

# Application of the Theory of Optimum Capital Accumulation

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TAX MEASURES FOR CONTROLLING investment expenditures by providing incentives or disincentives through tax credits and accelerated depreciation are now a permanent part of the fiscal policies of the United States and many other countries. The quantitative study of tax incentives, however, has lagged far behind the study of policies that operate directly upon income. For example, the multiplier effect of the tax cut of 1964 has been estimated with some care; much less is known about the quantitative effect of the investment tax credit of 1962. In view of the many current proposals to apply the tax incentive system in other sectors, notably low-cost housing, policies of this kind clearly call for extensive empirical study.

The effectiveness of tax policy in altering investment expenditures has been established in a qualitative sense by a number of authors; their argument can be stated in its essence as follows: If capital services cost less as a result of tax incentives, businessmen will employ more of them.<sup>1</sup> This

<sup>1</sup> The effects of tax policy on investment behavior are analyzed from this point of view by, among others, E. Cary Brown, "The New Depreciation Policy under the Income Tax: An Economic Analysis," *National Tax Journal*, Vol. 8 (March 1955), pp. 81-98; E. Cary Brown, "Tax Incentives for Investment," in American Economic Association, *Papers and Proceedings of the Seventy-fourth Annual Meeting, 1961* (*American Economic Review*, Vol. 52, May 1962), pp. 335-45; Norman B. Ture, "Tax Reform: Depreciation Problems," in American Economic Association, *Papers and Proceedings of the Seventy-fifth Annual Meeting, 1962* (*American Economic Review*, Vol. 53, May 1963), pp. 334-53,

view is not free of ambiguities even at the qualitative level. For example, a reduction in the tax rate would appear to reduce the burden of the corporate income tax and to act as a stimulus to investment. But as Samuelson has demonstrated, a reduction in the tax rate may make assets more attractive, less attractive, or equally attractive to the investor, depending on depreciation allowances for tax purposes.<sup>2</sup> At a further remove, a change in the tax rate may increase, decrease, or leave unchanged the prevailing cost of money.<sup>3</sup> The effect of a reduction in taxes depends on the responsiveness of saving, as well as that of investment, to the proposed change.<sup>4</sup>

Even where the qualitative implications of a tax change are clear and unambiguous, no answers are given to the important questions for economic policy: How much investment? When will it occur? A stimulus to investment may have large or small effects. The resulting investment expenditures may take place immediately or over a considerable period of time. To determine the effects precisely, a quantitative analysis of investment behavior is required. In two previous papers we have presented an econometric model designed specifically to study the effects of tax policy on investment behavior.<sup>5</sup> We have estimated the unknown parameters of this model from annual data on investment expenditures for the nonfarm sector of the United States beginning with 1929. Given the empirical results, we have

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esp. pp. 341-45; Sam B. Chase, Jr., "Tax Credits for Investment Spending," *National Tax Journal*, Vol. 15 (March 1962), pp. 32-52; and Richard A. Musgrave, "Effects of Tax Policy on Private Capital Formation," in Commission on Money and Credit, *Fiscal and Debt Management Policies* (Prentice-Hall, 1963), pp. 53-54 and 117-29.

<sup>2</sup> Paul A. Samuelson, "Tax Deductibility of Economic Depreciation To Insure Invariant Valuations," *Journal of Political Economy*, Vol. 72 (December 1964), pp. 604-06.

<sup>3</sup> The effects of changes in the corporate tax rate on the cost of capital is the subject of much controversy. Recent contributions to the discussion include those of Robert J. Gordon, "The Incidence of the Corporation Income Tax in U.S. Manufacturing, 1925-62," *American Economic Review*, Vol. 57 (September 1967), pp. 731-58; Marian Krzyzaniak and Richard A. Musgrave, *The Shifting of the Corporation Income Tax: An Empirical Study of Its Short-run Effect upon the Rate of Return* (Johns Hopkins Press, 1963); and John G. Cragg, Arnold C. Harberger, and Peter Mieszkowski, "Empirical Evidence on the Incidence of the Corporation Income Tax," *Journal of Political Economy*, Vol. 75 (December 1967), pp. 811-21.

<sup>4</sup> Little is known about the effects of the rate of return on saving. Harberger assumes that changes in the rate of return leave saving unchanged. See Chap. 7.

<sup>5</sup> Robert E. Hall and Dale W. Jorgenson, "Tax Policy and Investment Behavior," *American Economic Review*, Vol. 57 (June 1967), pp. 391-414, and Robert E. Hall and Dale W. Jorgenson, "The Role of Taxation in Stabilizing Private Investment," in Vincent P. Rock (ed.), *Policymakers and Model Builders: Cases and Concepts* (Washington: Gordon and Breach, 1969).

calculated the effects of tax policy on investment behavior in the postwar period. Specifically, we have studied the effects of the adoption of accelerated depreciation in 1954, new lifetimes for depreciation in 1962, the investment tax credit in 1962 and its modification in 1964, and suspension of the investment tax credit in 1966–67.

The purpose of this study is similar to that of our previous work. We first reestimate our econometric model of investment behavior, taking into account data that have become available since our earlier work. We have revised our econometric technique to take advantage of recently developed methods of estimation. With these changes we obtain a new set of investment functions for the nonfarm sector of the United States, which we employ to characterize the effects of the various measures adopted between 1954 and 1967. We calculate both the impact of the suspension of the tax credit actually in effect from October 1966 to March 1967 and the hypothetical results of the originally proposed suspension through December 1967.

The evolution of tax policies during the postwar period provides a broad range of experience for a quantitative study of their effects on investment behavior. On the basis of our analysis, we conclude that tax policy has been highly effective in changing the level and timing of investment expenditures. It has also affected the composition of investment expenditures in the nonfarm sector. The adoption of accelerated methods for depreciation and the reduction in depreciation lifetimes for tax purposes increased investment expenditures substantially. They also resulted in a shift in the composition of investment away from equipment toward structures. Limited to equipment, the investment tax credit has been a potent stimulus to the level of investment; it has also shifted the composition of investment toward equipment.

An econometric model of investment behavior has a decisive advantage over a purely qualitative analysis of the effects of tax policy as a basis for policy making. At the same time our study has significant limitations that must be made explicit at the outset. Our calculations are based on a partial equilibrium analysis of investment behavior. A general equilibrium analysis would be required to determine the full effects of a change in tax policy. We calculate the effects of tax policy on investment behavior given the prices of investment goods, the cost of financial capital, and the level and price of output. Obviously, the results derived from a complete econometric model—incorporating our econometric model of investment and an explanation of the prices of investment goods, the cost of financial capital, and the level and price of output—could differ substantially. Since no econometric

model of this scope is currently available, such a general equilibrium analysis of tax policy, however desirable, is not now feasible. For quantitative analysis we are forced to choose between an econometric model of investment behavior that adequately reflects the direct effects of tax policy on investment and general equilibrium analysis based on the more traditional ad hoc explanations of investment behavior. This important gap in the study of macroeconomic models could be remedied by combining our model of investment behavior with an explanation of the supply of investment goods, the supply of and demand for consumer goods, and the supply of saving.

## Theory of Investment Behavior

Our econometric model of investment behavior is based on the theory of optimal capital accumulation. This theory can be approached from two alternative and equivalent points of view.<sup>6</sup> In the first, the objective of the firm may be taken as the maximization of its market value. Given a recursive description of technology—output depending on the flow of current input and of capital services, and capital depending on the level of investment and the past value of capital—maximization of the market value of the firm implies that the marginal product of each current input is equal to its real price and the marginal product of each capital service is equal to its real rental. In the second approach, the objective of the firm is maximization of profit, defined as the difference between current revenue and current outlay less the rental value of capital services. The rental price of capital services is determined from the condition of market equilibrium that equates the value of an asset and the sum of discounted values of all capital services from that asset. These two approaches lead to the same theory of the firm. In this study, the maximization of profit is taken as the objective of the firm and an appropriate price of capital services determined from the price of capital assets. Tax policy affects investment behavior through the price of capital services.

There are, however, two objections to the theory of optimal capital accumulation as a basis for an econometric model of investment behavior. First, a substantial body of survey data suggests that “marginalist” considerations such as the cost of capital and tax policy are irrelevant to

<sup>6</sup> These alternative points of view on the theory of optimal capital accumulation are described in more detail in Dale W. Jorgenson, “The Theory of Investment Behavior,” in Robert Ferber (ed.), *Determinants of Investment Behavior*, a Conference of the Universities-National Bureau Committee for Economic Research (Columbia University Press for the National Bureau of Economic Research, 1967).

business decisions to invest. This evidence has, however, been carefully analyzed by William H. White, who concludes that the survey data are defective even by the standards of noneconometric empirical work and that no reliance can be placed on conclusions drawn from them.<sup>7</sup> A second objection is that previous attempts to analyze investment behavior on the basis of neoclassical theory have not been successful. This objection is valid so far as the first such attempts are concerned. Negative results have been reported by Jan Tinbergen, Charles Roos, and Lawrence Klein for models incorporating marginalist considerations.<sup>8</sup> However, an econometric model based on current formulations of the neoclassical theory provides a better explanation of investment expenditures than its competitors, the flexible accelerator model studied intensively by Robert Eisner, and the models containing combinations of capacity utilization, liquidity, and the rate of interest studied by Locke Anderson and by John Meyer and Robert Glauber.<sup>9</sup> Further, the predictive performance of the neoclassical model is as satisfactory as that of models based on alternative theories of investment behavior.<sup>10</sup>

<sup>7</sup> William H. White, "Interest Inelasticity of Investment Demand—The Case from Business Attitude Surveys Re-examined," *American Economic Review*, Vol. 46 (September 1956), pp. 565–87.

<sup>8</sup> Negative results were reported in Jan Tinbergen, *Statistical Testing of Business-Cycle Theories*, Vol. 1, "A Method and Its Application to Investment Activity" (Geneva: League of Nations, 1939). Subsequently, similar results were reported in Charles F. Roos, "The Demand for Investment Goods," in American Economic Association, *Papers and Proceedings of the Sixtieth Annual Meeting, 1947* (*American Economic Review*, Vol. 38, May 1948), pp. 311–20; Charles F. Roos and Victor S. Von Szeliski, "The Demand for Durable Goods," *Econometrica*, Vol. 11 (April 1943), pp. 97–122; Lawrence R. Klein, *Economic Fluctuations in the United States, 1921–1941* (Wiley, 1950), esp. pp. 14–40; Lawrence R. Klein, "Studies in Investment Behavior," in Universities-National Bureau Committee for Economic Research, *Conference on Business Cycles* (National Bureau of Economic Research, 1951).

<sup>9</sup> Robert Eisner, "Realization of Investment Anticipations," in James S. Duesenberry, Gary Fromm, Lawrence R. Klein, and Edwin Kuh (eds.), *The Brookings Quarterly Econometric Model of the United States* (Chicago: Rand McNally, 1965; Amsterdam: North-Holland, 1965); W. H. Locke Anderson, *Corporate Finance and Fixed Investment* (Harvard University, Graduate School of Business Administration, Division of Research, 1964); and John R. Meyer and Robert R. Glauber, *Investment Decisions, Economic Forecasting, and Public Policy* (Harvard University, Graduate School of Business Administration, Division of Research, 1964). See also Dale W. Jorgenson, Jerald Hunter, and M. Ishag Nadiri, "A Comparison of Alternative Econometric Models of Quarterly Investment Behavior," *Econometrica*, Vol. 38 (March 1970), pp. 187–212.

<sup>10</sup> The predictive performance of these alternative econometric models is compared in

In addition to the direct support for the neoclassical theory from econometric studies of investment behavior, indirect support overwhelmingly favorable to it is provided by econometric studies of cost and production functions.<sup>11</sup> Current empirical research emphasizes such technical questions as the appropriate form for the production function and the statistical specification of econometric models of production. As an example, Nerlove has recently surveyed the literature, running to more than forty references, devoted solely to estimation of the elasticity of substitution.<sup>12</sup> In it, the neoclassical theory of the firm is taken as a point of departure. The purpose of the empirical research reviewed by Nerlove is to give more precise results within the framework provided by neoclassical theory.

#### *Rental Price of Capital*

To be noted first in this detailed analysis of the relationship between tax policy and investment behavior is that the objective of the firm is to maximize profit. Profit  $Z'_B$  is defined in a special sense as the difference between current revenue and current outlay less the rental value of capital services:

$$Z'_B = pQ - wL - cK, \quad (2.1)$$

where

- $p$  = the price of output
- $Q$  = the quantity of output
- $w$  = the price of labor input
- $L$  = the quantity of labor input
- $c$  = the rental price of capital
- $K$  = the quantity of capital.

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Dale W. Jorgenson, Jerald Hunter, and M. Ishag Nadiri, "The Predictive Performance of Econometric Models of Quarterly Investment Behavior," *Econometrica*, Vol. 38 (March 1970), pp. 213-24.

<sup>11</sup> Three hundred forty-five references, almost all presenting the results of econometric tests of the neoclassical theory of the firm, are listed in a recent survey of the econometric literature on cost and production functions. See A. A. Walters, "Production and Cost Functions: An Econometric Survey," *Econometrica*, Vol. 31 (January-April 1963), pp. 1-66.

<sup>12</sup> Marc Nerlove, "Recent Empirical Studies of the CES and Related Production Functions," in Murray Brown (ed.), *The Theory and Empirical Analysis of Production* (Columbia University Press for the National Bureau of Economic Research, 1967).

