Complete Markets as an Approximation to the Bewley Equilibrium with Unemployment Risk*

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Abstract

I show that the full-insurance model of consumption choice is a reasonable approximation to the more realistic case of no insurance when the adverse shocks take the form of spells of unemployment. Further, my results support the assumption of the Mortensen-Pissarides model that the flow amenity value of unemployment—the value of not having to work—is a constant when stated in units of consumption. By accumulating precautionary balances, workers are able to keep the common value of the marginal utility of consumption and the marginal value of wealth reasonably close to constant, despite a realistic hazard of unemployment.

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1 Introduction

Unless intertemporal substitution is infinitely elastic, people should avoid transitory reductions in consumption when hit by transitory reductions in income. Unemployment is an important source of the latter. One way to insulate consumption from income lost on account of unemployment is through insurance. Another is build up a buffer stock of savings. I compare worker-consumer behavior in a regime with actuarially fair unemployment insurance to behavior in a Bewley regime lacking any insurance, with a capital market that allows saving but not borrowing against future income.

In reality, moral hazard and adverse selection constrain unemployment insurance. Absent full insurance, families, friends, and government unemployment benefits provide some help to the unemployed. The truth lies somewhere between strict individual self-reliance based on savings alone and the full insurance case. My investigation of the two extremes helps place bounds on the realistic case. An immediate purpose of this paper is to argue that the highly tractable full-insurance case is a reasonable approximation to reality.

I focus on the discrete choice between non-work and full-time work. I believe that the results would carry over to risks of temporary reductions in hourly wages, but have not studied that issue in a formal model.

I compare the distributions of consumption during unemployment spells implied by the two regimes. Because preferences are not separable between consumption and hours of work, but rather have the property that the marginal utility of consumption rises with hours of work, the unemployed choose to consume less than the employed. In the case of full insurance, this lower level of consumption is the same for all of the unemployed, irrespective of the length of the current spell or the individual's history of earlier spells. Without insurance, an unemployed worker chooses consumption based on the worker's savings. As a spell continues, the worker consumes less and less, as savings are depleted. A worker with past bad luck, who starts a spell with low savings, will consume correspondingly less at the outset of the new spell. I find that the distribution of consumption among the uninsured unemployed centers around the single level that the insured would choose, with little dispersion.

2 Preferences

I assume that preferences are time-separable. See Hall (2006) for a full discussion and review of the literature on the properties of preferences. I posit that individuals order consumption-hours pairs according to the period utility of Malin (2006),

$$U(c,h) = \frac{1}{1-\eta}\left[\frac{\underline{c}^{-(1/\sigma-1)} - c^{-(1/\sigma-1)}}{1/\sigma - 1} - \frac{\gamma}{1/\psi+1}h^{1/\psi+1}\right]^{1-\eta}. \quad (1)$$

The kernel inside the brackets governs the marginal rate of substitution between consumption and work within a month and state of the world. A key feature of the specification is to take a concave transformation of the kernel before adding across time periods or states. The parameter η controls the concavity—positive values of η correspond to complementarity of hours and work, with $U_{c,h} > 0$.

2.1 Parameter values

I use the same values for most parameters as in Hall (2006). The calibration of preferences takes a Frisch elasticity of consumption demand of -0.4 and of labor supply of 0.7 and a consumption decline from 1.00 to 0.85 if hours fall from 1.0 to 0. The corresponding values of the parameters of preferences are $\sigma = 0.48$, $\psi = 1.27$, and $\eta = 3.48$. I take the discount factor δ to be $0.95^{1/4}$.

The worker is either not working with $h_0 = 0$ hours of work and cash earnings of $y_0 = b = 0.11$ or working with $h_1 = 1$ and $y_1 = w = 1$. Time is in quarters. A worker who becomes unemployed at the beginning of the quarter spends that quarter unemployed and is re-employed at the beginning of the next quarter. The rate of job destruction is $s = 0.058$. Unemployment follows a two-state Markoff process with stochastic equilibrium

