CONSUMPTION TAXES VERSUS INCOME TAXES: IMPLICATIONS FOR ECONOMIC GROWTH

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The prevalence of tax systems whose yield exceeds a quarter or even a third of national income suggests that the study of the global economic implications of alternative taxes should have high priority. In fact, this is not the case. The study of the global effects of taxes has been largely confined to the area of stabilization, where it has reached a high degree of sophistication.[1] Relatively little attention has been paid to the following kind of question: What would happen, in the long run, if the federal income tax were replaced by a federal sales tax that maintained the same level of aggregate demand? This is a distinctly non-Keynesian question, since its answer is trivial in a simple Keynesian model. My purpose here is to focus on this question as representative of the many questions that could be asked about the global implications of alternative tax systems. In the course of this I will present a new interpretation of an important non-Keynesian model of economic equilibrium.

The difference between an income tax and a consumption tax is exactly the taxation of savings under the former but not under the latter. For this reason, most of my concern here is with the development of a theory of saving that can be applied to this problem. The actual application is quite straightforward and yields a surprisingly definite answer. We focus on the savings-consumption decision rather

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[1] For example, in the second Brookings Model volume, (5).

than the labor-leisure decision, not because the second is any less important, but because it is the first that is inherently intertemporal. Furthermore, there are no sharp differences in the effects of the two taxes on the labor-leisure choice, whereas the differences are striking for consumption and saving.

A single example is carried through the entire paper. Although the parameters of the model in the example were estimated in the most casual way, it is still intended to suggest the orders of magnitude of the equilibrium values of the variables involved.

1. *A Theory of Saving Appropriate for the Comparison of Taxes*

The starting point for this investigation is the well-known life-cycle hypothesis of Modigliani and his collaborators.[2] The important characteristic of this theory is that individuals save for a purpose, namely future consumption. It is crucially important that a model of savings behavior for our purpose take account of the motivation for savings, since the difference between the two taxes is in their treatment of savings. The analysis of the influence of taxes involves an examination of the reactions of savers to changes in the payoff to savings; this can only be done with an explicit examination of their motivation. Thus the simple Keynesian rule-of-thumb savings hypothesis, so useful in problems of stabilization, would be inappropriate here.[3]

In its simplest form, Modigliani's hypothesis holds that consumers allocate their total human and material wealth over the years of their lifetimes in constant proportions.[4] More precisely, the present value, say at age 21, of consumption at any future age is a fixed fraction of

[2] See Modigliani and Brumberg (10), Modigliani (8), and Ando and Modigliani (1). Irving Fisher is generally thought to have been the first to suggest this approach to intertemporal equilibrium. Two excellent presentations of the life-cycle model within the context of contemporary mathematical growth theory are those of Cass and Yaari (3) and Tobin (15). The present exposition follows these rather closely, differing mainly in its emphasis.

[3] This is not intended to imply that Keynes himself was not concerned with the influence of interest rates on savings. Rather, the thinking of Keynes and his successors (notably Patinkin) was confined to an individual savings function depending on income and the interest rate which could be blown up to get an aggregate savings function of the same form. Modigliani's key contribution (in this author's interpretation) was to point out that the individual savings function depends in an important way on the *age* of the individual, and that consequently the aggregate savings function could have a very different form than that of any individual savings function. The development of this view (which is the principal novel feature of this paper) is the main burden of this first section.

[4] This is equivalent to maximizing a Cobb-Douglas intertemporal utility function; on this, see Thompson (14).

total wealth at age 21; this fraction is independent of any market variables. It is natural to assume that consumers are *impatient,* so that the share of wealth going to consumption in early years is larger than that going to later years. This does not mean that consumption is less in later years; the wealth set aside at the beginning of the plan appreciates at the rate of interest until the consumption takes place. For convenience, we assume a fixed rate of impatience. Then if the rate of interest happens exactly to equal the rate of impatience, the consumption plan has equal consumption in all years of the individual's life, while if the rate of interest exceeds the rate of impatience, planned consumption will rise over the individual's lifetime. In both cases, the amount of wealth allocated to successive years' consumption declines over the lifetime.

It will be useful later to have this hypothesis formulated in algebraic terms. If c_t is the consumption of a representative individual in the t'th year of his life, W is his wealth measured at t = 1 (again, this might actually be age 21), r is the interest rate, and ρ is the rate of impatience, then the constant proportions hypothesis is

$$(1) \qquad \frac{\left(\frac{1}{1+r}\right)^{t-1} c_t}{c_1} = \left(\frac{1}{1+\rho}\right)^{t-1}.$$

Here the expression $\left(\frac{1}{1+r}\right)^{t-1} c_t$ is the amount of wealth set aside for future consumption in year t; compound interest at rate r will cause it to increase to c_t by that year.

The overall allocation of wealth is governed by a wealth budget constraint. In the absence of inheritances and bequests, the individual is constrained to spend exactly his total wealth over his lifetime; that is, the present value of his planned consumption must equal his total wealth:

$$(2)\ W = c_1 + \left(\frac{1}{1+r}\right) c_2 + \left(\frac{1}{1+r}\right)^2 c_3 + \cdots + \left(\frac{1}{1+r}\right)^{T-1} c_T,$$

for the individual with a life expectancy of T years. From equation 1 this can be written in terms of first-year consumption, c_1:

$$(3)\ W = c_1 + \left(\frac{1}{1+\rho}\right) c_1 + \left(\frac{1}{1+\rho}\right)^2 c_1 + \cdots + \left(\frac{1}{1+\rho}\right)^{T-1} c_1.$$

Thus first-year consumption is given by

$$(4)\qquad c_1 = \frac{W}{1 + \frac{1}{1+\rho} + \left(\frac{1}{1+\rho}\right)^2 + \cdots + \left(\frac{1}{1+\rho}\right)^{T-1}} = \frac{W}{G_T\left(\frac{1}{1+\rho}\right)};$$

here we have introduced a shorthand notation for the sum of a finite geometric series. The function $G_T(\cdot)$ is defined as $G_T(X) = 1 + X + X^2 + \cdots + X^{T-1} = \frac{1}{1-X}(1 - X^T)$. $G_T\left(\frac{1}{1+\rho}\right)$ can be thought of as the present value of a unit stream of income lasting from the present until year T, evaluated at interest rate ρ.

