally identical rule that turns out to be the Euler equation and its transversality condition. This, I think, makes the notion of a family utility function of the special form (15) more acceptable to those who reject the ideal of a family consciously maximizing a utility function on the grounds that there is no central authority within a family who makes and enforces consumption plans.

1.2 Consumption Demand for a Family That Lasts Forever

Economic intuition suggests that the behavior of a family that expects to last a thousand years should be only infinitesimally different from one that expects to last forever. The hypotheses on family behavior proposed in this section are not sufficiently strong to guarantee this irrelevance of the distant future, nor, in fact, are they strong enough to ensure that the criteria of reasonable family demand behavior are met when the family lasts forever. Paradoxically, demand functions that are efficient and reversible for any horizon T , no matter how far distant, may be inefficient or irreversible when the horizon is infinitely distant. Not surprisingly, this is closely related to the problem of impatience. A similar paradox has been observed in models of optimal economic growth (for example, Samuelson 1967), where it has been resolved by showing that impatience is a logical necessity if a true utility function is to exist (Diamond 1965). Thus we may immediately conclude that the inheritance hypothesis implies a surrogate family utility function if and only if $\rho(c) > n$ and $\rho'(c) = 0$ for all c .

The difficulty with respect to the properties of efficiency and reversibility is the following: If there is a c such that $\rho(c) < n$, then either

1. If reversibility holds, the consumption trajectory $c_t = c$ for all t is inefficient, because the implied interest rate is less than the rate of growth, and a debt incurred by any generation vanishes in the limit,¹ or

2. Reversibility fails. This will happen if $\rho'(c) < 0$; see section 1.3 in this regard.

Thus the inheritance hypothesis must be strengthened in the following way:

HYPOTHESIS ON IMPATIENCE: Either both efficiency and reversibility hold for the consumption demand of a family that lasts forever, *or* (equivalently) the family is always impatient: $\rho(c) > n$. Efficiency is the more important of the first two properties, since it alone implies that the competitive equilibrium involves an interest rate whose limit is at least as large as the rate of growth.

Now we are prepared to discuss the full family wealth allocation problem over infinite time. Stated formally, the problem is to find a first generation consumption c_1 so that for given initial family assets A_1 , $\lim_{t \rightarrow \infty} A_t \geq 0$, where future consumption and assets are predicted by the pair of difference equations

$$c_{t+1} = g(c_t, r_t) \quad (16)$$

$$A_{t+1} = \frac{1 + r_t}{1 + n} A_t + w_t - c_t. \quad (17)$$

As several authors have remarked, this problem may not have a sensible solution. For example, if

$$g(c_t, r_t) = \frac{1 + r_t}{1 + n} c_t \quad (18)$$

(this comes from a log-linear intergenerational utility), if r_t has the constant value $\tilde{r} > n$, and if $w_t = 0$ for all t , future consumption is

$$c_t = \left(\frac{1 + \tilde{r}}{1 + n} \right)^{t-1} c_1. \quad (19)$$

Family assets at time t are

$$A_t = \left(\frac{1 + \tilde{r}}{1 + n} \right)^{t-1} (A_1 - tc_1). \quad (20)$$

For any positive c_1 , $\lim_{t \rightarrow \infty} A_t = -\infty$. The fact that some interest-rate trajectories make it impossible for the family to allocate its wealth in a reasonable fashion should not cause us to reject this model of family behavior. Rather, it shows how the inheritance hypothesis restricts the form

of the competitive equilibrium. For example, if families have a fixed rate of impatience, the competitive equilibrium capital stock will approach a steady state over time such that the net marginal product of capital will exactly equal the rate of impatience. This is discussed at greater length in section 1.3.

