The Long Slump

Robert E. Hall*
Hoover Institution and Department of Economics,
Stanford University
National Bureau of Economic Research
rehall@stanford.edu; website: Google "Bob Hall"

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Abstract

In a market-clearing economy, declines in demand from one sector do not cause large declines in aggregate output because other sectors expand. The key price mediating the response is the interest rate. A decline in the rate stimulates all categories of spending. But in a low-inflation economy, the room for a decline in the rate is small, because of the notorious lower limit of zero on the nominal interest rate. In the Great Depression, substantial deflation caused the real interest rate to reach high levels. In the Great Slump that began at the end of 2007, low inflation resulted in an only slightly negative real rate when full employment called for a much lower real rate because of declines in demand. Fortunately the inflation rate hardly responded to conditions in product and labor markets, else deflation might have occurred, with an even higher real interest rate. I concentrate on three closely related sources of declines in demand: the buildup of excess stocks of housing and consumer durables, the corresponding expansion of consumer debt that financed the buildup, and financial frictions that resulted from the decline in real-estate prices.

*This paper is a draft of the version of my AEA presidential address that will appear in the April *American Economic Review*. See my website for related materials, including the animated slides and many data files and programs. I am grateful to John Cochrane, Steven Davis, Gauti Eggertsson, Martin Eichenbaum, Mark Gertler, Robert Gordon, Peter Klenow, Narayana Kocherlakota, Robert Lucas, N. Gregory Mankiw, Sergio Rebelo, Richard Rogerson, David Romer, Robert Shimer, Robert Solow, John Taylor, Susan Woodward, and seminar and conference participants at the University of Pennsylvania, the Society for Economic Dynamics, the Atlanta Federal Reserve Bank, Stanford, and the Minneapolis Federal Reserve Bank for comments.

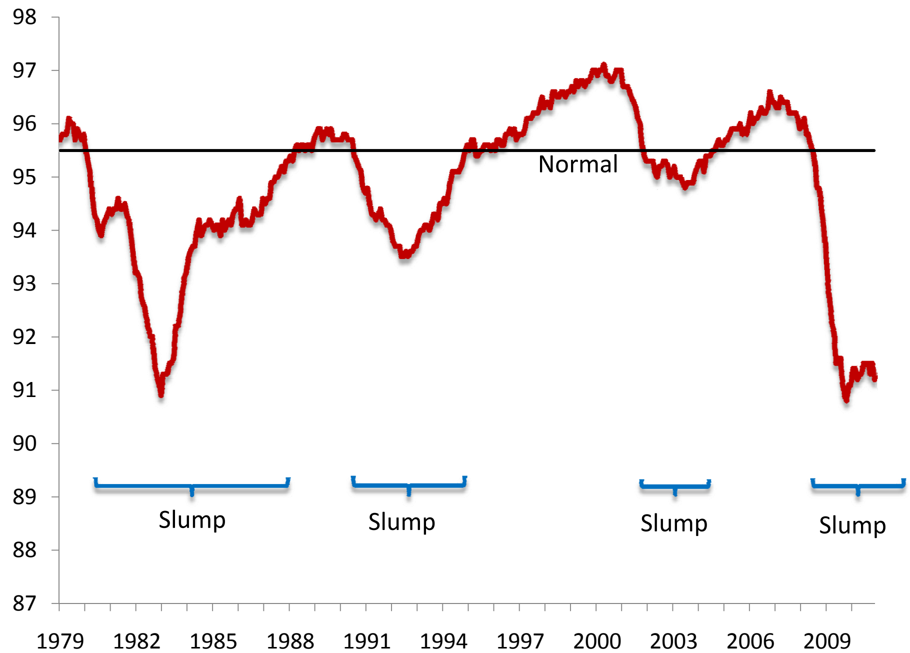

Figure 1: U.S. Employment Rate for Workers Aged 25 through 54, 1980 through 2010

1 Introduction

Slumps—extended periods of low resource utilization—are an enduring part of life in modern advanced economies. At the beginning of 2011, the U.S. and many other economies found themselves in slumps. The worst slump in U.S. history was the Great Depression, in which the economy contracted from 1929 to 1933 and failed to return to normal until the buildup for World War II. Figure 1 shows the employed fraction of the labor force aged 25 through 54 since the beginning of 1979 (the remaining fraction of the labor force is unemployed). Slumps are identified as periods when this measure of employment was less than its normal level of 95.5 percent of the labor force.

A slump begins with a contraction, often fairly brief, at least in comparison to the extended period of slow growth that follows the contraction. Relative to the vocabulary of peaks and troughs, a slump lasts from the time when employment falls below its normal level during the contraction to the time when employment regains its normal level during an expansion. Thus a slump spans the trough date. Usually most of the slump occurs after the trough, during the period of low but positive growth. Everybody but business-cycle specialists uses the term "recession" to describe a slump.

The more serious slumps in U.S. economic history have followed financial crises. In the case of the slump that began at the end of 2007 but became severe after the crisis of September 2008—the Great Slump—the origin in the financial events of the earlier years of the decade seems obvious. Years of stable and rising house prices made levered positions in real-estate-related assets seem quite safe. Regulators permitted increases in leverage, especially for investment banks and other financial entities thought to be free from government guarantees and the accompanying need for government supervision. The Securities and Exchange Commission eliminated capital requirements for investment banks in 2004 and their leverage rose. Risk analysis and loan underwriting used probability distributions that assigned zero probability to significant declines in housing prices. Credit became available to households who were denied access under earlier standards. The result was a bulge in homebuilding and sales of cars and other consumer durables, together with a corresponding bulge in consumer debt.

The wholly unexpected decline in housing prices brought the financial crisis. Commercial banks required help from the government that they had reason to expect. Events showed that the precarious conditions of other large financial entities—investment banks, insurance companies, and money-market funds—threatened the stability of the entire financial system, so they too received government assistance. The government's hands-off treatment of Lehman Brothers appeared to demonstrate the vulnerability of the financial system to the failure of an entity previously thought to be outside the class requiring protection and supervision. The crisis disabled the financial system in some dramatic and transitory ways, with low valuations of many types of financial claims and high valuations of others, notably claims on the U.S. government. These valuation effects soon stabilized, but financial intermediaries were left thinly capitalized. They responded with tightened lending standards and higher interest charges to borrowers relative to their own borrowing costs.

My topic is the aftermath of the crisis. Three adverse forces gripped the economy in the aftermath: (1) the overhang of housing and consumer durables resulting from the building and buying frenzy of the decade of the 2000s, (2) high consumer commitments to debt service, and (3) financial frictions from the crisis. These adverse forces were so destructive because the economy was unable to lower its interest rate to stimulate other kinds of spending to replace house construction and other affected components of spending. The result was the long and deep slump. Unemployment is a leading symptom of the poor performance of the economy. It lingered near 10 percent three years into the slump.

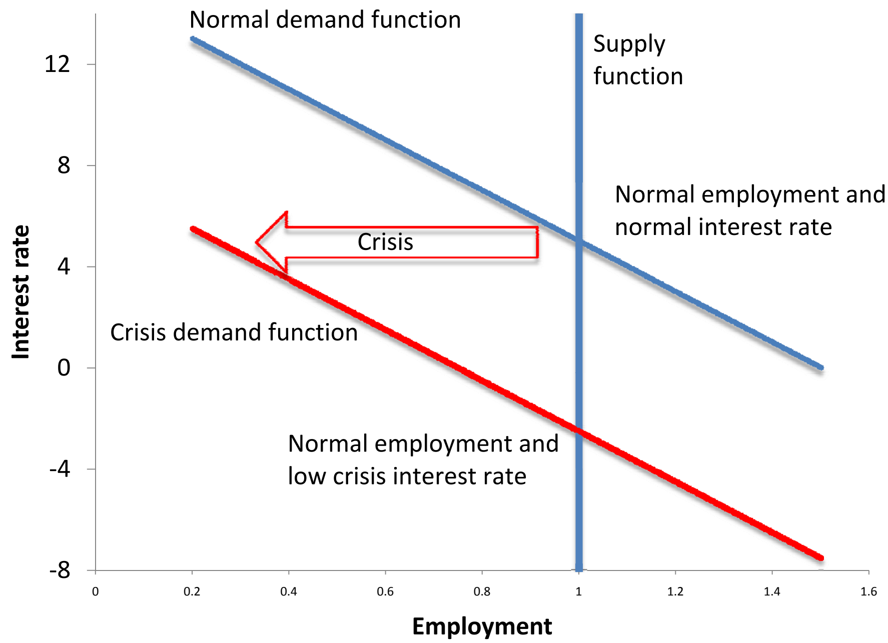

Figure 2: A Negative Interest Rate Preserves Full Employment

My discussion invokes a sequence of models to explain the central point about the failure of the interest rate to preserve full employment. Figure 2 is the starting point of the most stripped-down model. In this version, the real interest rate is completely flexible and can clear the labor market. The supply of employment is inelastic, a vertical line, and the demand for employment slopes downward. A high interest rate results in deferral of investment and consumption, and employment is correspondingly low. The crisis, operating through the factors I just listed, shifts the demand curve to the left. The new intersection of supply and demand occurs at the same level, full employment, but a much lower interest rate. Here, the interest rate has done its job to preserve full employment. For example, the collapse of house construction is offset by higher business investment and higher consumption of nondurables. Note that the story leaves the rest of the world out of the picture for simplicity.

The real interest rate did not fall to -2.5 percent in the Great Slump or in earlier slumps. Despite huge efforts in the Great Slump, the Fed could do no more than drive short term rates to zero and try to get longer-term rates down through non-standard policies. I'll deal only with the short rate, again for simplicity.

The lowest possible nominal interest rate is zero, so the lowest possible real interest rate is minus the rate of inflation. In the worst case—the Great Depression—the price level fell for several consecutive year and the real rate was pinned at a high positive level. Figure 3

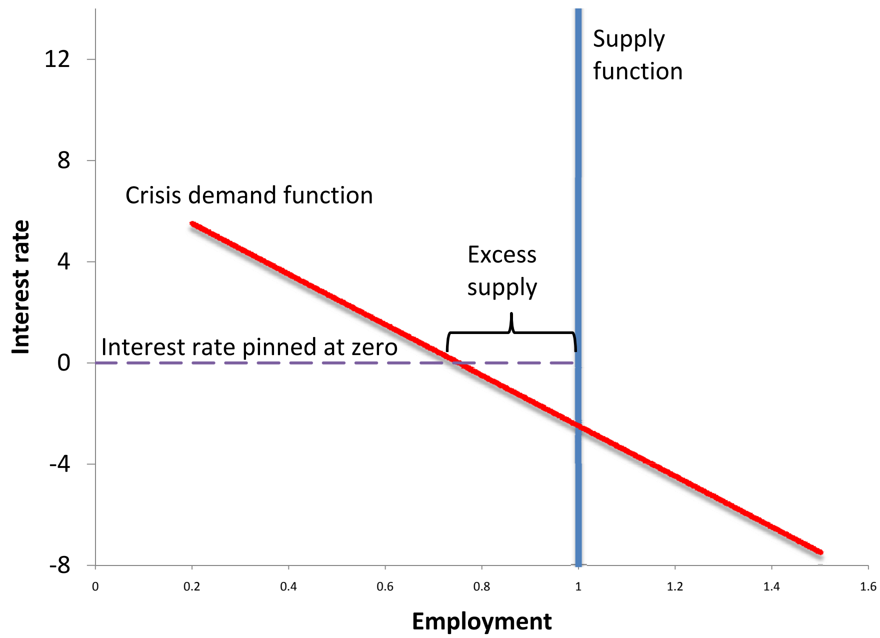

Figure 3: Excess Supply of Labor Resulting from a Real Interest Rate Pinned above Equilibrium

shows the simple implication of a real interest rate pinned at zero instead of falling to minus 2.5 percent. At the rate of zero, there is excess supply of labor. That excess supply translates into unemployment. Much of the analysis here involves building models that describe that translation.

I will proceed by documenting the conditions in the U.S. economy after the crisis, at the beginning of the Great Slump. Then I will discuss the effects in general equilibrium of an excessive real interest rate, first in a two-period setting and then in an infinite-horizon generalization. Finally, I will launch the infinite-horizon model from the initial conditions immediately post-crisis to demonstrate the large and persistent shortfall of economic activity that occurs in a model with a real interest rate pinned above its full-employment value when it starts with those initial conditions. I compare that outcome to the equilibrium in a counterfactual economy with a fully flexible real interest rate, where the effects are a low initial value of the real interest rate and mild changes in the composition of output but no decline of output below its full-employment value. I conclude with some observations on policies to fight the long slump.

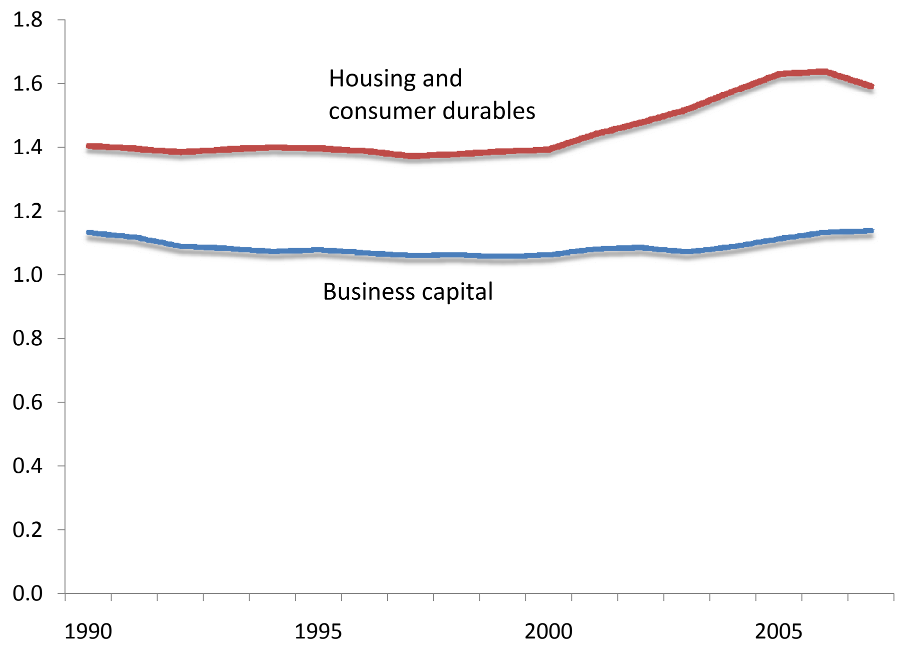

Figure 4: Ratios of Capital and Durables to GDP

2 The Overhang of Housing and Consumer Durables

Figure 4 shows the ratios of housing and consumer durables, on the one hand, and business capital (plant and equipment), on the other hand, over the past 20 years. Business capital was virtually constant as a fraction of GDP, and so was housing and durables in the 1990s, but a conspicuous bulge in household capital occurred in the 2000s. The economy reached the crisis with about 14 percent more housing and durables in relation to GDP than normal. Almost all economic models—and the historical performance of the U.S. economy—agree on the principle that what goes above normal will tend to return to normal. Thus the overhang of housing and consumer durables pointed toward lower future spending in these categories, no matter what happened to the economy.

I'll not say anything more about the fascinating topic of the causes of the splurge on houses and cars and the rise and fall in housing prices. I just take the abnormal level of household capital goods as a fact about the economy as of the end of 2008. Others are working on the topic of the house-price bubble. Macroeconomists have built asset-price crashes into general-equilibrium models—Burnside, Eichenbaum and Rebelo (2010) is a recent example where beliefs about appreciation spread among homeowners like an epidemic disease. See also Kocherlakota (2010) and Martin and Ventura (2010). No view about the origins of bubbles or crashes is yet firmly established.

3 Illiquid Households and Debt-service Commitments

A significant fraction of American consumers appear to be at corners in their intertemporal equilibria—they borrow as much as they can and hold almost no liquid assets. I take a family as liquidity-constrained if its holdings of net liquid assets are less than two months of income. Net liquid assets are the difference between holdings in savings accounts and the like and borrowing from credit cards and other unsecured forms. In the 2007 Survey of Consumer Finances, households illiquid by this standard earned 58 percent of all income. The fraction of households that were constrained—74 percent—is even higher because lower-income households are more likely to be constrained. Nonetheless, many quite prosperous families hold essentially no liquid financial assets. Their recourses in times of unexpected income losses include borrowing against unencumbered houses and cars, selling these assets, liquidating retirement accounts, and seeking help from friends and family. Blundell, Pistaferri and Preston (2008) find a rather smaller response of consumption to transitory income than is implied by the SCF data, suggesting that the other recourses may be important.

