

Optimal Contracts to Defend Upstream Monopoly

Robert E. Hall

Hoover Institution and Department of Economics
Stanford University

January 25, 2000

I. Introduction

Microsoft supplies a key input to the computer industry—the operating system. Local phone companies supply access services to long-distance carriers. These are two leading examples where monopoly sellers in an upstream market are in positions to extract profits from their downstream customers. I consider these vertical relationships in the framework of two-part pricing. Monopoly power is limited because downstream customers can avoid dependence on the upstream supplier by developing an alternative supply of the upstream product. For example, Dell Computer could develop an alternative operating system with the capabilities of Windows. Or a long-distance carrier could connect to its customers over wires it installed itself. In this setting, the upstream seller is constrained in its pricing in comparison to the case where no substitute is available. Intuition suggests that the form of the constraint is straightforward—the upstream supplier must price below the cost of the alternative. But that intuition does not do complete justice to the intricacies of market equilibrium. The monopolist can adjust the terms of its two-part pricing formula to prevent the profitable entry of self-suppliers in the downstream market. When there are only a few sellers in the downstream market who have chosen to be customers of the upstream monopolist, the optimal pricing formula sets a low—or even a negative—per-unit price, below the level that would be optimal without the threat of self-supply.

The issues of this paper arose in the trial of the government's case against Microsoft. Richard Schmalensee, appearing on behalf of Microsoft, argued that the potential availability of alternatives to Windows accounted for its \$65 price to computer makers, when the full monopoly price would be more than \$1000. The analysis of this paper supports the view that potential substitutes constrain the price of an upstream product even when the substitutes are only potential. It turns

out to be in the interest of the upstream seller to retain a position as the universal supplier and to moderate the terms under which it supplies its products as needed to achieve that goal, as against setting a higher price and permitting some downstream firms to make use of alternative sources for the upstream product. In this setting, market share would be a poor indicator of the constraining effect of competition.

This paper builds upon an extensive existing literature on patent licensing (see Kamien [1992] for a survey). The problem considered falls with the general class identified and studied by Segal [1999], where a principal uses contracts with a number of firms to correct externalities they impose on each other. As in the licensing literature, I use a two-stage model. In the first stage, the upstream seller auctions supply contracts to the downstream industry. The industry then plays an oligopoly game, where the customers of the upstream firm compete with each other and with firms that have chosen to use alternative suppliers.

I extend previous analysis in two important respects. First, I consider two-part pricing for the transaction between the upstream and downstream firm. The earlier literature has considered fixed-fee pricing and per-unit pricing as alternatives, but this analysis shows the need to consider the combination.¹ Second, I examine free-entry, zero-profit equilibria of the downstream market. Earlier work has taken the number of sellers to be constant. The free-entry assumption is compelling for the two leading applications—computers and long-distance phone service—where there are no significant barriers to entry and no evidence of profit for a potential entrant.

In my analysis, the upstream seller makes binding commitments prior to the production decisions of its downstream customers. As a result, many of the analytical intricacies of the recent literature on vertical relationships do not arise

¹ Two-part pricing is widely studied in the related literature on vertical contracting; see, for example, Bernheim and Whinston [1998].

(see Rey and Tirole [1997], Chemla [1999], and the many papers they cite). Although the issues raised by imperfect commitment and subsequent opportunism by the upstream seller are fascinating and potentially important in some settings, I believe the commitment framework provides a useful starting point for analysis in the applications I have in mind, such as Microsoft.

My analysis is complementary to a related paper by Fudenberg and Tirole [1999]. Their paper considers possible entry to a market for a product with network effects, such as computer operating systems. The entrant displaces the incumbent because it has a cost advantage. The incumbent—lacking a commitment mechanism—chooses a lower price to build a larger installed base in order to deter entry. Thus, Fudenberg and Tirole arrive at a limit pricing result by a quite different route.

I study Microsoft’s pricing of Windows within the framework of the analysis developed in the paper. The data suggest that the price of Windows is substantially lower than it would be without the possibility of entry. Potential competition controls the price of Windows—Microsoft is not an unconstrained monopolist. Nonetheless, the development cost facing a computer maker seeking to avoid licensing Windows is high—between \$3 and \$10 billion. This figure is the profit that a computer maker would achieve as the sole seller of PCs that were as good as Windows machines but did not require any payment to Microsoft. The analysis implies that Microsoft would set its pricing formula for Windows so that no single PC maker would find it advantageous to make the \$3 to \$10 billion investment.

I consider the potential benefits from an antitrust remedy that bars an upstream monopolist from using two-part pricing. A number of economists have proposed that Microsoft should be required to establish a single unregulated price at which any computer maker could purchase a Windows license for an incremental computer, and that this be the only way that Microsoft charges for

Windows. I show that the upstream monopolist will always deter entry in that setting, despite being deprived of the right to use a negative per-unit price. In other words, the remedy would not be successful in stimulating entry. Nonetheless, the remedy is beneficial to the consumer. Absent the remedy, with two-part pricing, the upstream monopolist will choose to have a single downstream customer. In the presence of the remedy, the upstream monopolist will enlist a large number of downstream customers. Although the monopolist will set a higher per-unit price with the remedy, the price reduction from higher competition in the downstream industry always exceeds the price increase from the more expensive input. But both effects are mediated by the change in the number of downstream sellers. In the personal computer business, already substantially competitive, the effect of the remedy would be small.

I use the Microsoft example to perform a sensitivity analysis of the approach developed in the paper. In addition to the Cournot model with linear demand, I use a Cournot model with constant-elastic demand and a differentiated-products Bertrand model calibrated to give the same amount of market power in the computer business. The comparison of the three models suggests that the results are not sensitive to the specification of oligopoly in the downstream market.

II. The Case of No Substitutes

In this section I consider the case where a downstream seller cannot avoid using the monopolist's product. The results in this section are completely standard (see Oi [1971] and Matthewson and Winter [1984]). They confirm the intuition that the upstream firm can create a monopoly in the downstream industry and extract the full value of the monopoly. The main purpose of the exposition is to develop some results that apply in the next section to the case where a sufficiently

large investment by a downstream firm can escape the need to purchase from the upstream supplier.