1.3 General Equilibrium in the Inheritance Model

Suppose that a neoclassical technology prevails, in which output per person, y , is given by a smooth convex function of capital per person:

$$y = f(k). \quad (21)$$

Capital deteriorates geometrically at a rate δ , so investment for replacement is δk , and investment to maintain the capital-labor ratio is nk . Thus net investment per person is

$$\Delta k_t = f(k_t) - c_{t+1} - (\delta + n)k_t, \quad \text{or} \quad (22)$$

$$k_{t+1} = (1 - \delta - n)k_t + f(k_t) - c_{t+1}. \quad (23)$$

The interest rate is equal to the net marginal product of capital:

$$r_t = f'(k_t) - \delta. \quad (24)$$

Consumption behavior is given by

$$c_{t+1} = g(c_t, r_t). \quad (25)$$

Finally, we have the fundamental budget constraint $A_{T+1} = k_T = 0$. The analysis of this system will be carried out in terms of consumption and the interest rate, although it could also be done in terms of any of several pairs of variables.

The (r, c) phase plane can be divided into two areas according to whether r is increasing or decreasing. The interest rate is unchanged from one period to the next only if k is unchanged, or

$$f(k) = (\delta + n)k + c. \quad (26)$$

In order to characterize this line in terms of r , we differentiate with respect to r :

$$f'(k) \frac{dk}{dr} = (\delta + n) \frac{dk}{dr} + \frac{dc}{dr} \Big|_{\Delta r=0}. \quad (27)$$

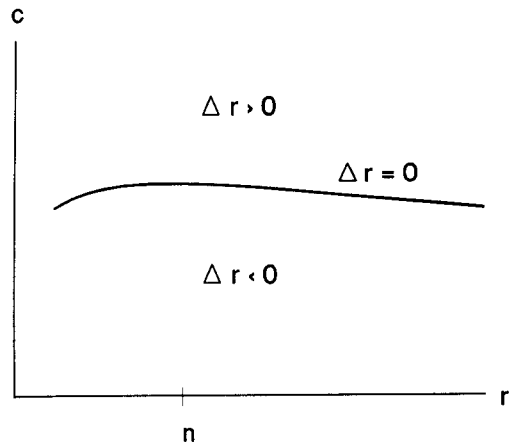


Figure 1.2

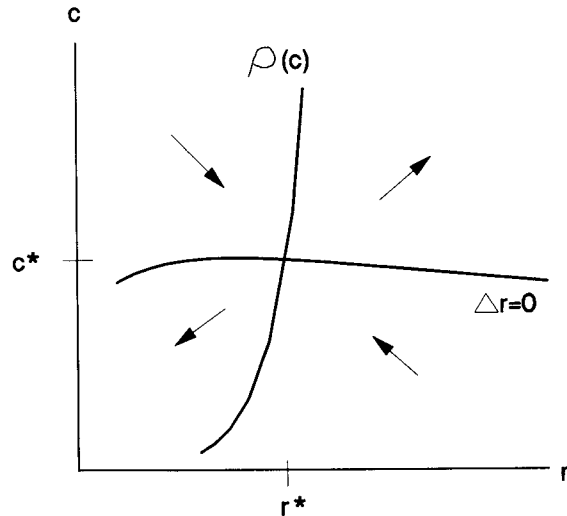


Figure 1.3

Now since $r = f'(k) - \delta$, $(dk/dr) = 1/f''(k)$, and

$$\begin{aligned} \left. \frac{dc}{dr} \right|_{\Delta r=0} &= \frac{f'(k) - \delta - n}{f''(k)} \\ &= \frac{r - n}{f''(k)} \end{aligned} \quad (28)$$

Thus the line slopes upward if $r < n$, reaches its maximum at $r = n$ (the golden rule), and declines if $r > n$; above the line capital is decreasing and r is increasing; the opposite occurs below the line (see figure 1.2):

A similar division of the phase plane is possible for the consumption equation: consumption is increasing whenever $r > \rho(c)$ and is decreasing whenever $r < \rho(c)$. Thus the phase diagram has the shown in figure 1.3.