I incorporate these facts into the model by dividing consumption into two parts. Consumption of unconstrained households obeys the standard life-cycle model, while consumption of constrained households is their earnings less their payments on outstanding credit. The payments are forced saving. To determine the amount of the required payments, I use the following logic: Let D_t be the outstanding debt of constrained households in quarter t. Constrained households always borrow the maximum allowed, so their debt is controlled by the borrowing limits imposed by lenders. The borrowing interest rate is r_D. The sources of funds for constrained households are income $\bar{y}_t$ and increased borrowing, $D_t - D_{t-1}$. Uses of funds are consumption $\bar{c}_t$ and payment of interest on earlier debt, $r_{D,t-1}D_{t-1}$. Thus consumption is

$$\bar{c}_t = \bar{y}_t + D_t - (1 + r_{D,t-1})D_{t-1}. \quad (1)$$

I let

$$s_t = r_{D,t-1}D_{t-1} - \Delta D_t, \quad (2)$$

the net burden on consumption relative to income associated with debt service. The level of debt is high enough that interest payments usually exceed new borrowing, so D_t is usually

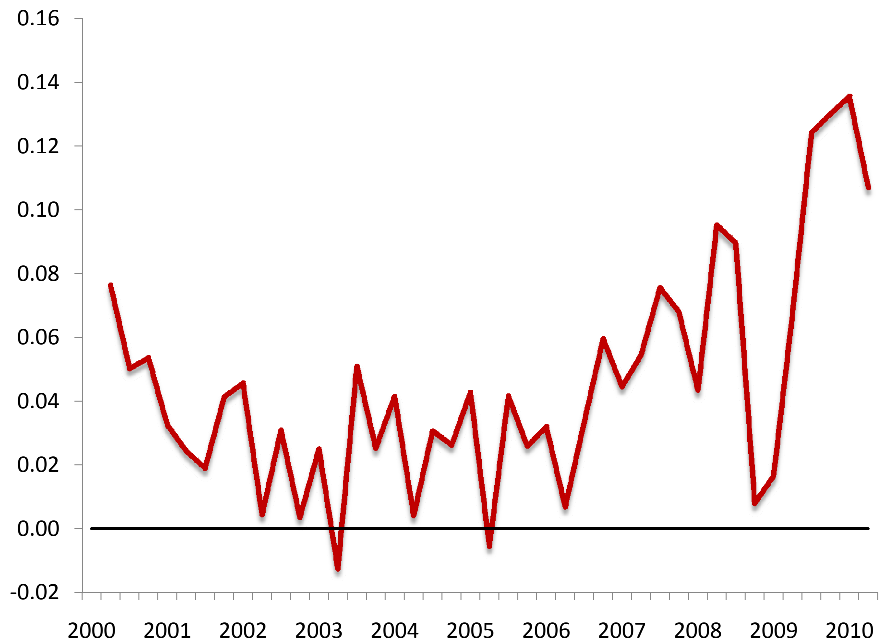

Figure 5: Burden of Debt Service, as a Fraction of GDP

positive. In a stationary setting, $D_t = \bar{D}$, a constant, debt repayments are zero, and household purchases of consumption goods are less than income by the amount of interest they pay, $r_D \bar{D}$.

Figure 5 shows the burden s_t, as a fraction of GDP, calculated from Flow of Funds data on consumer debt and NIPA data on household interest payments. Prior to 2007, the burden was close to zero—new borrowing came close to covering interest payments. Tightening of credit began even before the onset of recession at the end of 2007 and continued to the end of 2010.

In a full-employment economy containing unconstrained and constrained households, the tightening of credit as shown in Figure 5 would not have contributed to a slump. Rather, the interest rate would clear the output market, thanks to the absence of any lower bound on the interest rate. When constrained households cut back consumption spending, including purchases of new houses and consumer durables, low rates would induce unconstrained households to consume more by borrowing, thus offsetting the saving of constrained households.

Mian and Sufi (2010b) demonstrate large differences among states in the U.S. in durables purchases negatively correlated with indebtedness.

4 Financial Frictions

A financial friction drives a wedge between the returns that savers receive and the rates that borrowers pay. Agency problems in financial intermediaries are one source of those frictions. Adverse selection of borrowers is another. The financial crisis resulted in large declines in the equity values of financial intermediaries, which substantially worsened the agency problems. Loss of equity among household borrowers worsened adverse selection.

Hall (2010) describes a framework for studying financial frictions and Hall (2011) gives many details and cites to recent work on this topic. The framework in those papers shares many elements with the discussion here.

A widely studied setup—see Bernanke, Gertler and Gilchrist (1999)— generates frictions from the agency relationship between investors and financial intermediaries. See Gertler and Karadi (2009) for an application to the Great Slump. Investors lack the expertise to invest directly in productive enterprises. They place their funds with intermediaries who will abscond with some fraction of the funds unless their continuation values exceed the value of absconding. In equilibrium, the intermediary charges entrepreneurs more for credit than the amount paid to investors. The present value of the difference is the needed continuation value. The present value arises from spread between the intermediary's lending and borrowing rates and from the intermediary's equity. The setup has been influential in the theory of bank regulation, which holds that the value of a bank's franchise protects depositors and deposit insurers against excessive risk-taking by banks.

When assets held by intermediaries lose value—as from a decline in real-estate prices—the continuation value threatens to drop below the value from absconding. To prevent absconding, the investors accept a widening of the spread. Thus the behavior of credit spreads is central to this theory of variations in financial frictions.

Although the agency friction model has most often been applied to businesses, it is even more apt for consumers. Larger businesses escape the agency friction by sidestepping intermediaries and selling securities directly to providers of capital. Consumers borrow only from intermediaries, with the small exception of borrowing from family and friends.

Following the logic of the agency friction model, I consider a variable interpreted as a wedge between returns earned by savers and the cost of capital to businesses and households. The wedge is equivalent to a property tax on business capital and household durables, including houses. Increases in the wedge are potent sources of lower output and higher unemployment.

As a result of the real-estate crash, many financial intermediaries suffered severe depletion of their equity in long positions in real-estate-related assets. Measuring the resulting increase in frictions is a challenge. While it is easy to measure spreads for traded instruments, those spreads are not infected by the wedge suggested by the agency theory. The spread between BAA corporate bonds and Treasurys of the same maturity widened stunningly in late 2008 but the difference did not reflect profit accruing to any intermediary. Spreads of that type came back to normal way too fast to account for any of the persistence of the slump. These spreads may arise from transitory segmentation of asset markets—sudden loss of equity among one class of investors results in price declines for the assets they sell. Quickly, other classes of investors develop the expertise to recognize the undervaluation of the assets and bid their prices back to the normal level relative to safe Treasurys.

Intermediation spreads are potentially instructive about frictions. One important challenge is to distinguish the part of a spread that arises from the probability of default from the part that is a true friction. The ideal measure of the spread for bank loans would be the lending rate for new loans, less the best forecast of risk-adjusted default losses and less the bank's borrowing rate. The difficulty is determining the forward-looking expected loss from default. Under the assumption that banks stabilize expected default losses for new loans, so that the default component is a constant, it is informative to study the total spread between lending and borrowing rates. Figure 6 shows the spread based on data from the Federal Reserve Board for business loans of $1 million or more. The spread is the loan rate reported by banks (not necessarily limited to new loans) less the federal funds rate as a measure of the borrowing rate. The spread rose by almost two percentage points and remained persistently high. I conclude that the evidence mildly supports the view that spreads of the type associated with frictions rose and remained high during the slump. I note that the spread remains persistently high at the same time that spreads for traded securities have returned to normal.

Figure 6 shows one measure of friction with respect to business investment, the spread between the rates that businesses pay and the rate that banks pay to fund loans, the fed funds rate. It widened dramatically after the crisis and was still widening even in 2010.

Figure 7 shows the spreads between the rate reported by the Federal Reserve Board for credit-card borrowing and the federal funds rate. There is just a hint of a decline at the end,

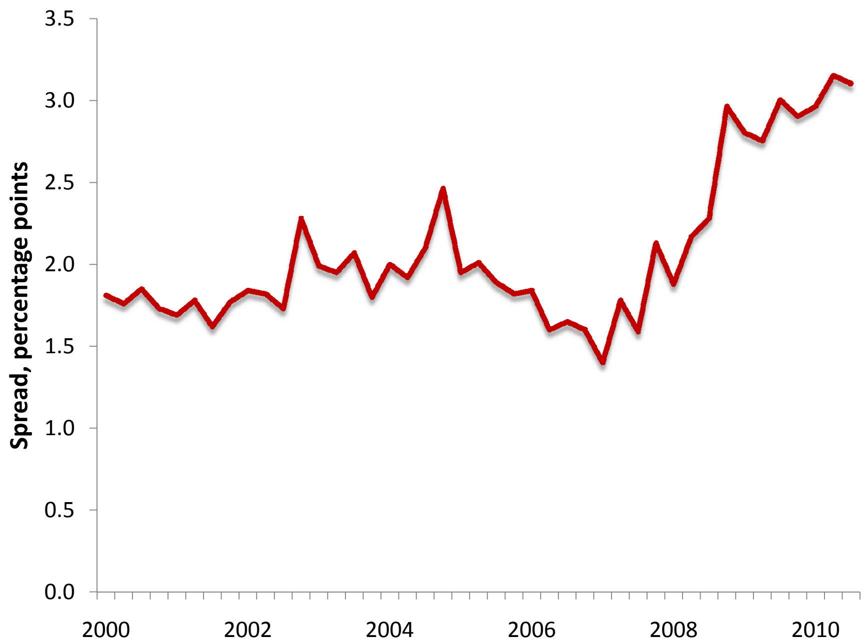

Figure 6: Spread, in Percentage Points, between Business Loan Rates and Banks' Borrowing Rate

but the spread definitely remained high for most of the slump to date.

The most important component of household credit is residential mortgages. Figure 8 shows the spread between the rate reported by the Federal Reserve Board for conventional (30-year fixed rate) mortgages and the yield on 10-year Treasury notes, a reasonable match to the actual duration of mortgage debt. The spread rose early in the slump until the Federal Reserve intervened and restored the spread to roughly normal levels. Mortgage underwriting practices have changed dramatically during the slump to try to limit losses on new mortgages.

My discussion of intermediation spreads has focused on the agency model, where spreads measure distortions but not actual losses of resources. Agency frictions have much the same effect as taxes—they create wedges and resulting inefficiencies, but do not consume output.

A different type of friction occurs when a borrower is unable to perform on a debt obligation. Another branch of the literature on financial frictions, starting from Townsend (1979), observes that debt contracts have the property that the lender need not consume resources monitoring the borrower unless the borrower fails to make good on the simple promise to repay the loan, an act that is costless to verify. When the borrower is unable to repay, the lender incurs substantial costs to recover value through a workout or bankruptcy. When borrowers suffer losses in asset values, the probability of default rises and total spreads rise.

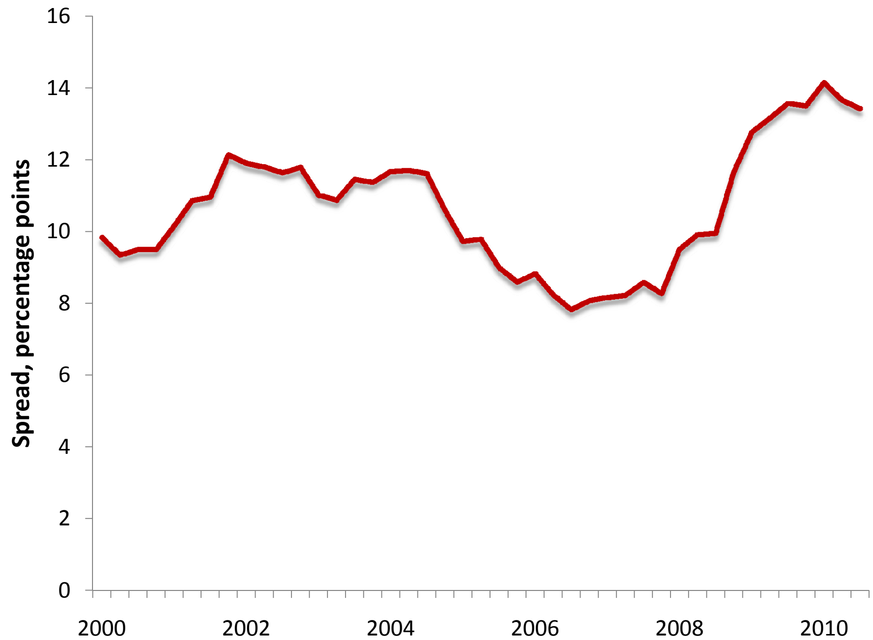

Figure 7: Spread, in Percentage Points, between Credit-Card Rates and Banks' Borrowing Rate

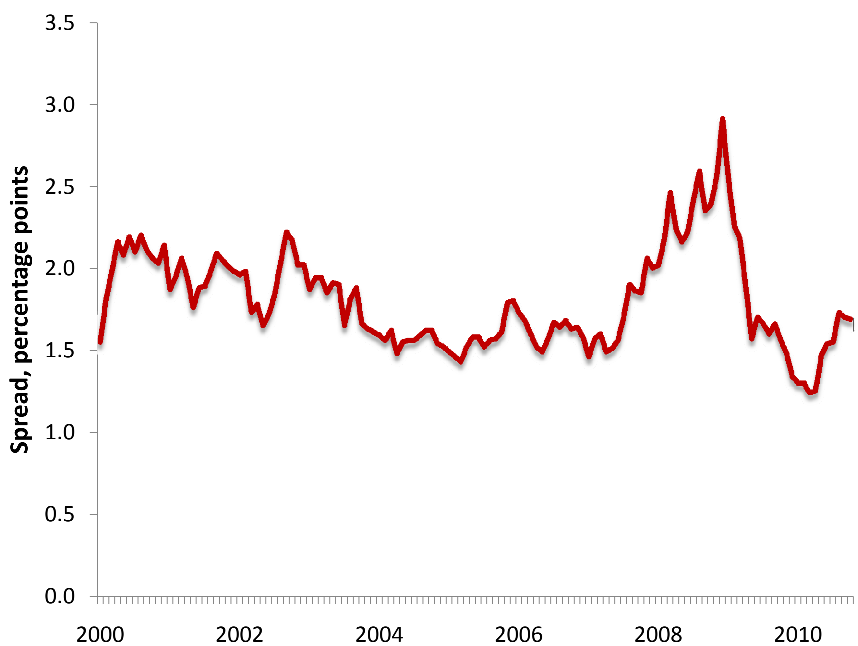

Figure 8: Spread, in Percentage Points, between Mortgage Rates and 10-year Treasurys

The friction is less than the spread, because one consequence of a default is a simple transfer of value from the lender to the borrower. The friction is the amount of the loss accruing to the borrower and lender jointly—it comprises bankruptcy costs, business interruption costs, and the like. But the friction is likely to move in proportion to observed spreads, so a widening of the total spread will usually indicate an increase in the financial friction.

4.1 Credit rationing and lending standards

Lenders always ration credit. They need to overcome substantial adverse selection problems. They set standards for borrower eligibility and spend resources verifying that borrowers meet the standards. One of the consequences of a slump, especially one with an initial financial crisis, is a tightening of lending standards for both businesses and households. Adverse selection becomes a more serious danger when more borrowers are close to the margin of failure.

Lending standards are increasingly based on credit scores and other metrics, but I am not aware of any systematic compilation of quantitative standards into an overall index. The Federal Reserve Board carries out a quarterly survey of senior loan officers of banks with respect to lending standards for a variety of types of loans. An example of a question in the survey is "Over the past three months, how have your bank's credit standards for approving applications for credit cards from individuals or households changed?" The permissible answers are (1) Tightened considerably, (2) Tightened somewhat (3) Remained basically unchanged (4) Eased somewhat, and (5) Eased considerably. Although the answers are qualitative, it appears possible to create an index of standards from the answers.

To this end, let x_t be an index of lending standards, interpreted as the mean across banks, where the change in the bank's own index is normal with mean $\Delta x_t + \mu$ and unit standard deviation. If a bank's own index change is in the interval $[-\nu, \nu]$, it reports that its standards "remained basically unchanged." The Federal Reserve reports the difference in the fraction of banks that reported a tightening of standards and the fraction that reported a loosening—this is called the net change. It is

$$\text{Net change } = \Phi(\Delta x_t + \mu - \nu) - \Phi(-\Delta x_t - \mu - \nu), \tag{3}$$

where Φ is the standard cumulative normal distribution (see the Appendix for details). To estimate the parameter ν, I observe that when $\Delta x_t + \mu = 0$, that is, when standards are not changing, the probability of the middle answer is the probability for $[-\nu, \nu]$, which is

$2\Phi(\nu) - 1$. For October 2010, a time of small net change, the Federal Reserve reports these probabilities, which are around 0.85. The corresponding value of ν is around 1.4. I calculate the time series for $\Delta x_t + \mu$ by solving equation (3) and estimate μ as the mean of the series, in the cases of business loans and credit cards. The identifying assumption is that Δx_t has mean zero. For mortgages, where data are available only for the crisis period, I use a mean of 0.15. My assumption that the cross-sectional standard deviation across banks is 1.0 amounts to a normalization of the units of the index.