The upstream seller provides an input to the downstream industry that is produced with zero marginal cost. This is the natural assumption for a pure license, such as Windows. The model would be easy to modify for a product such as telephone access service that has a positive marginal cost. The downstream industry is a symmetric Cournot oligopoly with M sellers. Industry demand is $p = \mathbf{a} - \mathbf{b}Q$. Producers incur a fixed cost K and a constant marginal cost c . The upstream firm quotes a price schedule with a lump-sum component L and a per-unit cost of r . The output of the representative firm is q . Its profit is $q[\mathbf{a} - \mathbf{b}(Q' + q) - c - r] - K - L$ where Q' is the output of all the other firms. The firm proceeds on the assumption that its own quantity decision does not affect Q' . The first-order condition for profit maximization of the firm is

$$-\mathbf{b}q + p - c - r = 0 \quad (2.1)$$

In the symmetric equilibrium, where all firms have the same output, the first-order condition and the demand function together imply

$$Q = \frac{M}{M+1} \frac{\mathbf{a} - c - r}{\mathbf{b}} \quad (2.2)$$

and

$$p = \frac{1}{M+1} \mathbf{a} + \frac{M}{M+1} (c + r) \quad (2.3)$$

The upstream seller sets the lump sum part of the pricing formula to extract all profit from each downstream firm:

$$L = q(p - c - r) - K \quad (2.4)$$

The upstream seller's revenue is

$$R = rQ + ML = rQ + Q(p - c - r) - MK = Q(p - c) - MK \quad (2.5)$$

That is, the upstream seller earns the entire amount of the industry's margin $Q(p - c)$ less the fixed costs of the M sellers. From this observation come two implications: First, the upstream firm should set the per-unit component of price, r , to the level that causes the Cournot equilibrium price to be the monopoly price, and, second, the upstream firm incurs redundant fixed costs if there is more than one downstream seller. On the second account, the upstream firm would prefer to deal with a downstream monopoly. In fact, it is possible for the upstream seller to engineer the optimum in a simple way, by setting the lump-sum element of the price equal to the monopoly profit and the per-unit component of the price to zero. In that case, only a single downstream seller can exist, it will be a monopoly, and it will surrender its entire monopoly profit to the upstream seller.

Notice that this conclusion rests on the assumption that the upstream seller has all of the bargaining power in its bilateral negotiation with a single downstream firm. The bargaining power results from the two-stage structure where the upstream monopolist offers take-it-or-leave-it opportunities to the downstream sellers. If the bargaining power were more symmetrically distributed, the upstream monopolist would favor more downstream customers. In the settings that I have in mind, however, I believe that the assumption that most of the bargaining power rests with the upstream firm is realistic. I will touch on this subject again later.

The upstream monopolist needs to use two instruments—the per unit price and the lump-sum charge—in order to satisfy two objectives—orchestrating monopoly in the downstream market and extracting all of the resulting monopoly profit. Both instruments will be used in general. In one polar case, a single downstream seller, the per-unit charge will be zero because there is no externality

to correct. In the other polar case, perfect competition in the downstream industry, the lump-sum will be zero.

By substituting equations (2.2) and (2.3) into the formula for upstream revenue in equation (2.5) and maximizing the resulting quadratic expression, one can derive a simple formula for the optimal per-unit price for M downstream firms:

$$r = \frac{M-1}{2M}(\mathbf{a} - c) \quad (2.6)$$

The optimal per-unit price rises from zero when there is a downstream monopoly to $\frac{1}{2}(\mathbf{a} - c)$ in the limit as M becomes large. When there is more than one Cournot player, the upstream seller needs to curb the tendency of the downstream rivals to compete, by adding to their costs and boosting the resulting price to the monopoly level. In Segal's [1999] framework, the per-unit charge corrects the externality caused by the fact that sales by one downstream seller reduce the sales of other sellers.

The upstream seller's revenue in the presence of more than one downstream seller is

$$R = \frac{(\mathbf{a} - c)^2}{4\mathbf{b}} - MK \quad (2.7)$$

This result confirms the earlier claim that the upstream monopolist can capture the entire monopoly profit less redundant fixed costs.

Now consider a Cournot model of the downstream market with entry. Its equilibrium occurs when M sellers can make a non-negative profit but $M+1$ sellers would each make a loss. When the upstream seller sets $L = \frac{(\mathbf{a} - c)^2}{4\mathbf{b}} - K$ and $r = 0$, the profit of one out of M downstream customers is

$$p_C = \frac{(a-c)^2}{b} \left[\frac{1}{(M+1)^2} - \frac{1}{4} \right] \quad (2.8)$$

Profit is zero at $M=1$ and negative for higher values, so the zero-profit equilibrium occurs at monopoly, and the upstream seller achieves the optimal outcome. Similarly, the upstream seller could choose a number of downstream firms, M , set the corresponding positive value of the per-unit price, r , from equation (2.6) and the value of the lump-sum component, L , to extract the right amount of profit, and enforce a zero-profit equilibrium with the M downstream firms. This strategy is optimal only if fixed cost, K , is zero.

In this setting, the principle of the “single monopoly profit” applies.² The upstream seller monopolizes the downstream market through its pricing of the controlling product. If downstream fixed cost, K , is zero, the upstream seller is indifferent to the number of downstream firms, M , as equation (2.7) shows.

III. The Case of Potential Substitution for the Upstream Product

Now I consider the possibility that a downstream firm can avoid the need to deal with the upstream supplier by incurring a fixed development cost D . To simplify the discussion, I will consider the case of self-supply. In a later section, I will interpret the results in the setting where the alternative supplier is actually a separate firm that is a rival to the upstream seller. There, I will comment on the possibility that the alternative supplier could have more than one downstream customer. In this section, however, I assume that the fixed cost must be incurred separately by each downstream self-supplier.

² Posner and Easterbrook [1981].

In general, I consider an asymmetric Cournot equilibrium with M sellers who purchase from the upstream supplier and another N who supply themselves. Because of my interest in the licensing situation—where there is no true per-unit cost for the product supplied to the downstream firms—I assume that there is also no per-unit cost associated with self-supply of the upstream input.

The upstream seller auctions M sales contracts to potential customers. Each contract gives the holder the right to buy any number of units at the price r and commits the seller not to issue more than the M contracts. In the absence of a contractual commitment, the upstream seller would have an incentive to enter into additional sales contracts once its original set of customers had paid their lump sums. M of the potential entrants make winning bids for the contracts based on their expectations of their profit in equilibrium. They are aware that other entrants can sell in the market by supplying themselves, each incurring the self-supply cost D . There is asymmetric Cournot rivalry among the M customers of the upstream supplier and the N self-supplying firms. In equilibrium, the profit anticipated by each self-supplying firm is positive.

Following Katz and Shapiro [1986], I choose the auction framework because it permits the upstream supplier to control the number of customers in the downstream market. Katz and Shapiro introduced an auction because control over the number of customers permitted an increase in the upstream seller's revenue. My motivation is different: With the auction, the upstream supplier can prevent equilibria that would otherwise exist in which the number of downstream purchasers is lower than the optimal number, given the chosen values of the lump sum and per-unit components of the price formula.

The assumption in this model about competition between the upstream firm and potential rivals (self-supplying downstream firms) is critical. The auction framework describes the position of a dominant incumbent. A developer of an alternative to the monopolist's product knows that no matter what it does, the

monopolist is committed to having M customers in the downstream market. The profit available to the potential rival supplier is limited by the competition of those M firms. Moreover, the monopolist can commit to a low or negative per-unit cost in order to stimulate the output of its downstream customers. In equilibrium, self-supply will be just short of profitable for a single firm and unambiguously unprofitable for more than one. The upstream firm dominates the market because its development costs are sunk and those of its rivals are not.

I believe that this setup is realistic for the two applications that I have in mind, Microsoft and local telephone companies. Microsoft gained the dominant position in personal computer operating systems as a result of its partnership with IBM when the first modern personal computer was launched. It has retained the position—if the analysis in this paper is on point—by pricing its operating systems with low effective per-unit costs and depriving rivals of adequate profit opportunities to support development of effective rival software.

