

# Quantifying Specification Error in Models with Expectations<sup>1</sup>

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## Introduction

In a broad class of interesting models in macroeconomics and finance, the random disturbance has two components. One is an expectations error, reflecting the new information not previously available to decision makers or traders. The other component is a traditional model error—it captures failures of the model to account precisely for the actual behavior of the dependent variable. The second component can be called specification error, or, in finance, noise.

We write the general class of models as

$$f(z_t, \beta) = \varepsilon_t - u_t$$

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Here  $f$  is a known scalar function,  $z_t$  is a random variable,  $\beta$  is a vector of parameters,  $\varepsilon$  is the expectation error, and  $u_t$  is the specification error. To focus on the issue at hand, we assume that the parameters  $\beta$  are known. We let  $y_t = f(z_t, \beta)$ . The fundamental identifying assumption we exploit is that  $\varepsilon_t$  is orthogonal to any vector of random variables,  $x_t$ , known to decision makers or traders at time  $t$ :

$$\text{Cov}(x_t, \varepsilon_t) = 0$$

The basic point we pursue is that any correlation of  $y_t$  with  $x_t$  arises from the specification error and not from the expectation error. To formalize this point, we let  $R_t(y, x)$  be the projection of  $y$  on  $x$  at time  $t$  (the fitted value of the regression of  $y$  on  $x$ ). The result behind all of the research we consider is

**Startz's Theorem.**  $V(u_t) \geq V(R_t(y, x))$

*Proof.* Let  $v$  be the residuals from the regression of  $y$  on  $x$ :

$$y_t = R_t(y, x) + v_t$$

Then we can write the specification error as

$$\begin{aligned} u_t &= -y_t + \varepsilon_t \\ &= -R_t(y, x) - v + \varepsilon_t \end{aligned}$$

and its variance is

$$V(u_t) = V(R_t(y, x)) + V(v_t - \varepsilon_t)$$

because both  $v_t$  and  $\varepsilon_t$  are orthogonal to  $x_t$ . Because the second variance can't be negative, the result follows.

In words, Startz's Theorem says that any correlation of the variable  $y_t$  with  $x_t$  must arise from its noise element because it can't come from the

expectation element. The fitted value from the regression is a conservative measure of specification error, in the sense that its variance cannot be less than the variance of the specification error.

The Theorem was stated and proved in a different but equivalent form by Richard Startz [1982] in an early paper unfortunately overlooked by subsequent authors.<sup>2</sup> Startz considered the general problem we stated above, in the context of the term structure of interest rates where the specification error is called an interest-rate premium. He made the additional assumption that the expectation error and premium are uncorrelated. Startz also obtained an upper bound on the variance of the expectation error and a lower bound on the variance of the expectation of the future short interest rate.

## Choice of explanatory variables

The bound depends on the choice of the explanatory variables,  $x$ . In most applications, the great bulk of the explanatory power comes from a single element of  $x$ . The easiest way to explain this point is to consider the application of the bound to the stock market. Let  $p_t$  be the price of a stock and let  $p_t^*$  be the realized present value achieved by holding the stock for some period following  $t$ . We let

$$y_t = p_t^* - p_t$$

This is the excess return over the holding period. Let  $p_t^e$  be the expectation of future returns over the holding period conditional on all information available to traders. The underlying model is one considered by Shiller [1981] and LeRoy-Porter [1981] extended to include an explicit specification error  $u_t$ :

$$p_t^* = p_t^e + \varepsilon_t$$

$$p_t = p_t^e + u_t$$

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<sup>2</sup>We did not become aware of Startz's work until seven years after our first work on this topic. The result in the Theorem was implicit in Theorem 3 of LeRoy and Porter [1981]; we will discuss their work in a later section.