Figure 9 shows the resulting indexes, the cumulations of the estimated changes. Because the indexes are constructed to start and end at zero, nothing should be read into the lack of trend, though it remains a reasonable assumption. Further, because each of the three indexes starts arbitrarily at zero, but in different years, the relative values of the indexes have no significance. Standards for business loans track the business cycle almost perfectly. Each recession saw tightening up to a peak that occurs around or a little after the trough of the cycle. Then standards began to ease, gradually during the 1990s and more rapidly in the 2000s. For credit cards, special factors not relevant for current purposes caused a tightening during the expansion of the 1990s. Starting from the peak in standards in 2003, credit card standards behaved similarly to business lending standards. The Federal Reserve added a question about mortgage lending standards in 2007, around the time of the low point in the other two indexes. The index of mortgage standards rose dramatically and has declined only slightly below its peak value.

Figure 10 provides further confirmation of a substantial and persistent increase in household rationing from the financial crisis. It shows an index from Google Insights of queries for the term "withdrawal penalty." The logic is that households suffering from income interruptions, who might normally have borrowing opportunities or other ways to deal with the need to fund continuing normal consumption, turn to expensive alternatives such as 401K withdrawals that are subject to a penalty tax. The index measures, in arbitrary units, the share of all search queries on Google for this term. The index jumped up in late 2008 and has remained high ever since.

Rationing through the application of lending standards enters the model in the same way as agency frictions. Borrowers behave as if credit were more costly than the interest rate they pay for the amounts they are actually allowed to borrow. The data in Figure 6 through Figure 10 suggest that agency frictions, default costs, and rationing all worsened during the

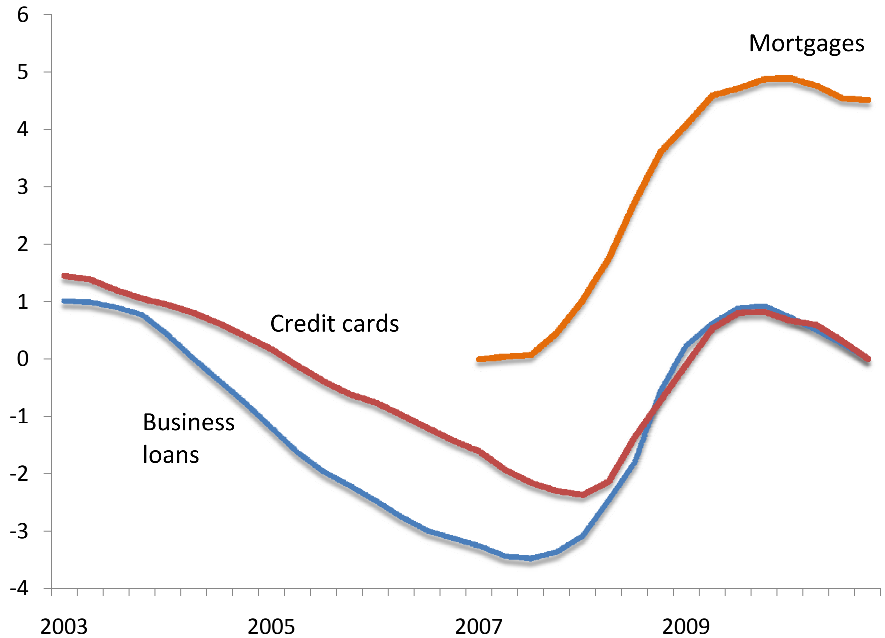

Figure 9: Indexes of Lending Standards Inferred from the FRB Senior Loan Officer Survey

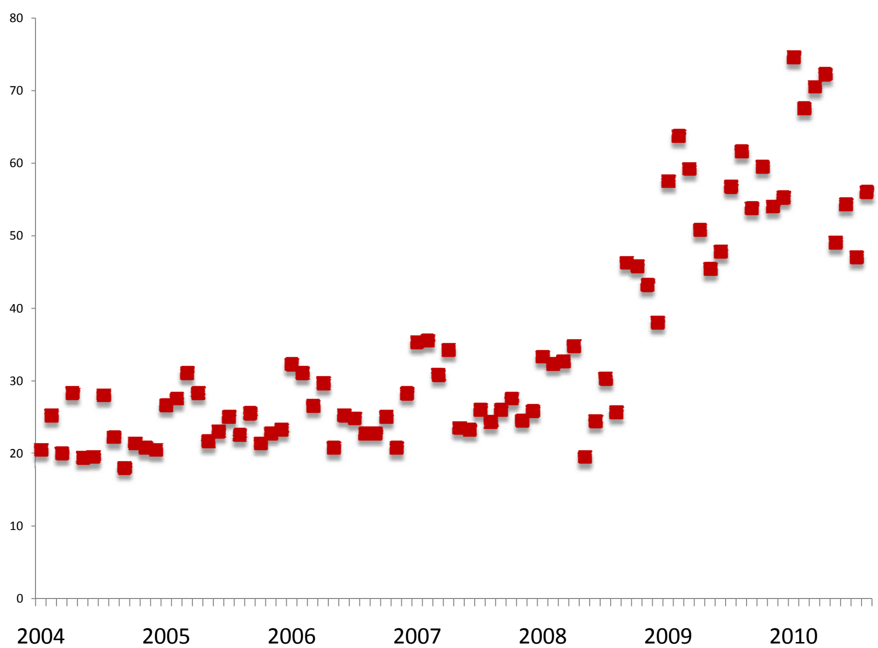

Figure 10: Share of Google Search Queries for the Term “withdrawal penalty”

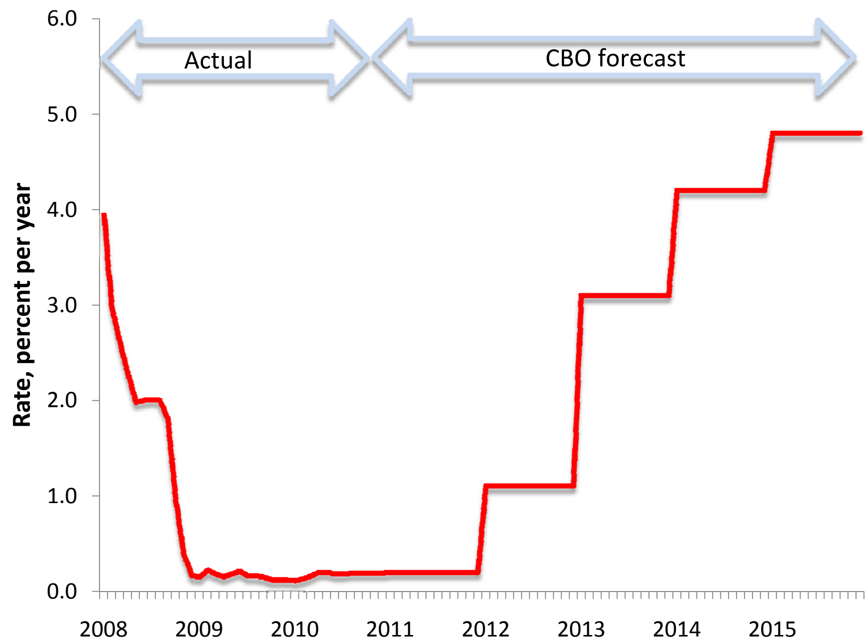

Figure 11: A Long Period with the Nominal Short Rate Pinned at Zero

crisis and have remained at levels close to their peaks during the crisis. I combine all of these factors into the wedge discussed earlier. Although the evidence seems strong that the wedge increased and has remained high, I have not found a basis for quantifying its rate. Rather, I will demonstrate that rates in a reasonable range have powerful negative effects on output and employment.

5 Nominal Interest Rate Pinned at Zero

Currency is a safe asset paying zero return. Currency becomes financially attractive if other safe assets pay negative returns. The Fed will always pay out currency in exchange for reserves. If the market return for a bond fell below zero, the owner could sell it, convert the proceeds to currency, and earn a safe higher return. Thus market prices of bonds would fall so that their returns rose to zero. Consequently, as long as the Fed will give currency in exchange for reserves dollar for dollar, the interest rate cannot be negative.

Figure 11 shows that the Fed pushed the nominal interest rate to almost zero immediately after the crisis and is expected to keep it there for a total of three full years at zero.

The term *real interest rate* is generic for a concept more precisely identified as an *own rate*. The latter is a rate measured in the physical units of some product. The own rates that matter in this analysis are those for consumption goods and services, for durables including houses, and for business capital. In the model I denote the own rate on output as r, which I call the real rate, and introduce the prices of other products relative to output as appropriate to generate the corresponding own return. I denote the safe nominal rate as r_n but for most of the analysis take $r_n = 0$. The real rate is the return measured in output units available from a one-period investment at the safe nominal rate:

$$r_t = (1 + r_{n,t}) \frac{p_t}{p_{t+1}} - 1, \tag{4}$$

where p is the dollar price of output.

Other real returns follow the same rule, all applying the same rate of inflation. I neglect any changes in risk, so when I assume that the safe short-term real return is roughly constant, I am assuming that returns for all assets of whatever risk are similarly constant. Although there is good evidence against the hypothesis of constant risk premiums, especially during crises, I do not believe that there is evidence of any significant difference between risk premiums in prolonged slumps and in normal times. In particular, slumps are not times of high volatility in asset markets or elsewhere in the economy.

The real rate is a basic price that clears the current labor and output markets. If the nominal rate is pinned at zero ($r_n = 0$), the real rate is minus the rate of inflation:

$$r_t = -\frac{p_{t+1} - p_t}{p_t} \tag{5}$$

6 Near-Exogeneity of the Rate of Inflation

The hypothesis of a given, unresponsive rate of inflation achieves a crucial simplification of macro modeling. The hypothesis only makes sense in an economy that had adapted to stable low inflation for many years. And it's only an approximation.

The U.S. entered the slump with a history of low and stable inflation. Cost-of-living increases were concentrated in a narrow band around two percent per year since the mid-1990s and were not much higher from the mid-1980s to 1990. An immediate issue following the sharp contraction in the last quarter of 2008 was whether inflation would fall and even turn into deflation as a result of the extreme slack that developed quickly. The answer, luckily, was no.

The Great Slump brought slacker product markets to the U.S. economy than had existed at any time since the depression in the 1930s. A line of thought rather deeply embedded in macroeconomics holds that product prices fall in slack markets. The logic is that sellers have much to gain by increasing output when output is low. On the reasonable assumption that marginal-cost curves slope upward, a contraction in output will cause a price-setting firm, irrespective of its market power, to cut its price in an attempt to take business away from its rivals.

Recent experience requires a fundamental reconsideration of the view that producers find it desirable to expand output by cutting prices. Their behavior across all industries suggests, to the contrary, that price-cutting is not the answer to any problem they perceive in a time of extreme slack.

Monthly inflation rates contain a good deal of noise from components of the price index with volatile prices, notably petroleum products and food. Practitioners have come up with a variety of ways of extracting a less noisy inflation signal from the monthly data. One approach is time aggregation—using annual or other multi-month changes. Another, currently the most widely used, is to study core inflation, price changes excluding the volatile food and energy components. The third—the one I favor—is to use inflation forecasts. The volatile components lose their unpredictable noise components but are not completely neglected in this approach. For the present purpose, forecasts seem the desirable approach, because it is expected inflation that matters for the real rate.

Figure 12 shows the one-year-ahead forecast of the GDP deflator from the Survey of Professional Forecasters, maintained by the Philadelphia Federal Reserve Bank, along with the unemployment rate. The period covered starts in 1987, the year that Alan Greenspan took command of the Federal Reserve. It contains three contractions marked by rapid increases in unemployment. Inflation fell in all three, but separating the response to slack from other determinants is a challenge. The decline in the inflation rate was greatest in the slump of 1991 through 1994, but the decline continued at about the same rate after the slump turned into a remarkable boom. Inflation did not flatten until unemployment reached 4.3 percent in 1999. Inflation was close to flat in the slump from 2001 through 2004, with just a hint of decline during the period when unemployment was rising. Finally, in the current slump, inflation took a discontinuous drop of about one percentage point early in the contraction, when unemployment was still fairly low, then stabilized at just over one percent per year when unemployment skyrocketed to the 9.5 percent level. Despite concerns that continuing extreme slack might result in further declines toward deflation, the rate of

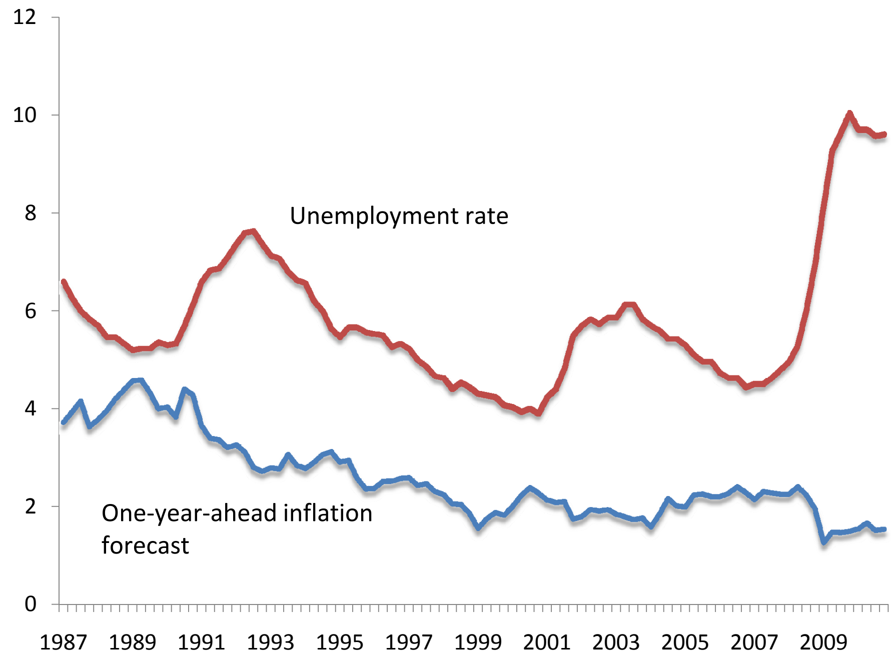

Figure 12: One-Year-ahead Inflation Forecast and Unemployment Rate, 1987 through 2010

inflation has remained remarkably stable at around one percent during the recent stable period.

The concept of the non-accelerating-inflation rate of unemployment or NAIRU has had a firm grip since Friedman (1968) formulated the concept, though he called it the natural rate. The idea is that there is a critical unemployment rate such that inflation will become greater and greater if unemployment is below the rate. Today, the relevant version is the non-decelerating-inflation rate of unemployment. By the theory underlying the concept, the rate of change of prices should fall more and more if the unemployment rate is above the critical rate. Generally the NAIRU is found to be around 6 percent. By this influential body of thought, month after month of unemployment over 6 percent should bring more and more deflation. Fortunately, the theory is wrong.

It is not news that NAIRU theory is a failure. Stock and Watson (2010) report that the best way to characterize the relation between inflation and unemployment is to measure downward pressure on inflation as the difference between the current unemployment rate and the lowest rate experienced in the previous 11 quarters. Thus, once a slump has lasted 11 quarters at, say, the same rate, no matter how high, unemployment loses its deflationary effect. That is exactly the opposite of the NAIRU theory. The finding is plainly consistent with the data in Figure 12. It's also plainly the case that the negative effect of unemployment

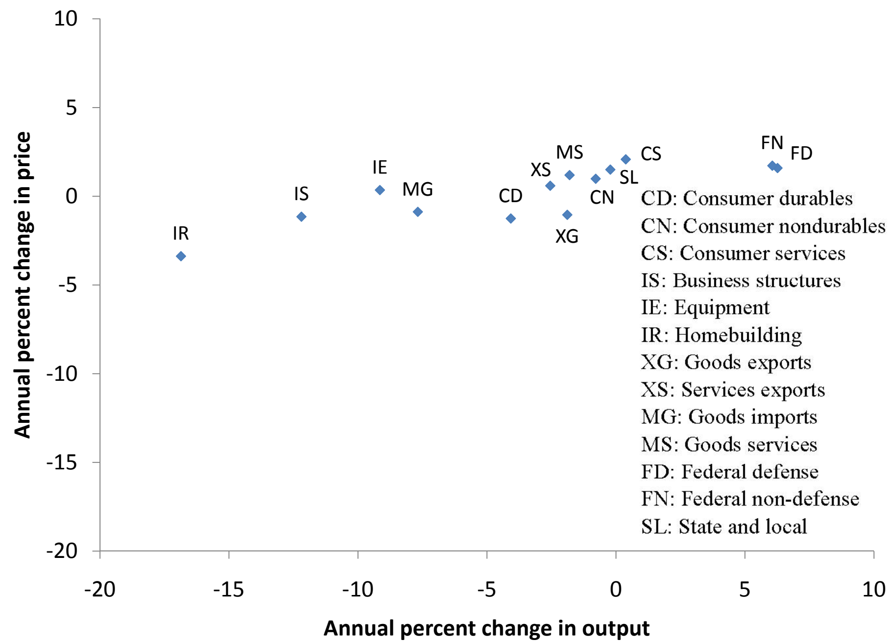

Figure 13: Annual Percent Changes in Output and Prices, 2007 Q4 to 2009 Q4

on inflation is small even during the time when it has any effect.