The nature of the entry barrier associated with contracts put in place by a dominant incumbent is described by Aghion and Bolton [1987]. The terms of the contract are crafted to attract the customer in the first place and to make it less likely that the customer will find it profitable not to purchase from the incumbent. The purchasers in Aghion and Bolton’s model, however, are final consumers, not sellers in a downstream market. Thus the monopolist is not concerned about influencing their production, as is the upstream monopolist in this model.

A. Cournot Equilibrium

Quantity and price in the asymmetric Cournot equilibrium with M and N downstream firms are:

$$Q = \frac{1}{b} \left[\frac{M+N}{M+N+1} (\mathbf{a} - c) - \frac{M}{M+N+1} r \right] \quad (3.1)$$

$$p = \frac{1}{M+N+1} \mathbf{a} + \frac{M+N}{M+N+1} c + \frac{M}{M+N+1} r \quad (3.2)$$

The profit margin per unit earned by the customers of the upstream firm is $p - c - r$ and the margin earned by the self-suppliers is $p - c$. The quantities sold are $\frac{p - c - r}{b}$ and $\frac{p - c}{b}$.

B. The Equilibrium Number of Self-Suppliers

I model the entry decision of the prospective self-supplier in the following way: Entry will occur if the profit of a self-supplier in the resulting Cournot equilibrium with one more downstream firm is positive, with the per-unit price of the upstream product, r , taken to be unchanged. In other words, the prospective entrant considers that the price of the downstream product will decline upon entry, according to the Cournot model, but does not anticipate a price response for the upstream product. This Bertrand-style assumption is implicit in the two-stage setup of the model, but does rule out familiar strategies for deterring entry. For example, the upstream monopolist could write contracts with its downstream customers with a vastly improved price contingent upon the entry of a self-supplier. I will discuss this point further in a later section.

From the results just stated for price and quantity, the profit of a self-supplying firm is seen to be

$$\mathbf{p}_S = (p - c) \frac{p - c}{b} - K - D = \frac{1}{b} \left(\frac{\mathbf{a} - c + Mr}{M + N + 1} \right)^2 - K - D \quad (3.3)$$

In general, were the upstream seller to set M and r at arbitrary levels, the equilibrium level of N would be the greatest integer less than the value that set equation (3.3) to zero. Because a higher per-unit charge r places the downstream customers at a greater cost disadvantage, the equilibrium number of self-suppliers is an increasing function of r .

In fact, the upstream firm will restrict its choices of r to those just below the level where r is high enough to permit the entry of another self-supplier. For analytical convenience—and without any substantive effect—I will assume that the last upstream seller does not enter if its profit would be exactly zero. The value of r that would cause zero profits with N self-supplying downstream firms is

$$r = \frac{\mathbf{d}(M+N+1) - (\mathbf{a} - c)}{M} \quad (3.4)$$

where $\mathbf{d} = \sqrt{\mathbf{b}(K+D)}$. Since the N th self-supplying firm does not actually enter at this price, the value of r that results in the entry of N firms is

$$r_0 = \frac{\mathbf{d}(M+N+2) - (\mathbf{a} - c)}{M} \quad (3.5)$$

C. Deterrence of Self-Supply

The lump-sum component of the price determined in the auction for the supply contracts will absorb all of the profit of the M firms that are the winning bidders. As in the case considered in the previous section, the result is that the upstream seller can orchestrate the monopolization of the downstream customers, but must bear the burden of redundant fixed costs. The revenue earned by the upstream supplier in the presence of M customers and N self-suppliers in the downstream market is

$$M \left\{ \frac{1}{\mathbf{b}} \left(\frac{1}{M+N+1} \right)^2 [\mathbf{a} - c + Mr] [\mathbf{a} - c - (N+1)r] - K \right\} \quad (3.6)$$

The revenue-maximizing price, r , analogous to equation (2.6), is

$$r_U = \frac{(M-N-1)(\mathbf{a} - c)}{2M(N+1)} \quad (3.7)$$

and the resulting level of revenue, analogous to equation (2.7), is

$$R = \frac{1}{4b} \frac{1}{N+1} (\alpha - c)^2 - MK \quad (3.8)$$

The upstream seller has a strong incentive to keep N at zero, that is, to deter all entry of self-suppliers to the downstream market.

Figure 1 shows the zero-profit function, (3.5), and the maximum-revenue function, (3.7), for four downstream customers ($M = 4$) and zero through 10 self-suppliers ($N = 0, \dots, 10$). The zero-profit function slopes upward—a higher price is consistent with zero profit for self-suppliers if there are more of them. The maximum-revenue function slopes downward—the upstream monopolist sets a higher price if there are fewer downstream self-suppliers.

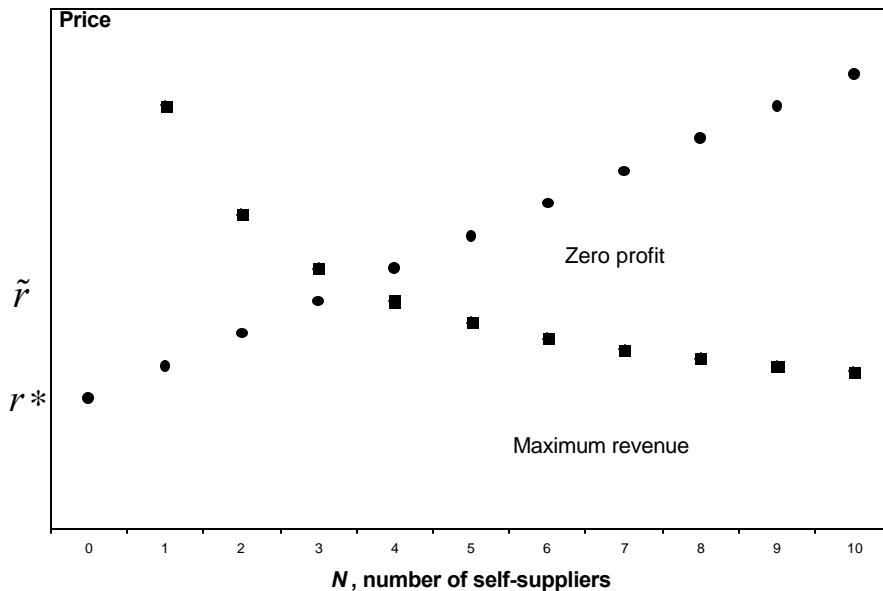


Figure 1. Zero Profit and Maximum Revenue Curves over N with Fixed M

Figure 1 shows the range of prices that the upstream seller might choose. The lowest, labeled r^* , would result in zero self-suppliers—it would completely deter entry of non-customers to the downstream market. However, that price is

well below the revenue-maximizing price in the face of zero self-suppliers. Entry deterrence is the controlling factor in the determination of the price. At higher prices, up to \tilde{r} , some entry would occur. Above \tilde{r} , the number of firms that would enter the downstream market as self-suppliers would be sufficiently large that the revenue-maximizing price would be below \tilde{r} . It would never be desirable for the upstream monopolist to set its price this high.