The fact that the interest rate does not appear in equation 4 establishes the following proposition: For a given level of total wealth, W, first-year consumption, c_1, is utterly insensitive to changes in the interest rate. This seems to suggest that the hope expressed at the beginning of this section that the life-cycle model can distinguish between consumption and income taxes is wholly illusory; if the interest rate does not affect consumption behavior in this model, how can the model predict different responses to taxes which differ only in their treatment of savings?

The answer to this question leads naturally into the discussion of the aggregative implications of the life-cycle hypothesis. Two comments should be made immediately. First, although for given values of W, c_1 is unaffected by changes in r, the same is not true for $c_2, \ldots, c_T$. The higher the interest rate, the higher is the value of the wealth set aside for consumption in years after the first. Thus the interest rate sensitivity of future consumption is positive and becomes larger as more distant consumption is considered. Second, it is misleading to take wealth as fixed when considering changes in the interest rate. A youth in the first period of his life has nothing but human wealth (the present value of his future wages) when he makes his consumption plan. The higher the interest rate, the lower is the present value of these wages. Further, under reasonable assumptions about the production side of the economy, there is an inverse relation between wages and the interest rate. This also causes W to drop when r rises. Taken together, these qualifications suggest that the overall interest rate sensitivity of an individual's consumption plan is negative at the outset and rises to a positive value for the last few years of the plan.

In order to obtain an exact aggregate view of the interest rate sensitivity of consumption and saving, it is necessary to add up the

consumption levels for all of the generations alive during any one year.[5] Under the assumption that the population grows at a constant annual rate (say, n), there are always more younger people than older people in the population. Since younger people have negative interest rate sensitivity, this suggests that the higher the rate of population growth, the lower (or more negative is the interest rate response of aggregate (steady-state) consumption. The interesting and important conclusion of the examination of the interest rate sensitivity of aggregate consumption is that for small rates of population growth and long lifetimes, this sensitivity can become indefinitely large even when the sensitivity of c_1 given W is zero. This fact has been overlooked in the theoretical literature on savings and consumption; its recognition makes possible a reconciliation of two divergent branches of that literature.[6] The relationship between the steady-state level of aggregate consumption and the interest rate is shown in a diagram in Figure 1.

Aggregate consumption reaches its minimum at an interest rate midway between the rate of growth of the population and the rate of impatience (throughout it is assumed that ρ exceeds n). At interest rates near ρ, consumption responds positively to increases in the interest rate. In this region, the consumption plan of each individual calls for roughly equal consumption in each year of his life. But one of the consequences of the fact that high powers of $\frac{1+r}{1+\rho}$ determine consumption in later years is that this consumption has a higher sensitivity to changes in the interest rate. Consumption in early years does not share this sensitivity. On the other hand, the income effect of increases in the interest rate is communicated proportionally to all years in the consumption plan. Thus at high interest rates, the large positive response of consumers in the later years of their lives dominates the response of aggregate consumption to changes in the interest rate, and the curve in Figure 1 slopes upward.

For low interest rates, however, consumption plans decline over each individual's lifetime. Most consumption is done by younger people with negative interest rate sensitivity, so the aggregate consumption curve slopes downward in this region. If this sounds somewhat farfetched, it is reassuring to note that the general equilib-

[5] This assumes that consumption plans are actually fulfilled. They are sure to be fulfilled if the economy is in a steady state. From this point forward, comparisons are always between steady states.

[6] Modigliani (8), Samuelson (11), Diamond (4), and Cass and Yaari (3) have studied intertemporal equilibrium with short lifetimes and have emphasized the anomalies that may arise there. Sidrauski (13), Arrow and Kurz (2), Shell and Stiglitz (12), and many others have studied models in which people effectively live forever, and have noted their favorable or optimal aspects.