Profit is maximized at each point of time subject to a production function,

$$Q = \Phi(L, K). \quad (2.2)$$

Investment  $I$  is the sum of changes in capital stock  $\dot{K}$  and of replacement. We assume that replacement is proportional to capital so that investment may be determined from the relationship,

$$I = \dot{K} + \delta K, \quad (2.3)$$

where  $\delta$  is the rate of replacement.

Necessary conditions for profit maximization are that the marginal product of current input is equal to its real price,

$$\partial\Phi/\partial L = w/p; \quad (2.4)$$

similarly, the marginal product of capital input is equal to its real rental,

$$\partial\Phi/\partial K = c/p. \quad (2.5)$$

Second, the price of new capital goods  $q$  must equal the present value of future rentals.<sup>13</sup> In the absence of direct taxation this relationship takes the form

$$q(t) = \int_t^{\infty} e^{-r(s-t)} c(s) e^{-\delta(s-t)} ds, \quad (2.6)$$

where

$r$  = the rate of return

$c$  = the rental price of capital input

$e^{-\delta(s-t)}$  = the quantity of capital input at time  $s$  resulting from the purchase of one unit of the capital asset at time  $t$ .

If prices of new investment goods are expected to remain stationary,<sup>14</sup>

$$c = q(r + \delta). \quad (2.7)$$

<sup>13</sup> Here we assume that investment is fully reversible. A discussion of the relationship between the price of capital goods and the present value of future rentals where investment is irreversible may be found in Kenneth J. Arrow, "Optimal Capital Policy with Irreversible Investment," in James N. Wolfe (ed.), *Value, Capital and Growth: Papers in Honour of Sir John Hicks* (Edinburgh: Edinburgh University Press, 1968).

<sup>14</sup> A detailed derivation is given in Jorgenson, "The Theory of Investment Behavior."

For the nonfarm sector of the U.S. economy, taxes are imposed on current revenue less outlay on current input and less certain deductions on capital account. As an approximation, taxation is represented in the nonfarm sector by the corporate income tax. It is assumed that business income is taxed at a constant marginal rate with deductions allowed for interest payments and for depreciation on capital assets. In addition a tax credit is allowed on the acquisition of new investment goods. Where the before-tax rate of return  $\rho$  reflects deductions of interest allowed for tax purposes, the relationship between the price of new capital goods and the present value of all future rentals and tax deductions becomes

$$q(t) = \int_t^{\infty} e^{-(1-u)\rho(s-t)} [e^{-\delta(s-t)}(1-u)c(s) + uq(t)D(s-t)] ds + kq(t). \quad (2.8)$$

The rental value of capital services after taxes is  $(1-u)c$ , where  $u$  is the tax rate. The depreciation formula  $D(s-t)$  gives the depreciation allowance per dollar of initial investment for tax purposes at time  $t$  on an asset of age  $(s-t)$ . Note that depreciation allowances depend on the price at which the asset is acquired  $q(t)$ , not the price of assets at the time depreciation is allowed as a charge against income  $q(s)$ . Finally, the tax credit is  $kq(t)$ , where  $k$  is the proportion of the value of the asset allowable as a credit against taxes; the tax credit is not deducted from the amount of depreciation to be claimed. This formulation is inappropriate to the tax credit for the years 1962 and 1963, during the period the Long amendment was in effect. Under this amendment the tax credit was deducted from allowable depreciation so that

$$q(t) = \int_t^{\infty} e^{-(1-u)\rho(s-t)} [e^{-\delta(s-t)}(1-u)c(s) + uq(t)(1-k)D(s-t)] ds + kq(t). \quad (2.9)$$

As before, it is assumed that the prices of new investment goods (and the rate of the investment tax credit) are expected to remain stationary. The relationship between the price of capital services  $c$  and the price of capital assets  $q$  after repeal of the Long amendment was

$$c = q[(1-u)\rho + \delta](1-k-uz)/(1-u), \quad (2.10)$$

where

$$z = \int_t^{\infty} e^{-(1-u)\rho(s-t)} D(s-t) ds \quad (2.11)$$

may be interpreted as the present value of depreciation deductions totaling



one dollar over the lifetime of the investment. Before repeal of the Long amendment, when the tax credit was deducted from the allowable depreciation base, the relationship was

$$c = q[(1-u)\rho + \delta](1-k)(1-uz)/(1-u). \quad (2.12)$$

Considering the impact of changes in the tax structure on the price of capital services, an increase in the investment tax credit  $k$  will always reduce the price of capital services. Where the investment tax credit is deducted from allowable depreciation, this credit has precisely the effect of a direct subsidy to the purchase of investment goods.<sup>15</sup> Second, an increase in the present value of depreciation deductions  $z$ , resulting from a reduction in lifetimes of investment goods allowable for tax purposes or from the use of accelerated depreciation formulas, reduces the price of capital services.

The effect of a change in the tax rate  $u$  on the price of capital services depends on the effect of such a change on the rate of return. If the before-tax rate of return  $\rho$  is held constant, a change in the tax rate is neutral in its effects on the price of capital services if the combined value of depreciation allowances and the investment tax credit is equal to the value of "economic" depreciation, where economic depreciation corresponds to

$$D(s-t) = \delta e^{-\delta(s-t)}. \quad (2.13)$$

The present value of economic depreciation  $z^*$  is

$$\begin{aligned} z^* &= \int_t^{\infty} e^{-(1-u)\rho(s-t)} \delta e^{-\delta(s-t)} ds, \\ &= \delta/(1-u)\rho + \delta. \end{aligned} \quad (2.14)$$

Provided that

$$k + uz = uz^*,$$

then

$$c = q(\rho + \delta),$$

so that the price of capital services is unaffected by changes in the tax rate.<sup>16</sup>

<sup>15</sup> A direct subsidy at the rate  $k$  results in a cost of acquisition of investment goods  $q(1-k)$ . With tax rate  $u$  and present value of depreciation  $z$ , the same formula is obtained for the rental price of capital as for the investment tax credit,  $c = q(1-k)[(1-u)\rho + \delta] \times (1-uz)/(1-u)$ .

<sup>16</sup> Similar results are given in E. Cary Brown, "Business-Income Taxation and Investment Incentives," in Lloyd A. Metzler and others, *Income, Employment and Public Policy: Essays in Honor of Alvin H. Hansen* (W. W. Norton, 1948); and in Samuelson, "Tax Deductibility of Economic Depreciation To Insure Invariant Valuations."

On the other hand, if the after-tax rate of return  $r = (1-u)\rho$  is held constant, a change in the tax rate is neutral if the combined value of depreciation allowances and the investment tax credit is equal to the value of immediate expensing of assets. Provided that

$$k + uz = u,$$

then

$$c = q(r + \delta);$$

thus the price of capital services is unaffected by changes in the tax rate.<sup>17</sup>

Therefore, if the before-tax cost of capital is fixed, changes in the tax rate have no effect on the price of capital services when the combined effect of the investment tax credit and depreciation allowances for tax purposes is equivalent to economic depreciation. Second, if the after-tax cost of capital is fixed, changes in the tax rate are neutral when the combined effect is equivalent to immediate expensing of assets. Thus the neutrality of changes in the tax rate depends on whether the burden of the tax is borne by the firm (before-tax cost of capital constant) or shifted (after-tax cost of capital constant). To resolve the controversy surrounding the incidence of the corporate income tax requires a general equilibrium analysis based on an econometric model including saving as well as investment.<sup>18</sup> Here it is assumed that the burden of the tax is borne by the firm, that is, that the before-tax rate of return is unaffected by changes in the tax rate.

#### *Depreciation Formulas*

**STRAIGHT LINE.** Prior to the Revenue Act of 1954, essentially the only depreciation formula permitted for tax purposes was the straight-line formula, with a constant stream of depreciation over the lifetime of the

<sup>17</sup> Similar results are given in Brown, "Business-Income Taxation and Investment Incentives"; in Richard A. Musgrave, *The Theory of Public Finance* (McGraw-Hill, 1959); and in Vernon L. Smith, "Tax Depreciation Policy and Investment Theory," *International Economic Review*, Vol. 4 (January 1963), pp. 80-91.

<sup>18</sup> For recent contributions to this controversy, see Gordon, "Incidence of the Corporation Income Tax in U.S. Manufacturing"; Krzyzaniak and Musgrave, *Shifting of the Corporation Income Tax*; and Cragg and others, "Empirical Evidence on the Incidence of the Corporation Income Tax." Our assumption of "no shifting"—that is, the before-tax rate of return is unaffected by changes in the tax rate—is supported by the results reported by Gordon. Alternative assumptions are suggested by Cragg and others and by Krzyzaniak and Musgrave. None of these empirical results is based on a complete econometric model appropriate to a general equilibrium analysis of the incidence of the corporate income tax.

asset. This formula can be expressed

$$D(\tau) = 1/T, \quad 0 \leq \tau \leq T, \quad (2.15)$$

where  $T$  is the lifetime and  $\tau = s-t$  is the age of the asset. The present value of depreciation deductions under the straight-line formula is

$$z = [1/(1-\mu)\rho T][1 - e^{-(1-\mu)\rho T}]. \quad (2.16)$$

Under the Revenue Act of 1954, three depreciation formulas were allowed for tax purposes. As alternatives to the straight-line formula, taxpayers were permitted to employ the sum-of-the-years-digits and declining-balance formulas. These two formulas are known as accelerated methods of depreciation because for a given lifetime and cost of capital they result in higher present values of depreciation deductions than the straight-line method.

**SUM-OF-THE-YEARS-DIGITS.** In the sum-of-the-years-digits method the deduction for depreciation declines linearly over the lifetime of the asset, starting at twice the corresponding straight-line rate; the depreciation formula is <sup>19</sup>

$$D(\tau) = 2(T-\tau)/T^2 \quad 0 \leq \tau \leq T. \quad (2.17)$$

The present value of depreciation deductions under this formula is <sup>20</sup>

$$z = [2/(1-u)\rho T]\{1 - [1 - e^{-(1-u)\rho T}]/[(1-u)\rho T]\}. \quad (2.18)$$

**DECLINING BALANCE.** In the declining-balance method of depreciation, the deduction drops exponentially over the lifetime of the asset starting at a fixed proportion of the straight-line rate. If this proportion  $\theta$  is 2, the method is referred to as double declining balance; if the proportion is 1.5 the method is called 150 percent declining balance. Tax provisions permit taxpayers to switch from the declining-balance to straight-line depreciation at any point during the lifetime of the asset. Obviously, the switchover point that maximizes the present value of the depreciation deduction  $T^*$  occurs where the flow of declining-balance depreciation equals the flow of straight-line depreciation after the switch. The declining-balance depreciation formula is

$$D(\tau) = \begin{cases} (\theta/T)e^{-(\theta/T)\tau}, & 0 \leq \tau \leq T^*, \\ \frac{e^{-(\theta/T)T^*}}{T-T^*}, & T^* \leq \tau \leq T. \end{cases} \quad (2.19)$$

<sup>19</sup> This is formula (7) in our earlier paper, Hall and Jorgenson, "Tax Policy and Investment Behavior," p. 394.

<sup>20</sup> See formula (8), *ibid.*

When (2.19) is solved for the optimal switchover point,

$$T^* = T[1 - (1/\theta)]. \quad (2.20)$$

The present value of depreciation under the declining-balance method is <sup>21</sup>

$$z = \frac{\theta/T}{(1-u)\rho + (\theta/T)} [1 - e^{-[(1-u)\rho + (\theta/T)]T^*}] + \frac{e^{-(\theta/T)T^*}}{(1-u)\rho(T - T^*)} [e^{-(1-u)\rho T^*} - e^{-(1-u)\rho T}]. \quad (2.21)$$

### *Production Functions*

Econometric implementation of a theory of investment behavior based on the neoclassical theory of optimal capital accumulation requires an appropriate form for the production function. The choice of the form has been the subject of much empirical research, which is currently focused on the choice of an appropriate value for the elasticity of substitution. Summarizing his recent survey of this research, Zvi Griliches finds that "the studies based on cross-sectional data yield estimates which are on the whole not significantly different from unity. The time series studies report, on the average, substantially lower estimates."<sup>22</sup>

In an attempt to reconcile this basic conflict between the estimates based on time series and on cross-sectional data, Griliches modifies in three ways the regression of output per employee on the real wage (both in logarithms) employed by Arrow, Chenery, Minhas, and Solow:<sup>23</sup> (1) measures of labor quality are introduced into the regression; (2) regional dummy variables are introduced to take account of possible differentials in price of output and labor quality by region; and (3) allowance is made for the possibility of serial correlation in the error term due to persistence of omitted variables.<sup>24</sup> The resulting cross-sectional estimates of the elasticity of substitution are similar to previous estimates. Only one (out of seventeen) of

<sup>21</sup> This result corrects an error in formula (9), *ibid.* Fortunately, this error did not affect any of the empirical results presented in that or the subsequent paper, Hall and Jorgenson, "Role of Taxation in Stabilizing Private Investment."

<sup>22</sup> Zvi Griliches, "Production Functions in Manufacturing: Some Preliminary Results," in Brown, *Theory and Empirical Analysis of Production*, p. 285.

<sup>23</sup> Kenneth J. Arrow, Hollis B. Chenery, B. S. Minhas, and Robert M. Solow, "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics*, Vol. 43 (August 1961), pp. 225-50.