Then

$$y_t = p_t^* - p_t = \varepsilon_t - u_t$$

The variable with uniquely high explanatory power is simply  $p_t$ . Scott [1985] pointed out the usefulness of testing the orthogonality of the realized return with the stock price. The projection of  $y_t$  on  $p_t$  is the projection of  $-u_t$  on  $p_t^e + u_t$ . In other words, the projection for placing a lower bound on noise is the projection of noise on itself plus the signal,  $p_t^e$ . If noise is at all large in relation to the signal, the projection will be informative about the amount of noise. In general, where there is an observed random variable of the form  $k_t + u_t$ , that variable is the most useful element of  $x$ . The variance bound in Startz's Theorem cannot be lower than  $V(R_t(-u, k + u))$ .

Because the single most useful variable in most cases will have the  $k + u$  form, it is useful to explore the conditions under which the bound will be most and least informative if we use only the single explanatory variable  $k + u$ . To this end, we let  $\rho$  be the correlation of  $k_t$  and the specification error,  $u_t$  and we let  $\lambda$  be the ratio of the variance of  $k$  to the variance of  $u$ . A page of straightforward algebra shows that

$$V(R(-u, z + u)) = \frac{(\rho\lambda + 1)^2}{\lambda^2 + 2\rho\lambda + 1} V(u)$$

Obviously, the lower is  $\lambda$ , the relative variance of the systematic variable,  $k$ , the closer the bound comes to the true value  $V(u)$ . The relation of the correlation to the usefulness of the bound is more subtle. Whenever the systematic element has greater variance than does the specification error (that is,  $\lambda > 1$ ), there is a value of  $\rho$  for which the bound is useless, namely  $\rho = -1/\lambda$ . If the systematic element is twice as important as noise for a stock price ( $\lambda = 2$ ), for example, and if noise has a negative correlation of 1/2 with the systematic element, then the noise will be invisible to the simple noise measurement procedure of regressing the excess return on the stock price. Other noise detection regressions may still be useful in these circumstances. If there are observed variables such as recent dividends that are good predictors of the

systematic part of the stock price, then their inclusion eliminates the problem just discussed. Intuitively, the inclusion of those variables is equivalent to generating a new noise detector  $z + u$  in which  $z$  has had the part predictable from observables removed, so its variance will be smaller and the bound more informative.

Even without special efforts whose effect is equivalent to lowering  $\lambda$ , the bound will generally be useful when the correlation of the systematic element and the specification error is zero or positive.

We can be reasonably definite about the choice of the explanatory variables for the calculation of the bound. First, it is essential to include the one variable, such as the current stock price, that consists of the specification error itself plus some other systematic element. Second, other observables should be included that help predict the systematic element, so that the loading on the specification error contained in the variable  $k + u$  can be as close to one as possible.

## Relation to research on testing present value models

For short measurement periods, the idea of projecting a statistic that, in theory, should be only an expectation error, on variables known in advance, first appeared in tests of the efficient markets hypothesis. Fama [1970] is a prominent example. Hall [1978] applied the same idea to consumption. The voluminous succeeding literature in both applications stressed testing the hypothesis of no noise or specification error, rather than quantifying the amount of noise or error. The literature also explored many modifications of finance and consumption models to eliminate the signs of specification error.

The examination of expectation errors over long periods began with LeRoy-Porter [1981] and Shiller [1981] (LRPS). Debate about the relative merits of short against long measurement periods is continuing even now. The original work on long periods considered applications in the stock market, in the framework discussed in the previous section. The excess return over a long period (infinite in the original papers),  $y_t$ , is the difference between the realized value,  $p_t^*$ , and the current price,  $p_t$ . The simplest proposition in the literature on long

periods is that the variance of  $p_t^*$  should be less than the variance of  $p_t$ , absent noise. LeRoy-Porter and many subsequent authors observed that this variance bound was less strict, in general, than a bound available from exactly the same data. The superior bound exploits the orthogonality of the expectation error and the current price and is the one obtained from Startz's Theorem by taking the forecasters,  $x$ , to be just the current price.