Although tolerating entry of self-suppliers into the downstream market allows the upstream firm to set a price closer to the revenue-maximizing level, it is never the rational policy, according to the following result:

Theorem 1 (Entry Deterrence). The monopolist's revenue is higher at the price, r^* , that deters all self-supply than at any higher price.

Proof: In Figure 1, it is possible that the self-supply deterrence constraint does not bind—the two curves do not intersect. In that case, the upstream firm sets the per-unit part of the price to its revenue-maximizing value, r_U , and the prospective profit of a single self-supplying firm is negative. Entry is deterred without affecting the price set by the upstream monopolist. Otherwise, the upstream firm chooses a value of N along the zero-profit curve. Revenue is

$$\frac{\mathbf{d}}{\mathbf{b}} \frac{M+N+2}{N+N+1} \left(\mathbf{a} - c - (N+1) \frac{M+N+2}{M+N+1} \mathbf{d} \right) - MK \quad (3.9)$$

Let $x = \frac{M+N+2}{M+N+1}$, a decreasing function of N . Note that $x[\mathbf{g} - xf(x)]$ is an increasing function of x if $f(x)$ is a decreasing function and $\mathbf{g} - 2xf(x) > 0$. From the condition that $r_0 < r_U$,

$$\frac{\mathbf{d}(M+N+2) - (\mathbf{a} - c)}{M} < \frac{(M-N-1)(\mathbf{a} - c)}{2M(N+1)} \quad (3.10)$$

This implies

$$\mathbf{a} - c - 2\mathbf{d} \frac{M+N+2}{M+N+1} (N+1) > 0 \quad (3.11)$$

as required. \square

D. Choosing the Number of Downstream Customers

With the number of downstream self-suppliers held at zero, it turns out that the optimal number of downstream customers of the upstream monopolist is one:

Theorem 2 (Downstream monopoly). The upstream monopolist's revenue is maximized at $M = 1$.

Proof: If self-supply deterrence is not a constraint on the price ($r_0 > r_U$), then revenue at the price r_U is $\frac{(\mathbf{a} - c)^2}{4\mathbf{b}} - MK$. The upstream firm captures the entire downstream monopoly profit, but incurs redundant fixed costs unless $M = 1$. If the constraint binds, revenue is

$$\frac{\mathbf{d}}{\mathbf{b}} \frac{M+2}{M+1} \left(\mathbf{a} - c - \frac{M+2}{M+1} \mathbf{d} \right) - MK \quad (3.12)$$

The demonstration that this is a decreasing function of M is essentially the same as the related one in the proof of Theorem 1. \square

An important feature of the optimum for the upstream monopolist is that it never earns revenue from its per-unit price. Rather, more than all the revenue is earned from the lump-sum component and the per-unit price, if needed, is used to subsidize the downstream customer in order to deter self-supplying rivals:

Theorem 3 (Subsidy). The upstream monopolist's per-unit price, r , is non-positive and is strictly negative if the deterrence constraint binds.

Proof: If self-supply is not profitable when the downstream market is an unconstrained monopoly, then equation (3.7) with $M=1, N=0$ shows that the price is zero. When the deterrence constraint binds, the zero-profit level for the price, r_0 , is less than the unconstrained price, $r_U=0$, and must therefore be negative. \square

The revenue earned by the upstream firm is strictly decreasing in the number of downstream customers, M , except in the case where fixed cost, K , is zero, and the entry deterrence constraint is not binding. However, the dependence is not strong unless fixed cost is high. It may be convenient for the upstream seller to select or at least tolerate a higher level of M than one, for another reason. The subsidy needed to deter self-supply when there is only one downstream customer can be substantially negative. When there is little competition in the downstream market, the upstream monopolist needs to stimulate the output of its customers by subsidizing their production on the margin; otherwise, there would be enough profit in the downstream market to support at least one self-supplier. The subsidy may be inconvenient or even impractical—the customer may claim sales that were never made in order to collect the subsidy. Further, in reality, downstream customers are not all the same size, and the lump-sum component of the price formula needs to be crafted to each customer separately. To the extent that this is done by looking at a customer's history of sales, the lump sum has more of the character of a per-unit charge. Finally, the once-and-for-all auction hypothesized in the model is an oversimplification, as I noted earlier. If the relationship between the upstream monopolist and its downstream monopoly customer were one of cooperative bargaining between equals, the upstream monopolist could not expect to extract the entire surplus—some of the surplus would go to the customer.

To avoid these problems, the upstream monopolist can set M high enough so that r does not need to be negative. Microsoft, for example, has accepted a substantially competitive personal computer industry at all times and has not tried to use the terms of the Windows license to limit the number of sellers.

Figure 2 shows the relation between the number of downstream sellers and the self-supply-deterring price for the same parameter values as in Figure 1.

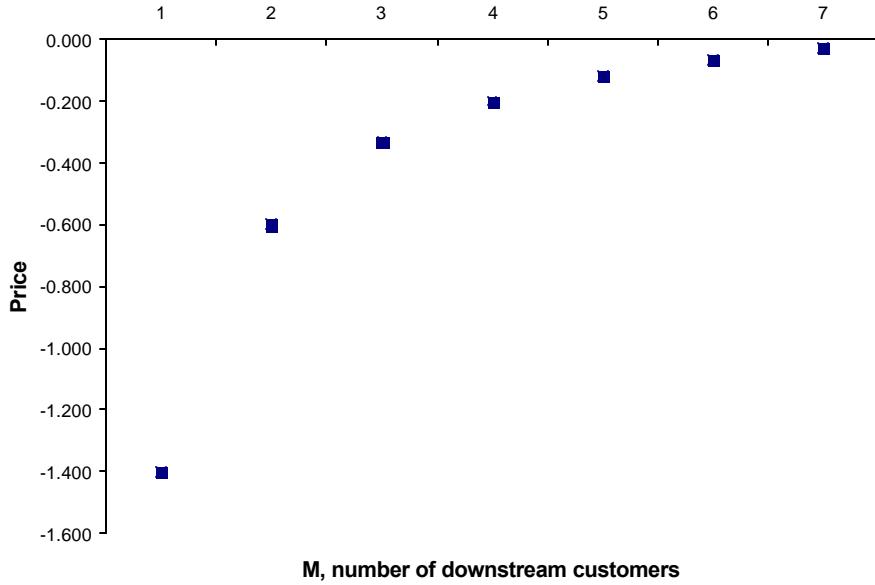


Figure 2. Relation between Price and Number of Downstream Customers

E. Conditions for Deterrence

Is it possible that the maximized revenue of the upstream monopolist is negative, so that the deterrence strategy developed earlier in this section is not worth the effort? Provided a downstream monopoly is profitable, that is, $\frac{1}{4b}(\mathbf{a}-c)^2 > K$, the answer is no: It is *always* profitable for the upstream monopolist to defend its position by precluding self-supply. Since I have already shown that the maximal revenue is always achieved with zero self-suppliers, it only

remains to show that the resulting revenue is positive, so that the monopolist does not shut down:

Theorem 4. (Universal deterrence). The upstream monopolist's maximized revenue is always positive.