<sup>24</sup> Griliches, "Production Functions in Manufacturing," p. 290.

these estimates of the elasticity of substitution is significantly different from unity, and that one is above unity.<sup>25</sup> Allowing for serial correlation of the errors in successive years, Griliches obtains estimates for successive cross sections that he characterizes as “. . . not . . . very different from unity, the significant deviations if anything occurring above unity rather than below it.”<sup>26</sup> “I do not intend to argue,” Griliches concludes from these and additional estimates of the elasticity of substitution, “that these results prove that the Cobb-Douglas [elasticity of substitution equal to unity] is the right form for the manufacturing production function, only that there is no strong evidence against it. Until better evidence appears, there is no reason to give it up as the maintained hypothesis.”<sup>27</sup> On the basis of the results presented by Griliches and the work that both he and Nerlove surveyed,<sup>28</sup> we adopt the Cobb-Douglas production function for our theory of investment behavior. This form was used in our earlier studies.<sup>29</sup>

#### *Capital Accumulation*

If there is no lag in the completion of investment projects, the level of investment appropriate for optimal capital accumulation may be determined from the conditions necessary for maximization of profit. In the theory of investment behavior described below, the assumption is that the actual level of capital stock may differ from the optimal level. More specifically, given capital stock, the levels of output and current input are assumed to be determined from the production function and the marginal productivity condition for current input. The desired level of capital is determined from the actual level of output, given the marginal productivity condition for capital input, while the actual level of capital is determined by past investment. Finally, time is required for the completion of new investment projects. Projects are initiated at every point in time so that the actual level of capital plus the backlog of uncompleted projects is equal to the desired level of capital.

If the production function has the Cobb-Douglas form, the marginal

<sup>25</sup> *Ibid.*, p. 292.

<sup>26</sup> *Ibid.*

<sup>27</sup> *Ibid.*, p. 297.

<sup>28</sup> Nerlove, “Recent Empirical Studies of the CES and Related Production Functions.”

<sup>29</sup> Hall and Jorgenson, “Role of Taxation in Stabilizing Private Investment”; and Hall and Jorgenson, “Tax Policy and Investment Behavior.”

productivity condition for capital input may be written

$$\alpha(Q/K^*) = (c/p), \quad (2.22)$$

where  $\alpha$  is the elasticity of output with respect to capital input and  $K^*$  is the desired level of capital. To solve for desired capital,

$$K^* = \alpha(pQ/c). \quad (2.23)$$

To represent the theory of investment, the proportion of investment projects initiated in time  $t$  and completed in period  $t+\tau$  was designated  $\mu_\tau$ . It is assumed that the sequence of proportions  $\mu_\tau$  depends only on the time elapsed between initiation of a project and its completion. New projects are held to be initiated in each period until the backlog of uncompleted projects is equal to the difference between desired and actual capital. Under this assumption new investment orders in each period are equal to the change in desired capital stock. In every period the level of actual net investment is a weighted average of projects initiated in previous periods,

$$I_t - \delta K_{t-1} = \mu_0[K_t^* - K_{t-1}^*] + \mu_1[K_{t-1}^* - K_{t-2}^*] + \dots, \quad (2.24)$$

where  $I_t$  is gross investment and  $\delta K_{t-1}$  is replacement investment.

To make the notation more concise, it is useful to use the lag operator  $S$ , defined as:

$$Sx_t = x_{t-1},$$

for any sequence  $x_t$ . With this notation, the expression for the level of net investment given above may be written more compactly as

$$I_t - \delta K_{t-1} = \mu(S)[K_t^* - K_{t-1}^*], \quad (2.25)$$

where

$$\mu(S) = \mu_0 + \mu_1 S + \dots, \quad (2.26)$$

is a power series in the lag operator.

### Summary

To summarize, investment in period  $t$  depends on the capital stock at the beginning of the period and changes in the desired level of capital in previous periods. The form of the relationship depends on the form of the distributed lag function and the rate of replacement. The desired level of capital depends on the level of output, the price of output, and the rental price of capital input. Tax policy affects investment behavior through the rental price of capital input. This price depends on the price of investment

goods, the cost of capital, the tax rate, the formulas for calculating depreciation allowances for tax purposes, and the level of the investment tax credit. A change in tax policy changes the rental price of capital input and consequently the desired level of capital stock. An increase in desired capital stock generates net investment; if the price of capital input and the other determinants of desired capital remain constant, net investment declines to zero as capital stock approaches its desired level. The change in tax policy continues to affect gross investment through replacement requirements for a permanently larger capital stock.

## Econometrics of Investment Behavior

Our theory of investment behavior implies a distributed lag relationship between net investment and changes in the desired level of capital. If this theory is to be implemented econometrically, restrictions must be imposed on the sequence of  $\mu_t$  coefficients. In previous studies we have employed the restriction that this sequence has a rational generating function. With this restriction the power series  $\mu(S)$  may be represented as the ratio of two polynomials in the lag operator, that is, a rational function of the lag operator,

$$\mu(S) = \gamma(S)/\omega(S). \quad (2.27)$$

The resulting rational distributed lag function may be written as a mixed moving average and autoregressive scheme in changes in the desired level of capital and net investment.<sup>30</sup> Second, a random component  $\varepsilon_t$  must be added to the distributed lag function,

$$\omega(S)[I_t - \delta K_{t-1}] = \gamma(S)[K_t^* - K_{t-1}^*] + \varepsilon_t. \quad (2.28)$$

Finally, an appropriate specification must be chosen for the stochastic component  $\varepsilon_t$ . In previous studies we have assumed that the random component is distributed independently and identically over time, a feature that we retain here. In addition we employ further restrictions on the sequence of coefficients  $\mu_t$  in order to economize on the number of parameters to be estimated.

The first assumption is that the distributed lag function may be represented as a finite moving average with an autoregressive error, that is

$$I_t - \delta K_{t-1} = \beta(S)[K_t^* - K_{t-1}^*] + v_t, \quad (2.29)$$

<sup>30</sup> For further discussion of this point, see Dale W. Jorgenson, "Rational Distributed Lag Functions," *Econometrica*, Vol. 32 (January 1966), pp. 135-49.

where  $\beta(S)$  is a polynomial in the lag operator and  $v_t$  is an autoregressive error. It is assumed that  $v_t$  is generated by an autoregressive scheme,

$$\omega(S)v_t = \varepsilon_t, \quad (2.30)$$

where  $\omega(S)$  is a polynomial in the lag operator and  $\varepsilon_t$  is distributed independently and identically over time. Multiplication of both sides of the distributed lag function by the polynomial  $\omega(S)$  results in an alternative form of the distributed lag function,

$$\begin{aligned} \omega(S)[I_t - \delta K_{t-1}] &= \omega(S)\beta(S)[K_t^* - K_{t-1}^*] + \omega(S)v_t \\ &= \omega(S)\beta(S)[K_t^* - K_{t-1}^*] + \varepsilon_t, \end{aligned} \quad (2.31)$$

which is a rational distributed lag function with independently and identically distributed error term, the specification employed in our earlier studies.

With the representation of the power series  $\mu(S)$  as the ratio of two polynomials in the lag operator, it is possible to write

$$\begin{aligned} \omega(S) &= \omega(S), \\ \gamma(S) &= \omega(S)\beta(S). \end{aligned}$$

The rational distributed lag function employed in our earlier studies is now further restricted in that the polynomial  $\gamma(S)$  is the product of two polynomials, one of them  $\omega(S)$ , the denominator of the original representation of the power series  $\mu(S)$ . If this restriction is valid, it may be used to reduce the number of parameters to be estimated. Further, the implied estimator of the power series  $\mu(S)$  reduces to an estimator of the polynomial  $\beta(S)$ , since

$$\begin{aligned} \mu(S) &= \frac{\gamma(S)}{\omega(S)} \\ &= \frac{\omega(S)\beta(S)}{\omega(S)} \\ &= \beta(S). \end{aligned}$$

This restriction overcomes a possible objection to an unconstrained estimator of the parameters of the power series  $\mu(S)$  for a rational distributed lag function. In some circumstances relatively small variations in the coefficients of the numerator of the power series may give rise to large variations in the coefficients of the power series itself, as Griliches has suggested.<sup>31</sup> Under

<sup>31</sup> Zvi Griliches, "Distributed Lags: A Survey," *Econometrica*, Vol. 35 (January 1967), pp. 16-49.



the restrictions proposed here, the estimator of the coefficients of the power series  $\mu(S)$  is independent of the estimator of the coefficients of the numerator  $\omega(S)$ .

As an example, if there are five terms in the original polynomial in the lag operator  $\beta(S)$ , the distributed lag function becomes

$$I_t - \delta K_{t-1} = \sum_{\tau=0}^4 \beta_{\tau} [K_{t-\tau}^* - K_{t-\tau-1}^*] + v_t. \quad (2.32)$$

If, further, the order of the polynomial  $\omega(S)$  is unity, that is, the disturbance has only first-order autocorrelation, both sides of the distributed lag function may be multiplied by  $\omega(S) = 1 + \omega_1 S$  to obtain

$$\begin{aligned} [I_t - \delta K_{t-1}] + \omega_1 [I_{t-1} - \delta K_{t-2}] &= \beta_0 [K_t^* - K_{t-1}^*] \\ &+ \sum_{\tau=0}^3 [\omega_1 \beta_{\tau} + \beta_{\tau+1}] [K_{t-\tau-1}^* - K_{t-\tau-2}^*] \\ &+ \omega_1 \beta_4 [K_{t-5}^* - K_{t-6}^*] + \varepsilon_t \\ &= \sum_{\tau=0}^5 \gamma_{\tau} [K_{t-\tau}^* - K_{t-\tau-1}^*] + \varepsilon_t. \end{aligned} \quad (2.33)$$

Satisfactory specifications of the distributed lag function between net investment and changes in the desired level of capital have been obtained using polynomials  $\omega(S)$  of low order. However, as many as five terms in the polynomial  $\beta(S)$  have been required to obtain a satisfactory specification. In order to economize further on the number of parameters to be estimated, we have employed an approximation proposed by Shirley Almon.<sup>32</sup> This method assumes that the polynomial in the lag operator  $\beta(S)$  has coefficients generated by a polynomial in the lag itself:

$$\beta_{\tau} = \pi_0 + \pi_1 \tau + \dots + \pi_k \tau^k. \quad (2.34)$$

To make this an approximation at all, of course, the order of the approximating polynomial must be less than the order of the polynomial in the lag operator.

If the order of the approximating polynomial is two and there are five terms in the original polynomial in the lag operator  $\beta(S)$ , the distributed lag function becomes

<sup>32</sup> Shirley Almon, "The Distributed Lag between Capital Appropriations and Expenditures," *Econometrica*, Vol. 33 (January 1965), pp. 178-96.

$$\begin{aligned}
I_t - \delta K_{t-1} &= \sum_{\tau=0}^4 \beta_\tau [K_{t-\tau}^* - K_{t-\tau-1}^*] + v_t, \\
&= \sum_{\tau=0}^4 [\pi_0 + \pi_1 \tau + \pi_2 \tau^2] [K_{t-\tau}^* - K_{t-\tau-1}^*] + v_t, \\
&= \sum_{i=0}^2 \pi_i \sum_{\tau=0}^4 \tau^i [K_{t-\tau}^* - K_{t-\tau-1}^*] + v_t. \tag{2.35}
\end{aligned}$$

On transformation this function becomes

$$\begin{aligned}
[I_t - \delta K_{t-1}] + \omega_1 [I_{t-1} - \delta K_{t-2}] &= \sum_{i=0}^2 \pi_i \sum_{\tau=0}^4 \tau^i [K_{t-\tau}^* - K_{t-\tau-1}^*] \\
&+ \omega_1 \sum_{i=0}^2 \pi_i \sum_{\tau=0}^4 \tau^i [K_{t-\tau-1}^* - K_{t-\tau-2}^*] + \varepsilon_t. \tag{2.36}
\end{aligned}$$

In this distributed lag function there are only four unknown parameters:  $\pi_0$ ,  $\pi_1$ ,  $\pi_2$ , and  $\omega_1$ .

Ordinary least squares may be employed to estimate the unknown parameters of a rational distributed lag function with independently and identically distributed error term  $\varepsilon_t$ . The resulting estimator is consistent; its asymptotic distribution may be characterized in precisely the same way as in our previous studies.<sup>33</sup>

#### *Estimating Procedure*

Provided that the restrictions on the coefficients we have proposed are valid, it is useful to take them into account in estimating the unknown parameters of the distributed lag function. First, approximation of the coefficients of the polynomial in the lag operator  $\beta(S)$  by a polynomial in the lag itself results in restrictions that are linear in the unknown parameters  $\beta_\tau$ . These restrictions are used to eliminate the parameters  $\beta_\tau$  and express the distributed lag function in terms of the parameters  $\pi_i$ . The constrained distributed lag function is still linear in the unknown parameters so that ordinary least squares may be applied directly. Secondly, generation of a rational distributed lag function by autoregressive transformation of a finite moving average results in a distributed lag function that is nonlinear in its parameters. To estimate such a function, Durbin's two stage least

<sup>33</sup> Further details on properties of the least squares estimator are discussed in Jorgenson, "Rational Distributed Lag Functions," pp. 142-43.

squares is employed.<sup>34</sup> It begins with application of an ordinary least squares estimator to the unconstrained rational distributed lag function, and continues with estimation of the parameters of the moving average  $\beta_\tau$  by applying least squares to the dependent and independent variables transformed in accord with the original autoregressive scheme. Parameters of the scheme  $\omega_\tau$  are set equal to their first-round estimates. The procedure results in estimates of the parameters  $\beta_\tau$ ,  $\omega_\tau$  that are asymptotically efficient.<sup>35</sup> It is easily seen to converge on successive iterations to the maximum likelihood estimator of the distributed lag function.

