Scalable Feedback Protocol for Achieving Sum-Capacity of the MIMO BC with Finite Feedback

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***Abstract*— We consider a feedback protocol with a limited amount of feedback that achieves the asymptotic sum-rate capacity. Time slots for channel feedback correspond not to users, but to a channel value; thus, users opportunistically access these slots based on their channel state information (CSI) measurement. We show analytically that the proposed SF protocol a) requires finite number of feedback slots upper bounded by a small number, b) is fully distributed, c) needs finite transmission energy during feedback and d) asymptotically achieves the sum-capacity of the MIMO BC. Numerical results show that the proposed feedback protocol performs close to a system with perfect CSI at the transmitter, with substantially less number of feedback bits compared with conventional CSI feedback methods because feedback requirements only grow as $\log_2(K)$ rather than linearly with K, where K is the number of users.**

I. Introduction

The capacity achieving strategy for MIMO BC's is dirty paper coding (DPC) and it was shown in [1] that the sum-capacity of the MIMO BC scales as $M\log(\log(K))$, where M is the number of transmit antennas at the base station, K is the total number of single-antenna users in the system and each entry of the vector channel is i.i.d zero mean circularly symmetric complex Gaussian (ZMCSCG) distributed for all the users. Thus the sum-capacity of the MIMO BC scales linearly with the number of transmit antennas (multiplexing gain) and only as $\log(\log(K))$ with the number of users (diversity gain). Although the sum-capacity of DPC with perfect CSIT can be approached by Tomlinson Harashima precoding [2], its large computational complexity hinders its application in practice. Recently, a number of techniques have been proposed [3], [4], [5], which are much simpler than dirty paper coding in terms of transmit complexity and still achieve the same $O(M\log(\log(K))$ asymptotic sum-capacity although they require perfect CSIT as well.

In order to deal with the limited-rate feedback problem, the users feedback only a quantized version of the CSI to the transmitter. For the random beamforming strategy it was shown in [6] that only one bit of feedback per user achieves the asymptotic sum-capacity. Also, recently in [7], it was shown that zero-forcing beamforming (which performs better than random beamforming for small number of users) also achieves the asymptotic sum-capacity with only a few bits of feedback per user. However, all these schemes require that all users feedback to the base station. Hence, although the sum-capacity grows only as $M\log(\log(K))$, the feedback rate required grows linearly with K. Due to this, eventually the feedback information itself may consume almost entire channel resources - bandwidth (frequency-bands) or time (time-slots), when K becomes very large.

It was shown in [8], a feedback method based on opportunistic p-persistent CSMA [9] can limit the number of users who feedback to be a constant that is independent of K in a single antenna channel, and this result is extended to the MIMO channel using random beamforming [10]. Unfortunately, the proof in [8] is only applicable to the Rayleigh fading channel (SISO channel), and only numerical results are shown for MIMO channels in [10]. Another random access scheme has been proposed in [11] for CSI feedback only by users having large CSI to limit the number of users who feedback. However, in their scheme a real number for the channel gain is fed back; whereas in [8] and the proposed scheme in this paper, channel gain is not fed back at all.

In this paper, scalable feedback (SF) protocols are considered for the MIMO BC that achieve the asymptotic sum-capacity with only a finite number of users (independent of K) feeding back their CSI to the base station. To generalize proofs in [8], a new method for calculating threshold is provided. In addition, we propose an SF protocol with ZFBF (SF-ZFBF) which uses semi-orthogonal user selection as proposed in [7] and achieves improved sum-rate for small K as compared to SF protocol with random beamforming (SF-RBF). For the proposed SF protocols, we show analytically that a) the expected number of feedback slots is independent of K and bounded above by a small constant which is a design parameter, b) the energy consumption during CSI feedback is independent of K as well and bounded by a small constant and c) the asymptotic sum-capacity of the MIMO BC is achieved. From a practical standpoint, time synchronization of short time slots during CSI feedback is a challenge. For the proposed SF protocols, it is shown that as long as the probability of out-of-synchronization transmission by any user is bounded above by a constant independent of K, asymptotic sum-capacity is still achieved. Numerical results show that the proposed SF protocols not only achieve the asymptotic sum-rate capacity $O(M\log(\log(K))$, but are actually close to sum-capacity with perfect CSIT; hence even the constants in the order notation $O(M\log(\log(K))$ are attained. Further, the numerical results follow the analytical expression derived very closely.