$$u = \frac{s}{s+1} = 5.5 \text{ percent}. \quad (2)$$

The transition matrix is

$$\pi = \begin{bmatrix} 0 & 1 \\ s & 1-s \end{bmatrix}. \quad (3)$$

3 Insured Workers

I consider first the case where workers have access to a market providing actuarially fair insurance against the personal risk of the labor market. Their discount factor δ equals the return factor for savings. Their choices obey the Borch-Arrow condition, equating the marginal utility of consumption across states of the world and across time periods to the common value λ (that is, the time-0 marginal utility is $\delta^t\lambda$). The value of earnings in utility units is λ as well. An unemployed individual chooses consumption to satisfy:

$$U_c(c_s, h_s) = \lambda,\ \ s = 0, 1. \tag{4}$$

3.1 Budget constraint

An individual's budget constraint, given actuarially fair insurance and a probability u of being unemployed, is

$$u \cdot (c_0(\lambda) - b) + (1 - u) \cdot (c_1(\lambda) - w) = 0. \tag{5}$$

Applying the budget constraint requires finding the root, λ, of this equation. The two values $c_0(\lambda)$ and $c_1(\lambda)$ fully specify behavior in the insured case.

3.2 Bellman values

In recursive form, the model with insurance attributes flow values to the two labor-market states:

$$v_s = U(c_s, h_s) - \lambda(c_s - y_s). \tag{6}$$

The model attributes asset or Bellman values to the job-seeker, V_0, and to the worker, V_1. In vector form, these are:

$$V = v + \delta\pi V, \tag{7}$$

so

$$V = (I - \delta\pi)^{-1}\, v. \tag{8}$$

4 Bewley Model

4.1 Bellman equation

Second, I investigate the case where the unemployed are uninsured (apart from benefits b) and cannot borrow against future income, so they face the constraint $c \leq W + y$. Let $V_s(W)$

be the worker's expected present value in state s with non-human wealth W, measured prior to this period's consumption and earnings. The worker's Bellman equation is:

$$V_s(W) = \max_c \left(U(c, h_s) + \delta \sum_{s'} \pi_{s,s'} V_{s'} \left(\frac{W - c + y_s}{\delta} \right) \right). \tag{9}$$

Let $W' = W - c + y_s$ be the wealth carried forward to the next period and define the flow value in state s to be

$$v_s(W') = U(c, h_s) - (V_s(W' + c - y_s) - V_s(W')), \tag{10}$$

The Bellman equation becomes

$$V_s(W') = v_s(W') + \delta \sum_{s'} \pi_{s,s'} V_{s'} \left(\frac{W'}{\delta} \right). \tag{11}$$

In the insured case, V_s is linear, with

$$V_s(W') = \alpha_s + \lambda W', \tag{12}$$

and the maximization in the Bellman equation satisfies the Borch-Arrow condition for insured consumption.

4.2 Solution

In addition to the binary state variable showing which job the worker currently occupies, the uninsured model has the continuous state variable, wealth, W. Following the dictates of Judd (1998), Chapter 12, I represent the value function as piecewise linear, with 100 nodes. Starting with $V_{s,T} = 0$, I iterate backwards, solving for the 100 nodal values of $V_{s,t}$ from the Bellman equation. I take $T = 160$ for a working life of 40 years.

To calculate the distribution of wealth at mid-career, $T/2$, I proceed as follows: From the value function at $T/2$, I compute a grid of 300 values of W satisfying the recursion,

$$W_n = \delta^{-1}(W_{n-1} - c_1 + w). \tag{13}$$

The sequence W_n describes the growth of wealth of a worker who stays employed indefinitely; its limit is an upper bound on W.

I define a worker's overall discretized state as $S = 2n + s$, where n is the bin containing the worker's wealth, W: $W_{n-1} < W \leq W_n$. I define the vector p as the joint distribution of the state s and wealth W, with the latter described by its discrete bin number, n. I