Proof: If the zero-profit constraint does not bind, then equation (3.8) shows that revenue is positive. If the constraint does bind, the constrained price charged by the upstream monopolist is negative, by Theorem 3. Thus, from equation (3.5),

$$\mathbf{a} - c - 3\mathbf{d} > 0 \quad (3.13)$$

By adding $\frac{3}{2}\mathbf{d}$ to each side and multiplying by $\frac{3\mathbf{d}}{2\mathbf{b}}$, this becomes

$$\frac{3}{2}\frac{\mathbf{d}}{\mathbf{b}}\left(\mathbf{a} - c - \frac{3}{2}\mathbf{d}\right) > \frac{9\mathbf{d}^2}{4\mathbf{b}} = \frac{9}{4}(K + D) > K \quad (3.14)$$

Then equation (3.9) for revenue shows that it is positive. \square

Although Theorem 4 has the implication that there can never be competition for the product supplied by the upstream monopolist, two points need to be kept in mind. First, self-supply or other duplication of the facilities needed to produce the product is socially wasteful, since it is produced with zero marginal cost. Second, the potential for self-supply lowers the downstream price below the monopoly level whenever the deterrence constraint binds. In that case, the upstream monopolist subsidizes the output of its one customer in order to deter the entry of a self-supplier.

IV. Why Characterize Entry as Self-Supply?

The model focuses on the possibility that one seller could supply itself instead of dealing with the upstream monopolist. The model could also be interpreted as one where a competitor might enter the upstream market but was limited to dealing with a single downstream customer. There are two reasons that this restriction is reasonable. The first is that a single downstream self-supplier captures a large fraction of the available profit. By construction, a single self-supplier earns a profit (before its development cost) of D . Two downstream suppliers in competition with a single customer of the upstream monopoly would earn $\frac{9}{8}D$. The analysis would hardly be changed if the upstream monopolist had to guard against entry by two or several self-suppliers who shared development cost. This point is less forceful if the monopolist has chosen a larger number of customers in the downstream market. With M customers, the profit accruing to a pair of self-suppliers is $2\left(\frac{M+2}{M+3}\right)^2 D$.

The second reason derives from the key assumption that the potential self-supplier does not anticipate that the upstream monopolist will lower its price upon the entry of the self-supplier. That assumption seems plausible for the case of a single self-supplier. The rational response to entry is not very large in any case. But the assumption does not ring true for the entry of a firm that actively seeks customers in direct competition with the upstream incumbent. Rather, in at least some markets, such as software, entry of that type triggers price wars. When there was rivalry for Microsoft DOS in the 1980s, Microsoft had the policy of meeting the price of any rival. The prospect of a price war is a powerful barrier to entry.

V. Microsoft Windows

In this section, I will investigate the price that Microsoft charges computer makers for Windows in the framework developed above. I also develop some variants of the model to investigate its sensitivity to various aspects of the specification. For further discussion of this approach to understanding the market for Windows, see Hall and Hall [2000].

My illustration is based on the following premises, offered as roughly realistic:

Microsoft sells 600,000,000 copies of Windows over a 5-year period.

Microsoft's charge for Windows to computer makers is \$65 per copy (lump sum plus per-copy charge).

Windows-based personal computers sell for \$1000.

The long-run elasticity of demand for personal computers is 2 at the observed level of sales.

The personal computer business is as competitive as a symmetric Cournot oligopoly with 11 sellers.

My calibration strategy is (1) use the zero-profit condition to find the cost parameter, c , and (2) use the downstream customer's first-order condition to find the per-unit charge r and, as a residual, the lump-sum charge, L .

Given a calibration, I solve the asymmetric oligopoly problem with one self-supplier, given the license terms from the calibration. The profit of the single self-supplier in that solution is the estimate of the development cost for a self-supplier.

The figures for the elasticity, the price, and the quantity of computers identify the demand function for computers:

$$p = 1500 - 8.3 \times 10^{-7} Q \quad (5.1)$$

From the principle that computer makers earn zero profit, the per-computer cost apart from Windows, c , is the difference between the selling price, \$1000, and the total Windows charge, \$65: $c = \$1000 - \$65 = \$935$.

In the symmetric Cournot equilibrium, each computer maker satisfies the first-order condition,

$$-\mathbf{b} \frac{Q}{M} + p - c - r = 0 \quad (5.2)$$

where p is the price of computers, M is the number of computer makers, \mathbf{b} is the slope of demand, c is the cost just derived, and r is Microsoft's price of Windows per incremental copy. This relation reveals the amount of the per-unit charge; it is \$20 per copy. The remainder of the \$65 per copy that Microsoft collects is a lump sum, in this calibration.

If there were no possibility of entry in the operating system market, Microsoft would arrange the pricing of Windows to monopolize the personal computer market. The monopoly price is

$$p = \frac{\mathbf{a} + c}{2} = \frac{1500 + 935}{2} = \$1218 \quad (5.3)$$

In Cournot equilibrium, the price in the presence of a per-unit price r is

$$p = \frac{1}{M+1} \mathbf{a} + \frac{M}{M+1} (c+r) \quad (5.4)$$

The per-unit price of Windows needed to achieve the monopoly price is $r = \$257$. Microsoft's revenue per unit is the full margin between price and cost, \$1218 – \$935 = \$283, so the lump sum charged to each computer maker is the difference, \$283 – \$257 = \$26. With 11 sellers, the PC industry is sufficiently close to fully

competitive that most of the value of Windows would be extracted by the per-unit element of pricing rather than the lump sum. The monopoly price found by these calculations is much lower than the one presented by Richard Schmalensee at the Microsoft trial based on constant-elastic demand.³ Monopoly occurs at a point on the demand curve that is much more elastic than the value assumed for the actual observed point. The calculation for linear demand should not be taken very seriously, as the demand function, equation (5.1), has the unrealistic property that the choke price where the demand for computers falls to zero is only \$1500. I will present calculations for the constant-elastic case shortly, which agree with Schmalensee's results. Because the monopoly price depends on the slope and position of the demand function rather far from the observed price and quantity, there is necessarily great uncertainty about the level of the monopoly price.

Now consider the profit that could be achieved by a PC maker who did not have to pay Microsoft for Windows because the maker had developed an operating system of equivalent appeal to computer users. In the resulting asymmetric Cournot equilibrium, the price is

$$p = \frac{1}{M+2} [a + (M+1)c + Mr] = \$995 \quad (5.5)$$

With 11 PC makers licensed by Microsoft at \$20 per incremental copy, a 12th seller would lower the price of a PC from \$1000 to \$995. The seller would avoid both the per-unit and lump-sum components of the Windows price. Its profit would be \$4.3 billion over the 5-year period. On the hypothesis that Microsoft sets the price of Windows to deny such a seller any profit net of the development costs of the Windows substitute, these calculations reveal that the development cost is approximately \$4.3 billion. Because Microsoft is pricing Windows far below the monopoly level, the development cost is a binding constraint.