Notation: $\mathbb{E}$ denotes statistical expectation. $P\{\mathcal{S}\}$ denotes the probability of an event $\{\mathcal{S}\}$. $x_{i:n}$ is called the i-th order statistic among n random variables. $\mathcal{C}^{x\times y}$ denotes the matrix of dimension $x\times y$ and with entries being complex numbers. $\|\mathbf{x}\|$ denotes the ℓ^2-norm of a vector $\mathbf{x}$. $f=O(\phi)$ and $f=$

$\Omega(\phi)$ mean that $|f| < A\phi$ and $|f| > A\phi$ respectively, for some constant A and for all n with a sequence f_n.

II. Broadcast Channel Model

We consider a MIMO broadcast channel with M transmit antennas at the base station and $K \geq M$ users. The system model is depicted in Figure 1.

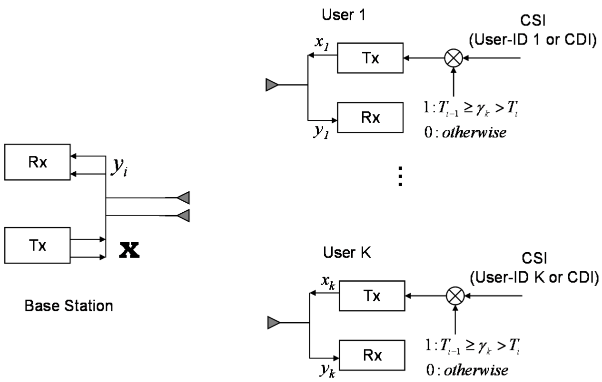

Fig. 1. MIMO BC with K users. During the data transmission period, $\mathbf{x}$ is transmitted and y_k is received at user k. During the feedback period, x_k is transmitted and y_i is recieved in slot i.

For simplicity, we assume users with single receive antenna. We assume users are homogenous and experience flat Rayleigh fading. The signal received by a user k can be represented as

$$y_k = \mathbf{h}_k \mathbf{x} + z_k, \quad , k = 1, 2, \ldots, K \tag{1}$$

where $\mathbf{h}_k \in \mathcal{C}^{1\times M}$ is the channel gain vector with i.i.d. ZMCSCG entries. Hence, the squared-norm of the channel or the channel gain is distributed as $Gamma(M, 1)$ for all the users[1]. The transmit vector is denoted by $\mathbf{x} \in \mathcal{C}^{M\times 1}$ which contains the information symbols for the chosen users, with an average power constraint $\mathbb{E}||\mathbf{x}||^2 \leq P$ and z_k is the additive Gaussian noise with unit variance. The set of chosen users is denoted by $\mathcal{S}$. To model temporal channel variations, a slowly-varying block-fading channel is assumed i.e. the channel vector is unchanged during a feedback period plus a BC packet transmission period (defined in Section III), and it changes independently of the previous channel realization after a packet transmission.

The transmit symbol vector $\mathbf{x}$ is related to information symbols $s_i, i \in \mathcal{S}$, via linear beamforming $\mathbf{x} = \sum_{i\in\mathcal{S}} \mathbf{w}_i s_i$. Therefore, the received symbol at one of the chosen users can be written as

$$y_k = (\mathbf{h}_k \mathbf{w}_k) s_k + \sum_{j\in\mathcal{S}, j\neq k} (\mathbf{h}_k \mathbf{w}_j) s_j + z_k, \quad k \in \mathcal{S} \tag{2}$$

The user set $\mathcal{S}$ is chosen to maximize the sum-rate as described in Section IV.

[1]Norm-squared of each entry of the channel vector is exponentially distributed and sum of exponential random variables is the *Gamma* distribution.

III. Feedback Protocol

This section proposes a feedback protocol, which will be shown in the next section to maintain the amount of feedback a constant and to achieve the asymptotic sum-rate capacity. We first describe the SF protocol for the single antenna case to achieve the diversity gain in a similar manner as in [8], and later propose two modifications to it, one using Random Beamforming (SF-RBF) (also shown in [10]) and the second one using Zero-Forcing Beamforming (SF-ZFBF) to achieve the multiplexing gain as well.

In Figure 2, we show the operation of the SF protocol for the SISO case.