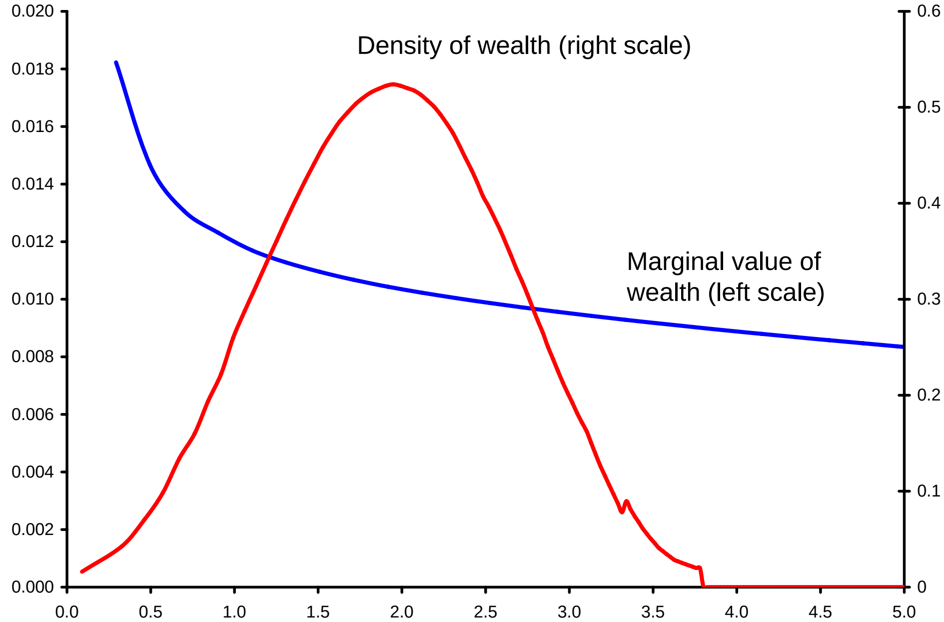

Figure 1. Slope of the Value Function and Stationary Distribution of Wealth

take the initial state of the worker to be unemployed at minimum wealth, so the initial distribution across states is $p_S = 0$ except $p_1 = 1$. I calculate $S_{s,s',n}$, the new state of a worker previously in job s with wealth W_n who has drawn new state s'. From the definition of the W-bin numbered n, $S_{1,1,n} = 2(n+1)+1$, at mid-career. For each quarter, I construct the 600 X 600 transition matrix from $S_{s,s',n}$ and the original transition matrix, $\pi_{s,s'}$. I then multiply p by the transition matrix to form the next distribution across states. I continue the multiplication until I reach mid-career. Finally, I calculate the marginal distribution of W at mid-career. These calculations give close to exact distributions that could have been approximated by simulating employment histories over hundreds of thousands of quarters.

Figure 1 shows the slope of the value function and the marginal distribution of wealth at mid-career. Because the utility discount and interest rate offset each other exactly, the only reason that people accumulate wealth is precautionary—absent the probability of occasional unemployment, workers would hold no wealth. Typical wealth holdings are fairly small, about two quarters of earnings. The interquartile range of wealth is from 1.4 to 2.4 quarters of earnings. Over this range, the marginal value of wealth, V', falls from 0.0121 to 0.0104. In the case of full insurance, the marginal value, λ, is constant at 0.0109.

Figure 2 shows the difference between the insured and uninsured cases in terms of the distribution of consumption while unemployed. With insurance, unemployment consumption