To see the relation between the variance bound and the orthogonality bound, let  $V^*$  be the variance of  $p^*$ ,  $V$  be the variance of  $p$ , and  $C$  be their covariance. The bound on noise variance from the regression of  $p^* - p$  on  $p$  is

$$\frac{(C-V)^2}{V}$$

If we consider the covariance,  $C$ , to be unknown, we could minimize this bound over admissible values of  $C$  to obtain a bound. The admissible values are those consistent with the Cauchy inequality,  $C^2 \leq VV^*$ . If the LRPS bound is satisfied ( $V^* \geq V$ ), then the minimizing value of  $C$  is  $V$  and the bound is uninformative. If the bound is violated, the minimizing value of  $C$  is  $\sqrt{VV^*}$ . Then the bound is

$$V(u) \geq \left( \sqrt{V} - \sqrt{V^*} \right)^2$$

This bound on the variance of the specification error is necessarily less informative than the regression bound. Because the covariance,  $C$ , is always observable if the two variances are, there is no reason to use the variance bound in place of the more informative regression bound that exploits the information in the covariance.

LeRoy-Porter explain their orthogonality test in a rather different way from our approach here, so it is probably worthwhile to show the relationship. The valuation model implies, absent noise,  $p_t^* = p_t + \varepsilon_t$ . Because  $\varepsilon$  is orthogonal to the observable  $p$ ,  $V(p^*) = V(p) + V(\varepsilon)$ , or  $V(p) = V(p^*) - V(\varepsilon)$ . This is a more informative restriction than the variance bound,  $V(p) \leq V(p^*)$ , because the variance of the expectation error,  $V(\varepsilon)$ , will generally be positive. Moreover, the orthogonality test is two-sided, whereas the variance bound test rejects only when

the variance of  $p$  is too high. Our projection approach has the same two advantages, because it is actually based on the same property. LeRoy-Porter's criterion is  $V(p) + V(\varepsilon) - V(p^*)$ ; if this quantity differs from zero, the model is falsified. But the model also implies

$$V(\varepsilon) = V(p^* - p) = V(p^*) + V(p) - 2\text{Cov}(p, p^*)$$

Thus the criterion can be expressed as  $2V(p) - 2\text{Cov}(p, p^*)$ , or  $2\text{Cov}(p - p^*, p)$ , which is exactly minus two times the numerator of the regression of the regression of  $p^* - p$  on  $p$ . Thus LeRoy-Porter's orthogonality bound is violated if and only if the regression coefficient for our projection is non-zero.<sup>3</sup>

Because a number of excellent survey and review articles have recently discussed the relationship between the long-horizon variance bounds tests and short-horizon orthogonality tests, we will not discuss the intervening literature here. See Cochrane [1991], Gilles and LeRoy [1991], and LeRoy and Steigerwald [1993]. We believe that a fair summary of current thinking is that the orthogonality of expectation errors to variables known at the time expectations are formed is generally regarded as the starting point for all tests; variance bounds are no more than a weak implication of orthogonality. The open questions today with respect to testing have to do with the duration of the period over which returns are measured.

## Duration of the measurement period

Because the random variable under consideration,  $y_t$ , has an expectation error, there must be some measurement period implicit in the construction of  $y_t$ . Different approaches to the specification error or noise issue may involve differences in the length of the period. We consider the relation between the specification error measures for short and long measurement periods in a model with a constant discount rate. We will use notation for the stock market, but the point applies to many other types of models where the future enters with

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<sup>3</sup>Cochrane [1991] covers this ground neatly and compactly.

geometrically declining weights. Consider the valuation model for  $T$ -period measurement:

$$p_t = \sum_{\tau=0}^T \beta^\tau E_t d_{t+\tau} + \beta^T E_t p_{t+T} + u_t$$

For 1-period measurement, we have

$$p_t = \beta(E_t d_{t+1} + E_t p_{t+1}) + \tilde{u}_t$$

By iterating the one-period equation, we can establish the relation between  $T$ -period noise  $u_t$  and one-period noise  $\tilde{u}_t$ :

$$u_t = \sum_{\tau=0}^T \beta^\tau E_t \tilde{u}_{t+\tau}$$

Because one-period noise enters the valuation model just like dividends, the rational-expectations relation between the level of the stock price and expected future noise is the same as for future expected dividends.<sup>4</sup>