Since the absolute value of $\rho'(c)$ is restricted to small values, it is reasonable to assume that the two stationary loci meet at a unique stationary point (r^*, c^*) . If so, $\rho(c)$ cuts $\Delta r = 0$ from below, the stationary point is a saddle point, and from the catenary properties of a saddle point, the following result is established.

COMPETITIVE TURNPIKE THEOREM. *As the end of the world becomes more distant (as the horizon T becomes large), the competitive equilibrium interest rate–consumption trajectory spends almost all of this time arbitrarily close to the point (r^*, c^*) , where the rate of time preference is equal to the stationary interest rate.*

Figure 1.4 illustrates trajectories for various T 's, starting with the same initial capital stock. Plotted against time, these have the appearance shown in figure 1.5. The case of an economy that lasts forever is a simple extension of the previous case. The only infinitely long (r, c) trajectories are those running along the top of the saddle, as seen in figure 1.6. From any initial capital stock, the economy eventually approaches indefinitely close to the steady-state point (r^*, c^*) .

It remains to show that these trajectories are truly competitive equilibria. That is, we must show that given the interest rate and wage trajectories derived from the phase-plane analysis, family demand would in fact be the consumption trajectory shown. Since these trajectories satisfy the difference equation (25), the conditions on the marginal rate of substitution between consecutive generations are clearly met. Furthermore, if the horizon is finite, terminal family assets are zero, and no generation can increase its consump-

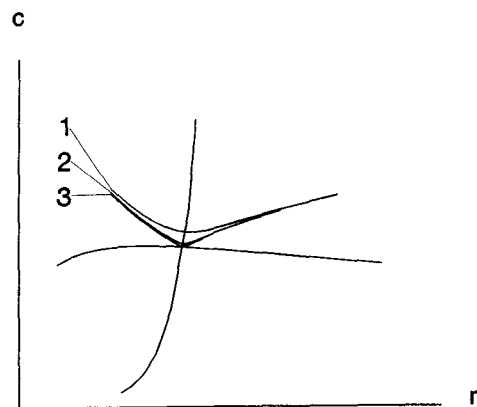


Figure 1.4

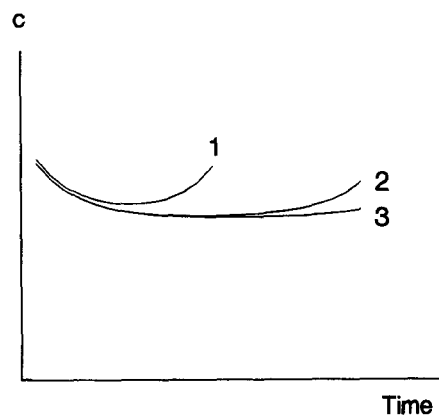


Figure 1.5

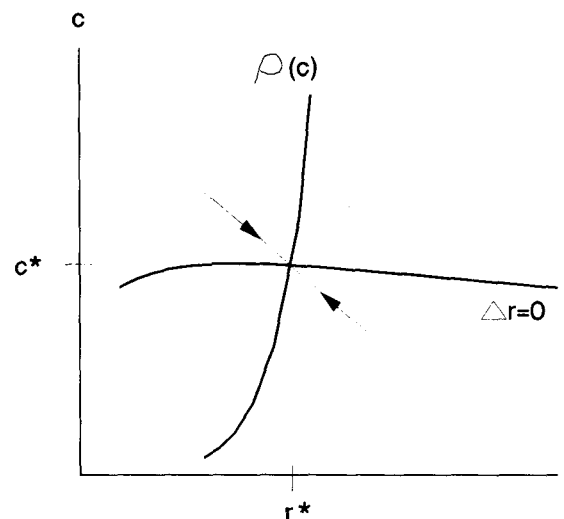


Figure 1.6

tion without decreasing the consumption of another generation. Thus the budget constraint is also met; we conclude that this trajectory is in fact the unique family demand. On the supply side, the assumption that the production function is convex guarantees that the supply of consumption goods is uniquely this trajectory.