The limited response of inflation applies at the level of components of output. Figure 13 shows annual rates of change of output and price for a number of components of GDP, over the two-year period from 2007 Q4 to 2009 Q4. The points lie along a line with a slightly positive slope—the line connecting the left-most observation to the right-most has a slope of 0.22 percentage points of price change per point of output decline. The most informative observation is for residential construction, where output declined at a 17-percent annual rate but price declined by only 3.4 percent per year. Construction is a good test case, because existing theories of sticky prices do not seem to apply to this component.

An adverse shift in the terms of trade may be an influence favoring unresponsive prices. If an increase in input prices occurs at the same time that product demand falls, product prices may hardly move at the same time that output falls. A spike in oil prices occurred in the summer of 2008. But the spike reversed by the end of 2008 and there was no meaningful shift in the terms of trade during the two years included in Figure 13. The ratio of the price indexes for imports and exports fell by 17 basis points per year during the period.

Most economic models of pricing derive a reasonably stable markup of price over cost. The dominant model of inflation embedded in practical macro models today hypothesizes that firms would like to set prices according to a markup theory, but only do so at random times. These models are inconsistent with the evidence above, because they imply that the NAIRU principle holds. They cannot explain the stabilization of inflation at positive rates in the presence of long-lasting slack.

The remarkable stability of the rates of change of price indexes conceals the high volatility and dramatic variations in trends of the prices of individual products. The prices of electronic products fall every year, while the prices of services provided by highly educated workers—notably health care—rise every year. An explosion of recent research on grocery-store prices and on the prices of individual products in the Consumer Price Index shows huge volatility. The stability of indexes cannot be explained by the stability of prices of individual products. In particular, the idea that sellers resist cutting individual prices is a complete non-starter. The explanation of stable overall inflation appears to involve factors that operate across products and not in individual product markets.

7 How a Pinned Interest Rate Causes a Slump

7.1 The non-existence of a standard equilibrium in an economy where the government attempts to trade at the wrong price

To start the analysis of the implications of an interest rate that is too high because it is pinned at zero, I'll use the Fisher diagram–the standard indifference curve-isoquant story applied to the intertemporal setting, shown in Figure 14. All households have the same preferences. The indifference curve describes preferences for consumption in the first and second periods. The isoquant expresses the fact that, by giving up current consumption, the household can invest in productive capital and consume more in the second period. The tangency of the isoquant and the indifference curve is the equilibrium of the economy. The slope of the tangent line is one plus the real interest rate at equilibrium. The economy achieves this equilibrium if there are no other opportunities open to households.

At the beginning of period 1, the household has capital k_0. Employment is n_1 and output at the end of the period is $y_1 = f(k_0, n_1)$, which is divided between immediate consumption c_1 and capital k_1 to carry forward to date 2, when the household will consume the resulting output and depreciated capital $y_2 = f(k_1, n_2)$ resulting from employment n_2. The household orders consumption pairs according to the utility function

$$U(c_1) + \beta U(c_2). \tag{6}$$

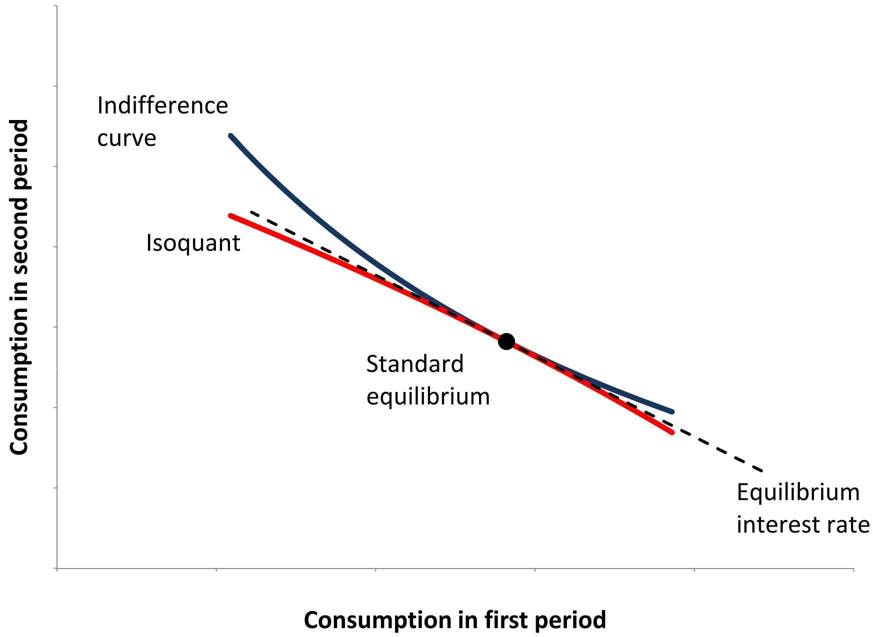

Figure 14: Fisher Diagram of Two-Period Equilibrium

The utility function U is concave and differentiable. The household incurs no disamenity from work, so it will normally choose to offer all of its time to the labor market: $n_1 = n_2 = \bar{n}$.

Figure 15 shows the standard equilibrium. At the equilibrium, the interest rate r^* is the common value of the marginal product of capital and the marginal rate of substitution between consumption in the first and second periods. The isoquant describes the full-employment tradeoff between c_1 and c_2:

$$c_2 = f(\bar{n}, f(\bar{n}, k_0 - c_1)). \tag{7}$$

Equilibrium definitions play a big role in this discussion. I start with

Definition: A *standard equilibrium* in the two-period economy is a pair of consumption levels (c_1, c_2) and an interest rate r satisfying the equality of marginal rate of substitution and marginal product of capital,

$$\frac{U'(c_1)}{\beta U'(c_2)} = \frac{\partial f}{\partial k_2} = 1 + r \tag{8}$$

and material balance at full employment:

$$c_2 = f(\bar{n}, f(\bar{n}, k_0 - c_1)). \tag{9}$$

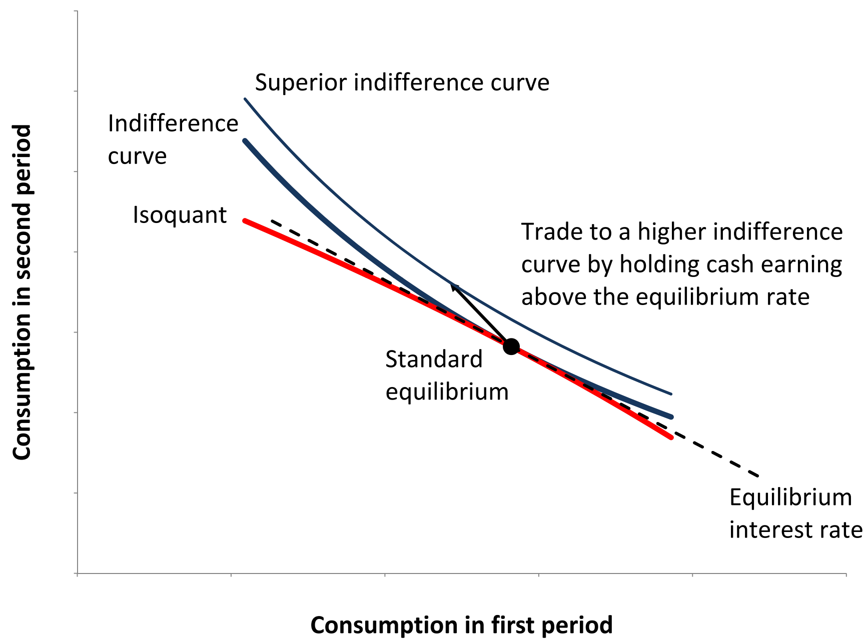

Figure 15: Fisher Diagram with Currency

It's obvious that r^* is the unique equilibrium in this economy.

But now suppose that the government issues a security with a guaranteed return that is *higher* than the equilibrium real interest rate. Rather than stay at the standard equilibrium, the big black dot, the household would like to hold that security and use it to trade along the arrow. This will take the household to a superior indifference curve.

Now consider the same economy with the addition of currency. I take currency as nothing more than a government security that provides a fixed real return of $\bar{r} \geq r^*$. It has no convenience yield or other monetary properties. This assumption is reasonable because central banks typically saturate the economy with currency when the nominal interest rate is zero. Saturation was the Fed's policy in the Great Slump. One unit of output converted to currency in the first period will buy $1 + \bar{r}$ units of output in the second period, because the price level falls by the factor $1/(1 + \bar{r})$ from period 1 to period 2. Currency pays a real return of $\bar{r}$.

Definition: A *standard equilibrium* in the two-period endowment economy with currency is a pair of consumption levels (c_1, c_2) and an interest rate r satisfying the equality of marginal rate of substitution and marginal product of capital,

$$\frac{U'(c_1)}{\beta U'(c_2)} = \frac{\partial f}{\partial k_2} = 1 + r, \tag{10}$$

the condition for optimal trading with the government,

$$\frac{U'(c_1)}{\beta U'(c_2)} = 1 + \bar{r}, \tag{11}$$

and material balance at full employment:

$$c_2 = f(\bar{n}, f(\bar{n}, k_0 - c_1)). \tag{12}$$

It's obvious that the only possible standard equilibrium with government trading occurs when the government trades at the equilibrium price for the economy without government trading, $\bar{r} \geq r^*$, in which case the government will not find any takers for its proposed trade.

A government issuing currency with a return of $\bar{r} \geq r^*$ is doing something fundamentally uneconomic that no private organization would do—it is overcompensating people who lend to it.

The conclusion:

Proposition: In the presence of currency yielding more than the equilibrium real interest rate, the economy has no standard equilibrium.

It is important to emphasize that this proposition says nothing about what mischief follows when the government goes ahead and offers to trade at a price above r^*. As I noted above, consumers would value the opportunity to trade at any price different from r^*. They would line up outside the government office where they could sign up for the deal. Something would happen in that economy. The key question is what kind of non-standard equilibrium results from the government's offer to trade at the wrong price.

Figure 15 also shows a line with a more negative slope than the dashed line corresponding to the equilibrium real interest rate. The household can trade along this line by holding currency and thus reach a superior indifference curve. The story behind that action is natural: The household finds it desirable to reduce first-period consumption and increase second-period consumption. Workers in the household are paid in currency, so all the household does is hold onto the currency rather than spend it on current consumption.

The point on the superior indifference curve cannot be an equilibrium under any reasonable definition, because the economy is physically incapable of supplying that combination of c_1 and c_2. On the other hand, the standard equilibrium is also not an equilibrium of the economy with high-yielding currency. Households will always break that equilibrium by hanging on to their currency.

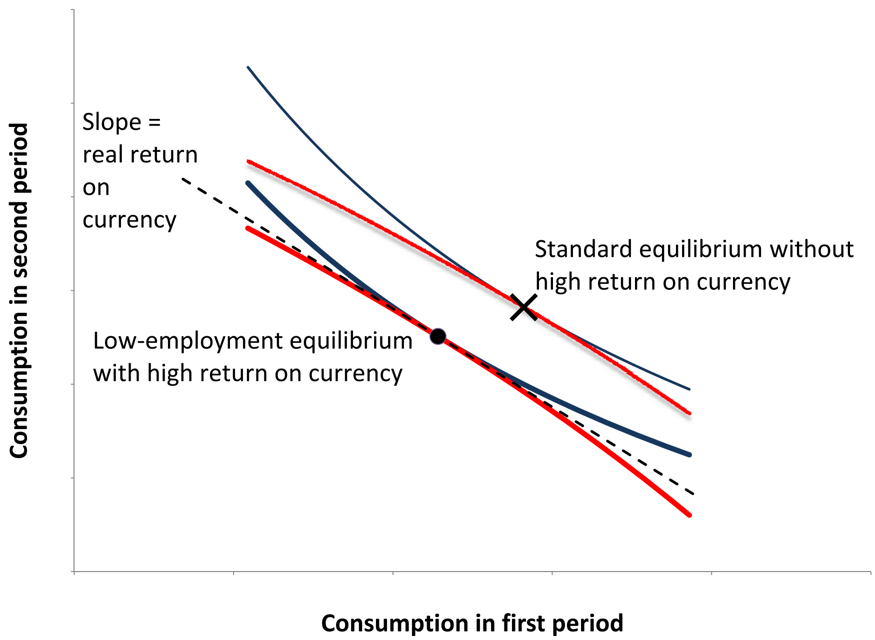

Figure 16: Low-Employment Equilibrium with Pinned Interest Rate

7.2 Low-employment equilibrium

The availability of currency breaks the standard equilibrium. What then happens in this economy? This question goes to the heart of the issue. The government is causing a major problem in the economy by offering too good a deal. People try to hold onto any currency that falls into their hands, rather than spending it right away.

Economies in slumps with interest rates pinned at zero have excess unemployment. That was true in the Great Depression, it's been true in Japan for a while, and it's true now in the U.S. So it seems appropriate that a model generate some kind of a low-employment equilibrium. Earlier work on this branch of macro model building including Krugman (1998), Eggertsson and Woodford (2003), and Christiano, Eichenbaum and Rebelo (2009) has made various assumptions that amount to letting unemployment account for the excess supply of labor, as in Figure 3.

Much of a story connecting the behavior induced by the inappropriately high return on currency to the outbreak of unemployment is still untold. The goal is clear—an equilibrium where some indifference curve is tangent to some isoquant. And the slope of the tangent line needs to be one plus the real rate the government is paying on money—the negative of the inflation rate. See Figure 16.

In the low-employment equilibrium, first-period consumption is quite a bit lower and second-period consumption—which the household hoped would be higher—is actually lower as well. The desire to trade to a better indifference curve, by hoarding currency, is thwarted in equilibrium.

In the equilibrium shown, unemployment occurs in the first period but not in the second. Less capital is carried from the first to the second period than in the standard equilibrium. In the second period, that capital is combined with all available labor. Because the labor/capital ratio is higher, the return to capital is higher and capital can compete with high-return currency.

To find an equilibrium of the economy with high-yielding currency, one needs an alternative concept of equilibrium. A long tradition of macroeconomics considers:

Definition: A *low-employment equilibrium* in the two-period economy at interest rate r is a consumption pair (c_1, c_2) and an employment pair (n_1, n_2), with $n_1 \leq \bar{n}$ and $n_2 \leq \bar{n}$, satisfying the equality of marginal rate of substitution and marginal product of capital at the interest rate r,

$$\frac{U'(c_1)}{\beta U'(c_2)} = \frac{\partial f}{\partial k_2} = 1 + r, \tag{13}$$

and material balance:

$$c_2 = f(n_2, f(n_1, k_0 - c_1)). \tag{14}$$

The two-period economy with currency yielding an excessive real return can have a low-employment equilibrium. Figure 16 shows one calculated with reasonable parameter values. First-period consumption is below its level in the standard equilibrium. That's not surprising, because the higher interest rate induces substitution to later consumption. But second-period consumption is lower as well. Because labor input is below full employment, the isoquant is closer to the origin.

Table 1 shows values of the variables in the two economies portrayed in Figure 16. Unemployment in the first period in the economy with high-yielding currency results in lower output. The amount of capital in use from period 1 to period 2, k_1, is lower—thus raising its rate of return—and first-period consumption is lower—thus raising the growth rate of consumption so as to satisfy the household's Euler equation at the higher real return. Notice that the economy has full employment in the second period. The labor/capital ratio that matters, n_2/k_1, is not depressed by lower employment; it is raised by lower capital.

Variable	*Notation*	*No currency*	*With currency*
Equilibrium concept		Standard	Low employment
First-period employment	n_1	1.000	0.874
Second-period employment	n_2	1.000	1.000
First-period consumption	c_1	0.541	0.514
Second-period consumption	c_2	0.541	0.526
First-period output	y_1	0.756	0.714
Second-period output	y_2	0.541	0.526
Capital stock from 1 to 2	k_1	0.215	0.200
Labor/capital ratio from 1 to	n_2/k_1	4.644	4.995
Interest rate from 1 to 2	r_1	0.005	0.050

Table 1: Comparison of the Standard Equilibrium without Currency and the Low-Employment Equilibrium with Currency

The existence of a low-employment equilibrium is not a given. The obstacle is the need to generate a labor/capital ratio sufficiently *higher* than in the no-currency equilibrium to make the rate of return on capital equal the higher level that currency earns, despite having less labor. In other words, the first-period capital stock k_1 needs to be proportionately lower by more than is the second-period employment level, n_2. In the two-period case, this condition is easy to satisfy—in the equilibrium shown in Figure 16, second-period employment is at its full-employment level. With lower first-period employment, the capital stock k_1 is lower because output is lower and because the difference in first-period consumption is smaller than is the difference in the capital stock.

7.3 Many periods

Economies lasting over many (or an infinity of) periods are less likely to have a low-employment intertemporal equilibrium. Two factors stand in the way of that equilibrium. First, in the two-period case, a reduction in first-period output can cause a meaningful proportional reduction in the capital stock carried from the first to the second period, k_1. The flow of output is a significant fraction of the stock of capital. As Table 1 shows, output is two or three times higher than capital. In a standard calibration of a long-horizon model—

and in data for the U.S. economy—output is only a fraction of the capital stock. Even a total collapse of output lowers the capital stock by a small percentage and may not be able to match the increase in the real return to capital needed to generate a low-employment equilibrium.