Our discussion of short- and long-term noise analysis will use a simpler notation that is also more general. Suppose that the construction of the long-term measure uses a sequence of coefficients  $\phi_\tau$ , so that

$$u_t = \sum_{\tau=0}^T \phi_\tau E_t \tilde{u}_{t+\tau}$$

Suppose further that the projection of short-term noise on  $x$  is

$$E(\tilde{u}_t | x_t) = x_t \delta$$

and that  $x$  obeys a VAR:

$$E(x_{t+1} | x_t) = x_t A$$

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<sup>4</sup>See Campbell and Shiller [1987] for a full development of this point.



Then the projection of long-term noise on  $x$  is

$$\begin{aligned} E(u_t | x_t) &= \sum_{\tau=0}^T \phi_{\tau} x_t A^{\tau} \delta \\ &= x_t \Phi(A) \delta \end{aligned}$$

Here  $\Phi$  is the generating function of  $\phi$ :

$$\Phi(z) = \sum_{\tau=0}^T \phi_{\tau} z^{\tau}$$

We conclude that the projection of long-term noise on the observables is

$$E(u_t | x_t) = x_t \gamma$$

where the parameters  $\gamma$  of the projection for long-term noise are related to the parameters  $\delta$  for short-term noise by

$$\gamma = \Phi(A) \delta$$

The relation between short-term and long-term noise is more complicated if the calculation of the statistic  $y_t$  does not use constant coefficients. For example, if the valuation model uses discounts derived from market values of risk-free securities, as in Hall and Hall [1993], the same discounts need to be applied to short-term noise in order to calculate long-term noise.

## Estimation

In the general notation used at the beginning of the paper, the estimation problem is to determine the projection  $R_t(y, x)$  from data on the realized discrepancy  $y$  and the predictor variables  $x$ . Under the assumption that the

projection is linear in  $x$ , the econometric problem is to estimate the coefficient vector  $\gamma$  in

$$y_t = -x_t\gamma + \varepsilon_t - v_t$$

Here  $v_t$  is the part of the specification error not correlated with  $x_t$ . In general, the composite disturbance  $\varepsilon_t - v_t$  will be serially correlated. First, if  $y_t$  is calculated from overlapping time periods, the expectation error  $\varepsilon_t$  will be serially correlated in a known way. Second, we have no general source of prior knowledge about the serial correlation of  $v_t$ . A simple way to proceed is to estimate  $\gamma$  from the ordinary least squares regression of  $y$  on  $-x$ , with standard errors calculated to reflect the serial correlation of the disturbance. These estimates of  $\gamma$  are consistent but not necessarily efficient.<sup>5</sup>

A second approach is to make a correction for serial correlation. For example, if the empirical serial correlation of the composite disturbance is  $\rho$ , one could form autoregressively transformed variables and estimate  $\gamma$  from them. The trick here is to preserve the orthogonality derived from timing. For this purpose, the transformation must be in reverse time:

$$\tilde{y}_t = y_t - \rho y_{t+1}$$

$$\tilde{x}_t = x_t - \rho x_{t+1}$$

The presence of  $x_{t+1}$  on the right side means that  $\tilde{x}_t$  is not orthogonal to  $\varepsilon_t$ . The natural solution is to use  $x_t$  as a set of instrumental variables for estimating  $\gamma$  from

$$y_t = -\tilde{x}_t\gamma + \varepsilon_t - \rho\varepsilon_{t+1} - (v_t - \rho v_{t+1})$$

The third approach is to restate the model for short, non-overlapping measurement periods, make inferences about short-term specification errors, and then calculate the implied time series for long-term specification errors.

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<sup>5</sup>Campbell [1993] shows that, under certain conditions, the inefficient regression gives rise to a more powerful test of the hypothesis  $\gamma=0$  than does the efficient regression. However, there is no reason to believe that his result implies that the inefficient procedure would give superior estimates of the parameters themselves.