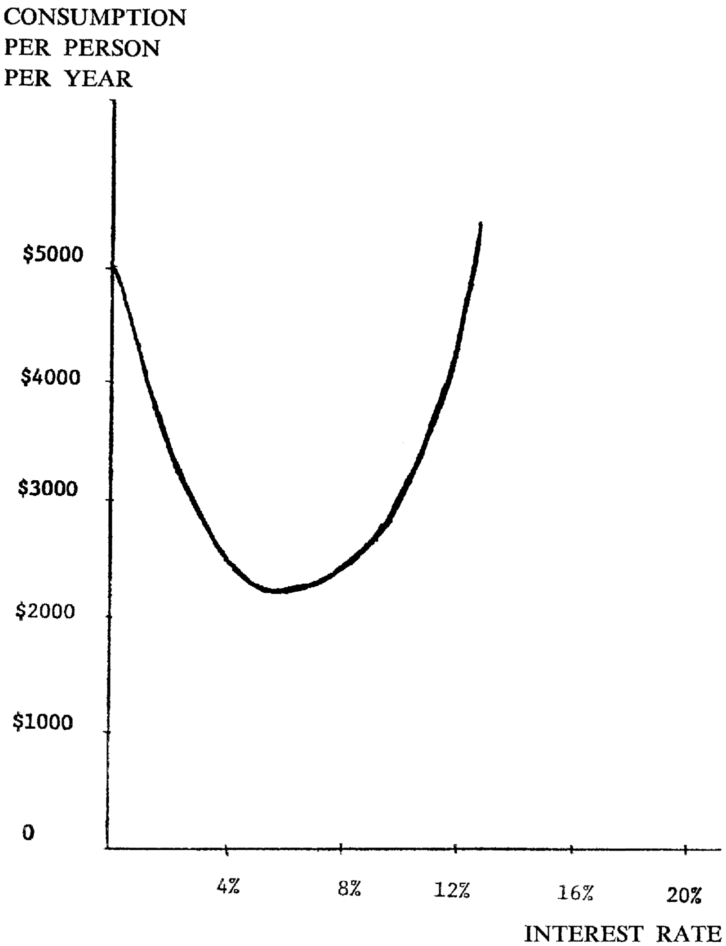

FIGURE 1. Steady-state Consumption Demand as a Function of the Interest Rate.

rium theory presented in the next section predicts that the steady-state interest rate will lie in the neighborhood of the rate of impatience, where consumers behave normally.

All of these assertions require support in terms of the algebra of the model. The first step is to derive a formula for aggregate consumption per capita given the consumption plan followed by consumers. If the population is growing at the constant rate, n, then at any time a fraction $\frac{(1+n)^{T-1}}{G_T(1+n)}$ of the population comprises people in the first year of their consumption plans, a fraction $\frac{(1+n)^{T-2}}{G_T(1+n)}$ comprises people in the second year, and so forth. These provide the appropriate weights for adding up $c_1, \ldots, c_t$ to get aggregate consumption per capita:

$$(5)\quad c = \frac{(1+n)^{T-1}}{G_T(1+n)} c_1 + \frac{(1+n)^{T-2}}{G_T(1+n)} c_2 + \cdots + \frac{1}{G_T(1+n)} c_T.$$

Now the constant proportions hypothesis expressed in equation 1 can be invoked to get c in terms of c_1 and the interest rate:

$$(6)\qquad c = \frac{(1+n)^{T-1}}{G_T(1+n)} c_1 + \frac{(1+n)^{T-2}}{G_T(1+n)} \frac{1+r}{1+\rho} c_1 + \cdots + \frac{1}{G_T(1+n)} \left(\frac{1+r}{1+\rho}\right)^{T-1} c_1.$$

The next step is to substitute for c_1 its expression in terms of wealth from equation 4:

$$(7)\qquad c = \frac{W(1+n)^{T-1}}{G_T\left(\frac{1}{1+\rho}\right) G_T(1+n)} \left[1 + \left(\frac{1+r}{(1+n)(1+\rho)}\right) + \left(\frac{1+r}{(1+n)(1+\rho)}\right)^2 + \cdots + \left(\frac{1+r}{(1+n)(1+\rho)}\right)^{T-1}\right]$$

Now $\frac{(1+n)^{T-1}}{G_T(1+n)} = \frac{1}{G_T\left(\frac{1}{1+n}\right)}$; inserting this and applying the G_T function to the summation inside the brackets gives

$$(8)\qquad c = W \frac{G_T\left(\frac{1+r}{(1+n)(1+\rho)}\right)}{G_T\left(\frac{1}{1+\rho}\right) G_T\left(\frac{1}{1+n}\right)}.$$

Were it not for the dependence of wealth, W, on the interest rate, equation 8 would describe the curve shown in Figure 1. The last step in setting down the algebra of the life-cycle hypothesis is to state explicitly the form of this dependence.

In the absence of inheritance, wealth at the beginning of any individual's career is purely human wealth—that is, the present value of future earnings. Under the simplifying assumption that in the steady state, all individuals receive the same wage, w, individual initial wealth W is

$$W = w\, G_T\left(\frac{1}{1+r}\right). \tag{9}$$

This relationship could be modified to take account of two salient characteristics of lifetime earnings ignored here: the tendency for earnings to rise over an individual's lifetime, and the tendency for people to spend a significant portion of their lifetimes in retirement.[7]

Finally, the description of the relationship between steady-state consumption and the interest rate can be completed by observing that there is a systematic inverse relationship between the steady-state wage and the steady-state interest rate; this can be written

$$w = \phi\,(r). \tag{10}$$

Collecting these results together gives the algebraic expression for the curve in Figure 1:

$$c = \phi(r)\,\frac{G_T\left(\frac{1}{1+r}\right) G_T\left(\frac{1+r}{(1+n)(1+\rho)}\right)}{G_T\left(\frac{1}{1+\rho}\right) G_T\left(\frac{1}{1+n}\right)}. \tag{11}$$

At two interest rates this formula has a simple form; these are $r = n$ and $r = \rho$, where $c = \phi(n)$ and $c = \phi(\rho)$ respectively. That is, at these interest rates, steady-state consumption per person is exactly equal to the wage—all wages are consumed and all property income is saved. Between these interest rates, consumption is less than the wage and net saving is even higher. At interest rates higher than the rate of impatience, $G_T\left(\frac{1+r}{(1+n)(1+\rho)}\right)$ begins to rise rapidly, especially if $1 + r$ exceeds $(1 + n)(1 + \rho)$, since in that region it is the sum of a *divergent* geometric series. Similarly, $G_T\left(\frac{1}{1+r}\right)$ rises rapidly as the interest rate approaches zero, for the same reason. The first of these effects is the mathematical embodiment of the observation made earlier that at high interest rates,

[7] This is done with considerable care in the previously cited study of Tobin, (15).

the positive interest sensitivity of older generations dominates the aggregate response of consumption to the interest rate. The second demonstrates that at low interest rates, the negative effect on wealth of increasing interest rates dominates the response of aggregate consumption.