With an infinite horizon, the limiting value of family assets is not zero, as it would be if the family could exhaust its wealth, but rather is the value of the steady-state capital stock k^* . However, if all debts must eventually be paid back (that is, if the present value of one unit of income, $\pi_{t=1}^r [(1+n)/(1+r_t)]$, goes to zero in the limit), no generation can increase its consumption by even the smallest amount without causing eventual family bankruptcy. Thus, even though family assets eventually always have a large positive value close to k^* , the budget constraint is met, and the trajectory is the true family demand. The requirement that debts must be paid back is crucial. For example, if there is a surrogate family utility function with zero rate of impatience, the present value function does not go to zero and there can be no competitive equilibrium. Given the interest-rate trajectory from the phase-plane analysis, the family will not choose the consumption trajectory shown there, but rather will choose one with higher consumption for one or more generations and for which the limiting value

of family assets is not k^* but zero. This is one of a variety of difficulties that arise in an economy in which the interest rate goes to n sufficiently quickly for the present value function not to have limit zero. Most of the hard problems of optimal growth theory are related to this problem; similarly, the interesting aspects of the study of the efficiency of an economy that lasts forever arise only in this case.

One property of the family's allocation problem deserves further attention; it is stated in the following theorem.

THEOREM ON THE IRRELEVANCE OF THE DISTANT FUTURE. *Along competitive equilibrium interest rate and wage trajectories the consumption of the present generation of a family becomes increasingly insensitive to their desired level of assets for generations t , as t increases, provided $\rho(c) > n$. Their sensitivity decreases with increasing impatience and increases with an increasing rate of change of impatience with respect to consumption.*

Proof. The sensitivity of the present generation to future asset levels is measured as the reciprocal of the derivative of A_t with respect to c_1 . The system of difference equations governing the allocation of family wealth is

$$c_{t+1} = g(c_t, r_t) \quad (29)$$

$$A_{t+1} = \frac{1 + r_t}{1 + n} A_t + w_t - c_t.$$

Differentiating with respect to c_1 , we get

$$\frac{dc_{t+1}}{dc_1} = \frac{\partial g(c_t, r_t)}{\partial c_t} \frac{dc_t}{dc_1} \quad (30)$$

$$\frac{dA_{t+1}}{dc_1} = \frac{1 + r_t}{1 + n} \frac{dA_t}{dc_1} - \frac{dc_t}{dc_1}, \quad (31)$$

with initial conditions $dc_1/dc_1 = 1$ and $dA_1/dc_1 = 0$. Now the function $g(c_t, r_t)$ is defined implicitly by

$$\frac{v'(c_{t+1})}{u'(c_t)} = \frac{1 + r_t}{1 + n} \quad (32)$$

so

$$\frac{v''(c_{t+1})}{u'(c_t)} \frac{\partial g}{\partial c_t} - \frac{v'(c_{t+1})u''(c_t)}{[u'(c_t)]^2}, \quad (33)$$

or

$$\frac{\partial g}{\partial c_t} = \frac{u''(c_t)v'(c_{t+1})}{v''(c_{t+1})u'(c_t)}. \quad (34)$$

Now c_t approaches the limit c^* , and from Hall (1967),

$$\rho'(c^*) = [1 + (c^*)] \left[\frac{v''(c^*)}{v'(c^*)} - \frac{u''(c^*)}{u'(c^*)} \right]. \quad (35)$$

Thus if $\rho'(c^*) < 0$, there is a T such that $t > T$ implies

$$\frac{u''(c_t)}{u'(c_t)} < \frac{v''(c_{t+1})}{v'(c_{t+1})}, \quad \text{or} \quad \frac{\partial g}{\partial c_t} > 1. \quad (36)$$

Similarly, if $\rho'(c^*) > 0$, eventually $\partial g/\partial c_t < 1$.