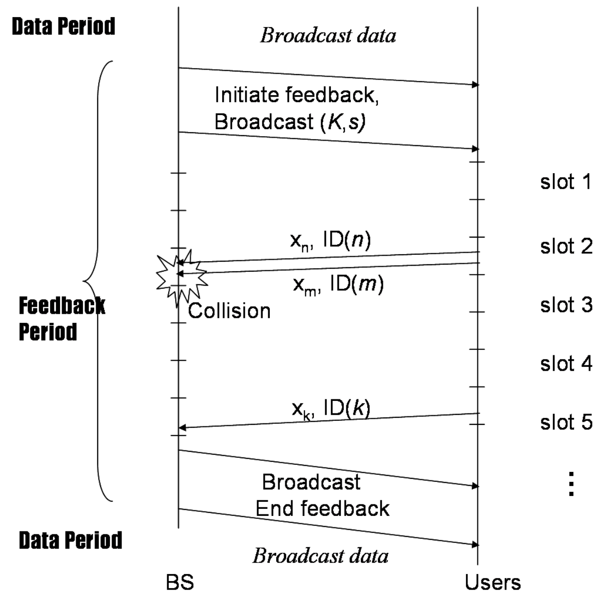

Fig. 2. The feedback period of the SF protocol.

The uplink feedback channel is divided into short slots. All the users are synchronized to the base station such that any feedback transmission initiates precisely at the beginning of a slot Slot duration is long enough to transmit one user's CSI x_k, $k = 1, 2, \ldots, K$ without packet errors. The base station broadcasts a training sequence, which is used by the user k to estimate the channel gain γ_k. Subsequently, the base station broadcasts an *Initiate Feedback* message to begin a feedback period. The total number of users K and the non-collision probability s are also attached to this message for the users to calculate the thresholds $T_i, i = 1, \ldots, N$. The relationship between T_i and K, s will be discussed later in this section. If no user sends a feedback message in a slot, the feedback period continues till the base station receives CSI from a user successfully, where success happens when there is no collision. At slot i, any user k compares its channel gain γ_k with the thresholds T_i and T_{i-1}. If $T_i < \gamma_k < T_{i-1}$, then user k transmits its CSI x_k in slot i. The CSI x_k is the unique user ID (for which the number of bits needed will be $\log_2(K)$). If two users, j and k, are such that $T_i < \gamma_j, \gamma_k < T_{i-1}$, then their feedback messages will collide. These collided messages are assumed to be irrecoverable for simplicity of the strategy, although the base station may decode both their messages by standard MAC decoding strategies like successive cancelation. If only one user transmits a feedback message in slot i, the base station decodes x_k successfully and thereby broadcasts an *End Feedback* message to stop the feedback period. Finally the base station transmits a data packet to the selected user u with a capacity achieving code using the channel gain T_i, where $T_i < \gamma_k < T_{i-1}$. If the probability of collisions reduces to 0 exponentially with K, the SF protocol 1) selects the user with the highest channel gain, 2) with its channel gain known to a high accuracy, in an autonomous and distributed manner. Though proved rigorously in Section IV, it can be noticed that if the number of thresholds (denoted by N) is large, both 1) and 2) are achieved.

If the distribution function for the channel gains (same for all users since homogenous fading) is given by $F_\Gamma(\gamma)$, then the probability that a user transmits its feedback message in the ith slot is given as

$$P\{T_i < \gamma < T_{i-1}\} = F_\Gamma(T_{i-1}) - F_\Gamma(T_i), \quad i = 1, 2, \ldots, N \tag{3}$$

To maintain the probability that a user transmits a feedback message in any slot to be p, we require $P\{T_i < \gamma < T_{i-1}\} = p, \forall i$. The transmission probability at the 1st slot is

$$P\{T_1 < \gamma\} = 1 - F_\Gamma(T_1) = p \tag{4}$$

and so the transmission threshold for any slot i is given as

$$T_i = F_\Gamma^{-1}(1 - ip), i = 1, 2, \ldots, N \tag{5}$$

It is easy to see that $N = \frac{1}{p}$ and the thresholds for a $Gamma(M, 1)$ distribution are shown in Figure 3.

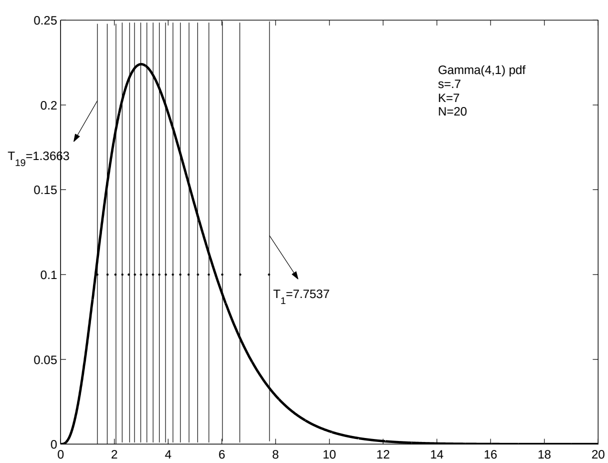

Fig. 3. Thresholds for the $Gamma(M, 1)$ pdf.