Exactly how responsive to changes in the interest rate is aggregate consumption? Figure 1 is drawn with the following assumptions: rate of impatience, $\rho = .10$, rate of population growth, $n = .015$, wages as a function of the interest rate, $\phi(r) = \frac{\$1500}{\sqrt{r + .15}}$ per man-year (the form of this function will be justified below), lifetime, $T = 70$ years. These are intended to approximate the contemporary American economy. Figure 1 suggests that if the life-cycle hypothesis holds, aggregate consumption is quite sensitive, indeed, to the interest rate. For example, if the interest rate were to rise by one percentage point, from .10 to .11, consumption per person per year would rise by close to 20 per cent, from \$3000 to \$3560. Thus the traditional conclusion, that the interest response of the consumption of a utility-maximizing individual can be either positive or negative but is sure to be small, cannot possibly be carried over to the case of aggregate consumption and a realistic lifetime. In the latter case, for the relevant range of interest rates (slightly above the rate of impatience), aggregate consumption rises very steeply with the interest rate.

So far we have dealt with consumers' intertemporal decisions in terms of the resulting aggregate consumption. The analysis can also be stated in terms of aggregate asset accumulation. High interest rate sensitivity of consumption turns out to imply high sensitivity of aggregate asset holdings.

Now with a low interest rate, impatience causes individuals to consume more in the early years of their lives than in the late years. This early consumption is financed by borrowing against future earnings. The result is that each consumer is always in debt; his indebtedness reaches a peak around middle age and then declines, reaching zero at his death. Consequently, aggregate asset holdings of consumers are always negative if the rate of interest is less than the rate of impatience. If the interest rate is exactly equal to the rate of impatience, consumers neither save nor dissave, and aggregate assets are zero. Finally, in the realistic case where the interest rate exceeds the rate of impatience, consumers accumulate assets in the early parts of their careers in anticipation of higher consumption in later years. Whenever r exceeds ρ, aggregate asset holdings are positive.

These conclusions could be supported in terms of the algebra of the life-cycle model, by finding individual asset holdings as a function of age and aggregating by the age distribution of the population, exactly as was done above for consumption. Fortunately, this is made unnecessary by the availability of an easy short-cut. If steady-state consumption is different from the wage, the difference must be accounted for by interest receipts on asset holdings. This relation can be turned on its head to give asset holdings in terms of aggregate consumption; we already know how the latter is related to the interest rate.

In the steady state, assets per person are the same in one year as they were in the previous year. Now what remains of last year's assets is $(1 + r) A + w - c$, where A denotes assets per person. But this year there are $1 + n$ people for each person last year. Thus the relation among these variables which defines the steady state is

$$A = \frac{(1 + r) A + w - c.}{1 + n} \tag{12}$$

Solving this for A and substituting $\phi(r)$ for w gives

$$A = \frac{c(r) - \phi(r).}{r - n} \tag{13}$$

By c(r) we mean the consumption demand function shown in Figure 1. If the interest rate is less than the rate of population growth, the numerator of equation 13 is positive and the denominator is negative. If the interest rate lies between n and ρ, the numerator is negative and the denominator is positive. In both cases, aggregate assets are negative. Finally, if r exceeds ρ, both the numerator and denominator are positive, and aggregate assets are positive. This establishes the assertions made above. In Figure 2 we present the relation between assets per person and the interest rate implied by the curve in Figure 1.

2. *General Equilibrium with Life-cycle Savings*

We assume that the standard one-sector growth model technology prevails. Net output per person, y, depends on capital per person, k, according to a production function with diminishing returns to capital, $y = f(k)$. Capital deterioration is accounted for in $f(k)$, so the interest rate will equal the net marginal product of capital, $r - f'(k)$, provided credit markets are competitive. Steady states in this technology are achieved when the part of output not consumed

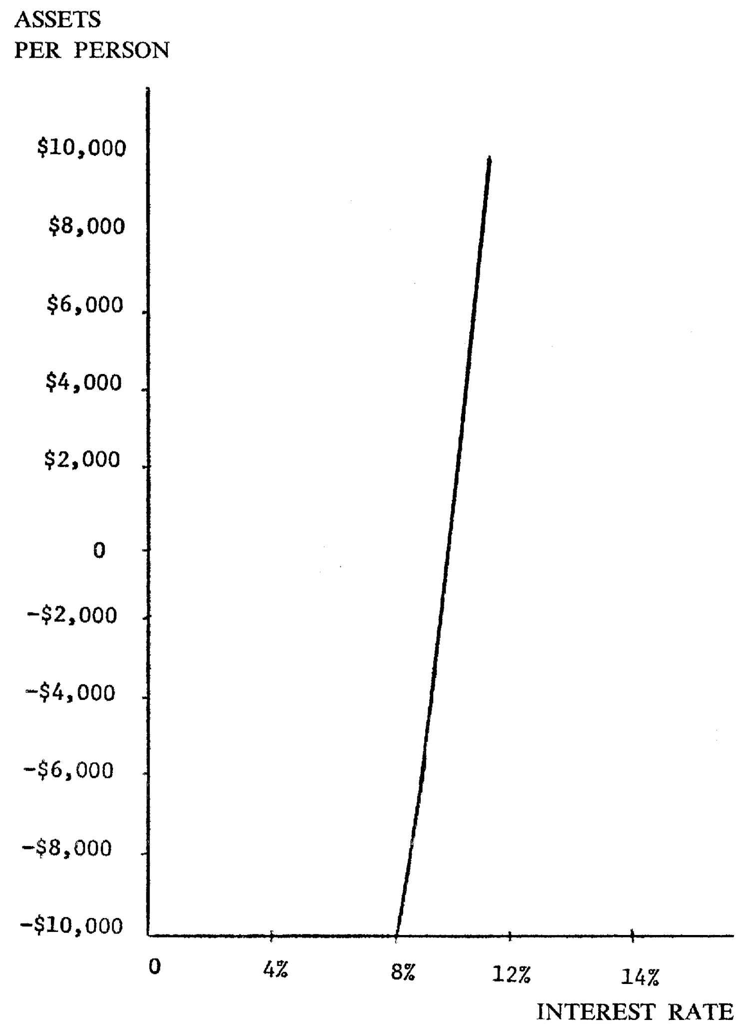

FIGURE 2. Asset Demand Per Person as a Function of the Interest Rate.