Estimation for the short-term model would apply our first suggestion; serial correlation should not be severe unless the unexplained part of the short-term specification error, say  $\tilde{v}_t$ , is highly serially correlated, which seems unlikely. One would estimate the parameters  $\delta$  of the regression of short-term noise on the observables and then construct  $\gamma$  by multiplying by  $\Phi(A)$ , as discussed in the previous section.

In the case where  $y$  is constructed from fixed coefficients and  $x_t$  obeys a VAR, the two approaches just discussed—serial correlation correction and instrumental variables on long-term variables and ordinary least squares on short-term variables with inference of long-term noise—give essentially identical results. The results will be literally identical if the autoregressive transformation is

$$\tilde{y}_t = \Phi^{-1}(F)y_t$$

$$\tilde{x}_t = \Phi^{-1}(F)x_t$$

Here  $F$  is the forward shift operator. This transformation exactly removes serial correlation from the expectation error,  $\varepsilon_t$ . The inverse transformation of  $y$ , and therefore of  $u$ , does not result in short-term noise  $\tilde{u}$ . However the result does have the same projection on the observables:

$$E(\tilde{y}_t | x_t) = E(\tilde{u}_t | x_t)$$

Under our second method, we apply instrumental variables to estimate the coefficients  $\gamma$ . It is convenient to think about the IV estimator as two-stage least squares, where we substitute the fitted value  $E(\Phi^{-1}(F)x_t | x_t)$  for  $\Phi^{-1}(F)x_t$ . It is easy to show that the fitted value is

$$x_t \Phi^{-1}(A)$$

Thus the second method amounts to regressing  $u$  on  $x_t \Phi^{-1}(A)$  to estimate the long-run parameters  $\gamma$  directly. Plainly the resulting estimates satisfy  $\gamma = \Phi(A)\delta$ , so the two methods give the same results.

Although it is never possible to calculate measures for infinite  $T$ , some further light on the choice between the two alternative methods can be shed by looking at that case. For infinite  $T$  and constant discounting at rate  $\beta$ ,  $\Phi(F) = \frac{1}{1-\beta F}$  and  $\Phi^{-1}(F) = 1 - \beta F$ . The choices then are to regress the one-period return on the observables and then build up estimates of long-term noise by forming  $\frac{1}{1-\beta F} x_t \delta$ , or to regress one-period returns on  $x_t(I - \beta A)$  and then form estimates of long-term noise as  $x_t \gamma$ . Here the second method seems simpler.

For finite  $T$ , the first method looks better. It is relatively easy to deal with  $\Phi(A) = I + A + \dots + A^{T-1}$  and difficult to manage its inverse, which has an infinite number of terms. However, for larger values of  $T$ , a simple backward first-order autoregressive transformation will come very close to dealing fully with serial correlation, so that estimation by two-stage least squares on the transformed data is easy and yields an estimate of  $\gamma$  which can then be applied to  $x$  to get very nearly efficient estimates of long-term noise.

## Illustrative application to the stock market

In this application,  $p_t$  is the S&P 500 index, measured monthly, and  $p_t^*$  is the realized present discounted value of dividends as of month  $t$  for an investor purchasing in month  $t$  who sells in December 1992. Both are stated in 1987 dollars as defined by the implicit deflator for GDP. In the first version, a dollar of dividends in month  $t + \tau$  is discounted to month  $t$  by  $\beta^\tau f_t / f_{t+\tau}$ , where  $f_t$  is the deflator. The monthly discount factor  $\beta$  is 0.99486, or 6.2 percent per year. Figure 1 shows the actual price and the present discounted value of dividends.