The second factor that may block a low-employment equilibrium is keeping the level of employment high enough to elevate the labor/capital ratio to the point needed to match the high return to currency. On the one hand, a low-employment equilibrium needs to produce a low level of output so as to cut the capital stock and raise the rate of return to capital. On the other hand, the equilibrium needs to keep employment high to generate a high enough labor/capital ratio. In the two-period low-employment equilibrium, there is no conflict—the cut in employment occurs in the first period, while employment is at its maximal level in the second period, so the labor capital ratio can be high. But in cases where the availability of excessive returns on currency last multiple periods, no similar pattern can generate a high enough labor/capital ratio.

7.4 Interpretation of no equilibrium

By equilibrium in a model, I mean a solution of its equations. Some macroeconomists speak of "disequilibrium" models, meaning models that eliminate standard market-clearing equilibrium conditions, especially in the labor market. I find that term confusing, but there's no substantive disagreement here, only a matter of taste.

Non-existence of equilibrium is a defect of a model, not a statement about the behavior of the economy. Something always happens in the economy and it is the job of the model to replicate what happens. Non-existence of a low-employment equilibrium calls for consideration of other mechanisms. The challenge is the following: At times when the real return to currency is high enough to cause problems—such as the U.S. and many other countries since late 2008 or in the Great Depression—the return to capital must match the return to currency, including the appropriate risk premium. But the collapse of the economy, especially the decline in employment, seems to point in the direction of a low return to capital.

7.5 Capital utilization

One element that may help in building a model of a slump that has an equilibrium is a decline in capital utilization. This idea is an important element of the New Keynesian model. If x

is the fraction of capital in productive use, the economy's technology is

$$y = n^{\alpha}(xk)^{1-\alpha}. \tag{15}$$

The return to capital is the marginal product of capital. One notion of the marginal product is

$$\frac{\partial y}{\partial k} = (1-\alpha)x\left(\frac{n}{xk}\right)^{\alpha}. \tag{16}$$

The factor x appears because of the implicit assumption that incremental capital suffers the same utilization rate as existing capital. The presence of that factor implies that a decline in capital utilization will lower the return to capital unless accompanied by an increase in employment. A model incorporating the assumption that incremental capital is only partly utilized is completely unpromising in delivering an equilibrium when high-yielding currency is available.

Under the alternative assumption that investment occurs only when full utilization is expected, the marginal product of fully utilized capital as an increment to a partially utilized stock is

$$\frac{\partial y}{\partial (xk)} = (1-\alpha)\left(\frac{n}{xk}\right)^{\alpha}. \tag{17}$$

In this case, if the decline in capital utilization is proportionally greater than the decline in labor utilization, so $n/(xk)$ rises, the return to capital can increase even though the economy has entered a slump, with low utilization of both factors, even without a decline in k.

7.6 Role of adjustment costs

My discussion so far assumes costless adjustment of the capital stock. Under the more realistic assumption of positive adjustment costs, the return to capital differs from the marginal product of capital. In the case of costs that increase smoothly with the magnitude of the adjustment, Tobin's q model applies; q is the market value of installed capital. The return to capital is the marginal product of capital plus the capital gain in its market value. But it turns out that the insights from the simple case of no adjustment cost carry over reasonably accurately to the more complicated case of positive adjustment costs.

The effect of adjustment costs is to cut the immediate response of the capital stock to shocks and to spread the effect over time. With adjustment costs, the decline in installed capital that would raise the return to capital happens slowly. But another factor replaces the quantity adjustment. Tobin's q falls discontinuously when a negative shock occurs. From that time on, q rises back to normal, resulting in capital gains on installed capital.

8 Long-Horizon Fully Specified Dynamic Model

Now I'll move to a more complete model that builds around this picture of intertemporal equilibrium. It looks into the indefinite future. It's basically a Solow growth model in terms of technology, but has life-cycle consumption for some households. Like the Solow model, it has inelastic labor supply. It permits unemployment along the lines I just discussed and capital utilization is proportional to employment, so capital can be unemployed too. The model incorporates the stock of houses and consumer durables as well as business capital, with adjustment cost for both kinds of capital. The Diamond-Mortensen-Pissarides model governs the labor market. Some households are liquidity-constrained and have debt service commitments. Financial friction drives a wedge between the return that households earn from savings and the rate at which businesses and households borrow.

The economy in the model lasts for many years and households last as long as the economy. Households consume nondurable goods and services and the services of durables, including housing. Output is divided among three uses: nondurables consumption, investment in new durables and housing, and investment in business capital. The length of a period is a calendar quarter. Because the real interest rate is bounded by the negative of the rate of inflation, a low-employment equilibrium may occur in the product market in any quarter. Both types of investment incur standard quadratic adjustment costs, captured by Tobin's model of investment. As before, household preferences have constant intertemporal elasticity of substitution.

Uncertainty is not an important element in the model. In particular, the model lacks any second-moment effects. Decision-makers have perfect foresight. I don't regard this as a realistic assumption, but experience has shown that perfect-foresight models give surprisingly good accounts of what happens in a dynamic model once a major surprise becomes known.

I solve in two phases. First, for the model when the interest-rate bound does not bind, I find the consumption function $C(z)$, where z is a vector of endogenous and exogenous state variables. The elements of z are the two types of capital, business capital (plant and equipment) k and household capital (houses and consumer durables) d, the level of committed debt service s, and the level of the financial friction f. To find the function, I solve the model for many different values of the vector z and fit an interpolation function. The Appendix reports the function and explains its derivation.

For T early periods, the lower bound on the real interest rate associated with government currency is binding. I solve this part of the model for the values of all variables in all of the binding periods. Thus I treat all the values of all the variables as one big vector of 5T-8 unknowns and solve that many nonlinear equations jointly for their exact values. The model requires that the last value of consumption match the consumption function $C(z)$ for that period's values of the state variables z. The Appendix supplies more information about the solution process.

8.1 Technology, adjustment costs, rental prices, and capital demand

The technology for producing output y is Cobb-Douglas with labor elasticity α:

$$y_t = n_t^{\alpha}(x_t k_{t-1})^{1-\alpha}. \tag{18}$$

Here x_t is the utilization rate for capital. Output is the production of goods, which are used to make capital, houses, and consumer durables, or are consumed directly. The price of output is $p_{y,t}$.

The value of the marginal product of labor is

$$m_t = \alpha p_{y,t}\left(\frac{x_t k_{t-1}}{n_t}\right)^{1-\alpha}. \tag{19}$$

Capital installation occurs up to the point where the marginal adjustment cost equals the difference between the price of installed capital q_k and the price of uninstalled capital, $p_{y,t}$:

$$p_{y,t}\kappa_k\frac{k_t - k_{t-1}}{k_{t-1}} = q_{k,t} - p_{y,t}. \tag{20}$$

The parameter κ measures capital adjustment cost—if $\kappa_k = 0$, q_k is always $p_{y,t}$ and there are no adjustment costs. Housing installation follows a similar equation with subscript d replacing k.

The rental prices of capital and durables include the financial frictions $f_{k,t}$ and $f_{d,t}$:

$$p_{k,t} = (1 + r_{n,t-1})(1 + f_{k,t})q_{k,t-1} - (1 - \delta_k)q_{k,t} \tag{21}$$

and

$$p_{d,t} = (1 + r_{n,t-1})(1 + f_{d,t})q_{k,t-1} - (1 - \delta_d)q_{d,t}. \tag{22}$$

Here $r_{n,t-1}$ is the nominal rate of interest for borrowing at the end of period $t-1$ and repaying at the end of period t.

The market-clearing condition for capital equates the value of the marginal product of capital to the rental price:

$$(1-\alpha)\frac{p_{y,t}y_t}{x_t k_{t-1}} = p_{k,t}. \tag{23}$$

8.2 Household product demand

Households fall into two categories, unconstrained ones who follow the standard life-cycle intertemporal model, and constrained ones who are at the corner of their intertemporal choice resulting from an inability to engage in unsecured borrowing beyond a modest limit. Both types of households have active choices about the division of spending between consumption of nondurable goods and services on the one hand and the services of durable goods including housing, on the other hand. A tilde (˜) denotes unconstrained households and a bar (¯) denotes constrained ones.

Consumption is a Cobb-Douglas composite of consumption of standard output, $c_{y,t}$, and the services of durables, d_{t-1}:

$$\tilde{c}_t = \tilde{c}_{y,t}^{\phi}\tilde{d}_{t-1}^{1-\phi}, \tag{24}$$

and similarly for constrained households. The price of composite consumption is

$$p_{c,t} = \phi^{-\phi}(1-\phi)^{-(1-\phi)}p_{y,t}^{\phi}p_{d,t}^{1-\phi}. \tag{25}$$

Here $p_{d,t}$ is the rental price of durables, as above. The unconstrained household's demand for the goods component of consumption satisfies:

$$p_{y,t}\tilde{c}_{y,t} = \phi p_{c,t}\tilde{c}_t \tag{26}$$

and similarly for constrained households. Total nondurable consumption is:

$$p_{y,t}c_{y,t} = \phi p_{c,t}(\tilde{c}_t + \bar{c}_t) \tag{27}$$

Unconstrained households order their paths of composite consumption according to the intertemporal utility function

$$\sum_t \beta^t \frac{\tilde{c}_t^{1-1/\sigma}}{1-1/\sigma}, \tag{28}$$

where σ is the intertemporal elasticity of substitution.

Constrained households' consumption is

$$p_{c,t}\bar{c}_t = p_{y,t}(\omega y_t - s_t y_t), \tag{29}$$

where ω is the fraction of constrained households and s_t is the burden of interest and debt repayments of constrained households as a fraction of output.

Consumption of durables services is:

$$p_{d,t}d_{t-1} = (1-\phi)p_{c,t}(\tilde{c}_t + \bar{c}_t), \tag{30}$$

8.3 The price of output

I take the price of output, $p_{y,t}$, to be an upward trend at rate π:

$$p_{y,t} = \bar{p}(1-\pi)^{-t}. \tag{31}$$

8.4 Employment

As in the standard Diamond-Mortensen-Pissarides model, all workers desire to work a standard number of hours. The only source of variation in aggregate hours of work arises from unemployment.

Hall (2009) gives a compact summary of the search-and-matching model whose canon is Mortensen and Pissarides (1994). My approach generalizes wage determination relative to the Nash bargain in that paper. Also, I simplify the treatment of labor-market dynamics by considering only the stochastic equilibrium of labor turnover, which means that the employment rate n measures the tightness of the labor market. The vacancy rate enters the picture only in fast transitional dynamics of the matching process, which can be ignored in a quarterly model without losing much. Thus the recruiting success rate is a function $q(n)$ of the employment rate. Success is higher when employment is lower.

Without loss of generality, the wage paid to the worker can be decomposed into two parts, corresponding to a two-part pricing contract (the decomposition is conceptual, not a suggestion that actual compensation practices take this form). The worker pays a present value J_t to the employer for the privilege of holding the job and then receives a flow of compensation equal to the worker's marginal revenue product.

The cost of recruiting (holding a vacancy open) is γ per period, taken to be constant in output terms. The zero-profit condition for recruiting equates the expected benefit of recruiting to its cost:

$$q(n_t)J_t = p_{y,t}\gamma. \tag{32}$$

Thus unemployment rises if J falls. I take

$$J_t = J(m_t), \tag{33}$$

an increasing function of the marginal revenue product of labor, m_t, so that, in slack markets with high x_t and thus lower m_t, a worker pays less for a job. I solve for employment as a function of m_t and take the function as constant-elastic:

$$n_t = \bar{n}\left(\frac{m_t}{\bar{m}}\right)^{\psi}, \tag{34}$$

where the elasticity ψ is positive, $\bar{m}$ is the normal level of m_t which I take to be its full-employment value,

$$\bar{m} = \alpha p_{y,t}\left(\frac{k_{t-1}}{\bar{n}}\right)^{1-\alpha}. \tag{35}$$

Solving,

$$n_t = \bar{n}x_t^{(1-\alpha)\psi}. \tag{36}$$

The contraction in J when the marginal product of labor falls can be interpreted as wage stickiness, or, more accurately, compensation stickiness. If total compensation is sticky and the net benefit falls, then J must fall, because J is the present value of the difference between the net benefit and the worker's actual compensation.

The separation or turnover rate is a fraction ν of employment, a constant. The cost of filling a vacancy, γ/q, is 14 percent of a quarter's earnings, according to Silva and Toledo (2008).

8.5 The financial market

Only unconstrained households participate in asset markets on the margin. They price assets with returns measured in units of output by the discounter,

$$\mu_t = \beta\frac{p_{c,t}/p_{y,t}}{p_{c,t+1}/p_{y,t+1}}\left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right)^{-1/\sigma}. \tag{37}$$

I use the term *real interest rate* to mean the own interest rate on output. The optimal choice of consumption growth results in a discounter that discounts the market real interest rate to one:

$$(1+r_t)\mu_t = 1. \tag{38}$$

The real and nominal interest rates are related as

$$1 + r_t = (1 + r_{n,t}) \frac{p_{y,t}}{p_{y,t+1}}. \tag{39}$$

Thus the zero lower bound on the nominal rate $r_{n,t}$ implies

$$r_t \geq \bar{r}_t = \frac{p_{y,t}}{p_{y,t+1}} - 1. \tag{40}$$

From the earlier assumption in equation (31),

$$\bar{r}_t = -\pi. \tag{41}$$

8.6 Material balance

At the beginning of a period, the stock of installed capital is k_{t-1} and the stock of housing is d_{t-1}. At the end of the period, output y_t becomes available and is allocated to consumption of goods $c_{y,t}$, and investment in capital and housing, including adjustment cost, resulting in the new capital stock, k_t and new housing stock d_t. Firms expend $\gamma n_t / q(n_t)$ in recruiting cost. The equation for the economy's material balance is

$$k_t + \frac{\kappa_k}{2} \frac{(k_t - k_{t-1})^2}{k_{t-1}} + d_t + \frac{\kappa_d}{2} \frac{(d_t - d_{t-1})^2}{d_{t-1}} = (1-\delta_k) k_{t-1} + (1-\delta_d) d_{t-1} + y_t - c_{y,t} - \nu\gamma \frac{n_t}{q(n_t)}. \tag{42}$$

8.7 Standard v. low-utilization periods

The cases that I consider all have equilibria in which the interest bound binds and the model is in low-utilization equilibrium for $t \in [1, T]$ and in standard equilibrium in later quarters. Although one could imagine an economy that went in and out of the interest bound, I only consider cases involving a single transition away from the bound.

8.8 Parameter values

Table 2 gives the parameter values I use in the base case and their sources. I choose the elasticity of employment with respect to the marginal revenue product ψ to make capital utilization x_t move in proportion to employment:

$$\psi = \frac{1}{1 - \alpha}. \tag{43}$$

This choice is analogous to the high wage elasticity found necessary to rationalize the observed volatility of employment in all types of macro models.

Parameter	*Description*	*Value*	*Source*
α	Labor elasticity of production function	0.646	NIPA income share
κ_k	Capital adjustment cost	8	Hall (2004)
κ_d	Durables adjustment cost	8	See text
δ_k	Capital depreciation rate	0.0188	NIPA Fixed Asset Tables
δ_d	Durables depreciation rate	0.0129	NIPA Fixed Asset Tables
ϕ	Nondurables consumption share	0.82	NIPA
β	Utility discount factor	0.9950	Derived from real federal funds rate
σ	Intertemporal elasticity of substitution	0.5	Hall (2009)
ω	Fraction of constrained consumption	0.58	See text
$\bar{n}$	Normal employment rate	0.945	Average, 1948-2007
ψ	Elasticity of employment function	2.8	See text
γ/q	Job-filling cost	.14 of quarterly wage	Silva-Todeto (2007)
υ	Separation rate	0.12	JOLTS, adjusted

Table 2: Parameter Values and Sources

9 The Model's Implications for an Economy Hit by Adverse Forces when the Real Interest Rate is Pinned

9.1 Scenario

I focus on a particular scenario to illustrate the principles. The scenario is only loosely connected to actual events in the current slump. I consider an economy with a fixed inflation rate at just below zero ($\pi = -0.12$ percent per year decline). The stock of housing and consumer durables is 14 percent above normal at the outset, which I take to be roughly late 2008. The stock of business capital k starts at its normal, stationary level. 58 percent of consumption is in liquidity-constrained households with debt-service commitments of 6.7 percent of GDP that are gradually declining by 2 percent per quarter ($s_t = (0.067)(0.98^t)$). There is a financial friction f equivalent to a property tax on both types of capital at 2 percent per year, gradually declining at the same rate as for s.