is exactly large enough to equip next year's larger labor force with the same amount of capital as this year's. Combinations of steady-state levels of consumption and the interest rate meeting this requirement form the curve shown in Figure 3. At high interest rates, there is a capital shortage, and steady-state consumption is low. At low interest rates, the capital stock per person is so large that most output is devoted to equipping new workers. The maximum point on this curve is at an interest rate equal to the rate of growth of the population; this is the familiar theorem of the Golden Rule.

The production function used in drawing Figure 3 is a Cobb-Douglas function with capital share ⅓ and 15% of capital input lost as deterioration:

$$y = f(k) = 248\, k^{1/3} - .15\, k. \tag{14}$$

Some facts about this function are useful to set down. First, the interest rate corresponding to a lend, k, of capital stock per person is

$$r = f'(k) = k^{-2/3} - .15. \tag{15}$$

Conversely, the amount of capital per person can be calculated from the inverse of equation 15:

$$k = \left(\frac{r + .15}{83}\right)^{-3/2} \tag{16}$$

The wage can be calculated most easily from the fact that factor shares of *gross* output are constant for a Cobb-Douglas production function:

$$w = 165\, k^{1/3}. \tag{17}$$

Our $\phi(r)$ function is obtained by substituting the formula for k in terms of r from equation 16;

$$w = \phi(r) = 1500\,(r + .15)^{-1/2}. \tag{18}$$

Armed with these results, we can look at steady-state general equilibrium in two equivalent ways. First, in a graph with consumption on the vertical axis and the interest rate on the horizontal axis, we can find the equilibrium as the intersection of the consumption demand curve implied by the life-cycle savings hypothesis and the supply curve defined the technology. Equivalently, we could put capital and assets on the vertical axis and the interest rate again on the horizontal axis; then the equilibrium lies at the intersection of the asset demand curve and the marginal product curve giving the relation between capital stock and the interest rate. The logic of the second view is that in equilibrium, the demand for assets must equal the capital stock, since there is no other way to hold wealth in this economy. These two views are illustrated in Figure 4.

On the left side of Figure 4 there is a second intersection at $r = n$ in addition to the expected one at $r > \rho$. Is it possible for this economy to have a steady-state equilibrium at the Golden Rule?

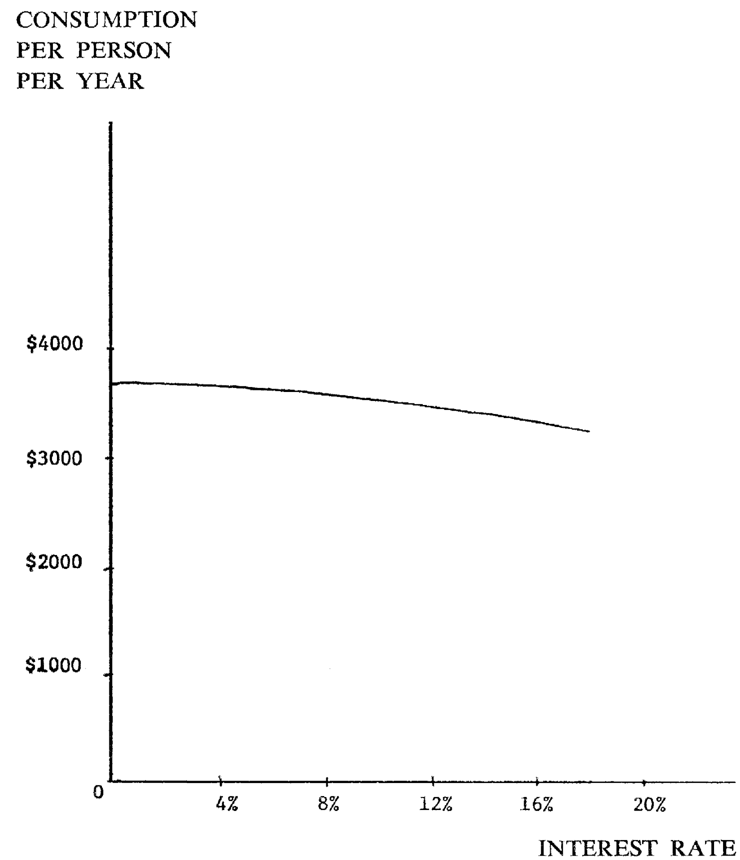

FIGURE 3. Steady-state Consumption Supply as a Function of the Interest Rate in the One-sector Technology.

Here we have a failure (in a sense) of the intertemporal form of Walras' Law — clearing of the output market does not imply clearing of the capital market, since there is no corresponding intersection in the right-hand side of Figure 4. The precise reason for this anomaly need not detain us here; the curious reader should consult Cass and Yaari (3) for an explanation. The capital market diagram makes it clear that the Golden Rule cannot be a steady-state equilibrium in the life-cycle model.