Durbin's two stage procedure may be characterized as follows: First, the parameters of the rational distributed lag function equation (2.33) are estimated without constraints— $\omega_1$ ,  $\gamma_0 \dots \gamma_5$ —by ordinary least squares. Second, least squares are applied to the relationship

$$[I_t - \delta K_{t-1}] + \hat{\omega}_1 [I_{t-1} - \delta K_{t-2}] = \sum_{i=0}^2 \pi_i \sum_{\tau=0}^4 \tau^i \{ [K_{t-\tau}^* - K_{t-\tau-1}^*] + \hat{\omega}_1 [K_{t-\tau-1}^* - K_{t-\tau-2}^*] \} + \hat{\epsilon}_t, \quad (2.37)$$

where

$\hat{\omega}_1$  = the first-round estimator of the autocorrelation parameter

$\hat{\epsilon}_t$  = the error in the distributed lag function plus the error in the first stage estimator  $\hat{\omega}_1$ , times the corresponding variables and parameters.

Since the first stage estimator is consistent, the error in it does not affect the asymptotic properties of the estimator of the remaining parameters  $\pi_0$ ,  $\pi_1$ , and  $\pi_2$ .

To test the validity of the two constraints we have proposed, we begin with the unconstrained least squares estimator of the unknown parameters of the distributed lag function. This is the first stage in Durbin's two-stage estimator. We then impose the constraints, obtaining an estimator satisfying the restrictions that  $\gamma(S) = \omega(S)\beta(S)$  and that  $\beta(S)$  has coefficients that may be approximated by a polynomial in the lag itself. A test statistic  $\mathcal{F}$

<sup>34</sup> James Durbin, "Estimation of Parameters in Time-series Regression Models," *Journal of the Royal Statistical Society, Series B*, Vol. 22 (1960), pp. 139-53.

<sup>35</sup> See *ibid.*, pp. 150-53.

that is asymptotically equivalent to a likelihood ratio test is:

$$\mathcal{F} = \frac{(\hat{\epsilon}'_0 \hat{\epsilon}_0 - \hat{\epsilon}'_1 \hat{\epsilon}_1)/(m_1 - m_0)}{\hat{\epsilon}'_1 \hat{\epsilon}_1/(m_2 - m_1)}, \quad (2.38)$$

where

$\hat{\epsilon}'_0 \hat{\epsilon}_0$  = the constrained estimators

$\hat{\epsilon}'_1 \hat{\epsilon}_1$  = the unconstrained estimators

$m_1$  = the number of parameters to be estimated without constraints

$m_0$  = the number of parameters to be estimated with constraints taken into account

$m_2$  = the number of observations.

This statistic is asymptotically equivalent to the statistic associated with the likelihood ratio test of this hypothesis. In the example above there are seven unknown parameters in the unconstrained distributed lag function so that  $m_1 = 7$ . In the constrained estimator there are only four, so that  $m_0 = 4$ . It should be noted that acceptance of the null hypothesis at conventional levels of significance is not in itself justification for imposing the constraints; it is merely an indication that there is no strong evidence contradicting the constraints.

## Estimates of the Parameters of the Investment Functions

### *Data Sources*

The econometric model of investment behavior outlined in previous sections has been fitted to data on investment expenditures based on the 1966 capital goods study of the Office of Business Economics (OBE).<sup>36</sup> Data are available for structures and equipment separately for both the manufacturing and nonfarm nonmanufacturing sectors of the U.S. economy for the years 1929–65. The data are derived by allocating commodity flow data on gross private domestic investment from the national product accounts among sectors of destination. The investment data used in this study differ from those employed in our earlier studies in two ways: (1) They reflect revisions in commodity flow estimates of gross private domestic investment resulting from revisions of the U.S. national income and product

<sup>36</sup> These unpublished data, collected by the U.S. Department of Commerce, Office of Business Economics, were kindly made available to us by Robert Wasson of the Office of Business Economics.

accounts,<sup>37</sup> and (2) they incorporate estimates of government-owned capital used in private production and some other minor adjustments made by Gordon.<sup>38</sup>

Published price indexes for gross private domestic investment are biased because they are based in part on the price of inputs to the capital goods industries rather than the price of output. To overcome this bias we used the Bureau of Public Roads price index for structures in our previous studies. Here we use an index constructed by Gordon, based on price indexes for the output of structures from the 1966 capital goods study. In our previous study we replaced the implicit deflator for producers' durables by a deflator for consumers' durables. To avoid a possible bias resulting from differences in the cyclical behavior of consumers' and producers' price indexes, it was decided not to attempt to correct the bias in the producers' durables price index, which, in any case, is not very substantial.<sup>39</sup> Accordingly, the implicit deflator for producers' durables from the national product accounts is employed in this study. All price indexes have a 1965 base.

Capital stock for equipment and structures in both industry groupings is obtained from the recursive relationship,

$$K_t = I_t + (1 - \delta)K_{t-1},$$

where  $I_t$  is investment in period  $t$ , derived as outlined above, and  $\delta$  is the rate of replacement, taken to be 2.5 times the inverse of the Bureau of Internal Revenue's *Bulletin F* lifetime.<sup>40</sup> Similar rates of replacement were used by the Office of Business Economics in its 1966 capital goods study. The values of  $\delta$  are the same as those employed in our previous studies:

Manufacturing equipment	0.1471
Manufacturing structures	0.0625
Nonfarm nonmanufacturing equipment	0.1923
Nonfarm nonmanufacturing structures	0.0694

<sup>37</sup> U.S. Department of Commerce, Office of Business Economics, *The National Income and Product Accounts of the United States, 1929-1965: Statistical Tables* (1966).

<sup>38</sup> Robert J. Gordon, "Problems in the Measurement of Real Investment in the U.S. Private Economy" (Ph.D. dissertation, Massachusetts Institute of Technology, 1967).

<sup>39</sup> Our original estimate of the rate of growth of this bias was 0.651 percent per year, or about one-third the bias for structures. See Hall and Jorgenson, "Tax Policy and Investment Behavior," p. 399.

<sup>40</sup> U.S. Treasury Department, Bureau of Internal Revenue, *Income Tax Depreciation and Obsolescence: Estimated Useful Lives and Depreciation Rates*, Bulletin F (revised January 1942); referred to as *Bulletin F*.

Initial values for capital stock in 1929 were estimated by cumulating net investment over the whole period for which data are available for each asset.

The desired level of capital stock depends on the value of output. As a measure of output and prices  $pQ$ , we have used gross value added at factor cost, in current dollars, defined as gross product originating in each industry less indirect business taxes. For the years 1929 to 1946 these data are identical to those of our previous studies. For the years 1947 to 1965 data were obtained from the OBE study of gross product originating in each sector.<sup>41</sup>

The desired level of capital also depends on the rental price for capital services. Through 1953 the rental price is that appropriate to straight-line depreciation. Since 1954 the rental price is that appropriate to sum-of-the-years-digits depreciation.<sup>42</sup> From October 1966 to March 1967 the appropriate rental price for structures is that for 150-percent-declining-balance depreciation.

The investment tax credit was introduced in 1962 at a rate nominally equal to 7 percent of the value of investment in equipment. In practice certain limitations on the applicability of the investment tax credit reduce its effective rate to 6 percent for manufacturing equipment and 5.8 percent for nonfarm nonmanufacturing equipment.<sup>43</sup> For 1962 and 1963, under the Long amendment, the base for depreciation was reduced by the amount of the tax credit; after 1964, with the repeal of the amendment, the base for depreciation is not reduced by the amount of the credit. From October 1966 to March 1967, the investment tax credit was suspended.

The rental price of capital services also depends on the tax rate  $u$ , the after-tax rate of return  $r$ , the investment goods price  $q$ , the rate of replacement  $\delta$ , and the lifetime of capital goods allowable for tax purposes. We took the tax rate to be the statutory rate prevailing during most of each year. We did not allow for excess profits taxes during the middle thirties or the Korean war. For all years we took the rate of return before taxes  $\rho$  to be constant at 20 percent. This value is higher than the value of 14 percent used in our previous studies, but it is consistent with the results of Jorgenson

<sup>41</sup> See Jack J. Gottsegen, "Revised Estimates of GNP by Major Industries," *Survey of Current Business*, Vol. 47 (April 1967), pp. 18-24.

<sup>42</sup> Depreciation under the sum-of-the-years-digits formula has a higher present value for the range of lifetimes and rates of return of interest for this study. See Hall and Jorgenson, "Tax Policy and Investment Behavior," Table 1, p. 395.

<sup>43</sup> These estimates of the effective rate of the tax credit are based on data from tax returns for 1963. See U.S. Treasury Department, Internal Revenue Service, *Statistics of Income—1963, Corporation Income Tax Returns* (1968).

and Griliches.<sup>44</sup> Under the assumption of a constant before-tax rate of return, the after-tax rate  $r = (1-u)\rho$  varies with the tax rate.

The investment goods price is the same as that used to deflate investment expenditures in current prices and the rate of replacement is the same as that used to calculate capital stock. Estimates of lifetimes of assets allowable for tax purposes were based on a special Treasury study,<sup>45</sup> and are the same as those employed in our previous studies:

<i>Period</i>	<i>Asset lifetimes</i> (years)	
	<i>Equipment</i>	<i>Structures</i>
1929-54	17.5	27.8
1955	16.3	25.3
1956-61	15.1	22.8
1962-65	13.1	22.8

#### *Equation Estimates*

The previous section described a statistical technique for fitting our econometric model to data on investment expenditures. In summary this technique is based on the application of least squares in two stages. First, an unconstrained rational distributed lag function is fitted to data on net investment and changes in the desired level of capital for each class of asset for each sector. The independent variables include lagged values of net investment and current and lagged changes in the desired level of capital. We have designated the lag operators  $\beta(S)$  and  $\omega(S)$  as fourth- and first-order polynomials, respectively, so that one lagged value of net investment and current and five lagged changes in desired capital are included among the independent variables. The results of the first stage regressions, of the form of equation (2.33), for the periods 1935-40 and 1954-65 are presented in Table 2-1. The coefficient  $-\hat{\omega}_1$  is associated with lagged values of net investment and is an estimate of the autocorrelation of the disturbances. The coefficients  $\hat{\alpha}\hat{\gamma}_0 \dots \hat{\alpha}\hat{\gamma}_5$  are associated with changes in the ratio of the value of output to the rental price of capital services.

<sup>44</sup> Dale W. Jorgenson and Zvi Griliches, "The Explanation of Productivity Change," *Review of Economic Studies*, Vol. 34 (July 1967), pp. 249-81.

<sup>45</sup> U.S. Treasury Department, Internal Revenue Service, "Statistics of Income—1959: Supplementary Depreciation Data from Corporation Income Tax Returns, with Special Appendix—Depreciation Methods and Amortization Data, 1954 through 1961" (mimeograph, June 1965).

**TABLE 2-1. Fitted Investment Functions for Equipment and Structures, by Industrial Sector, 1935-40 and 1954-65, First Stage Regressions**

Sector and asset class	$\hat{\omega}_1$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$	$R^2_N$	$R^2_I$	$S_e$	DW
Manufacturing											
Equipment	-0.4753 (0.2276)	0.0123 (0.0052)	0.0190 (0.0058)	0.0071 (0.0079)	0.0034 (0.0075)	0.0001 (0.0069)	0.0015 (0.0057)	0.801	0.969	0.658	2.277
Structures	-0.6109 (0.3255)	0.0036 (0.0042)	0.0055 (0.0040)	0.0030 (0.0045)	0.0035 (0.0048)	0.0015 (0.0046)	0.0001 (0.0043)	0.524	0.815	0.585	1.447
Nonfarm nonmanufacturing											
Equipment	0.0916 (0.3319)	0.0317 (0.0117)	0.0389 (0.0198)	0.0229 (0.0163)	0.0143 (0.0138)	0.0054 (0.0132)	0.0202 (0.0133)	0.820	0.965	1.255	1.837
Structures	-1.0065 (0.0953)	0.0057 (0.0037)	0.0082 (0.0040)	-0.0010 (0.0044)	-0.0025 (0.0039)	-0.0029 (0.0039)	-0.0046 (0.0038)	0.987	0.967	0.531	1.931

Source: Equation (2.33).  
Note: t-statistics are shown in parentheses.



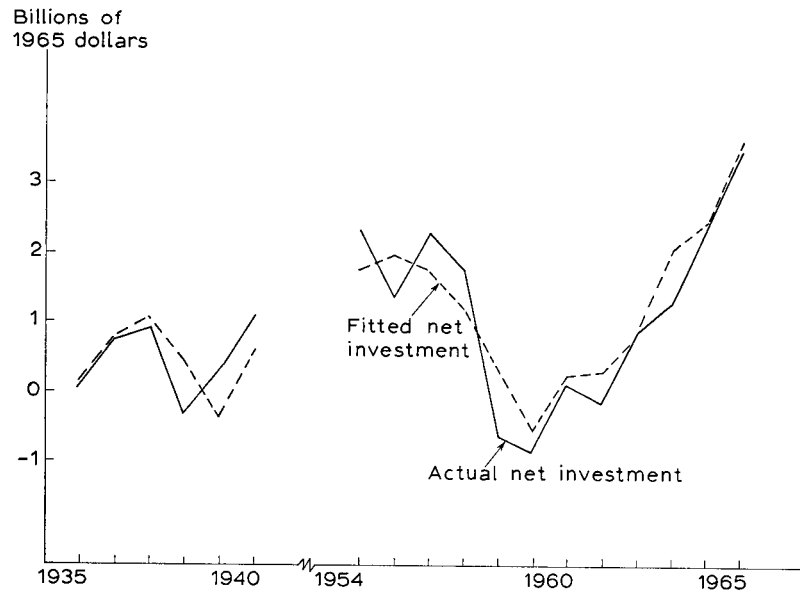
Measures of goodness of fit of the first stage regressions are also given in Table 2-1. Goodness of fit is measured in two ways: (1) the ratio of the explained sum of squares to the total sum of squares for gross investment,  $R_G^2$ , and (2) the ratio of the explained sum of squares to the total sum of squares for net investment,  $R_N^2$ . Neither ratio is corrected for degrees of freedom. While net investment is the dependent variable in the regression, gross investment is the variable of primary interest for policy considerations. The standard error of estimate  $S_e$ , corrected for degrees of freedom, is also presented for each of the regressions. The standard error is the same for gross and net investment. Autocorrelation of errors has already been taken into account in the generation of the distributed lag function underlying our econometric model. A test for autocorrelation may be performed by combining first and second stage results. For completeness the Durbin-Watson ratio  $DW$  is presented for each regression. The usual test for autocorrelation based on this ratio is, of course, biased toward randomness.<sup>46</sup>

Actual and fitted values of net investment from the first stage regressions are plotted in Figures 2-1–2-4. The overall goodness of fit is superior to that of our previous investment functions for 1931–41 and 1950–63, except for manufacturing structures. This improvement is mainly due to the change in time period and to revisions of the basic investment data; however, it is also due partly to the change in specification of the distributed lag function. The addition of three lagged changes in desired capital improves the result to some extent.