The simple noise measurement regression is

$$p_t^* - p_t = 1.29 - .61p_t \quad (0.35)$$

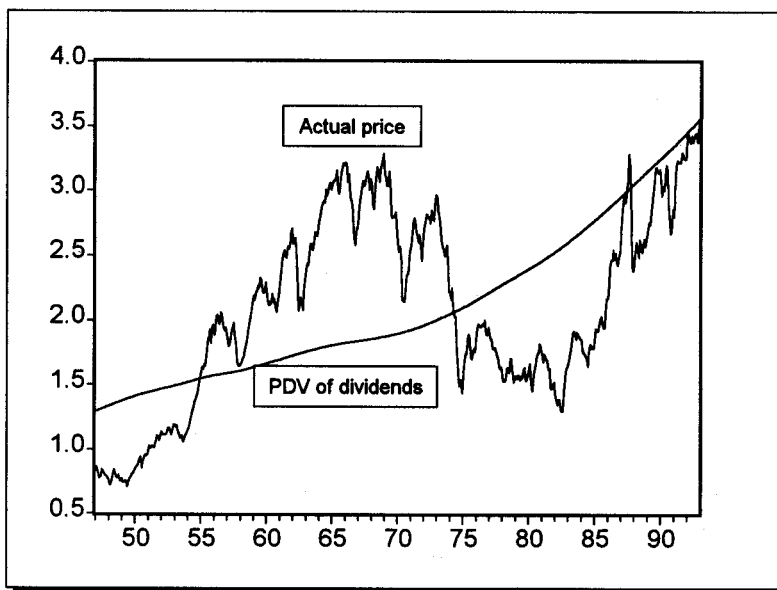


Figure 1. Actual S&P 500 index, deflated by the GDP deflator, and present discounted value of dividends, with constant real discount rate of 6.2 percent per year.

The standard error of the slope coefficient is calculated on the assumption that the composite disturbance,  $\varepsilon_t - v_t$ , is AR(1) with serial correlation measured as the sample value from the residuals, .9985. The covariance matrix is estimated as  $(X'X)^{-1}X'\Omega X(X'X)^{-1}$ , where  $X$  is the matrix of data on the right-hand side of the regression and  $\Omega$  is the covariance matrix of the AR(1) process.

The breakdown of variance from this regression is

Total variance of $p^* - p$	0.48
Measured noise variance (lower bound)	0.20
Expectation error variance (upper bound)	0.28

Noise is at least 40 percent of the total variance of the discrepancy between current price and realized value; expectation errors about dividends are no more

than about 60 percent. Figure 2 shows the time series behavior of fitted noise. In this simple specification, fitted noise is proportional to the current stock price.

The alternative approach is to measure noise over short periods and infer long-term noise. In the constant-real-discount framework, the monthly excess return is

$$w_t = d_t + \beta p_{t+1} - p_t$$

The regression of  $w_t$  on  $p_t$  is

$$w_t = 0.0175 - .0079p_t$$

(.0043)

Figure 3 shows the cumulation of noise as measured from this regression. The fitted specification error is fairly similar to what is shown in the level regression in Figure 2.

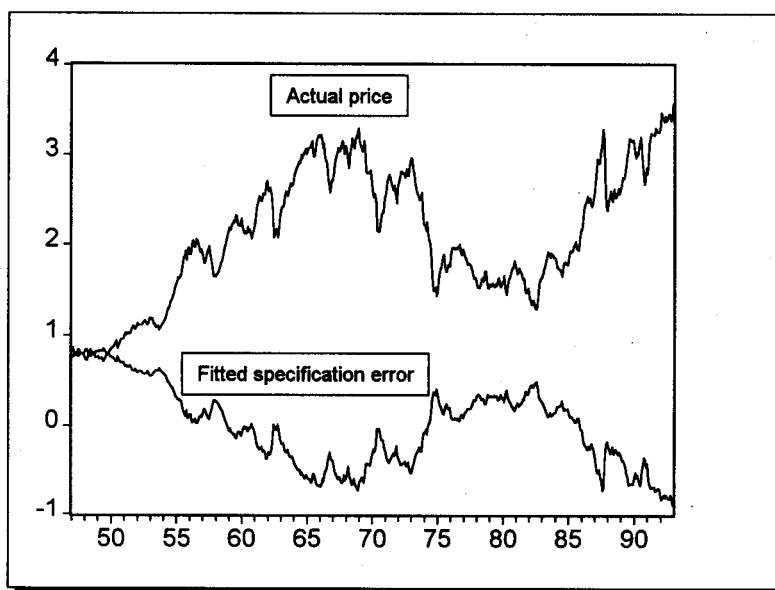


Figure 2. Actual S&P 500 deflated price and specification error from level regression.