3. *Taxes in the Life-cycle Model of Intertemporal Equilibrium*

A proportional consumption tax [8] can be illustrated most easily on the consumption-interest rate diagram. If the vertical axis is taken to be the actual amount of consumption (as opposed to consumption expenditures, which exceed consumption by the amount of the tax), then the effect of the tax on the steady-state consumption demand curve is to shift it downward proportionally, so that the level of consumption expenditures after the tax is the same, at any interest rate, as the level of consumption before the tax. This follows from the observation that a rational consumer, faced with a proportional tax on consumption, allocates consumption *expenditures* over his lifetime in precisely the same way that he would allocate consumption in the absence of the tax.[9]

On the supply side, we need to derive a supply function that is net of government purchases, since these represent a diversion from consumption. Our assumption is that government purchases are a constant fraction (equal to the tax rate) of the total supply to final demand at any interest rate. This assumption guarantees that the government budget is balanced no matter where the equilibrium falls; it could be replaced without affecting the analysis by any other assumption about government expenditures that resulted in a balanced budget at the particular interest rate characteristic of equilibrium. To be perfectly rigorous, we must also require that neither production nor consumption activities be affected by government expenditures — military expenditures are the archtype in our view of government purchases. Under these assumptions, the supply curve shifts downward proportionally, so that at any interest rate, the net supply after the imposition of a consumption tax and corresponding expenditure program is less than supply in the absence of the expenditures by the amount of the consumption tax yield.

These supply and demand curves are shown in Figure 5. Since both curves shift downward by the same proportion, the after-tax steady-state equilibrium occurs at exactly the same interest rate as the before-tax equilibrium. This establishes the well-known proposition that a consumption tax is intertemporally neutral in the sense that total output is unaffected by the tax and expenditure program — there is no deadweight loss associated with a consumption tax.

This neutrality is not shared by the income tax. As we shall see, total output is reduced by the imposition of an income tax (and balanced-budget expenditure policy); the reduction in consumption

[8] By which we mean an expenditures tax, a sales tax on consumption goods, or a value-added tax with a deduction for investment expenditures.

[9] A rigorous justification of this assertion would require an appeal to the foundation of the life-cycle hypothesis in intertemporal utility maximization.

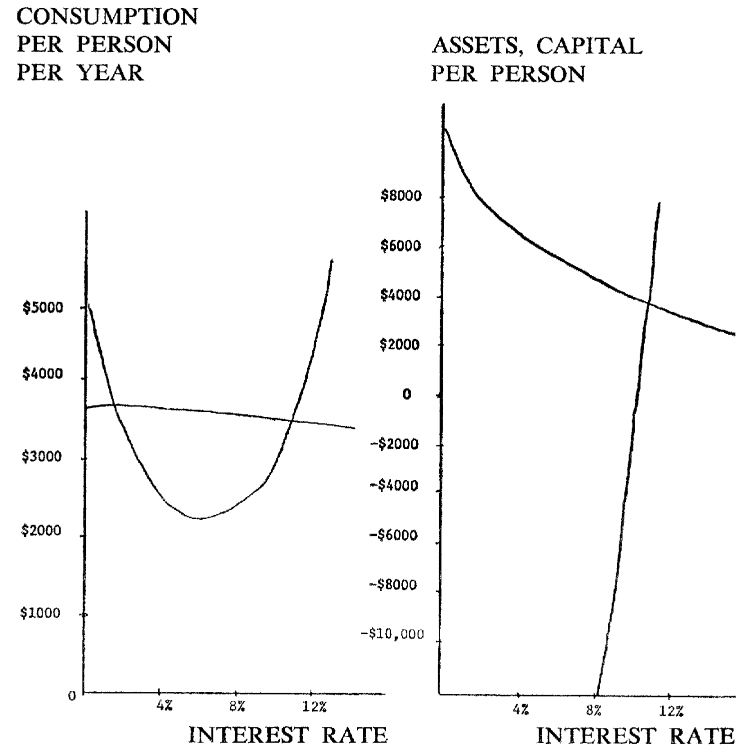

FIGURE 4. Alternative Views of Steady-state Equilibrium in the Life-cycle Savings Model.

exceeds the yield of the tax. This defect of the income tax led Irving Fisher, and later Nicholas Kaldor (7), to very forceful support of consumption taxes to replace income taxes. The argument of the present paper supports the claims of the proponents of a consumption tax in that the general equilibrium analysis confirms their basically partial equilibrium criticism of the income tax. On the other hand, our results suggest that the magnitude of the deadweight loss from an income tax is hardly large enough to cause serious concern.

An income tax has two effects on the life-cycle behavior of consumers. Like the consumption tax, it reduces demand proportionally at equivalent interest rates because of the need to pay the tax. But unlike the consumption tax, the income tax causes the consumer to use a lower interest rate in his consumption planning than the producer uses in his planning. Herein lies the source of the inefficiency of the income tax. It is the familiar double taxation of savings inherent in a tax falling on both wages and interest receipts that accounts for the deadweight loss.

If we continue to put the interest rate paid by producers to their consumer-bondholders on the horizonal axis of the supply and demand diagram, then we must look to the left on the before-tax diagram to find consumption demand after taxes, since the corresponding after-tax interest rate is proportionally lower. Thus the demand curve after taxes will lie proportionally to the *right* of the before-tax curve, as well as below it. Since the demand curve slopes upward quite steeply, it is practically guaranteed that the after-tax interest rate will exceed the before-tax rate by an amount close to the tax rate. In a sense, the question of the effect of the income tax on the interest rate is the same as the question of the shifting of the corporate income tax, in that one way of imposing an income tax is as a combination of a corporate profits tax (assuming all businesses are corporations) and a wage tax.[10] The life-cycle model gives a rather definite answer to the shifting question — the interest rate rises almost enough in the presence of an income tax to maintain the after-tax interest rate at its previous level. That is, tax shifting is close to 100% in this model.