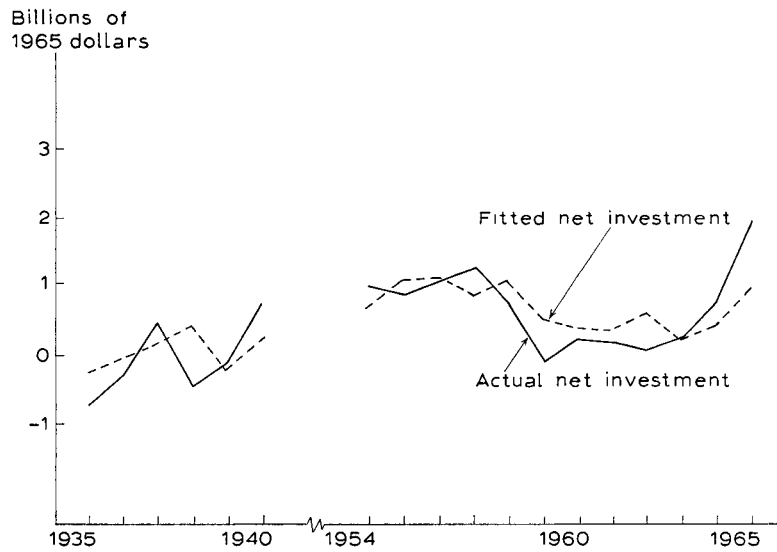
The second stage of our statistical procedure is to transform all variables in accord with the estimated autoregressive scheme of the errors from the first stage. We approximate the polynomial in the lag operator  $\beta(S)$  by a polynomial in the lag itself. We have chosen a second-order polynomial for this purpose so the lag function is a parabola. The dependent variable is now net investment plus  $\hat{\omega}_1$  multiplied by lagged net investment, while the independent variables are weighted sums of changes in desired capital plus  $\hat{\omega}$  multiplied by the corresponding lagged value. The weights depend on the lags. The derived estimates of the parameters  $\alpha\beta_0 \dots \alpha\beta_4$  are presented in Table 2-2. Measures of goodness of fit similar to those presented for the unconstrained distributed lag functions are also given in Table 2-2. It should be noted that  $R^2$  for these regressions is a measure of the degree of explanation of the autoregressively transformed values of net investment. The only

<sup>46</sup> See Griliches, "Distributed Lags," and Edmond Malinvaud, "Estimation et Prévision dans les Modèles Economiques Autorégressifs," *Revue de l'Institut International de Statistique*, Vol. 29, No. 2 (1961), pp. 1–32.

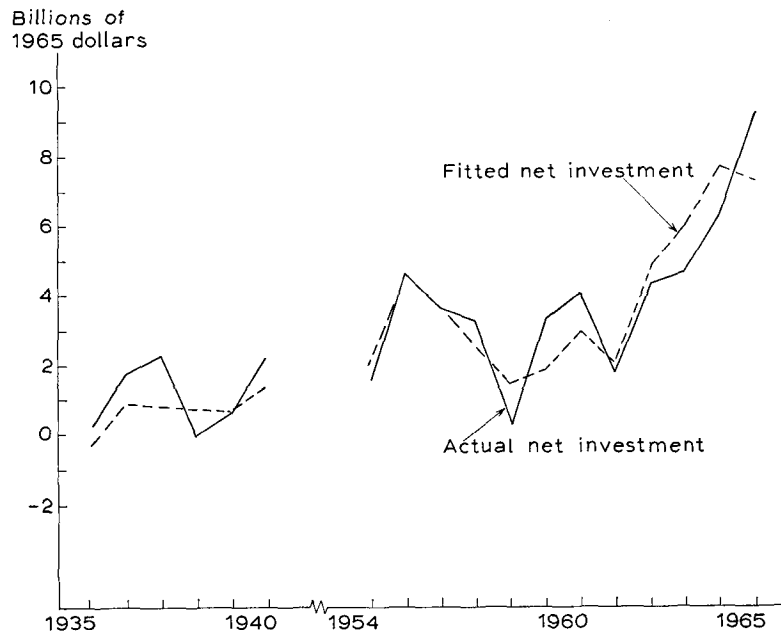
**FIGURE 2-1. Net Investment in Manufacturing Equipment, 1935-40 and 1954-65, First Stage Regressions**



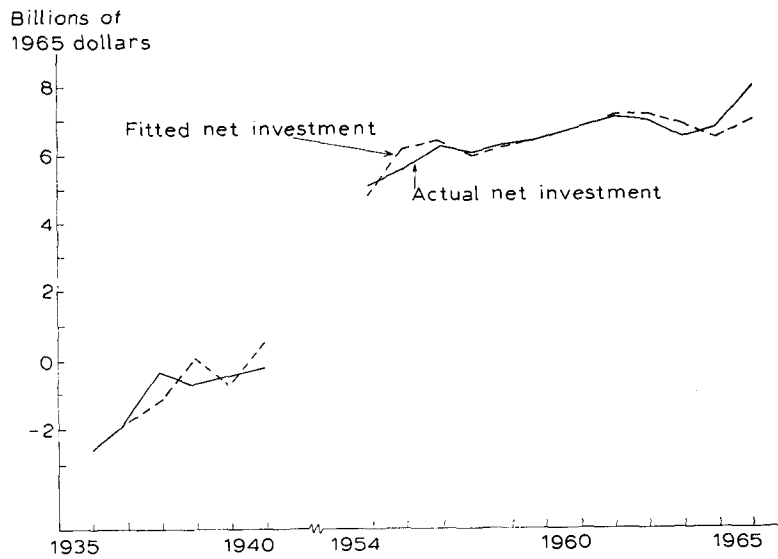
**FIGURE 2-2. Net Investment in Manufacturing Structures, 1935-40 and 1954-65, First Stage Regressions**



**FIGURE 2-3. Net Investment in Nonfarm Nonmanufacturing Equipment, 1935-40 and 1954-65, First Stage Regressions**



**FIGURE 2-4. Net Investment in Nonfarm Nonmanufacturing Structures, 1935-40 and 1954-65, First Stage Regressions**



**TABLE 2-2. Fitted Investment Functions for Equipment and Structures, by Industrial Sector, 1935-40 and 1954-65, Second Stage Regressions**

Sector and asset class	$\hat{\alpha}\beta_0$	$\hat{\alpha}\beta_1$	$\hat{\alpha}\beta_2$	$\hat{\alpha}\beta_3$	$\hat{\alpha}\beta_4$	R <sup>2</sup>	S <sub>e</sub>	DW
Manufacturing Equipment	0.0130 (0.0047)	0.0200 (0.0034)	0.0208 (0.0040)	0.0153 (0.0040)	0.0036 (0.0053)	0.602	0.620	2.099
Structures	0.0041 (0.0033)	0.0082 (0.0030)	0.0093 (0.0035)	0.0073 (0.0032)	0.0024 (0.0034)	0.186	0.513	1.304
Nonfarm nonmanufacturing Equipment	0.0374 (0.0083)	0.0282 (0.0038)	0.0211 (0.0063)	0.0160 (0.0047)	0.0129 (0.0090)	0.800	1.190	1.724
Structures	0.0059 (0.0034)	0.0105 (0.0032)	0.0118 (0.0035)	0.0098 (0.0031)	0.0046 (0.0029)	0.169	0.506	1.825

Source: Equation (2.37).  
Note: t-statistics are shown in parentheses.

measure of goodness of fit comparable to those in Table 2-1 is the standard error of estimate  $S_e$  for each of the regressions. This standard error is uniformly lower for all regressions, reflecting the fact that loss in explanatory power due to reduction in the number of parameters to be estimated is more than compensated for by the reduction in the number of degrees of freedom required for estimation. Actual and fitted values of net investment from the second stage regressions are plotted in Figures 2-5-2-8. The actual values in these plots are net investment, not the transformed net investment series that served as the left-hand variable in the second stage. The fitted values were calculated by substituting the parameter estimates from the second stage into the first stage regression equation. It would not be meaningful to plot the actual and fitted values directly from the second stage because of the autoregressive transformation.

We have generated the distributed lag function for our econometric model of investment behavior by using two restrictions: (1) The distributed lag is finite (that is, the error is autoregressive); and (2) the coefficients of the polynomial  $\beta(S)$  lie along a second degree polynomial in the lag itself. The statistic derived above, based on sums of squared residuals with and without constraints, is used to test the validity of these restrictions. The resulting test statistic  $\mathcal{F}$  is presented in the first column of Table 2-3. When the very low values of this statistic are compared with the critical value of the  $\mathcal{F}$ -ratio at the 0.05 level, 3.59, the null hypothesis is easily accepted for all regressions. We conclude that the distributed lag is finite

**TABLE 2-3. Fitted Investment Functions for Equipment and Structures, by Industrial Sector, 1935-40 and 1954-65, Derived Results**

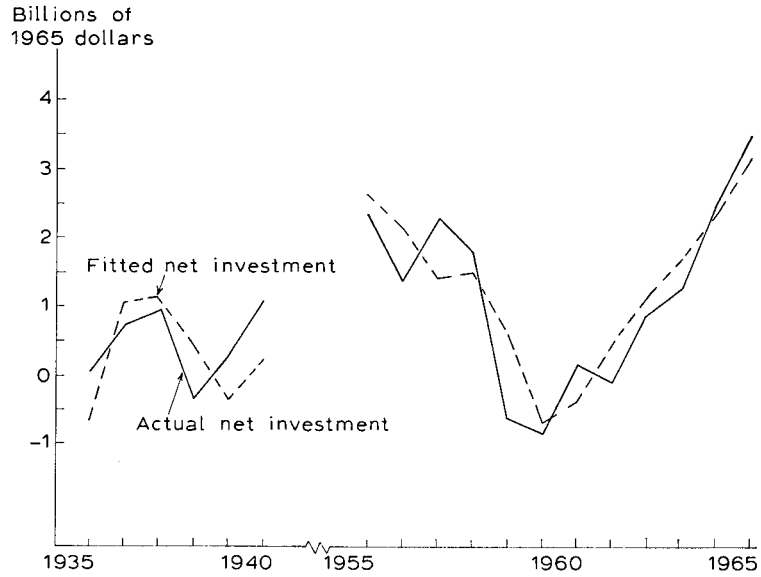
Sector and asset class	$\mathcal{F}_1^a$	$\mathcal{F}_2^b$	$\hat{\alpha}^c$	Mean lag (years)
<b>Manufacturing</b>				
Equipment	0.577	3.912	0.0727	1.67
Structures	0.138	4.764	0.0312	1.86
<b>Nonfarm nonmanufacturing</b>				
Equipment	0.623	0.004	0.1160	1.47
Structures	0.655	156.681	0.0426	1.92

<sup>a</sup>  $\mathcal{F}$ -statistic for the null hypothesis that the distributed lag is finite and has a parabolic shape. The critical value of  $\mathcal{F}$  with 3 and 11 degrees of freedom is 3.59 at the 0.05 level.

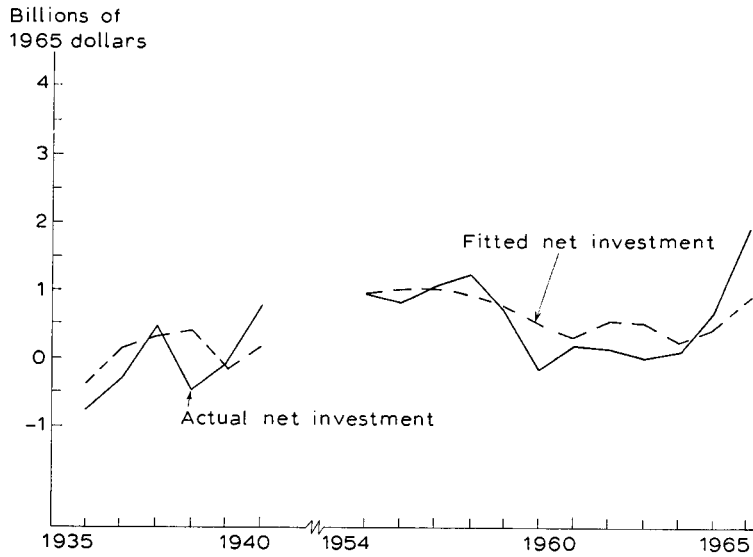
<sup>b</sup>  $\mathcal{F}$ -statistic for the null hypothesis that there is no autocorrelation. The critical value of  $\mathcal{F}$  with 1 and 14 degrees of freedom is 4.60 at the 0.05 level.