³ See also Reddy *et al.* [1998].

My impression is that the development cost of a PC operating system narrowly conceived is rather less than \$1 billion. But Microsoft is not held to the low price that would be required to deter self-supply if it cost only, say, \$500 million. The direct development cost is a small part of the total cost of providing an operating system equally as appealing as Windows. People buy an operating system because of the application software that it will run. Inducing the development of compatible applications is costly and time-consuming. Other types of network externalities also inhibit the development of a new system equivalent to Windows, such as the widespread knowledge of the idiosyncrasies of Windows.

The \$4 billion strikes me as a completely reasonable figure for the cost of a direct attack on Microsoft's Windows monopoly. The methods described in this paper permit Microsoft to generate \$39 billion in licensing revenue by deterring an entry threat that would cost \$4 billion. The two key features are (1) the stimulus to high output and low prices in the computer business through a low per-unit price of \$20 and (2) the limitation of the form of entry to self-supply, thanks to the barrier to more extensive entry created by the prospect of Bertrand competition in an open market for operating systems.

If the self-development cost were reduced to \$1.5 billion, the price of PCs would fall to \$973 on account of Microsoft's efforts to deter self-supply. Microsoft's revenue per unit would be \$38 in comparison to \$65 in the earlier calculations. The pricing formula would be \$48 in a lump sum, per unit actually sold, which would be partly offset by a per-unit subsidy of \$10.

VI. Defending Upstream Market Power without Two-Part Pricing

As the results developed earlier in this paper demonstrated, two-part pricing is advantageous to the upstream monopolist. The per-unit element is set at a low or negative level to deter entry. A self-supplying entrant perceives

inadequate profit despite avoidance of the purchase from the monopolist because the monopolist uses a low or negative per-unit price to stimulate output in the downstream industry. The monopolist extracts most or even more than all of its earnings from the lump-sum component of the pricing formula. An interesting question is whether the prohibition of lump-sum pricing would improve the disciplinary power of potential entry.

Two related strands in the literature and practice concerning the enforcement of competition bear on this question. First, contracts have been challenged as exclusionary when they have provisions that make the cost that a customer avoids by shifting its business from one seller to another less than the average amount paid by the customer.⁴ The two-part contracts considered in this paper are one example of the challenged class of contracts. Second, pricing below cost has been challenged as predatory. In the setting of this paper, where the upstream monopolist has zero marginal cost, a negative per-unit price would fall into this category.

A. Pricing without the Possibility of Self Supply

Consider first the way that an upstream monopolist without concern for self-supply would set its price if it were confined to a per-unit price as the only element of its pricing formula. For simplicity, I will consider the case where there are no fixed costs in the downstream industry: $K = 0$. In the two leading cases that motivate this paper, it appears that fixed costs are not an important feature of the technology of the downstream industries. The multiplicity of computer makers and long-distance carriers suggests that fixed costs are small in those industries.

⁴ See Aghion and Bolton [1987] in general and Baseman, Warren-Boulton, and Woroch [1995] in the context of Microsoft's contracts.

In the symmetric Cournot equilibrium with M sellers, each paying the per-unit price r , output is $\frac{M}{M+1} \frac{\mathbf{a}-c-r}{\mathbf{b}}$ and the monopolist's revenue is r times this amount. The value that maximizes revenue is $r_U = \frac{\mathbf{a}-c}{2}$. The resulting revenue is $\frac{M}{M+1} \frac{(\mathbf{a}-c)^2}{4\mathbf{b}}$. As the number of downstream customers, M , becomes large, the revenue approaches the monopoly revenue of equation (2.7). In the presence of two-part pricing, the upstream monopolist is indifferent to the number of downstream customers; it can engineer the monopoly profit for any number. By contrast, when the monopolist is restricted to per-unit pricing, it would strictly prefer more downstream sellers.⁵

In the earlier analysis, where the monopolist was permitted to extract a lump sum from each customer, it did so with an auction. The same idea does not work in the case where the lump sum component is excluded. If the monopolist offered to make sales contracts to the customers who bid the highest per-unit price, the price would be bid up to eliminate the entire difference between the choke price, \mathbf{a} , and cost c , and at that point output and the monopolist's revenue would be zero. Instead, the upstream monopolist should set the number of downstream customers, M , at a high value and give each one a sales contract permitting unlimited purchases at the price r . Each of these contracts would have a positive value as considered in the earlier analysis, so the monopolist would be leaving a small amount of money on the table.

⁵ Ordover and Panzar [1982] reach the same conclusion by another route: They demonstrate that an upstream monopolist gains no advantage from two-part pricing when the downstream industry is competitive.

B. Pricing to Deter Self-Supply

From equation (3.5), the price set by the upstream monopolist to deter self-supply that would otherwise occur is $\mathbf{d} - \frac{\mathbf{a} - \mathbf{c} - 2\mathbf{d}}{M}$. This price is an increasing function of M provided that $\mathbf{a} - \mathbf{c} - 2\mathbf{d} > 0$. It is straightforward to show that this inequality holds whenever the deterrence constraint binds.

The output of the downstream industry when the deterrence constraint binds and the upstream monopolist charges $r = \mathbf{d} - \frac{\mathbf{a} - \mathbf{c} - 2\mathbf{d}}{M}$ is, according to equation (3.1), with $N = 1$, $\frac{1}{\mathbf{b}} \left(\mathbf{a} - \mathbf{c} - \frac{M+2}{M+1} \mathbf{d} \right)$. This too is an increasing function of the number of customers, M . Consequently, the upstream monopolist's revenue, which is the product of the price and the quantity, is an increasing function of the number of customers. As in the case considered earlier, where the deterrence constraint did not bind, the upstream monopolist will choose a large number of downstream customers, M .

C. Benefit to the Consumer from Barring Two-Part Pricing

In the revenue-maximizing configuration, with an infinite number of downstream customers, the price set by the upstream monopolist is $r = \mathbf{d}$. The perfectly competitive downstream market sets price equal to cost: $p = c + \mathbf{d}$. By contrast, the point of revenue maximization when two-part pricing is permitted is with a single downstream customer, $M = 1$, and a price of $r = 3\mathbf{d} - (\mathbf{a} - \mathbf{c})$, from equation (3.4). The resulting price of the downstream product, according to equation (3.2), is $p = c + \frac{3}{2}\mathbf{d}$. Thus, imposing the restriction that the upstream monopolist may not use two-part pricing reduces the markup of price over cost by 50 percent, from $\frac{3}{2}\mathbf{d}$ to \mathbf{d} . Notice that all of the benefit operates through the number of customers. For a given number of customers, the per-unit price, r , is the same for one-part and two-part pricing, according to equation (3.4). If the

number of customers remains the same and is finite, eliminating a lump-sum element from the pricing formula simply transfers a rent of that amount from the upstream seller to each downstream customer.