I solve the model over a long horizon (40 years) with the initial conditions just described. Macroeconomists often describe a dynamic model's properties in terms of impulse response functions showing how an unexpected shock affects the key variables starting from the time of the shock. My approach is different, because the forces that became so harmful in the Great Slump did not strike as a shock, except for the financial friction, but rather built up over about half a decade. Appendix D discusses the relation between my approach and impulse response functions.

9.2 Results

Figure 17 shows the response of the unemployment rate to the combined effects of the three adverse forces. Unemployment starts at a very high rate above 30 percent and gradually declines during the four years that the interest rate is blocked from declining below 0.12 percent. Once the economy is free of the limit on the interest rate, the unemployment rate drops immediately to its normal 5.5 percent.

Figure 18 shows the response of the consumption levels of the two types of households. Consumers who are able to level consumption by adjusting saving have flat profiles, with no discontinuity at the point when the interest rate is no longer pinned at too high a level. On the other hand, liquidity-constrained households consume more when their incomes rise as unemployment drops to normal.

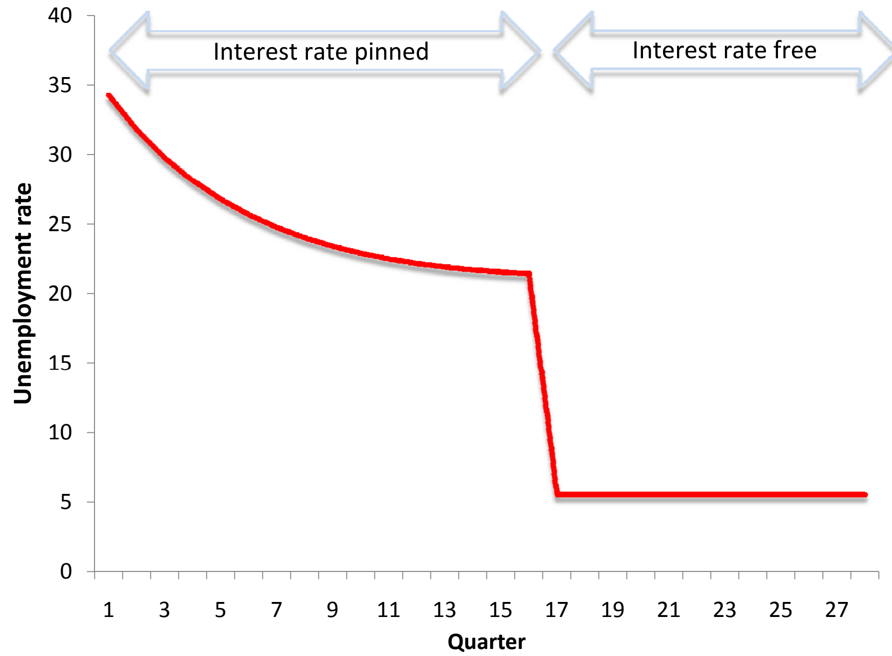

Figure 17: Response of the Unemployment Rate to All Three Adverse Forces Jointly

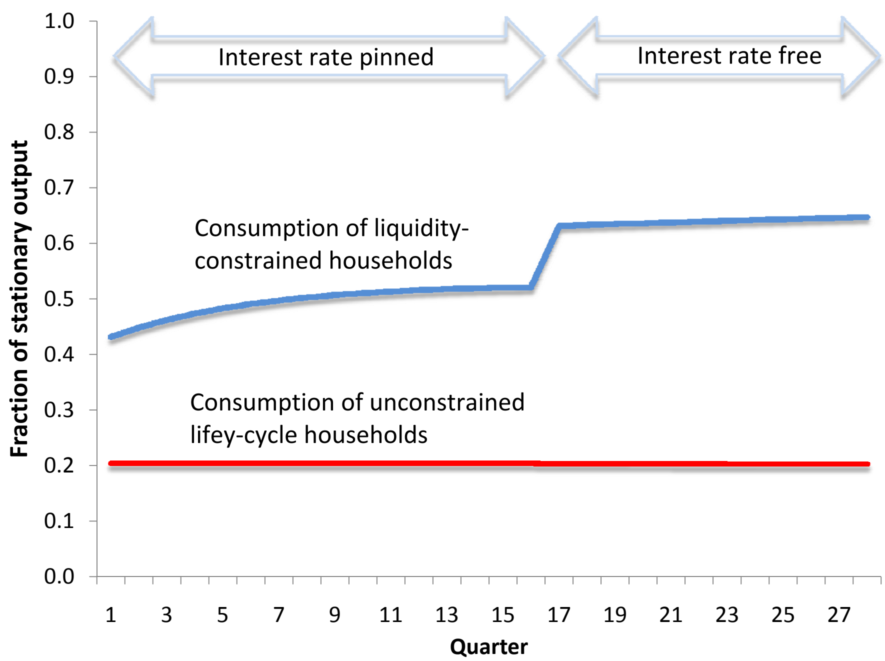

Figure 18: Response of Consumption to all three adverse forces jointly

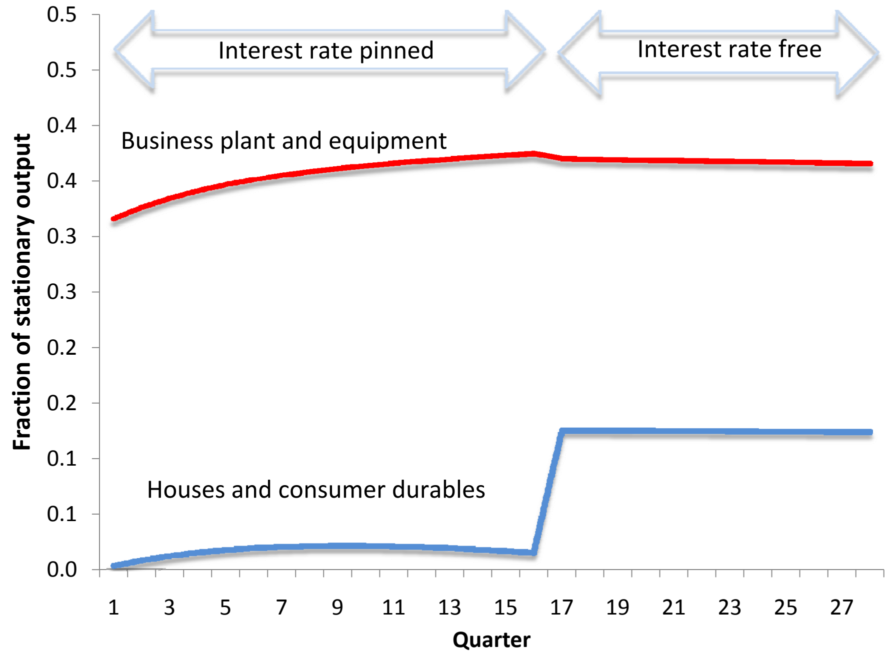

Figure 19: Response of Investment to All Three Adverse Forces Jointly

Figure 19 shows the response of investment. Business investment takes a modest hit at the outset, then recovers gradually. Investment in houses and consumer durables falls to zero immediately and does not recover much until the interest rate is unpinned, when it jumps up to normal levels.

Figure 20 shows the response of the interest rate. For 4 years, it is pinned at 0.12 percent. At the end of the period of high interest, the rate drops discontinuously to -0.20 percent. That is enough, at that time, to restore full employment. The rate then gradually rises as the adverse forces dissipate.

Figure 21 shows the responses of unemployment to other combinations of adverse forces. The blue line takes the financial friction out of the mix. The initial increase in unemployment is not as severe, but the recovery is not as fast. Taking tight credit out as well isolates the effect of the bulge of housing and durables, which by itself pushes unemployment up to about 13 percent.

9.3 Responses of a similar economy without a pinned real interest rate

What if the economy had been hit by the same adverse forces but the interest rate was completely free to do its job? Figure 22 shows what the model implies for that counterfactual.

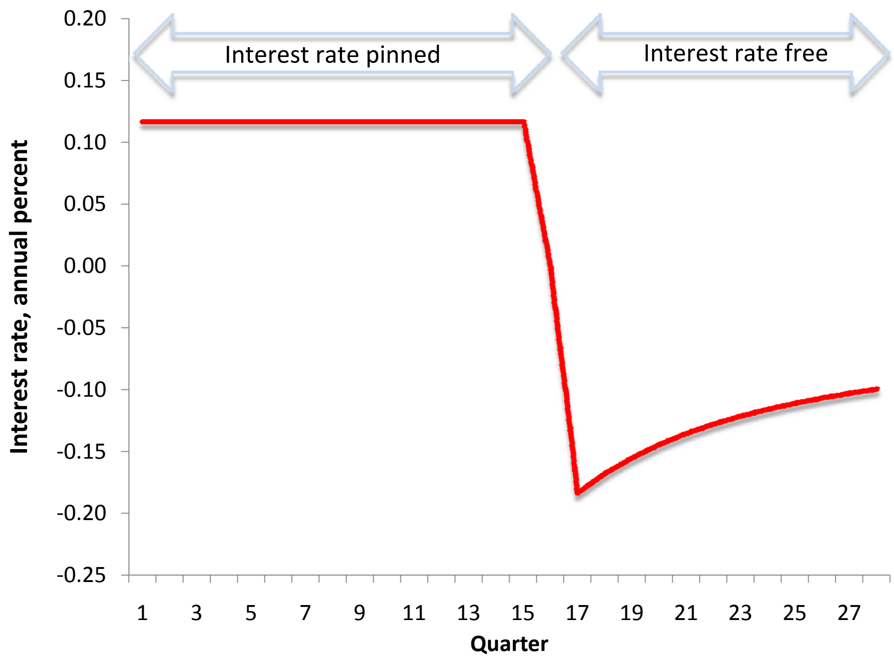

Figure 20: Response of the Interest Rate to All Three Adverse Forces Jointly

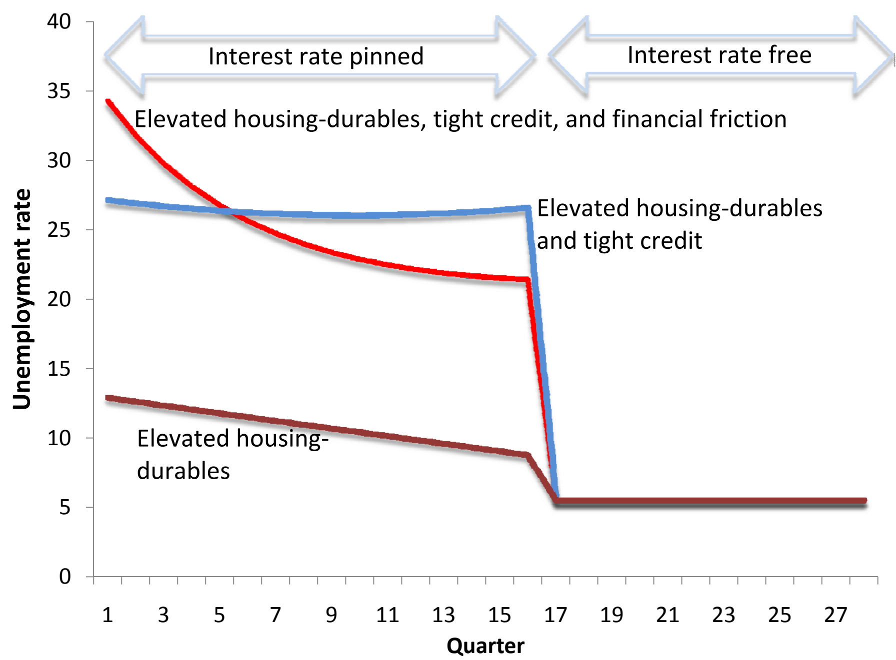

Figure 21: Responses of the Unemployment Rate to the Three Adverse Forces

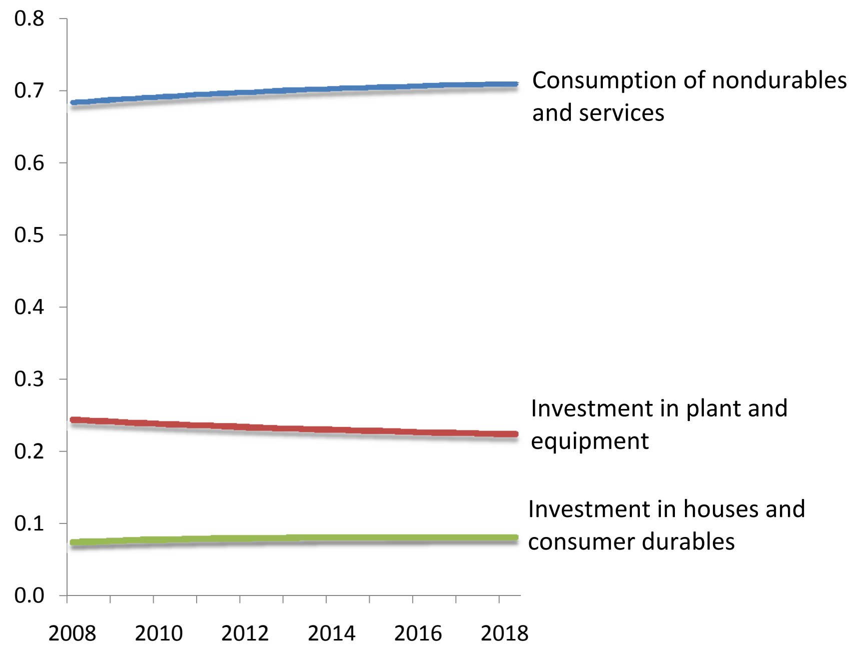

Figure 22: Shares of Output with Flexible Interest Rate

Consumption would start out a bit low, because of the inheritance of heavy debt burdens on constrained households. Business investment would be above normal, because resources released from consumption and investment in houses would flow into business capital. Unemployment would stay at its normal level of 5.5 percent. All this because, as Figure 23 shows, the interest rate would have started out at a dramatically negative level. The negative interest rate would stimulate business investment, would offset most of the decline in housing investment that actually occurred, and would stimulate the consumption of unconstrained households.

9.4 Conclusions from the model

These results demonstrate the potential extreme sensitivity of economic activity in a low-inflation economy whose real interest rate has inadequate room to decline to offset a force that lowers the equilibrium real rate. The same economy without a lower limit on the real rate is completely stable and always operates at full employment.

The model shares a feature with almost all dynamic general-equilibrium models—it has almost no ability to generate long slumps endogenously. The persistence of the driving forces controls the persistence of low employment. In the case of the overhang of housing and consumer durables, the model generates only moderate persistence. If that were the

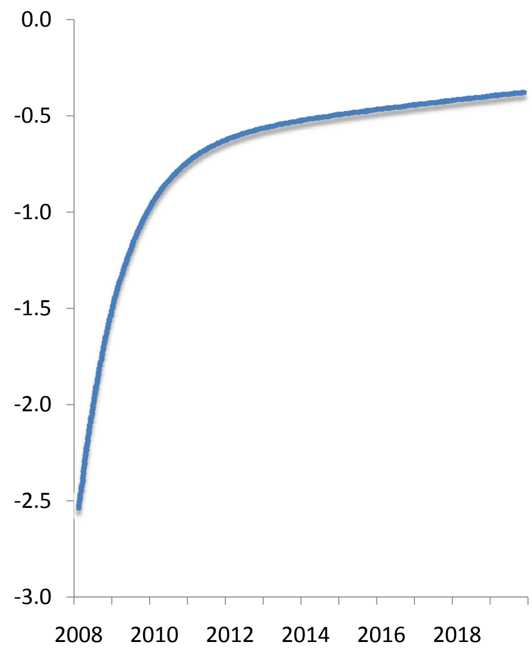

Figure 23: Market-Clearing Real Interest Rate

only factor explaining the slump, the economy would be well on the way to recovery as of the end of 2010.

Evidence on the persistence of the two credit-related driving forces—consumer credit standards and the financial friction—is mixed. This topic needs a lot more investigation.

The model solution here corresponds qualitatively with Romer's 1992 account of the Great Depression and subsequent recovery. She shows that real interest rates were cruelly high during the period from 1929 to 1933, when deflation was rampant. Then the resumption of inflation and the continuation of zero nominal rates through 1941 resulted in quite negative real rates except around the recession of 1937. The economy expanded rapidly during the periods of negative real rates. She attributes the inflation not to conscious acts of the Fed, but to its passive response to gold inflows resulting from the increase in the pegged price of gold and from events in Europe. Thus, as in the model solution, a period of a pinned real interest rate caused a huge contraction in activity and the release from the high real rate caused a rapid expansion of activity.

10 Simple Sticky-Price and Sticky-Wage Models

The model developed here has three important properties that are departures from standard economic thinking: (1) the rate of inflation in the price of output is close to exogenous, (2) capital utilization falls in slumps, and (3) real wages are sticky. Various forms of these departures have been found necessary in a number of modern dynamic general-equilibrium macro models. In this section, I investigate whether any simpler specifications, closer to standard economic principles, might deliver reasonable accounts of the facts about persistent slumps.