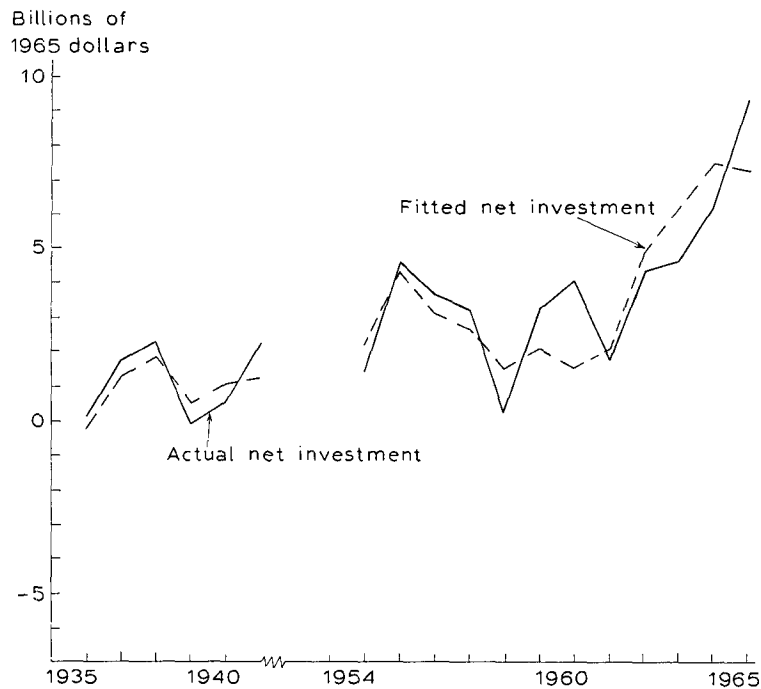
<sup>c</sup> Estimate of the elasticity of output with respect to the capital input.

**FIGURE 2-5. Net Investment in Manufacturing Equipment, 1935-40 and 1954-65, Second Stage Regressions**



**FIGURE 2-6. Net Investment in Manufacturing Structures, 1935-40 and 1954-65, Second Stage Regressions**



**FIGURE 2-7. Net Investment in Nonfarm Nonmanufacturing Equipment, 1935-40 and 1954-65, Second Stage Regressions**

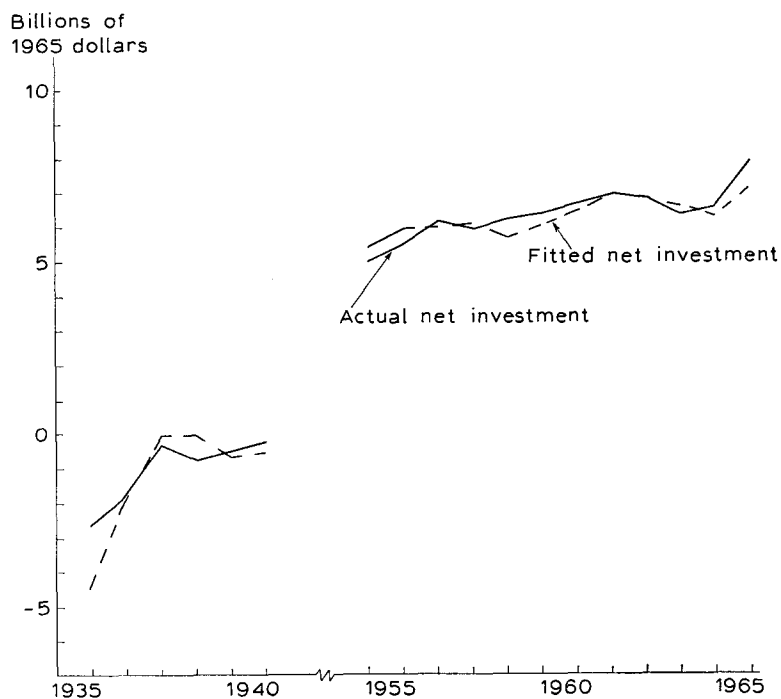
and that the coefficients of  $\beta(S)$  lie along a second degree polynomial. Accordingly, we employ the second stage regressions for further analysis of the distributed lag function.

Also presented in Table 2-3 (column 2) are the results of testing the null hypothesis of no autocorrelation in the finite parabolic distributed lag model. The  $\mathcal{F}$ -statistic for this test is

$$\mathcal{F} = 14(\hat{\varepsilon}'_0 \hat{\varepsilon}_0 - \hat{\varepsilon}'_1 \hat{\varepsilon}_1) / (\hat{\varepsilon}'_1 \hat{\varepsilon}_1),$$

where 14 is the number of degrees of freedom in the unconstrained regression,  $\hat{\varepsilon}'_1 \hat{\varepsilon}_1$  is the sum of squared residuals in that regression, and  $\hat{\varepsilon}'_0 \hat{\varepsilon}_0$  is the sum of squared residuals in the constrained regression. The unconstrained regressions are those reported in Table 2-2. The constrained regressions are of precisely the same form as those in Table 2-2 except that the variables have not been subjected to the autoregressive transformation. As can be seen, there is evidence of autocorrelation in all sectors except nonmanu-

**FIGURE 2-8. Net Investment in Nonfarm Nonmanufacturing Structures, 1935-40 and 1954-65, Second Stage Regressions**



facturing equipment. The null hypothesis is rejected in both equations for structures. These results are exactly in accord with the regression results for the first stage regressions reported in Table 2-1. The very high autocorrelation in the equation for nonmanufacturing structures suggests the possibility of specification error.

#### *Distributed lags*

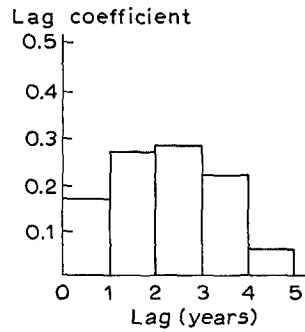
The parameters of the distributed lag function  $\mu_t$  may be estimated by employing the constraint that the sum of the coefficients of this function must be unity to estimate the parameter  $\delta$ .<sup>47</sup> The resulting estimates are given in Table 2-3. The derived estimates of the parameters of the distributed lag function are plotted in Figures 2-9-2-12. The mean lag for each function

<sup>47</sup> For detailed discussion of this restriction and its use in estimating the parameter  $\alpha$ , see Jorgenson, "Rational Distributed Lag Functions," pp. 135 and 147-48.

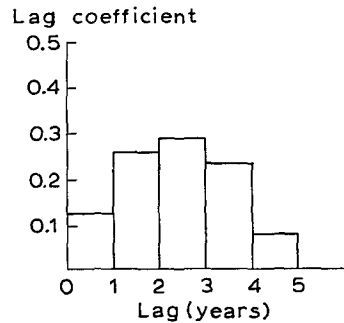


is also given in Table 2-3. When these mean lags are compared with estimates from our earlier studies, the new estimates are found to be very similar for investment in equipment. The mean lag is now estimated to be slightly lower for manufacturing equipment and slightly higher for nonfarm nonmanufacturing equipment. For structures, however, the new estimates differ

**FIGURE 2-9. Estimated Lag Function  $\beta_t$ , for Manufacturing Equipment**

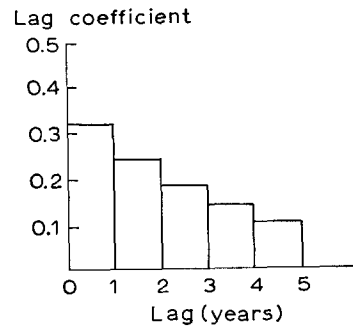


**FIGURE 2-10. Estimated Lag Function  $\beta_t$ , for Manufacturing Structures**

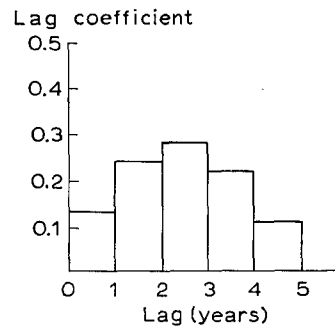


substantially from the old. The old estimate of the mean lag for manufacturing structures was 3.84 years, whereas the new estimate is 1.86; the old estimate of the mean lag for nonfarm nonmanufacturing structures was 7.49 years, while the new estimate is 1.92. For both sets of results the lags are estimated to be longer for structures than for equipment.

**FIGURE 2-11. Estimated Lag Function  $\beta_\tau$  for Nonfarm Nonmanufacturing Equipment**



**FIGURE 2-12. Estimated Lag Function  $\beta_\tau$  for Nonfarm Nonmanufacturing Structures**



A disturbing feature of our earlier results is that the lag pattern fails to agree with the substantial body of evidence from studies by Jorgenson and Stephenson at the level of two-digit industries and by Jorgenson and Siebert at the level of the individual firm.<sup>48</sup> For manufacturing, Jorgenson and Stephenson estimate the average lag at about two years, while results from individual industries range from six to eleven quarters and cluster in the neighborhood of the overall average. The results for individual firms are characterized by more variability than the results for industries, as

<sup>48</sup> Dale W. Jorgenson and James A. Stephenson, "The Time Structure of Investment Behavior in United States Manufacturing, 1947-1960," *Review of Economics and Statistics*, Vol. 49 (February 1967), pp. 16-27; and Dale W. Jorgenson and Calvin D. Siebert, "Optimal Capital Accumulation and Corporate Investment Behavior," *Journal of Political Economy*, Vol. 76 (November-December 1968), pp. 1123-51.

would be expected. The average lags estimated by Jorgenson and Siebert range from less than a year to over three years, with values between one and two years predominating. Based on a survey he made, Mayer's estimate of the average lag from the decision to undertake investment to the completion of the project for manufacturing is seven quarters.<sup>49</sup> We conclude that our new estimates agree closely with Mayer's results and with estimates derived from investment functions for industry groups and for individual firms. Our previous estimates of the average lags for structures are evidently biased by specification errors in the underlying distributed lag functions and should be replaced by our new estimates.

### Impact of Tax Policy on Investment Behavior

If desired capital stock is increased by a change in tax policy, through a consequent change in the rental price of capital services, additional net investment is generated; if the determinants of investment then remain at stationary levels, this net investment eventually brings actual capital stock up to the new desired level. The initial burst of net investment increases gross investment at first, but this effect gradually declines to zero as the gap between desired and actual capital stock is eliminated. However, gross investment is permanently increased by the higher levels of replacement associated with higher levels of capital stock. If desired capital stock is decreased by tax policy, these effects are precisely reversed.

The qualitative features of the response of investment to a change in tax policy are essentially the same for all changes. To evaluate the effects of particular tax measures, it is useful to assess the response of investment quantitatively. Accordingly, we calculate the effects of changes in tax policy that have taken place in the United States in the postwar period. The calculations are based on a partial equilibrium analysis of investment behavior. All determinants of investment expenditures except tax policy are held equal to their actual values. We then measure the impact of tax policy by substituting into our investment functions parameters of the tax structure—tax rate, depreciation formulas, tax credit, and depreciation lifetimes—appropriate to alternative tax policies. The difference between investment resulting from actual tax policy and investment that would have resulted from alternative tax policies is our measure of the impact of tax policy.

<sup>49</sup> Thomas Mayer, "Plant and Equipment Lead Times," *Journal of Business*, Vol. 33 (April 1960), pp. 127–32.

We present estimates of the impact of the adoption of accelerated depreciation in 1954 and of new lifetimes for depreciation of equipment and the investment tax credit in 1962, the tax cut of 1964, and the suspension of the tax credit for equipment and restriction of the use of accelerated depreciation for structures in 1966-67. The tax measure of 1964 reduced the corporate tax rate from 52 percent to 48 percent and also restored the tax credit to the depreciation base for tax purposes. In our earlier studies we presented calculations of the effects of all these changes in tax policy. In view of the substantial revisions in the underlying investment data and the alterations in our specification of the investment functions, we provide a complete set of estimates based on our new results.

In the new calculations both investment and capital stock are measured in 1965 prices. We estimate the impact of all changes in tax policy through 1970. In order to make these estimates, we employed a rough set of projections of the determinants of investment. No great precision was required in these projections, since the estimates of the differential impacts of alternative policies are not at all sensitive to the assumed level of investment. The projected levels of gross value added and the price deflators for investment goods are shown in Table 2-4.<sup>50</sup> Although the projections of gross value added are in current dollars and are likely to be serious underestimates because of the relatively rapid rate of inflation that has developed recently, this will not affect the results, since only the ratio of gross value added and the investment deflator enter the calculations. Finally, all tax variables were assumed to stay at their 1965 values, except for the brief suspension of the investment tax credit and accelerated depreciation in 1966-67; the treatment of this suspension is described in detail below.

As a basis for comparison with alternative tax policies, Table 2-5 presents data on the actual levels of gross investment, net investment, and capital stock, for 1950-65. Also included are extrapolated values calculated from the fitted investment functions for 1966-70 for plant and equipment, for both the manufacturing and nonfarm nonmanufacturing sectors.

#### *Accelerated Depreciation, 1954*

The first change in tax policy we attempt to evaluate is the adoption of accelerated methods of depreciation for tax purposes in 1954. As an alternative policy we suppose that only the straight-line formula was permitted from

<sup>50</sup> These are crude extrapolations of previous trends, modified by fragmentary data available in October 1967, the time of the computations.