As I noted earlier, revenue maximization by the upstream monopolist in the face of a threat of self-supply involves a single downstream customer, but requires the upstream monopolist to subsidize the customer's purchases on the margin. If the monopolist chooses to have more than one customer, then the price will be $p = c + \frac{M+2}{M+1}d$. There will always be an advantage to the consumer of the downstream product from forbidding two-part pricing, but the advantage will be smaller if the upstream monopolist decides to enlist more customers.⁶

Notice that the principle of universal deterrence applies to constrained pricing: No matter how small is the self-development cost, the monopolist can price below it in order to deter self-supply. The consumer gets the benefit of the possibility of self-supply without the social burden of the duplicative investment that would occur if there were self-supply in equilibrium.

VII. Other Oligopoly Models

In this section, I will explore the sensitivity of the results to the special assumptions of the model. I consider two other models of competition in the downstream market—Cournot with constant-elastic demand and differentiated-product Bertrand—and two pricing formulas—two-part pricing and one-part pricing. In all cases, I start by calibrating the demand function to the observed combination of price and quantity and to the assumed market elasticity. I repeat

⁶ Woroch *et al* [1996] chide the Justice Department for not barring two-part pricing in the 1995 Consent Decree, but my analysis suggests that the effects on the price of computers would be small.

the calculations relating to Windows in section V for the new specifications. Details of the calculations appear in the Appendix.

Recall that my calibration strategy is in the case of two-part pricing is (1) use the zero-profit condition to find the cost parameter, c , (2) use the downstream customer's first-order condition to find the per-unit charge r , and (3) take the value of the lump-sum charge, L , to be the amount of the resulting profit to the computer maker. I assume that fixed costs are zero. For one-part pricing that approach would be over-identified. The per-unit charge is observed directly and the lump-sum component is zero by assumption. Further, although it is in the interest of the upstream monopolist to have a large number of downstream sellers when there are no lump-sum charges and no fixed cost, as a practical matter the computer industry is not perfectly competitive. To interpret the resulting equilibrium when the upstream monopolist cannot imposed fixed costs on its downstream customers, I need to invoke fixed costs. Consequently, my strategy is (1) use the first-order condition to determine the marginal cost parameter, c , and (2) use the zero-profit condition to find the level of fixed cost, K . As a result, the calculations for one-part pricing use somewhat different marginal and fixed costs than those for two-part pricing.

The demand function with constant elasticity for the second Cournot model is

$$p = \mathbf{a}Q^{-\frac{1}{b}} = 44.7 \times 10^6 Q^{-1/2} \quad (7.1)$$

For the differentiated products Bertrand model, each seller quotes its own price, p_i . The demand facing seller i is

$$\frac{e^{-\mathbf{q} p_i}}{\sum e^{-\mathbf{q} p_j}} \left(\frac{p_i}{\mathbf{a}} \right)^{-b} \quad (7.2)$$

In effect, customers choose the brand by comparing prices across brands, with choice shares governed by a logit.⁷ They choose the quantity of that brand according to a constant-elastic demand function. The elasticity of demand facing seller i is $\mathbf{b} + (1 - s_i) \mathbf{q} p_i$, where s_i is the logit share. I use the same value for the market elasticity, \mathbf{b} , as in the Cournot models, namely 2. I choose the parameter \mathbf{q} to equate the own-price elasticity to the value that generates the same profit margin or Lerner ratio as in the Cournot models, namely $\mathbf{q} = \frac{M\mathbf{b}}{p}$.

The calculations for the Cournot model with constant elasticity of demand are similar to those for the linear demand function and are relegated to the appendix. For the differentiated products Bertrand model, the key calculation is the asymmetric equilibrium when M sellers pay the per-unit charge r and one seller is free from that charge. Their prices are \hat{p} and p' . The first-order conditions for maximizing profit under the Bertrand hypothesis of no resulting price changes from rivals are

$$\left[\mathbf{b} + \mathbf{q} \hat{p} \left(1 - \frac{e^{-\mathbf{q} \hat{p}}}{M e^{-\mathbf{q} \hat{p}} + e^{-\mathbf{q} p'}} \right) \right] = \frac{\hat{p}}{\hat{p} - c - r} \quad (7.3)$$

and

$$\left[\mathbf{b} + \mathbf{q} p' \left(1 - \frac{e^{-\mathbf{q} p'}}{M e^{-\mathbf{q} \hat{p}} + e^{-\mathbf{q} p'}} \right) \right] = \frac{p'}{p' - c} \quad (7.4)$$

Table 1 shows results for the three specifications for all of the calculations reported earlier in Section V. In interpreting this table, one should bear in mind that the results on the right for one-part pricing are based on a different pair of cost parameters than those for two-part pricing on the left. Thus a comparison of

⁷ The demand specification is similar to one developed by Werden and Froeb [1994].

a cell on the right to the corresponding one on the left involves not just the difference between the pricing formula but also the cost parameters.

Table 1. Calculations for Three Oligopoly Models

Two-part pricing				One-part pricing		
	Cournot, linear demand	Cournot, constant- elastic demand	Differen- tiated products Bertrand	Cournot, linear demand	Cournot, constant- elastic demand	Differen- tiated products Bertrand
<i>Inferences about actual values</i>						
Marginal cost, c	935	935	935	890	890	890
Fixed cost, K (billions)	0	0	0	2.5	2.5	2.5
Per-unit charge for Windows, r	20	20	20	65	65	65
Lump-sum charge, L/q	45	45	45	0	0	0
Development cost of Windows alternative (billions), D	4.3	4.3	3.4	10.0	10.1	6.1
<i>Monopoly</i>						
Computer price, p	1218	1870	1870	1195	1779	1779
Per-unit charge for Windows, r	257	850	888	277	809	797
Lump sum charge, L/q	26	45	47	0	0	0
<i>With development cost of \$1.5 billion</i>						
Computer price, p	973	972	959	952	947	957
Per-unit charge for Windows, r	-10	-7	-21	13	14	22
Lump-sum charge, L/q	48	44	66	0	0	0
Total charge	38	37	45	13	14	22

The parsing of the \$65 total charge for Windows into per-unit and lump-sum components on the left side of the table is the same for all three models. This conclusion follows from the calibration setting the markup of price over cost to the same value for all three models. The inference of the actual development cost of a replacement for Windows is virtually the same for the constant-elastic and linear demand Cournot models. The curvature of the constant-elastic demand function is of little importance over the range relevant for this calculation.

The monopoly prices for computers are the same for the constant-elastic Cournot and Bertrand models, because the two models share the same market demand function. The prices are far higher than for linear demand, for the reason noted earlier—the monopoly calculation considers a point on the linear demand function where demand is much more elastic than at the observed point. The monopoly prices differ between the two-part and one-part pricing formulas only because of the difference in the marginal cost parameter.

The bottom panel of Table 1 uses the oligopoly models to restate the market equilibrium in a setting where the cost to a computer maker of developing an alternative to Windows is \$1.5 billion. The computer price falls to a range of \$959 to \$973. With two-part pricing, the three models agree that the optimal per-unit charge for Windows is negative, coupled with a positive lump-sum charge. There is fairly close agreement that the total charge for Windows will be in the range from \$37 to \$45 per unit. Thus computer makers would pay \$20 to \$28 less for Windows if Microsoft believed that it cost only \$1.5 billion rather than \$3.4 to \$4.3 billion to develop an alternative. With one-part pricing, the charge for Windows is in the range of \$13 to \$22, a larger saving. In addition to my earlier observation that the one-part results cannot be compared directly to the two-part results because the cost parameters are different, I should add that the reduction in development costs in the one-part pricing case is much larger, because the estimated current levels are in the range from \$6 to \$10 billion.