In the asset equation, eventually $(1 + r_t)/(1 + n) > 1$, since r_t approaches the limit $\rho(c^*)$, which is greater than n . Because of its simple recursive form, the properties of the system (30) and (31) may be seen by inspection. Since $\partial g/\partial c_t$ is always positive and $(1 + r_t)/(1 + n)$ is eventually strictly greater than one, dA_t/dc_1 becomes indefinitely negative with increasing t . If $\rho'(c^*) < 0$, the contribution of the term $-dc_t/dc_1$ also becomes indefinitely large, whereas in the opposite case, its contribution is eventually zero. But in either case, dc_1/dA_t has the limiting value zero, and the theorem is established.

The property stated in this theorem is also observed in all optimal growth problems with catenary motions; it is sometimes referred to as instability, but this is extremely misleading, since its behavioral implication is one of stability, not instability.

The assumption that the family faces interest-rate and wage trajectories that will turn out to be the equilibrium trajectories is crucial in this theorem. Along other trajectories, there may be a solution to the allocation problem. This will almost always be true if $\rho'(c) < 0$, since any stationary point of equations (29) is an unstable node, with roots

$$\lambda_1 = \frac{1 + r}{1 + n} \quad \lambda_2 = \frac{\partial g}{\partial c},$$

both of which exceed 1. Usually no trajectory can reach such a stationary point of an asymptotically autonomous system. Along an equilibrium trajectory, however, the interest rate changes over time exactly fast enough to allow the (A_t, c_t) trajectory to reach the unstable node. This means that a *tâtonnement* procedure would probably not be able to find the equilibrium, since it would require families to solve the allocation problem with interest rate trajectories that are not equilibria. In fact, computational experiments have indicated that the family must take account of the effect of its allocation of wealth (and the identical allocation of all other families) on the interest rate in order to obtain a *tâtonnement* procedure that is likely to work. It is possible to show that there is an interest-rate adjustment of a simple form that can be applied by each family and that guarantees convergence to the competitive equilibrium—this adjustment converts the family's allocation problem into one with strictly catenary properties.

1.4 Some Implications of the Inheritance Hypothesis

The most important difference between this and other models of competitive equilibrium with individually directed saving is that each person is required to see some distance into the future because he is sensitive to future economic developments. This has a number of important implications. First, since the family has a rate of preference for the parents' consumption at least equal to the rate of growth, the possibility of an inefficient competitive equilibrium is ruled out.

Second, in this model the equilibrium is independent of the size of the government's debt, so there is no burden of the debt. In Diamond's model, the equilibrium is sensitive to the size of the debt because the market capitalizes all the interest payments that a bond yields, but the individual takes account of only the tax payments to finance the interest that are levied during his lifetime. This asymmetric effect makes him spend more and save less, driving up the interest rate. Under the inheritance hypothesis there is no asymmetry because the individual does not distinguish between his own wealth and the wealth of the future generations of his family. The equilibrium is independent of any transfer of wealth between generations and in particular is independent of the transfer implied by taxing and paying interest.

Third, this model resolves a perplexing question about the competitive equilibrium price for an asset that cannot be produced, such as land.

Nothing in the equilibrium condition for the market for such an asset prevents an upward speculative movement in its price that lasts forever. That is, if p_t is the equilibrium price for the asset, then

$$p_t + s_0 \prod_{r=0}^T (1 + r_r) \quad (37)$$

is also an equilibrium price, where s_0 is any positive constant. Under any hypothesis, however, *no* price that goes to infinity is a general equilibrium price, because families would then have infinite wealth in the limit, allowing additional consumption for at least one generation. In this way, speculative booms in nonreproducible assets can be ruled out.

Notes

1. The effect on the limit of family assets per person of one additional unit of consumption by generation t is

$$\lim_{T \rightarrow \infty} \left(\frac{1 + \rho(c)}{1 + n} \right)^{T-t} = 0.$$

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