**TABLE 2-4. Projected Levels of Gross Value Added and Price Deflators for Manufacturing and Nonfarm Nonmanufacturing, 1966-70**

Year	Gross value added (billions of current dollars)		Price deflators (1965=1.000)		
	Manufacturing	Nonfarm non- manufacturing	Manufacturing equipment	Nonfarm non- manufacturing equipment	Structures, both sectors
1966	198.4	363.6	1.031	1.031	1.043
1967	209.1	383.2	1.068	1.068	1.079
1968	221.6	406.2	1.103	1.095	1.100
1969	234.9	430.6	1.142	1.121	1.121
1970	249.0	456.4	1.183	1.150	1.144

Source: See text, p. 44.

1954 to 1970 and that all other determinants of investment are unchanged. The levels of the annual rental price of capital services, and the reduction brought about in 1955 (the first full year) through the adoption of accelerated methods of depreciation, were:

	<i>Without accelerated depreciation</i>	<i>With accelerated depreciation</i>	<i>Percentage decrease</i>
Manufacturing equipment	0.293	0.267	8.9
Manufacturing structures	0.229	0.208	9.2
Nonfarm nonmanufacturing equipment	0.375	0.341	9.1
Nonfarm nonmanufacturing structures	0.239	0.217	9.2

Estimates of the increase in gross investment, net investment, and capital stock resulting from the adoption of accelerated depreciation in 1954 are given in Table 2-6.

The effects of the adoption of accelerated depreciation are very substantial. Although the same pattern prevails in all four classes of assets, it is useful to trace out the quantitative impact of tax policy on net investment, gross investment, and capital stock for each class. The peak effect on net investment for manufacturing equipment is attained in 1956 with a level of \$744 million, or 32 percent of net investment in that year. By 1959 the effect is essentially nil; however, the adoption in 1962 of new equipment lifetimes for tax purposes and of the investment tax credit provides an additional stimulus from the use of accelerated methods of depreciation.

**TABLE 2-5. Gross Investment, Net Investment, and Capital Stock in Equipment and Structures, by Industrial Sector, Actual, 1950-65 and Projected, 1966-70**  
(In billions of 1965 dollars)

Year	Manufacturing						Nonfarm nonmanufacturing					
	Equipment			Structures			Equipment			Structures		
	Investment	Net	Capital stock	Investment	Net	Capital stock	Investment	Net	Capital stock	Investment	Net	Capital stock
Gross			Gross			Gross			Gross			
Actual												
1950	5.553	0.627	33.488	1.949	-0.252	35.220	14.469	4.079	54.028	9.323	3.986	76.903
1951	7.265	2.247	34.115	2.776	0.591	34.968	14.659	3.485	58.107	8.895	3.281	80.889
1952	8.291	2.942	36.362	2.888	0.666	35.558	14.421	2.577	61.592	8.775	2.934	84.170
1953	8.578	2.796	39.304	3.091	0.827	36.224	15.020	2.680	64.169	10.348	4.303	87.104
1954	8.544	2.351	42.100	3.276	0.960	37.051	14.327	1.472	66.849	11.345	5.001	91.407
1955	7.927	1.388	44.451	3.222	0.846	38.011	17.699	4.561	68.321	12.195	5.504	96.408
1956	9.034	2.291	45.840	3.526	1.097	38.858	17.714	3.699	72.882	13.264	6.191	101.912
1957	8.875	1.795	48.131	3.751	1.254	39.955	17.973	3.246	76.581	13.445	5.943	108.104
1958	6.726	-0.618	49.926	3.321	0.745	41.209	15.595	0.244	79.827	14.125	6.210	114.046
1959	6.423	-0.830	49.308	2.490	-0.132	41.954	18.671	3.273	80.072	14.740	6.394	120.256

1960	7.299	0.168	48.477	2.821	0.207	41.822	20.090	4.063	83.345	15.459	6.669	126.651
1961	7.067	-0.089	48.645	2.786	0.159	42.029	18.617	1.809	87.408	16.190	6.937	133.320
1962	8.040	0.897	48.557	2.681	0.044	42.188	21.475	4.319	89.216	16.563	6.829	140.258
1963	8.550	1.275	49.454	2.836	0.196	42.233	22.641	4.654	93.535	16.582	6.374	147.087
1964	9.941	2.479	50.729	3.353	0.701	42.429	25.175	6.293	98.189	17.251	6.601	153.461
1965	11.333	3.506	53.208	4.659	1.963	43.130	29.323	9.231	104.482	18.977	7.869	160.061
Projected												
1966	11.434	3.092	56.714	4.462	1.644	45.093	28.337	6.470	113.713	19.557	7.903	167.930
1967	11.259	2.462	59.806	4.306	1.385	46.737	28.149	5.038	120.183	20.127	7.924	175.833
1968	11.138	1.978	62.268	4.216	1.208	48.122	29.198	5.118	125.221	20.841	8.088	183.757
1969	11.044	1.593	64.246	4.165	1.082	49.330	29.598	4.534	130.339	21.610	8.296	191.845
1970	11.104	1.419	65.839	4.222	1.071	50.412	30.243	4.307	134.873	22.446	8.556	200.141

Source: Unpublished data from the 1966 capital goods study of the U.S. Department of Commerce, Office of Business Economics. The capital stock figures are derived from unrounded data and will not necessarily equal the sum of the preceding year's capital stock and net investment.

**TABLE 2-6. Estimated Changes in Gross and Net Investment in Equipment and Structures, and in Capital Stock, Resulting from Accelerated Depreciation, 1954-70**  
(In millions of 1965 dollars)

Year	Manufacturing						Nonfarm nonmanufacturing					
	Equipment			Structures			Equipment			Structures		
	Investment	Capital stock	Net	Investment	Capital stock	Net	Investment	Capital stock	Net	Investment	Capital stock	Net
1954	209	0	209	79	0	79	847	847	847	204	0	204
1955	627	209	596	280	79	275	1,871	1,708	1,708	649	847	635
1956	862	805	744	432	354	410	1,756	1,265	1,265	944	2,555	886
1957	878	1,549	650	450	764	402	1,723	988	988	1,023	3,820	903
1958	621	2,199	298	323	1,166	250	1,662	737	737	867	4,808	684
1959	395	2,497	27	181	1,416	92	1,519	453	453	584	5,545	354
1960	369	2,524	2	153	1,508	59	1,294	141	141	448	5,998	194
1961	406	2,522	35	169	1,567	71	1,386	205	205	478	6,139	214
1962	487	2,557	111	180	1,638	78	1,573	353	353	430	6,344	201
1963	545	2,668	152	172	1,706	65	1,779	491	491	473	6,697	178
1964	660	2,820	245	165	1,771	53	2,090	708	708	440	7,188	134
1965	733	3,065	282	159	1,824	45	2,049	531	531	384	7,896	70
1966	727	3,347	235	93	1,869	24	2,067	446	446	206	8,427	-114
1967	696	3,582	170	41	1,845	74	2,068	362	362	113	8,873	-205
1968	682	3,752	131	97	1,771	14	2,213	437	437	258	9,235	-46
1969	702	3,883	131	193	1,757	83	2,214	354	354	471	9,672	169
1970	728	4,014	138	287	1,840	172	2,309	377	377	675	10,049	362

8



We estimate that 17.5 percent of the net investment in manufacturing equipment over the period 1954–70 may be attributed to the change in methods for calculating depreciation. Similarly, the peak effect for nonfarm nonmanufacturing equipment is \$1.708 billion in 1955, or 37.4 percent of the net investment that took place in that year. Over the seventeen-year period, 15.4 percent of the net investment in nonfarm nonmanufacturing equipment may be attributed to the change in depreciation rules in 1954.

Although the average lag in response of investment is longer for structures than for equipment, the effects of accelerated depreciation are broadly similar. For manufacturing structures the peak effect on net investment occurs in 1956 with \$410 million, or 37.3 percent of the net investment that took place in that year. For the 1954–70 period the increase in net investment in manufacturing structures due to accelerated depreciation is estimated at 15.0 percent of the total. For nonfarm nonmanufacturing structures, the peak effect on investment occurs in 1957 with \$903 million, or 15.2 percent of the net investment that took place in that year. Over the whole period, 4.5 percent of the net investment in nonfarm nonmanufacturing structures may be attributed to the adoption of accelerated methods for depreciation in 1954.

Capital stock is a cumulation of net investment so that its behavior is implied by that of net investment. For both manufacturing and nonfarm nonmanufacturing equipment, two phases in the response of capital stock can be distinguished. First, the immediate impact of adoption of accelerated depreciation was to raise desired capital substantially above actual capital. By 1957 more than half the gap resulting from accelerated depreciation was eliminated; by 1959 none remained. Second, adoption of accelerated depreciation in 1954 resulted in additional stimulus from subsequent changes in lifetimes for tax purposes and from adoption of the investment tax credit. Half the total rise in the stock of manufacturing equipment from 1954 to 1970 took place by 1958, while half the rise in nonfarm nonmanufacturing equipment took place by 1959. The patterns for structures in both the manufacturing and nonfarm nonmanufacturing sectors are qualitatively similar to those for equipment but without a clear demarcation between successive phases. As in equipment, half the total rise in the stock of manufacturing structures over the period as a whole took place by 1958, while half the rise in nonfarm nonmanufacturing structures took place by 1959.

Gross investment is the sum of net investment and replacement; further, replacement rises in proportion to capital stock. By 1958 replacement had become the dominant component in the response of gross investment in

equipment to the adoption of accelerated depreciation for both the manufacturing and nonfarm nonmanufacturing sectors. For manufacturing the peak response of gross investment occurred in 1957 with a change of \$878 million. By 1970, added replacement requirements are expected to maintain gross investment at near-peak levels of \$728 million. Similarly, gross investment in nonfarm nonmanufacturing equipment reached a peak of \$1.871 billion in 1955, declined for several years, and will rise to a new peak of \$2.309 billion by 1970, propelled by rising replacement requirements. For manufacturing structures the high of \$450 million was attained in 1957; by 1970 it is estimated the level will reach \$288 million. The general pattern of response for investment in nonfarm nonmanufacturing structures is similar in timing but different in magnitude. The largest response of gross investment was \$1.023 billion in 1957; the level in 1970 is estimated at \$672 million.

The total effect of the adoption of accelerated depreciation in 1954 on gross investment during the whole period from 1954 to 1970 may be assessed by comparing investment resulting from the new methods of depreciation with investment that would have taken place under the old methods. For equipment, 6.7 percent of gross investment in manufacturing and 8.0 percent of the gross investment in nonfarm nonmanufacturing may be attributed to accelerated depreciation over the period in question. For structures, the percentages are 5.7 for manufacturing and 3.0 for nonfarm nonmanufacturing. By 1970 we estimate that 6.6 percent of gross investment in manufacturing equipment, and 7.7 percent of gross investment in nonfarm nonmanufacturing equipment, will be due to the adoption of accelerated depreciation in 1954. The corresponding percentages for structures are 6.8 for manufacturing and 3.0 for nonfarm nonmanufacturing.

#### *Depreciation Life Guidelines, 1962*

The adoption of new guidelines for the determination of lifetimes allowable for tax purposes in 1962<sup>51</sup> affected only equipment lifetimes. The levels of the annual rental price of capital services, and the reductions brought about through the adoption of the 1962 depreciation guidelines, were:

	<i>Without guidelines</i>	<i>With guidelines</i>	<i>Percentage decrease</i>
Manufacturing equipment	0.315	0.307	2.5
Nonfarm nonmanufacturing equipment	0.384	0.374	2.6

<sup>51</sup> U.S. Treasury Department, Internal Revenue Service, *Depreciation Guidelines and Rules*, Publication No. 456 (July 1962).

Estimates of the increase in gross investment, net investment, and capital stock in equipment resulting from adoption of the new guidelines are given in Table 2-7.

*Investment Tax Credit and Long Amendment, 1962*

A second change in tax policy during 1962 was the adoption of an investment tax credit of 7 percent for equipment in the Revenue Act of 1962. As has been noted, various limitations on the applicability of the tax credit reduce the effective rate to 6 percent for manufacturing and 5.8 percent for nonfarm nonmanufacturing. Furthermore, the imposition and subsequent repeal of the Long amendment first eliminated the tax credit from the depreciation base in 1962 and 1963 and then restored it in 1964 and subsequent years. The levels of the annual rental price of capital services for 1963, and the reductions brought about by adoption of the tax credit with the Long amendment, were:

	<i>Without credit</i>	<i>With credit</i>	<i>Percentage decrease</i>
Manufacturing equipment	0.316	0.297	6.0
Nonfarm nonmanufacturing equipment	0.383	0.361	5.7

Estimates of the increase in gross investment, net investment, and capital stock in equipment resulting from the investment tax credit are given in Table 2-7. The impact of both of these policies is substantial, although the effect of the investment credit is several times larger than that of the depreciation guidelines. For the guidelines, the peak response in manufacturing industries of net investment in equipment took place in 1964, when they accounted for 12.5 percent of total net investment. The peak response to the investment credit took place a year later in 1965; in that year it accounted for 28.4 percent of net investment in equipment in the manufacturing sector. In nonmanufacturing industries, both peak responses took place earlier—17.2 percent for the guidelines in 1962 and 36.8 percent for the investment credit in 1964—reflecting the shorter lag in equipment investment in that sector.