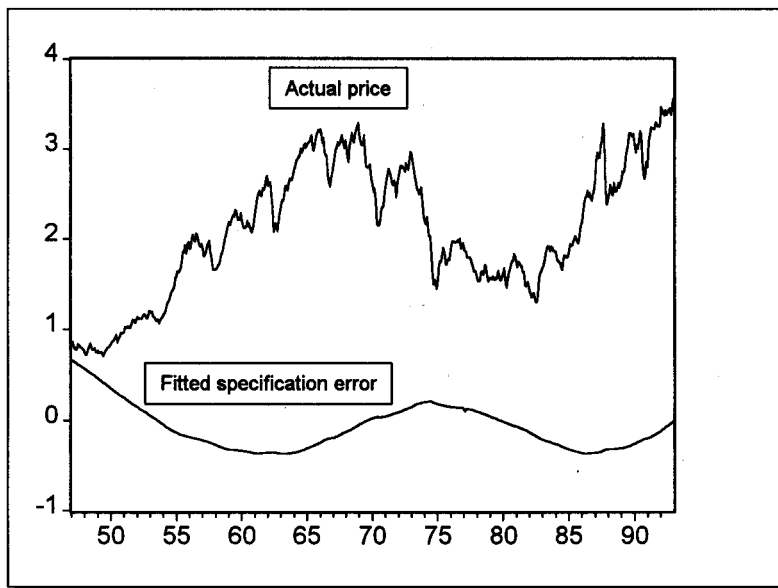


Figure 3. Actual deflated S&P 500 index and fitted specification error in level form from monthly return regression.

The breakdown of variance is:

Total variance of $p^* - p$	0.48
Measured noise variance (lower bound)	0.06
Expectation error variance (upper bound)	0.44

The two components do not sum to the total variance because they have a slight negative correlation.

Although the evidence is not statistically completely unambiguous, the constant-real-discount model has evidence of important specification error. An obvious source is the assumption of a constant real discount rate. An alternative

model holds that the discount applied in month  $t$  to expected future nominal dividends in month  $t + \tau$  is

$$\left( \frac{1}{1+r_t+\delta} \right)^\tau$$

where  $r_t$  is a risk-free nominal interest rate and  $\delta$  is a constant risk premium. We use the 3-year constant-maturity Treasury note rate. This procedure gives an excellent approximation to the use of the full term structure of forward interest rates, because the yield curve rarely slopes enough to make much difference in the present discounted value of dividends. Figure 4 shows the resulting series for discounted dividends, deflated by the GDP implicit deflator.

There is a huge valuation puzzle in the late 1940s and 1950s, which disappears around 1960. During this period, the nominal interest rate was exceptionally low relative to the dividend yield of the S&P 500. After 1960, the actual value of the S&P tracks discounted dividends reasonably well, and far better than does the constant-real-discount model of Figure 1. These visual impressions are confirmed by noise projections in levels for the entire period and for the period starting in 1960:

$$1947 - 92 : p_t^* - p_t = 2.47 - .96p_t \quad (.34)$$

$$1960 - 92 : p_t^* - p_t = 0.52 - .23p_t \quad (.23)$$



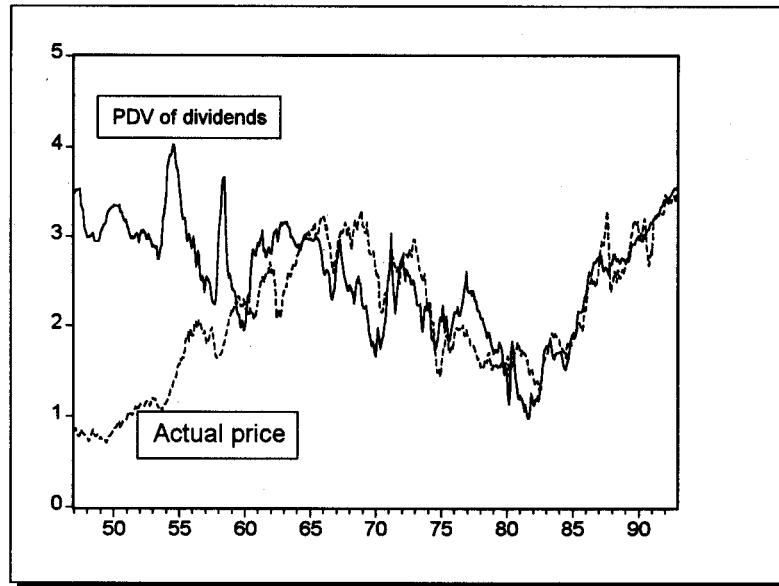


Figure 4. Actual S&P 500 deflated price and present discounted value of dividends, with fixed premium of 4.7 percent per year over 3-year Treasury note rate.

The variance decompositions are:

	1947- 92	1960- 92
Total variance of $p^* - p$	0.88	0.16
Measured noise variance (lower bound)	0.50	0.02
Expectation error variance (upper bound)	0.38	0.14

The one-month return in this framework is

$$w_t = d_t + \frac{1}{1+r_t+\delta} p_{t+1} - p_t$$

Projections of the return on the level of the price for the two periods are:

$$1947 - 92 : w_t = 0.019 - .0091p_t$$

(.0042)

$$1960 - 92 : w_t = 0.017 - .0088p_t$$

(.0070)

and the variance decompositions in levels are:

	1947- 92	1960- 92
Total variance of $p^* - p$	0.88	0.16
Measured noise variance (lower bound)	0.08	0.05
Expectation error variance (upper bound)	0.74	0.23

Figure 5 shows the modest amount of long-term noise implied by the regression for 1960-92.

Further reduction of specification error needs to concentrate on the immediate postwar period, when the stock market conspicuously failed to appreciate the opportunities available from borrowing at very low interest rates and investing the proceeds in a high-yielding stock market.

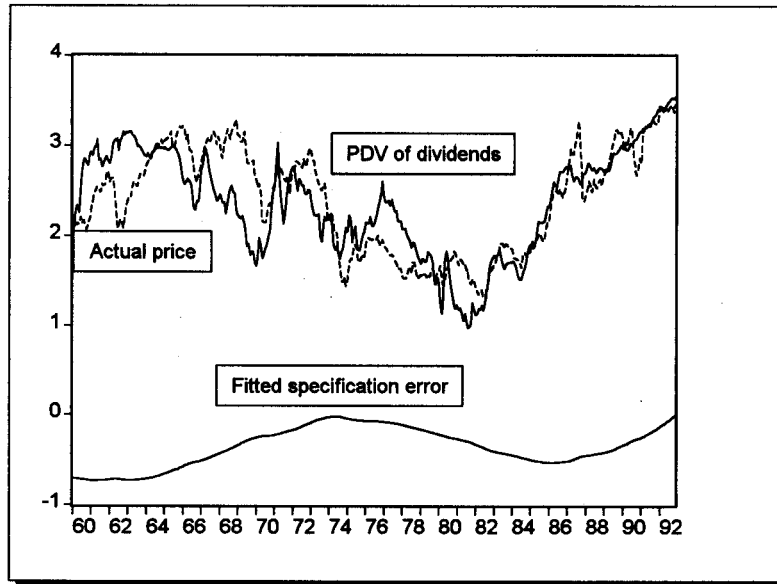


Figure 5. Actual deflated price of the S&P 500, present discounted value of dividends, and fitted specification error cumulated from return regression, 1960-92.

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