One possibility is to drop property (3), sticky real wages, in favor of normal equilibrium in a competitive labor market without search frictions. In the model, this alternative corresponds to setting the parameter ψ, the elasticity of employment with respect to the marginal product of labor, to zero. In effect, the model with these two modifications is a real business cycle model, which determines the real interest rate. Forcing the interest rate upward by imposing the zero lower bound places the model out of equilibrium. A standard solution to this problem is to drop labor-market equilibrium and require intertemporal equilibrium in the output market, which amounts to assuming a sticky real wage—it generates an implicit flat labor supply relation.

Another possibility is to drop properties (1) and (2), and instead to take the output market to be perfectly competitive and always in equilibrium. The simplest version of the model assumes that labor supply is perfectly elastic at a fixed real wage. Shimer (2010) takes this approach. The resulting model generates persistent slumps if some force causes a reduction in the full-employment marginal product of labor. A fall in productivity would have that effect. Because productivity rose at only slightly below normal rates during the contraction phase of the slump, explanations of the slump based on declining productivity are not plausible.

Shimer proposes a fall in the business capital stock as a driving force. Because that event drives down the full-employment marginal product of labor, the loss of capital causes an immediate contraction. The level of employment falls by the same proportion as the capital stock, thus restoring the marginal product of labor to the level of the rigid real wage. A slump ensues until capital accumulation and productivity growth raise the full-employment marginal product of capital back to the level of the fixed real wage.

Data on business capital show no decline around 2008. Thus Shimer's model in its stated form does not explain the current slump unless there has been a decline in the effective capital stock not captured in the standard data. My assumption of declining utilization of the capital stock would have that effect. At the end of the paper he suggests another approach that might have this implication.

Shimer's model differs from the one developed here mainly in its lack of concern with the decline of the safe short-term nominal interest rate to zero during the current U.S. slump and in other slumps, such as the one that began 20 years ago in Japan. An extension of his model to this topic would require adding features that deal with the nominal interest rate. The assumption of a competitive output market, together with almost any standard specification of the determination of the price level, would probably imply that the zero lower bound on the nominal interest rate does not matter. The reason is that, in flexible-price monetary models, the immediate effect of an adverse shock is a discontinuous drop in the price level followed by a gradual rise. The expected rate of inflation rises the moment the shock hits. Thus the nominal interest rate rises and the lower bound of zero is irrelevant.

Adding sticky inflation to Shimer's model would also require the model to take a stand on what happens when the intertemporal output market has excess current supply. The simplest answer—embodied in Krugman (1998) and Christiano et al. (2009)—is that the quantity transacted is the amount demanded. When combined with the assumption of a fixed real wage, this results in a model that accounts nicely for slumps. It is a fairly close cousin of the model developed here, which differs mainly in its addition of a DMP-style model of the labor market.

11 How Keynesian is the Model?

Modern macroeconomics views John Maynard Keynes as one of the major contributors to current thinking, along with others, including Robert Solow, Milton Friedman, Franco Modigliani, James Tobin, and Robert Lucas. I still have and read the copy of the *General Theory* that I bought in 1962. By far Keynes's most important contribution to macroeconomic modeling was his insistence that unemployment be a central element of our thinking about the business cycle. Keynes successfully disputed the notion that market clearing describes the labor market. The reigning theory of the labor market in macroeconomics today, created by Peter Diamond, Dale Mortensen, and Christopher Pissarides (Mortensen and Pissarides (1994)), replaces the cleared labor market with sequences of idiosyncratic transactions between workers and employers. Unemployment arises because job-seekers have to wait until they are able to make a transaction.

Keynes embodied his approach to modeling unemployment in a concept called *aggregate demand*. Although schools of thought are unimportant in macroeconomics today, macroeconomists do fall into two camps. One disregards aggregate demand and builds models that may or may not include unemployment. The other thinks that aggregate demand is really important, but does not try to formalize it. A group in the middle—including me—thinks that Keynes was onto something, but that it's better to build fully specified models that try to get to the underlying issues than to try to create a variable labeled aggregate demand. I've already considered this issue without mentioning it. The demand function in Figure 2 might be seen as an aggregate demand function. If so, it's not the function that Keynes had in mind, because in Chapter 13 of the *General Theory*, Keynes explicitly rejected the view embodied in that figure, that the interest rate is the price that balances current purchases against future purchases.

Keynes was the leader in recognizing the dependence of consumption on current income, though his view is now considerably altered and improved by the life-cycle-permanent-income model of Friedman and Modigliani.

Keynes's position on wage and price flexibility is hard to figure out from the *General Theory*, but the idea that prices and nominal wages are sticky has come to be a central feature of many models of aggregate fluctuations. In the New Keynesian model (Christiano, Eichenbaum and Evans (2005)), the slow adjustment of wages and prices leads to persistent departures from full employment and sensitivity of output to purely monetary shocks. In the model I have presented here, the only role—but a central one—for price stickiness is to turn the zero lower bound on the nominal interest rate into a limitation on the real interest rate. It's not stickiness of prices that matters, but stickiness of expected price *changes*.

Outside the macroeconomics profession, Keynes is seen mainly as an advocate of government purchases to cure slumps. There's essentially no dispute within that profession that GDP rises and unemployment falls if the government buys more stuff. But the failure of the government to buy more stuff in 2009, when the need seemed most intense and the party favoring that policy had firm control of the government, seems to make government purchases irrelevant to stabilization policy. It's not that purchases are ineffective but that the government is incapable of executing a rapid and large increase in purchases.

Keynes's thinking informed and advanced macroeconomics. The analysis of slumps builds on and advances his ideas and those of his successors. Paul Samuelson's "neoclassical synthesis" is alive and well in macroeconomics.

12 Macro Policy

Are there any policy moves available now that would speed up the slow recovery? Monetary policy has gone to its limit in pushing interest rates down. The government seems to lack the logistical tools to expand government expenditures significantly and the political wind is blowing in the wrong direction to push that lever very hard.

Earlier commentary, starting with Krugman (1998), has suggested that central banks could overcome the problem of high real interest rates in slumps by raising expected inflation. One popular proposal is to make the goal of monetary policy one of keeping the price *level* on a growth path, rather than stabilizing inflation. Under this policy, the inflation rate would rise to correct a shortfall in the price level that developed during a slump. Then every episode of inadequate inflation would automatically generate expectations of corrective higher inflation and the problem of excess real interest rate from low inflation would be self-correcting. The Fed has declined to embrace this formulation of policy. Starting the policy in the conditions of the beginning of 2011 seems futile, given the lack of any policy tool that seems capable of changing the rate of inflation under these conditions. Adopting a price-level target as a longer-term formulation of monetary policy has merit, however.

Buiter (2009) discusses ideas dating from the Great Depression that would depress the return on currency during periods of deflation and permit adequately negative real interest rates. He considers a number of alterations in currency policy that could have the same effect in a modern economy, such as the abolition of government-issued currency. None of these policies seem even remotely likely of adoption.

Tax policies that emulate the effect of low real rates could ameliorate the burden of high real rates by making current purchasing cheaper than future. A key feature of these policies is to defer the time when the policy reverses itself until after full employment prevails. The cash-for-clunkers program in 2009 induced a significant bulge in car purchases, but because it lasted only a few months, it only deferred purchases for that many months and did little to shift purchases from a time of full employment to the present, according to Mian and Sufi (2010a). The effective program would place a high subsidy on current purchasing and phase out the subsidy, eventually becoming a consumption tax that financed the earlier subsidy. The shrinkage rate of the subsidy would amount to a negative real interest rate in consumer purchasing decisions.

One idea is to phase in a tax that adds to the prices of goods, such as a value-added tax (Feldstein (2002)). The anticipation of higher later prices incorporating higher tax rates would have the same accelerating effect on spending as a negative interest rate. Income taxes could be phased out at the same time, with no net budgetary effect. Britain did this when it joined the European Union at a time when the resulting inflation was harmful. Now that we need some inflation, the idea looks better. A second virtue is that a value-added tax is a consumption tax, with well-known efficiency benefits.

For a complete discussion involving a full set of fiscal instruments, but in the specific context of the New Keynesian model, see Correia, Farhi, Nicolini and Teles (2010).

The most important policy lesson is to prevent the repetition of the poor oversight of government-protected financial institutions that gave us too much housing, too many cars, and too much debt during the past decade.

13 Concluding Remarks

An economy with a disabled real interest rate is in deep trouble when one type of spending—homebuilding and consumer durables in the current slump—declines. A slump will last until the affected spending resumes its normal level. Consequently, the slump may last many years.

The analysis and calculations in this paper assume that the gradual price adjustment described by the Phillips curve does not occur. Inflation remains at the same rate. If inflation declines and turns into growing deflation, the slump will worsen, as the real interest rate rises. So far in the current slump, notwithstanding episodes of grave concern, no slide into deflation has occurred.

References

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, "The Financial Accelerator in a Quantitative Business Cycle Framework," in John Taylor and Michael Woodford, eds., *Handbook of Macroeconomics, Volume 1B*, Amsterdam: Elsevier, 1999, chapter 21, pp. 1341 – 1393.

Blundell, Richard, Luigi Pistaferri, and Ian Preston, "Consumption Inequality and Partial Insurance," *American Economic Review*, December 2008, *98* (5), 1887–1921.

Buiter, Willem H., "Negative Nominal Interest Rates: Three Ways to Overcome the Zero Lower Bound," *The North American Journal of Economics and Finance*, 2009, *20* (3), 213 – 238.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo, "Understanding Booms and Busts in Housing Markets," November 2010. Department of Economics, Northwestern University.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Jounal of Political Economy*, 2005, *113* (1), 1–45.

—— , —— , and Sergio Rebelo, "When Is the Government Spending Multiplier Large?," October 2009. NBER Working Paper 15394.

Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles, "Policy at the Zero Bound," December 2010. Banco de Portugal.

Eggertsson, Gauti B. and Michael Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, *2003* (1), pp. 139–211.

Feldstein, Martin, "Commentary: Is There a Role for Discretionary Fiscal Policy?," *Economic Policy Symposium, Federal Reserve Bank of Kansas City*, 2002, pp. 151–162.

Friedman, Milton, "The Role of Monetary Policy," *American Economic Review*, 1968, *58* (1), pp. 1–17.

Gertler, Mark and Peter Karadi, "A Model of Unconventional Monetary Policy," April 2009.

Hall, Robert E., "Measuring Factor Adjustment Costs," *Quarterly Journal of Economics*, August 2004, *119* (3), 899–927.

——, "Reconciling Cyclical Movements in the Marginal Value of Time and the Marginal Product of Labor," *Journal of Political Economy*, April 2009, *117* (2), 281–323.

——, "Why Does the Economy Fall to Pieces after a Financial Crisis?," *Journal of Economic Perspectives*, Fall 2010, *24* (4), 3–20.

——, "The High Sensitivity of Economic Activity to Financial Frictions," *Economic Journal*, 2011. Forthcoming.

Kocherlakota, Narayana, "Two Models of Land Overvaluation and Their Implications," October 2010. Federal Reserve Bank of Minneapolis.

Krugman, Paul R., "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, September 1998, (2), 137–205.

Krusell, Per and Jr. Smith Anthony A., "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 1998, *106* (5), pp. 867–896.

Martin, Alberto and Jaume Ventura, "Theoretical Notes on Bubbles and the Current Crisis," September 2010. NBER Working Paper 16399.

Mian, Atif R. and Amir Sufi, "The Effects of Fiscal Stimulus: Evidence from the 2009 'Cash for Clunkers' Program," 2010.

—— and ——, "Household Leverage and the Recession of 2007−09," *IMF Economic Review*, July 2010, *58* (2), 74–117.

Mortensen, Dale T. and Christopher Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 1994, *61* (0), 397–415.

Romer, Christina D., "What Ended the Great Depression?," *The Journal of Economic History*, 1992, *52* (4), pp. 757–784.

Shimer, Robert, "Wage Rigidities and Jobless Recoveries," November 2010. Department of Economics, University of Chicago.

Silva, Jose A. and Manuel Toledo, "Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment," February 2008. Universitat Autonoma de Barcelona and Universidad Carlos III de Madrid.

Stock, James H. and Mark W. Watson, "Modeling Inflation after the Crisis," *Economic Policy Symposium, Federal Reserve Bank of Kansas City*, 2010. Forthcoming.

Townsend, Robert M., "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory*, 1979, *21* (2), 265 – 293.

Appendixes

A Constructing the Index of Lending Standards

The model of bank i's change in standards is

$$\delta_i = \Delta x + \mu + \epsilon_i, \tag{44}$$

where ϵ_i has the standard normal distribution. Then

$$\begin{aligned}
\Pr[\text{Bank } i \text{ eased}] &= \Pr[\delta_i < -\nu] && (45)\\
&= \Pr[\Delta x + \mu + \epsilon_i < -\nu] && (46)\\
&= \Pr[\epsilon_i < -\Delta x - \mu - \nu] && (47)\\
&= \Phi(-\Delta x - \mu - \nu) && (48)
\end{aligned}$$

and, by similar logic,

$$\Pr[\text{Bank } i \text{ tightened}] = \Phi(\Delta x + \mu - \nu). \tag{49}$$

Thus

$$\begin{aligned}
\text{Net change} &= \Pr[\text{Bank } i \text{ tightened}] - \Pr[\text{Bank } i \text{ eased}] && (50)\\
&= \Phi(\Delta x + \mu - \nu) - \Phi(-\Delta x - \mu - \nu) && (51)
\end{aligned}$$

B Simple Dynamic Model with Fixed Interest Rate

The simple dynamic model has preferences taking the form of the many-period analog of equation (28). In the full-employment version of the model, employment n_t is always $\bar{n} = 1$. Output is $y_t = k_{t-1}^{1-\alpha}$. Figure 24 shows the standard phase-diagram analysis for the economy without a bound on its interest rate. Consumption and the capital stock evolve along the converging arms to the economy's stationary point. The arms form a function $c = C(k)$, the economy's consumption function.

B.1 The effect of an interest-rate bound with fixed capital utilization

Next I consider the dynamic equilibrium in the same economy subject to a lower bound on its real interest rate. The economy can have a low-employment equilibrium. The utilization

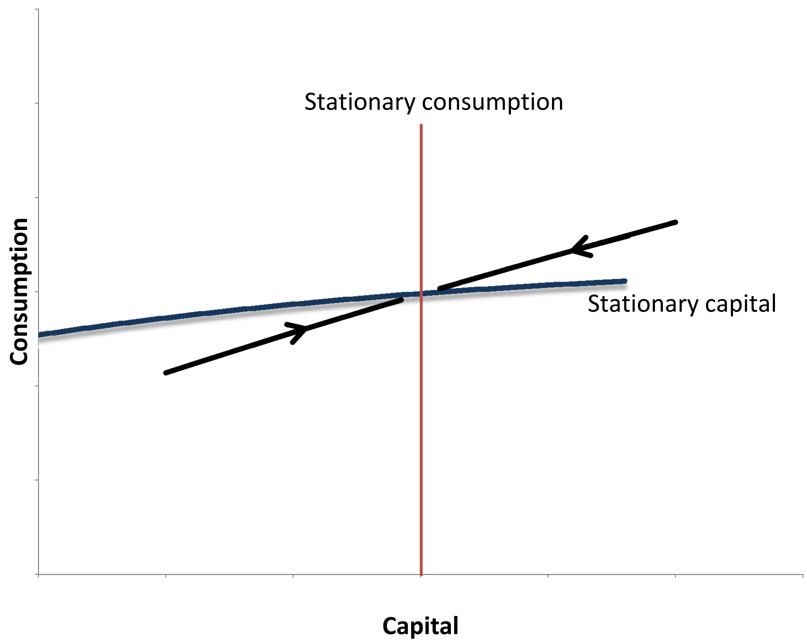

Figure 24: Equilibrium of the Simple Dynamic Model with Full Employment

rate of capital in this version is fixed. The bound on the interest rate drops discontinuously at time T to an irrelevant negative value.

Consumption evolves according to

$$c_t = c_1[\beta(1+\bar{r})]^{\sigma(t-1)}. \tag{52}$$

Thus the path of consumption is exogenous during the period when the zero lower bound binds the interest rate—no variable during that period affects consumption unless it causes the interest rate to exceed the bound.