An assumption about government expenditures that parallels our assumption in the consumption tax case is the following: At any interest rate, government expenditures are set equal to the yield of the income tax at the level of income generated by the technology at that interest rate. Again, this assumption is made for convenience in drawing the graphs; the analysis would remain unchanged for any program of government expenditures which had the same level at the equilibrium interest rate.

Supply and demand curves for the case of an income tax of 25% are shown in Figure 6. The new steady-state interest rate is 14.3%, compared to 10.9% in the absence of the tax, implying a rate of shifting of 98.3%. Consumption drops from \$3,487 to \$2,533 per person per year after the imposition of the tax. The yield of the tax (and level of government expenditures) is \$862 per person per year, so the deadweight loss associated with a 25% income tax is \$3487 − (\$2533 + \$862) = \$92 per person per year. By contrast, the 33% consumption tax yields \$872 per person per year and permits consumpion of \$2615 per person per year.

[10] The empirical literature on tax shifting has concentrated almost exclusively on the supply side of the capital market. Some caution is called for in comparing the idea of tax shifting mentioned here in the context of a model with perfect markets, and the usual idea of tax shifting as a characteristic of highly imperfect markets.

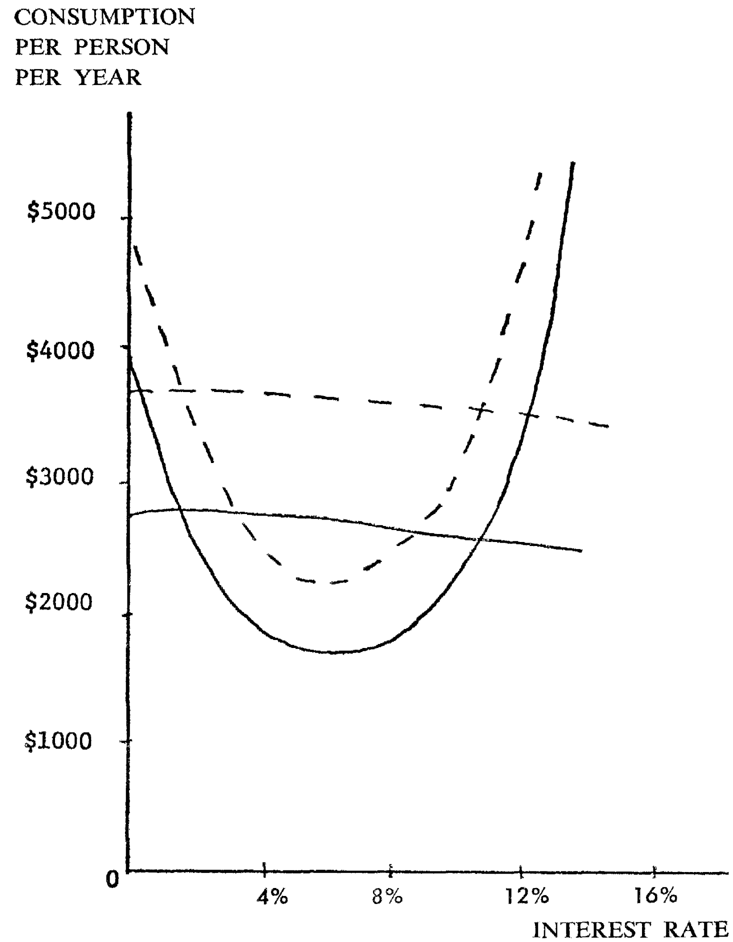

FIGURE 5. Effect of a 33% Tax on Consumption in the Economy of Figure 4. Broken lines represent the economy in the absence of the tax; solid lines represent it with the tax.

Thus we see that the dramatic effects of an income tax are largely confined to a substantial increment in the interest rate. Compared to a neutral alternative source of government revenue (the consumption tax), the income tax is more than 97% efficient in its effect on consumption. What accounts for its good performance? The potentially harmful effect of the income tax is in driving a wedge between consumers' and producers' interest rates. In the diagram this is seen as the deflection of the consumption demand curve to the right. The consequences of this deflection depend on the downward slope of the supply function; if it is steep, it is costly to the economy to have the interest rate driven up. But under realistic assumptions about production, the supply curve turns out to be practically flat. This is basically a consequence of the relatively small share that capital actually has in production. If capital were a more important factor, then the cost of inefficiency in its accumulation would be larger. As it happens, however, the case for a consumption tax over an income tax cannot be made very forcefully on the grounds of efficiency in savings and investment.

4. *Implications for Economic Growth*

In the long run, the rate of growth of the economy is the subject of purely technological determination. Without technical progress, the steady-state rate of growth of output cannot exceed the rate of growth of the population. In view of this limitation, the very long-run effects on the rate of economic growth of alternative tax systems are all the same. But this is by no means the end of the story. As we have seen, different taxes imply different *levels* of economic activity. Any change in the tax system that takes the economy to a new steady-state level causes a temporary (but possibly rather long-lasting) change in the growth rate.

Thus if the federal government were to switch to a sales tax in place of the personal income tax, our analysis would suggest that the rate of growth would be higher for a few years, allowing consumption to rise to make up for the deadweight loss of the current income tax. Calculations based on a dynamic model not presented here indicate that the adjustment rate is high enough to close two-thirds of the gap within five years or so.[11] It is unlikely, however, that the growth would register in any but the most sensitive economic indicator. An overall change in output of three per cent, distributed over several years, would hardly be noticeable in an economy whose movements are not otherwise particularly smooth.