The responses to the investment credit in both sectors show a dip resulting from its suspension in 1966–67. A smaller dip appears in the estimated effect of the depreciation guidelines during the same period, especially in the nonmanufacturing sector. This is explained by the fact that after the repeal of the Long amendment, investment credit and depreciation policies enhanced each other's effect. Thus the depreciation guidelines had a smaller

**TABLE 2-7. Estimated Changes in Gross and Net Investment in Equipment, and in Capital Stock, Resulting from 1962 Depreciation Guidelines and Investment Tax Credit, by Industrial Sector, 1962-70**  
(In millions of 1965 dollars)

Year	Depreciation guidelines										Investment tax credit									
	Manufacturing equipment					Nonfarm nonmanufacturing equipment					Manufacturing equipment				Nonfarm nonmanufacturing equipment					
	Investment		Capital stock		Gross	Investment		Capital stock		Gross	Investment		Capital stock		Gross	Investment		Gross	Net	Capital stock
	Gross	Net	Gross	Net		Gross	Net	Gross	Net		Gross	Net	Gross	Net		Gross	Net			
1962	165	165	0	743	743	743	0	0	185	185	185	0	804	804	0	804	804	0	0	
1963	292	268	165	770	627	743	743	743	527	500	500	185	1,711	1,556	804	1,556	804	804	804	
1964	375	311	433	845	582	1,370	1,370	1,370	992	891	891	685	2,773	2,319	2,319	2,773	2,319	2,319	2,360	
1965	369	260	744	816	441	1,952	1,952	1,952	1,229	997	997	1,576	2,726	1,826	1,826	2,726	1,826	1,826	4,679	
1966	260	112	1,004	828	368	2,393	2,393	2,393	1,015	637	637	2,573	2,069	818	818	2,069	818	818	6,505	
1967	215	51	1,116	641	110	2,761	2,761	2,761	724	252	252	3,210	2,017	609	609	2,017	609	609	7,323	
1968	211	39	1,167	687	135	2,871	2,871	2,871	635	126	126	3,462	2,516	991	991	2,516	991	991	7,932	
1969	218	41	1,206	687	109	3,006	3,006	3,006	723	195	195	3,588	2,283	567	567	2,283	567	567	8,923	
1970	226	43	1,247	716	117	3,115	3,115	3,115	884	328	328	3,783	2,330	505	505	2,330	505	505	9,490	

impact during the period of the suspension of the investment credit for equipment.

*Corporate Tax Cut, 1964*

In analyzing the effect of the reduction from 52 percent to 48 percent in the corporate tax rate in 1964, we assume that the before-tax rate of return was left unchanged. Under this condition the effect of a change in the tax rate on the rental price of capital services is neutral provided that depreciation for tax purposes is equal to economic depreciation.<sup>52</sup> Under the conditions actually prevailing in 1964, depreciation for tax purposes was in excess of economic depreciation for both plant and equipment in the manufacturing and nonfarm nonmanufacturing sectors. Accordingly, the rental price of capital services resulting from the tax cut was actually greater than the rental price before the cut. Following are the results for the annual rental prices for 1965, the first full year of the tax cut:

	<i>Without tax cut</i>	<i>With tax cut</i>	<i>Percentage increase</i>
Manufacturing equipment	0.296	0.299	1.0
Manufacturing structures	0.237	0.240	1.3
Nonfarm nonmanufacturing equipment	0.352	0.355	0.9
Nonfarm nonmanufacturing structures	0.247	0.250	1.2

Our estimates of the change in gross investment, net investment, and capital stock resulting from this change are given in Table 2-8. In general, the effects of the rate reduction are small and negative. It should be emphasized that these estimates depend on the level of output actually resulting from the tax cut; quite clearly the overall effect of the tax cut was to stimulate investment by increasing output.

*Repeal of Long Amendment, 1964*

A second, little-noticed change in tax policy in 1964 was the repeal of the Long amendment, which restored the tax credit to the depreciation base for tax purposes. Under the amendment, the effective rate of the tax credit

<sup>52</sup> For further discussion of tax-neutral depreciation, see pp. 17–18 above and Brown, “Business-Income Taxation and Investment Incentives”; Musgrave, *Theory of Public Finance*; Samuelson, “Tax Deductibility of Economic Depreciation To Insure Invariant Valuations”; and Smith, “Tax Depreciation Policy and Investment Theory.”

**TABLE 2-8. Estimated Changes in Gross and Net Investment in Equipment and Structures, and in Capital Stock, Resulting from the Tax Cut of 1964, by Industrial Sector, 1964-70**  
(In millions of 1965 dollars)

Year	Manufacturing						Nonfarm nonmanufacturing					
	Equipment			Structures			Equipment			Structures		
	Investment		Capital stock	Investment		Capital stock	Investment		Capital stock	Investment		Capital stock
	Gross	Net		Gross	Net		Gross	Net		Gross	Net	
1964	-49	-49	0	-20	-20	0	-135	-135	0	-44	-44	0
1965	-136	-129	-49	-62	-61	-20	-267	-241	-135	-125	-123	-44
1966	-181	-155	-178	-88	-83	-81	-217	-145	-376	-167	-155	-167
1967	-186	-137	-333	-87	-77	-164	-223	-123	-521	-168	-146	-322
1968	-155	-86	-470	-66	-51	-241	-274	-150	-644	-142	-110	-468
1969	-120	-38	-556	-43	-25	-292	-245	-92	-794	-99	-59	-578
1970	-119	-32	-594	-43	-23	-317	-213	-43	-886	-88	-44	-637

was approximately 6 percent; repeal raised it to almost 10 percent. The levels of the annual rental price of capital services, and the resulting reductions, are:

	<i>With Long amendment</i>	<i>Without Long amendment</i>	<i>Percentage decrease</i>
Manufacturing equipment	0.302	0.293	3.0
Nonfarm nonmanufacturing equipment	0.363	0.352	3.0

Estimates of the increase in gross investment, net investment, and capital stock resulting from this change are given in Table 2-9.

**TABLE 2-9. Estimated Changes in Gross and Net Investment in Equipment, and in Capital Stock, Resulting from Repeal of the Long Amendment, 1964-70**

(In millions of 1965 dollars)

Year	<i>Manufacturing equipment</i>			<i>Nonfarm nonmanufacturing equipment</i>		
	<i>Investment</i>		<i>Capital stock</i>	<i>Investment</i>		<i>Capital stock</i>
	<i>Gross</i>	<i>Net</i>		<i>Gross</i>	<i>Net</i>	
1964	238	238	0	1,042	1,042	0
1965	400	365	238	958	758	1,042
1966	412	329	567	706	360	1,800
1967	349	217	896	750	335	2,160
1968	229	67	1,113	1,021	541	2,495
1969	236	64	1,180	761	177	3,036
1970	297	115	1,244	792	174	3,213

These increases are quite substantial. The peak effect for manufacturing equipment took place in 1965, when the net investment in equipment attributable to the repeal was 10.4 percent of total net investment. In nonfarm nonmanufacturing, the peak effect for equipment came in 1964, accounting for over \$1 billion and 16.6 percent of net investment in that sector. Once again, the diminution of the impact of this policy change can be seen in 1966 and one or two years after, resulting from the suspension of the investment credit. The lag structure in the nonmanufacturing sector makes the dip much more noticeable there than in the manufacturing sector.

**TABLE 2-10. Estimated Changes in Gross and Net Investment in Equipment and Structures, and in Capital Stock, Resulting from Suspension of the Investment Tax Credit for Equipment and Accelerated Depreciation for Structures from October 10, 1966, through March 8, 1967, 1966-70**  
(In millions of 1965 dollars)

Year	Manufacturing						Nonfarm nonmanufacturing					
	Equipment			Structures			Equipment			Structures		
	Investment Gross	Net	Capital stock	Investment Gross	Net	Capital stock	Investment Gross	Net	Capital stock	Investment Gross	Net	Capital stock
1966	-177	-177	0	-46	-46	0	-762	-762	0	-119	-119	0
1967	-271	-245	-177	-89	-86	-46	-599	-452	-762	-200	-192	-119
1968	-153	91	-422	-60	-52	-132	69	303	-1,214	-126	-104	-311
1969	9	66	-513	11	12	-184	51	226	911	-11	18	-415
1970	157	223	-447	74	75	-172	18	150	685	111	139	-397



*Investment Credit Suspension, 1966*

In 1966 an important objective of economic policy was to restrain investment. After a number of alternative changes in tax policy were considered and rejected,<sup>53</sup> the investment tax credit for equipment was suspended beginning October 10, 1966; at the same time accelerated depreciation for structures was replaced by 150 percent declining-balance depreciation. Originally, the suspension was to remain in effect until the end of 1967, or almost fifteen months, but it was lifted on March 9, 1967, so that the period was a little less than five months. The effects of the suspension on the annual rental price of capital in 1967 were the following:

	<i>Without suspension</i>	<i>With suspension</i>	<i>Percentage increase</i>
Manufacturing equipment	0.320	0.351	9.7
Manufacturing structures	0.259	0.276	6.6
Nonfarm nonmanufacturing equipment	0.379	0.414	9.2
Nonfarm nonmanufacturing structures	0.270	0.287	6.3

Our estimates of the effects of the suspension on gross investment, net investment, and capital stock are given in Table 2-10.

For all categories of assets, the suspension had a restraining effect on the level of investment in 1967, which we estimate continued into 1968 except for nonfarm nonmanufacturing equipment. For both sectors the restoration of the original tax credit for equipment and accelerated depreciation for structures will result in a stimulus to investment in 1969 and 1970. For no class of assets is the level of capital stock as high at the end of 1970 as it would have been in the absence of the suspension. The total gross investment for the five-year period 1966-70 is considerably lower than it would have been in the absence of the five-month suspension.

If the suspension of the investment tax credit for equipment and accelerated depreciation for structures had continued for fifteen months, the impact on the level of investment would have been much more substantial, as our estimates in Table 2-11 reveal. For investment in structures the restraining effect of the suspension would have continued into 1969 in both sectors, although the impact would have been very slight in that year. For investment in equipment, as well as in structures, the magnitude of

<sup>53</sup> Policies under consideration during early 1966 and their potential impact on investment expenditures are discussed in Hall and Jorgenson, "Role of Taxation in Stabilizing Private Investment."

**TABLE 2-11. Estimated Changes in Gross and Net Investment in Equipment and Structures, and in Capital Stock, Resulting from Hypothetical Suspension of the Investment Tax Credit for Equipment and Accelerated Depreciation for Structures from October 10, 1966, through December 31, 1967, by Industrial Sector, 1966-70**

(In millions of 1965 dollars)

Year	Manufacturing						Nonfarm nonmanufacturing					
	Equipment			Structures			Equipment			Structures		
	Investment		Capital stock	Investment		Capital stock	Investment		Capital stock	Investment		Capital stock
	Gross	Net		Gross	Net		Gross	Net		Gross	Net	
1966	-177	-177	0	-46	-46	0	-762	-762	0	-119	-119	0
1967	-872	-846	-177	-250	-247	-46	-3,190	-3,043	-762	-614	-606	-119
1968	-567	-416	-1,023	-234	-216	-293	208	940	-3,805	-475	-425	-725
1969	-181	31	-1,439	-63	-32	-509	171	722	-2,865	-154	-74	-1,150
1970	270	477	-1,408	117	152	-541	093	505	-2,143	193	278	-1,224

the impact would have been much greater. As a result, the stimulus from restoration of the tax credit and accelerated depreciation would have been correspondingly increased.

## Conclusion

The objective of this study is to assess the effects of tax policy on investment behavior. For this purpose we have presented an econometric model of investment behavior based on the neoclassical theory of optimal capital accumulation. This model differs from a version used in two earlier studies<sup>54</sup> mainly in the imposition of further restrictions on the parameters of the underlying distributed lag function. These restrictions enable us to improve our specification of the lag structure and to economize on the number of parameters to be estimated. The resulting numerical estimates of the unknown parameters of our econometric model reflect the alterations in our statistical technique and incorporate data that have become available since our earlier studies. The lag structure derived from our new estimates suggests that the average lag between changes in the determinants of investment and actual expenditures for structures is shorter than that derived from our previous estimates. The new results are in much better agreement with evidence on the lag structure from sample surveys and from econometric models of investment fitted to data for industry groups for individual firms.

Our overall conclusion is the same as that in our previous studies: Tax policy can be highly effective in changing the level and timing of investment expenditures. Qualitatively speaking, a change in tax policy that reduces the rental price of capital services will increase the desired level of capital stock. This increase will generate net investment that eventually brings actual capital up to the new desired level. Gross investment follows the course of net investment at first, but gradually replacement requirements resulting from the higher level of capital stock come to predominate. Even if all the determinants of desired capital remain stationary at their new levels, gross investment is permanently increased by the higher levels of replacement associated with higher levels of capital.

From a quantitative point of view the tax measures we consider have substantially different impacts. The investment tax credit, essentially a subsidy to the purchase of equipment, has had a greater impact than any of the other changes in tax policy during the postwar period, especially

<sup>54</sup> *Ibid.*, and Hall and Jorgenson, "Tax Policy and Investment Behavior."

after repeal of the Long amendment made it even more effective. The shortening of lifetimes used in calculating depreciation for tax purposes and the use of accelerated methods for depreciation have been very important determinants of levels of investment expenditure since 1954. Suspension of the investment tax credit and accelerated depreciation from late 1966 to early 1967 had an important restraining effect on the level of investment; if this suspension had been allowed to remain in force for fifteen months rather than five, the impact would have been substantially greater. Of all the tax measures, only the reduction of the corporate tax rate in 1964, in our view, has had little impact on the level of investment expenditures. The reason for this is that tax depreciation and economic depreciation were virtually equal by 1964 so that any change in the tax rate would have been neutral in its effects on the price of capital services. The much-acclaimed tax cut of 1964 affected investment, but its main direct impact was through the enhanced effectiveness of the investment tax credit; reduction in the tax rate had a small but clearly negative impact on the level of investment.