The equality of the marginal product of capital to the rental price of capital, with a rate of deterioration of capital of δ_k,

$$(1-\alpha)\left(\frac{n_t}{k_{t-1}}\right)^{\alpha} = \bar{r}+\delta \tag{53}$$

implies that

$$n_t = \left(\frac{\bar{r}+\delta}{1-\alpha}\right)^{1/\alpha} k_{t-1}, \tag{54}$$

for the periods when a decision is to be made about capital: $t > 1$. The equation does not constrain the initial employment level n_1. The capital stock follows the law of motion

$$k_t = k_{-1}t^{1-\alpha} + (1-\delta)k_{t-1} - c_t. \tag{55}$$

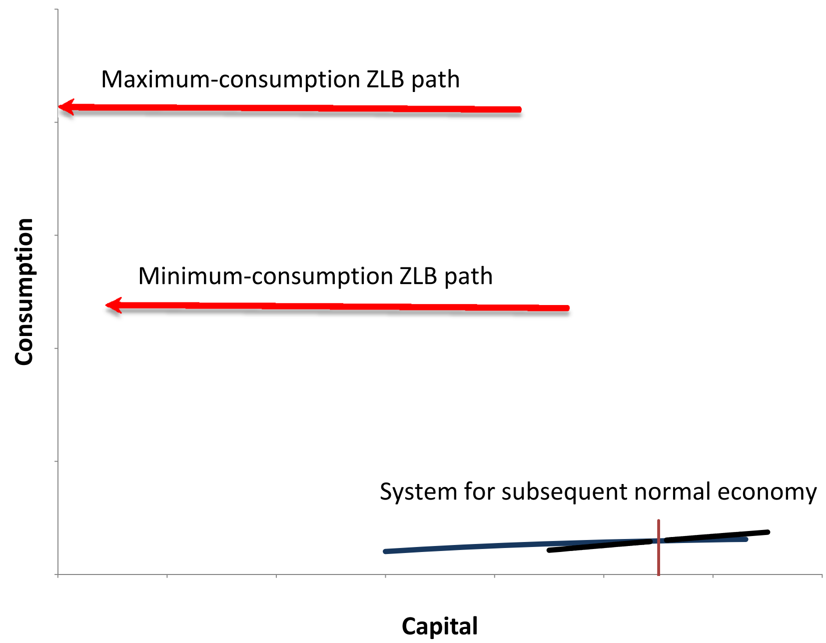

Figure 25: Phase Diagram Including an Initial Period of Bounded Interest with Fixed Capital Utilization

I use generally accepted parameter values in the simple model: The elasticity of output with respect to labor input is $\alpha = 0.6$, the utility discount is $\beta = 0.995$ at a quarterly rate, capital deterioration is $\delta_k = 0.0188$ per quarter, and the intertemporal elasticity of substitution is $\sigma = 0.5$.

Because consumption and capital evolve smoothly in a perfect-foresight economy, the economy must land on one of the converging arms at the end of the period when the interest bound is in effect. Figure 25 portrays an economy with 5 periods under the interest-rate bound. It shows that a simple model with standard parameter values cannot have an equilibrium starting from its stationary capital stock. The left-pointing arrows show the highest and lowest paths, in terms of consumption, that start from the stationary level of capital, for an interest rate held one percent per year above the stationary level. Any consumption path above the top one drives the capital stock to zero prior to the 5th period. Any consumption path below the bottom one results in employment, from equation (54) above the upper limit of one. None of the admissible paths for the economy with a bounded interest rate comes anywhere close to landing on the converging arms of the subsequent economy with a free interest rate.

I conclude that the model needs something more than the possibility of low employment to achieve equilibrium in the presence of a binding lower bound on the interest rate.

B.2 The case of variable capital utilization

A particularly simple specification for capital utilization takes the utilization rate x as proportional to employment:

$$x_t = \frac{n_t}{\bar{n}}. \tag{56}$$

Under the assumption that incremental capital is fully utilized, the marginal product of capital is

$$\frac{\partial y}{\partial (xk)} = (1-\alpha)\left(\frac{n_t}{x_t k_t}\right)^{\alpha} = (1-\alpha)\left(\frac{\bar{n}}{k_t}\right)^{\alpha}. \tag{57}$$

With the interest rate held at $\bar{r}$, capital has the constant value

$$\bar{k} = \left(\frac{1-\alpha}{\bar{r}+\delta_k}\right)^{1/\alpha}\bar{n}. \tag{58}$$

Output is

$$y_t = n_t\left(\frac{\bar{k}}{\bar{n}}\right)^{1-\alpha} \tag{59}$$

and equals product demand,

$$y_1 = c_1 + \bar{k} - (1-\delta_k)k_0 \text{ and } y_t = c_t + \delta_k\bar{k}, t > 1. \tag{60}$$

Employment is

$$n_t = \left(\frac{\bar{n}}{\bar{k}}\right)^{1-\alpha} y_t. \tag{61}$$

As before, the consumption path ends at $C(\bar{k})$ at the moment the interest-rate bound expires. Consumption begins at

$$c_1 = [\beta(1+\bar{r})]^{-\sigma(T-1)}C(\bar{k}). \tag{62}$$

The economy subject to the constant interest rate evolves along a simple path, with a constant capital stock and either rising or falling output and consumption depending on the sign of $\beta(1+\bar{r})-1$.

Figure 26 shows the path in a magnified section of the phase diagram of Figure 24. Along the vertical part of the path with the interest-rate bound, employment starts in quarter 1 at 42 percent of normal, then bounces back to 99 percent for the rest of the bounded episode. This path is for a bound that is 20 basis points at an annual rate above the stationary rate, r^*. If the bounded rate is much higher, the equilibrium does not exist because employment in the first period would have to be negative.

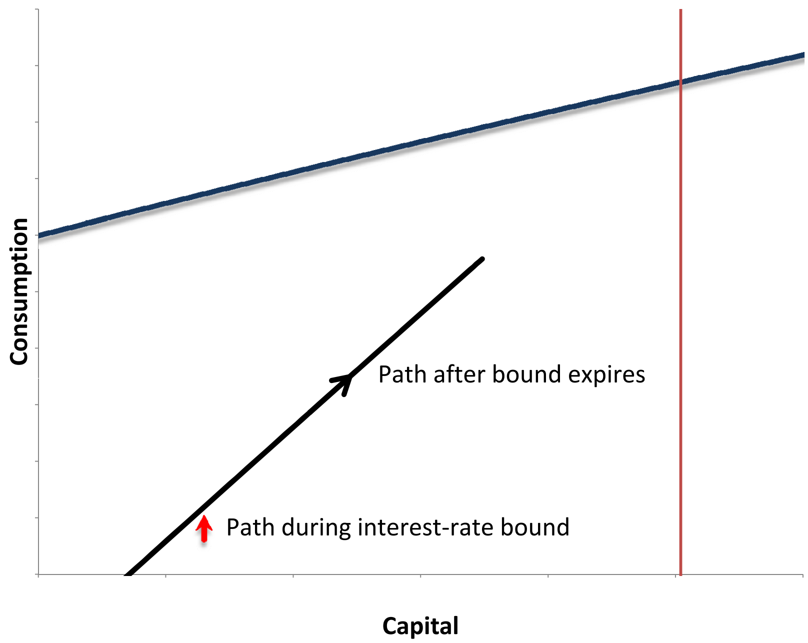

Figure 26: Phase Diagram with Bounded-Interest Path Connecting to Converging Arm of the Standard Model

In the simple dynamic model, the government purchases multiplier is exactly one during the period of the ZLB. With the addition of government purchases, g_t, to the GDP identity of equation (60), it becomes

$$y_t = c_t + \delta_k \bar{k} + g_t. \tag{63}$$

Because the other two terms on the right-hand side are exogenous, the derivative of y_t with respect to g_t is one.

B.3 Conclusions from the simple dynamic model

In the simple model with standard parameter values, no low-employment equilibrium exists if capital remains fully utilized. If capital and labor utilization rates are equal—so the response to excess product supply is equal reductions in utilization of both factors—a low-utilization equilibrium exists for interest-rate bounds that are not too tight. These conclusions carry over to the more complicated and realistic dynamic model of the next section.

Finding a dynamic low-utilization equilibrium is only the first step in modeling the economy in the thrall of a zero nominal interest rate and latent excess supply of output. High unemployment and low capital utilization is presumably the result of the behavior of decision makers in response to the latent excess supply. Models have not yet made significant progress in explaining why those decisions result in low utilization rather than, for example, vigorous price cuts by suppliers to take business away from each other.

In the flexible-price equilibrium of the simple model, the price level would fall discontinuously and then start rising. Expected inflation would be at a level that made the bound irrelevant. Full employment and full capital utilization would prevail at all times.

C Model Details

To facilitate understanding of the Matlab code for the model, I distinguish between core equations, which form the system of equations to be solved, and auxiliary equations, which yield the values of variables that can be calculated once the core variables are known. The solution process uses fsolve.m in Matlab. A Matlab function delivers the discrepancies associated with trial values of the core variables to fsolve.m, which then finds the values of the core variables that set the vector of discrepancies to zero.

C.1 Stationary model

Auxiliary:

$$r = \frac{1}{\beta} - 1 \tag{64}$$

$$n = \bar{n} \tag{65}$$

$$p_k = r + \delta_k \tag{66}$$

$$p_d = r + \delta_d \tag{67}$$

$$c_y = p_c c \phi \tag{68}$$

Core:

$$y = n^{\alpha} k^{1-\alpha} \tag{69}$$

$$k = (1 - \alpha) \frac{y}{p_k} \tag{70}$$

$$p_d d = (1-\phi)p_c c \tag{71}$$

$$y = c_y + \delta_k k + \delta_d d + \gamma\nu\bar{n} \tag{72}$$

$$p_c = \phi^{-\phi}(1-\phi)^{-(1-\phi)} pd^{1-\phi} \tag{73}$$

C.2 Dynamic model during period of pinned real interest rate

Because Matlab starts vectors at location 1 rather than 0, the observation numbering convention here is one higher than in the body of the paper—initial values are dated 1 and the first calculated values are dated 2.

Timing: Output in t uses workers n_t and end-of-period capital k_{t-1}. Consumption occurs at the end of t and comes out of output at the end of t, y_t. The real interest rate from t to $t+1$ is r_t. The rental price for capital used during t is $p_{k,t}$. The net return to capital in use during period t is r_{t-1}. That capital was acquired in $t-1$ at price $q_{k,t-1}$. The discounter from t to $t+1$ is μ_t. All prices are restated here are restated in output units; they correspond to prices divided by $p_{y,t}$ in the body of the paper.

Core variables: $p_d, t \in [2,T](T-1)$ values, $\tilde{c}, t \in [2,T](T-1)$ values, $n, t \in [2,T](T-1)$ values, $k, t \in [2,T-1](T-2)$ values, and $d, t \in [2,T-1](T-2)$ values. Total number of values to solve for: $5T-6$

Auxiliary:

$$y: \ y_t = n_t^{\alpha}(x_t k_{t-1})^{1-\alpha}, t \in [2,T] \tag{74}$$

$$x: \ x_t = \frac{n_t}{\bar{n}}, t \in [2,T] \tag{75}$$

$$p_k: \ p_{k,t} = (1-\alpha)\left(\frac{x_t k_{t-1}}{n_t}\right)^{-\alpha}, t \in [2,T] \tag{76}$$

$$p_c: \ p_{c,t} = \phi^{-\phi}(1-\phi)^{-(1-\phi)} p_{d,t}^{1-\phi}, t \in [2,T] \tag{77}$$

$$\mu: \ \mu_t = \beta\frac{p_{c,t}}{p_{c,t+1}}\left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right)^{-1/\sigma}, t \in [2,T-1] \tag{78}$$

$$\tilde{c}_T = C(k_T, d_T, \omega, s_T, f_{k,T}, f_{d,T}) \tag{79}$$

$$\bar{c}: \; p_{c,t}\bar{c}_t = \omega y_t - s_t, t \in [2, T] \tag{80}$$

$$c_y: \; c_{y,t} = \phi p_{c,t}(\tilde{c}_t + \bar{c}_t), t \in [2, T] \tag{81}$$

$$q_k: \; \kappa_k \frac{k_t - k_{t-1}}{k_{t-1}} = q_{k,t} - 1, t \in [2, T] \tag{82}$$

$$q_d: \; \kappa_d \frac{d_t - d_{t-1}}{d_{t-1}} = q_{d,t} - 1, t \in [2, T] \tag{83}$$

$$r: \; r_t = -\pi, t \in [2, T-1] \tag{84}$$

Core discrepancies:

$$\text{Uses: } k_t + \frac{\kappa_k}{2}\frac{(k_t - k_{t-1})^2}{k_{t-1}} + d_t + \frac{\kappa_d}{2}\frac{(d_t - d_{t-1})^2}{d_{t-1}} + c_{y,t} + \nu\gamma\frac{n_t}{q(n_t)} \tag{85}$$

$$\text{Sources: } y_t + (1 - \delta_k)k_{t-1} + (1 - \delta_d)d_{t-1} \tag{86}$$

$$\text{Uses} - \text{Sources }, t \in [2, T](T-1) \tag{87}$$

$$(1 + r_t)\mu_t - 1, t \in [2, T-1](T-2) \tag{88}$$

$$p_{k,t} - [(1 + r_{t-1})(1 + f_{k,t})q_{k,t-1} - (1 - \delta_k)q_{k,t}], t \in [3, T](T-2) \tag{89}$$

$$p_{d,t} - [(1 + r_{t-1})(1 + f_{d,t})q_{d,t-1} - (1 - \delta_d)q_{d,t}], t \in [3, T](T-2) \tag{90}$$

$$p_{d,t}d_{t-1} - (1 - \phi)p_{c,t}(\tilde{c}_t + \bar{c}_t), \; t \in [2, T](T-1) \tag{91}$$

Total core discrepancies: $T - 1 + T - 2 + T - 2 + T - 2 + T - 1 = 5T - 8$

Initial conditions: $k_1 = k^*$ and $d_1 = 1.14d^*$.

C.3 Dynamic model with free real interest rate

The model is the same as above except that the initial values of the state variables (k_T, d_T, ω, $s_T, f_{k,T}, f_{d,T}$) are supplied as the arguments of the function C, equation (79) and equation (84) are omitted, and employment is determined by

$$n_t = \bar{n}. \tag{92}$$

C.4 The pasting function $C(k_T, d_T, \omega, s_T, f_{k,T}, f_{d,T})$

I solved the model for 16 values of the state variables, with all possible combinations of one of two values for each variable (taking $f_{k,T} = f_{d,T}$). I used separate equations for $\omega = 0$ and $\omega = 0.58$. I regressed the calculated value of c_2 on the initial state variables and all multiplicative interaction pairs, to get the approximation shown in Table 3. The R^2 for both equations is 1.000000.

D Characterizing the Properties of a Dynamic Model

A dynamic model imposes a relationship on a vector of variables y_t of the form

$$y_t = F(y_{t-1}, \epsilon_t). \tag{93}$$

Here y_t is a vector of exogenous and endogenous variables and ϵ_t is a vector of random shocks. The relationship is a reduced form that derives from behavioral principles expressed in structural equations, including the life-cycle principle that calls for consumers to formulate consumption plans by looking into the indefinite future. In the limited class of models with a single representative agent and no wedges or externalities, which can be reformulated as dynamic programs, the function F comprises the policy functions and laws of motion of the dynamic program.

The sequence of derivatives

$$\frac{\partial F}{\partial \epsilon_t}, \; \frac{\partial F}{\partial y_t}\frac{\partial F}{\partial \epsilon_t}, \; \frac{\partial F}{\partial y_{t+1}}\frac{\partial F}{\partial y_t}\frac{\partial F}{\partial \epsilon_t}, \cdots \tag{94}$$

is the set of *impulse response functions* of the model. It shows how the variables in the model respond in period t and later to shocks in period t. Elements representing the response of exogenous variables to shocks in equations that determine endogenous variables are zero by definition.

	ω=0	ω=0.58
k	0.0154	0.0055
	(0.000)	(0.000)
d	0.0179	0.0053
	(0.000)	(0.000)
f	17.2026	2.4512
	(0.173)	(0.101)
s	2.3405	1.1249
	(0.017)	(0.010)
kd	0.0000	0.0000
	(0.000)	(0.000)
kf	0.2535	0.1091
	(0.002)	(0.001)
ks	0.0067	0.0049
	(0.000)	(0.000)
df	0.3428	0.1141
	(0.002)	(0.001)
ds	0.0053	0.0036
	(0.000)	(0.000)
fs	32.1424	30.3103
	(0.180)	(0.105)
Intercept	2.1441	0.1663
	(0.007)	(0.004)

Table 3: Coefficients of the Approximation to $C(k_T, d_T, \omega, s_T, f_{k,T}, f_{d,T})$

The sequence of derivatives

$$\frac{\partial F}{\partial y_0},\ \frac{\partial F}{\partial y_1}\frac{\partial F}{\partial y_0},\ \frac{\partial F}{\partial y_2}\frac{\partial F}{\partial y_1}\frac{\partial F}{\partial y_0},\cdots \tag{95}$$

describes how the variables respond to the initial conditions y_0. Most authors do not consider these responses and they do not have a widely used name. But the nature of the current investigation gives them an important role. I will call them *initial condition response functions.*

I noted earlier that the model does not treat uncertainty explicitly, even though it is obvious that households and producers lack anything like perfect foresight. I exploit the

Law of Dynamic Responses

The initial condition response functions of the perfect-foresight version of a model are surprisingly good estimates of the impulse response functions of a stochastic version of the same model, where the perfect-foresight version is solved as if the realization of a random shock had just entered the initial conditions of the model.

The law is on the same footing as Krusell and Smith's (1998) finding that a small number of well-chosen moments can stand in for an entire distribution in a dynamic general-equilibrium model. It is less than a universal truth.

E Sources Cited in Table 2

Hall (2004), Hall (2009), Silva and Toledo (2008)