On the other hand, this policy change would have conspicuous effects on economic variables other than output. The interest rate would decline by essentially the full amount of the tax, in reflection of the more generous treatment of savings under a consumption tax. As a result, investment would be stimulated during the period of transition, until the marginal product of capital was driven down to

[11] The dynamic model, but not the calculations, appears in (6).

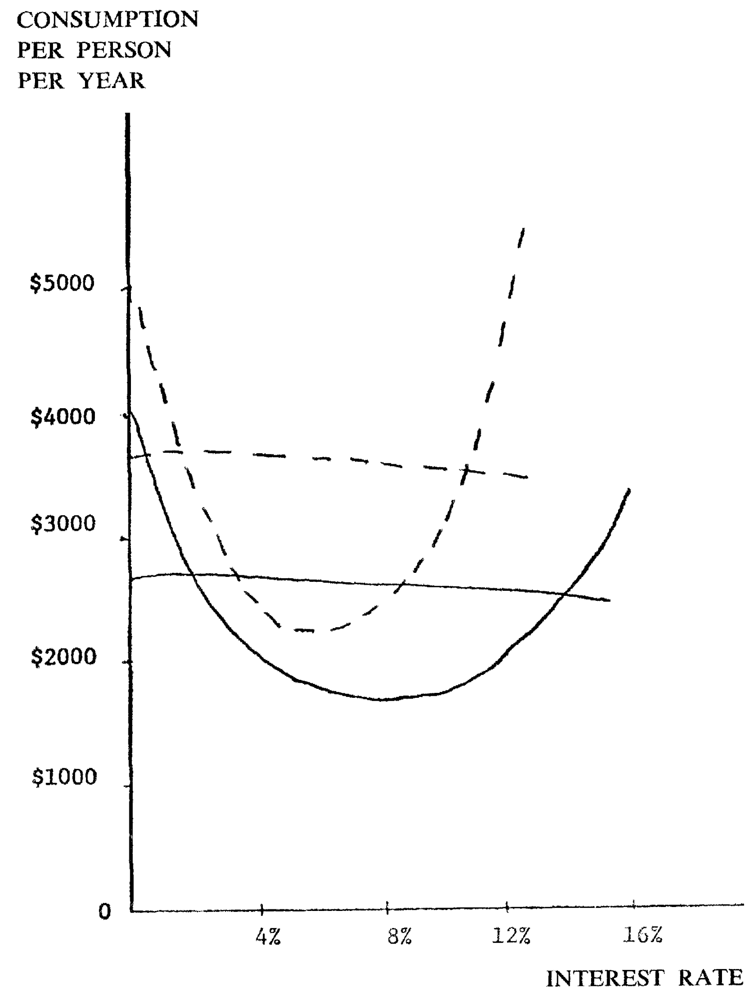

FIGURE 6. Effects of a 25% Income Tax on the Economy of Figure 4. Broken lines represent the economy in the absence of the tax; solid lines represent it with the tax.

the new lower interest rate. More investment requires more saving, which in turn implies a *lower* level of consumption, at least at the beginning of the transition. A higher steady-state level of consumption is obtained only by sacrificing consumption at the beginning of the program. Tax reform, just as any program to bring about growth by stimulating investment, has an immediate cost as well as a long-run benefit.

Our conclusion can hardly be other than that the argument from economic efficiency for a consumption tax is weak indeed. A careful examination of the life-cycle model of intertemporal equilibrium has shown that while there is an inefficiency associated with an income tax, its steady-state value is quite small, and can only be escaped by a temporary further reduction in consumption.

Having stated the conclusions (and, in fact, the whole analysis) in rather a bold fashion, we are obligated to discuss some of the qualifications that necessarily attach to this kind of economics. We shall be particularly concerned with qualifications that suggest a weakening of our conclusions in the sense that they suggest a greater loss associated with a non-neutral tax like the income tax.

The assumption of our model that largely determines our conclusions is that the supply curve for steady-state consumption is almost flat. It would be more steeply downward-sloped if either the elasticity of output with respect to capital were larger, or if substitutibility between capital and labor were larger. On both counts our assumptions were generous. One-third is surely an outside estimate for the elasticity of gross output with respect to capital. The unitary elasticity of substitution of the Cobb-Douglas production function also seems about as high as most economists are likely to agree upon.

The substantive conclusion for the present purpose emerging from the study of the life-cycle savings hypothesis is that the steady-state demand curve for consumption as a function of the interest rate is upward-sloping (though possibly very steep). The fact that it cannot slope downward limits the rate of shifting of the interest tax component of the income tax to 100% or less, and thereby limits the interest rate response to a value less than or equal to the tax rate. Do the realistic qualifications to an admittedly rather stylized model suggest that the demand curve might slope downward? Among the most important of these qualifications is that people lack the predictive ability to make lifetime plans of the kind described in the model. But the principal hypothesis underlying our conclusion that the demand curve slopes upward is that the consumption level of the typical individual rises over his lifetime, and that the interest-sensitivity of older people should be greater than that of younger people (in our usual sense of the comparison of steady states). The first of these is likely to withstand almost any attempt to make the theory more realistic since it is a matter of empirical fact. The second is more debatable; presumably the effect of uncertainty and other com-

plications is to reduce interest sensitivity of people of all ages. As a first approximation, however, it seems unlikely that they might cancel out the formidable influence of interest compounded over the length of a lifetime. Furthermore, we should be cautious about overstating the degree and influence of uncertainty in what is predominantly a middle-class society. Uncertainty about future income is surely greater than uncertainty about how wealth should be spread over a lifetime, yet it is the latter which is important for our conclusion.

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