The results in Table 1 suggest that the analysis in this paper is not sensitive to the choice of oligopoly model for the downstream industry. In particular, the Bertrand differentiated products model gives results similar to those for the Cournot models, despite the rather different assumptions it rests on.

VIII. Concluding Remarks

Consumers receive a large fraction of the benefits of competition, in the circumstances described in this paper, even though the supply of a key input such as the PC operating system is in the hands of a monopoly supplier. Limit-pricing does its job in this setting, because self-suppliers are invulnerable to Bertrand eradication. And the economy is spared the burden of duplicate investments.

Nothing in this analysis rules out further benefits from reducing the cost of entry. Windows is cheap at \$65 because of the possibility of self-supply at \$3 to \$10 billion, but it would be even cheaper if that figure could be lowered. Controlling conduct that creates artificial barriers to entry might deliver a lower price, in particular.

References

- Aghion, Phillippe, and Patrick Bolton. 1987. "Contracts as a Barrier to Entry" *American Economic Review* 77:388-401. June.
- Baseman, Kenneth C., Frederick R. Warren-Boulton, and Glenn A. Woroch. 1995. "Microsoft Plays Hardball: The Use of Exclusionary Pricing and Technical Incompatibility to Maintain Monopoly Power in Markets for Operating System Software" *Journal of American and Foreign Antitrust and Trade Regulation* vol. XL, Number 2, Summer.
- Bernheim, B. Douglas, and Michael Whinston. 1998. "Exclusive Dealing" *Journal of Political Economy* 106: 64-103. February.
- Chemla, Gilles. 1999. "Downstream Competition, Foreclosure, and Vertical Integration" unpublished, University of British Columbia. June.
- Hall, Chris E. and Robert E. Hall. 2000 "Towards a Quantification of the Effects of Microsoft's Conduct" forthcoming, *American Economic Review Papers and Proceedings*.
- Kamien, Morton I. 1992. "Patent Licensing" in R.J. Aumann and O. Hart (eds.) *Handbook of Game Theory with Economic Applications*, vol. 1 (Amsterdam: North-Holland) pp. 331-354.
- Kamien, Morton I., and Yair Tauman. 1986. "Fees versus Royalties and the Private Value of a Patent" *Quarterly Journal of Economics* 101:471-492. August.
- Katz, Michael L., and Carl Shapiro. 1986. "How to License Intangible Property" *Quarterly Journal of Economics* 101:567-590. August.

- Matthewson, G. and R. Winter. 1984. "An Economic Theory of Vertical Restraints" *Rand Journal of Economics*. 15: 27-38.
- Oi, Walter Y. 1971. "A Disneyland Dilemma: Two-part Tariffs for a Mickey Mouse Monopoly" *Quarterly Journal of Economics*. 85(1): 77-96. February.
- Ordover, Janusz A., and John C. Panzar. 1982. "On the Nonlinear Pricing of Inputs" *International Economic Review*. 23(3): 659-675. October.
- Posner, Richard A., and Frank H. Easterbrook. 1981. *Antitrust: Cases, Economic Notes, and Other Materials*. 2nd ed. St. Paul: West Publishing
- Reddy, Bernard J., David S. Evans, and Albert L. Nichols. 1998. "Why Does Microsoft Charge So Little for Windows?" NERA. October 9.
- Rey, P., and Jean Tirole. 1997. "A Primer on Foreclosure" *Handbook of Industrial Organization*. Forthcoming.
- Segal, Ilya. 1999. "Contracting with Externalities" *Quarterly Journal of Economics* 114:337-388. May.
- Warren-Boulton, Frederick. 1974. "Vertical Control with Variable Proportions" *Journal of Political Economy* 82: 783-802. No. 4. Jul.-Aug.
- Werden, Gregory J., and Luke M. Froeb. 1994. "The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy" *Journal of Law, Economics, and Organization*. 10: 407-426.
- Woroch, Glenn A., Kenneth C Baseman, and Frederick R. Warren-Boulton. 1996. "Exclusionary Behavior in the Market for Operating System Software: The Case of Microsoft" unpublished. February.

Appendix: Details for the Two Other Oligopoly Models

The key calculation is the amount of profit that would be earned by a self-supplier. This requires calculating p' , the price that the self-supplier would command, and q' , the quantity it would sell. Then $D = (p' - c)q' - K$. This calculation leads directly to the estimate of self-development costs from the calibrated model. In addition, for a specified value of D , such as the \$1.5 billion in Table 1, the equation $D = (p' - c)q' - K$ can be solved for p' and then the supporting value of the per-unit charge, r , can be calculated. Finally, the price in the symmetric solution given r can be calculated, and from this, the lump-sum charge, which is the profit of each seller in the symmetric equilibrium.

Cournot with Constant Elasticity of Demand

Calibration:

The first-order condition for the downstream firm is:

$$-\frac{p}{M\epsilon} + p - c - r = 0 \quad (\text{A.1})$$

where ϵ is the elasticity of demand. This provides the value of c in the case of 2-part pricing and of r in the case of 1-part pricing.

Asymmetric equilibrium with one self-supplier:

$$p' = \frac{\epsilon(M+1)}{\epsilon(M+1)-1}c + \frac{\epsilon M}{\epsilon(M+1)-1}r \quad (\text{A.2})$$

$$q' = \epsilon \left(\frac{p' - c}{p'} \right) \left(\frac{p'}{a} \right)^{-\epsilon} \quad (\text{A.3})$$

Symmetric equilibrium with no self-supplier:

$$p = \frac{\mathbf{e}M}{\mathbf{e}M - 1}(c + r) \quad (\text{A.4})$$

$$q = \frac{1}{M} \left(\frac{p}{\mathbf{a}} \right)^{-\mathbf{e}} \quad (\text{A.5})$$

Monopoly:

$$p = \frac{\mathbf{e}}{\mathbf{e} - 1}c \quad (\text{A.6})$$

Solve for the supporting r from equation (A.4).

Bertrand Differentiated Products with Logit Demand

The calibration of the cross-effect parameter \mathbf{q} is described in the text. The value of p' is obtained by solving equations (7.3) and (7.4) for \hat{p} and p' . Then q' can be obtained from the demand function. To determine the value of r consistent with a prescribed value of D , find the roots \hat{p} , p' , and r of equations (7.3), (7.4), and $D = (p' - c)q' - K$. Then the common price in the symmetrical equilibrium is the root of:

$$\mathbf{b} + \mathbf{q}p(1 - M) = \frac{p}{p - c - r} \quad (\text{A.7})$$

The monopoly price is the same as in the constant-elasticity Cournot case. Equation (A.7) provides the value of r that supports the monopoly price with M downstream sellers.