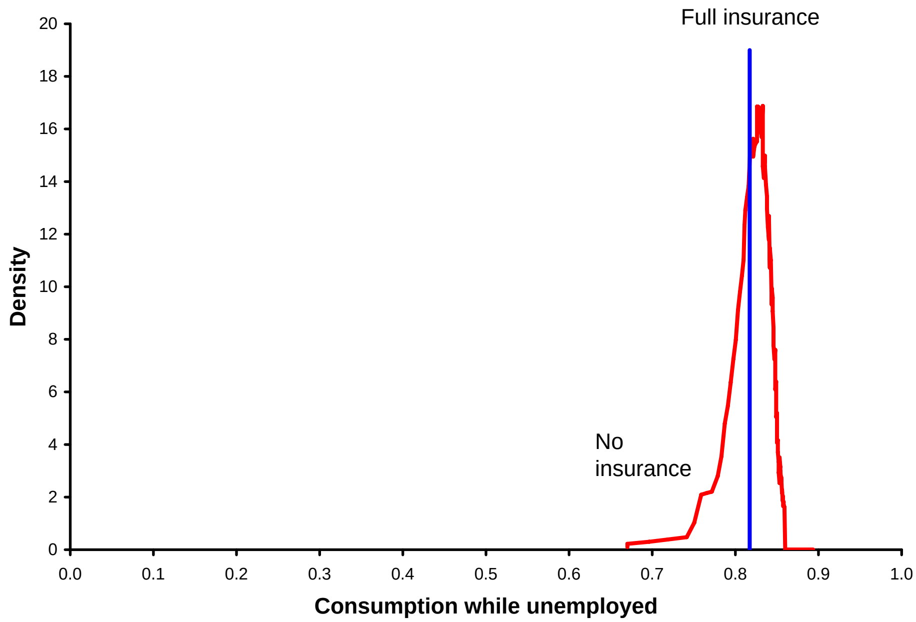

Figure 2. Cumulative Distributions of Consumption While Unemployed, with and without Insurance

is always 82 percent of employment earnings (It is 85 percent of employment consumption, which is slightly below earnings in order to generate savings for unemployment spells). Without insurance, unemployment consumption is less than that amount if unemployment hits when wealth is low and greater if wealth is high. The interquartile range of unemployment consumption is from 79 percent of earnings to 83 percent. The ability to save (but not borrow) gives people a powerful tool to stabilize consumption when cash income drops to 11 percent of its normal level.

Figure 3 compares the full-insurance and no-insurance cases in terms of the distributions of the flow value of unemployment, z. I calculate z in the no-insurance case from equation (33) in Hall (2006), substituting the state-dependent marginal value of wealth, $V_0'(W)$ for the constant marginal value λ. Again, the distribution centers fairly tightly around the value of z in the fully insured case, $z = 0.55$, so using a flow value z based on a constant value of wealth is probably a good approximation.

5 Conclusion

I conclude that the full-insurance model is reasonably close to the more realistic no-insurance model.

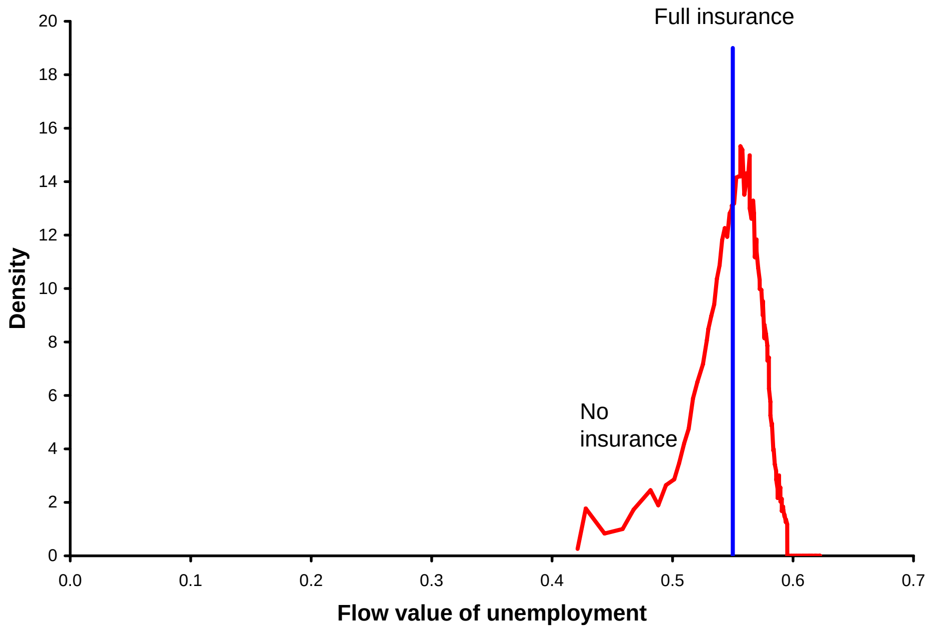

Figure 3. Cumulative Distributions of Flow Value While Unemployed, with and without Insurance

References

Hall, Robert E., "Work-Consumption Preferences, Cyclical Driving Forces, and Employment Volatility," August 2006. stanford.edu/∼rehall.

Judd, Kenneth, *Numerical Methods in Economics*, Cambridge: MIT Press, 1998.

Malin, Benjamin, "Lower-Frequency Macroeconomic Fluctuations: Living Standards and Leisure," February 2006. Department of Economics, Stanford University.