Notice the duality between the conventional CSI feedback systems as used in [3], [4], [5], [6] and the proposed SF protocol. In conventional systems, a feedback slot is reserved for the user and the channel gain value (quantized or unquantized) is fed back. The feedback resources thus scale linearly in K. In the SF protocol, a feedback slot is reserved for the channel gain value and the user ID is fed back. Since, the expected number of feedback slots is a small constant independent of K (proved in Section IV), the feedback resources essentially scale as $\log_2(K)$, the number of bits required for user ID. This is just for the asymptotic capacity part; in general, the SF protocol leads to higher rates as channel gain value is known sufficiently accurately as well.

We now consider the MIMO system. The multiplexing gain (linear increase of sum-rate with M) is achieved by the virtue of being able to transmit M independent data streams. When using simple linear beamforming strategies for transmission (which is the case here), in order to get multiplexing gain the transmitter has to find M orthogonal channel directions so that there is no multiuser interference, which is possible with high probability (w.h.p.) when the number of users K is large since all users have independent channels. The SF protocol can be easily modified to find the orthogonal transmission directions in two ways.

The SF-RBF employs the random beamforming in [3] in conjunction with the proposed feedback method. To obtain the threshold, the effective SINR is used in place of SNR, which is defined as

$$\begin{aligned} \text{SINR}_{k,m} &= \frac{\mathbf{w}_m^{\text{RBF}*}\mathbf{h}_k}{\frac{1}{\rho} + \sum n \neq m \mathbf{w}_n^{\text{RBF}*}\mathbf{h}_k}, \\ k &= 1, \ldots, K, m = 1, \ldots, M \end{aligned} \tag{6}$$

where $\rho = \frac{P}{M}$. To select M users for M different beams, the SF protocol in this section is repeated for M times. Analysis that the SF-RBF protocol achieves the asymptotic sum-capacity is provided in Section IV.

A. SF-ZFBF Protocol

The SF-ZFBF protocol uses ZFBF and the semi-orthogonal user selection procedure as described in [4]. As shown in [4], the random beamforming scheme does not perform well for small K because for small K, the selected random orthogonal beams may not be well aligned with any of the user's actual current channel realizations. Hence, we find users with high current channel gains and who are also semi-orthogonal and construct beams based on the selected user's channels.

In the SF-ZFBF protocol, a feedback period consists of M sub-feedback periods. The very first feedback period is exactly the same as that for the single-antenna case, only that

$$\gamma_k = \frac{\rho||\mathbf{h}_k||^2 \cos^2(\theta_k)}{1 + \rho||\mathbf{h}_k||^2 \sin^2(\theta_k)} \tag{7}$$

where $\rho = \frac{P}{M}$ and θ_k is the quantization error during channel direction information (CDI) quantization using B bits and random vector quantization (RVQ). The base station broadcasts a training sequence, which is used by user k to estimate the effective SNR γ_k as in Eqn (7). Subsequently, the base station broadcasts an *Initiate First Feedback* message to begin the first feedback period. The total number of users K, the non-collision probability s and an semi-orthogonality parameter ϵ are also attached to this message for the users to calculate the thresholds $T_i, i = 1, \ldots, N$. Based on its own γ_k, when the user k transmits in slot i, in addition to transmitting the user ID, user k also transmits its channel direction information CDI or the direction of $\mathbf{h}_k$ quantized to B bits as detailed in [7]. When the base station decodes x_k (comprising of the user ID and CDI information) successfully, it broadcasts an *End First Feedback* message to stop the first sub-feedback period. In the second sub-feedback period, the base station broadcasts the *Initiate Second Feedback* message with $\hat{\mathbf{h}}_{u_1}$, the quantized CDI for the user u_1 selected at the end of the first sub-feedback period. The users then calculate

$$\phi_k = |\hat{\mathbf{h}}_k \hat{\mathbf{h}}_{u_1}^*| \tag{8}$$

Unlike the first sub-feedback period, now only the users who have $\phi_k < \epsilon$ transmit x_k (comprising of the user ID and CDI information) in a feedback slot. This assures that the only the semi-orthogonal users transmit in the second sub-feedback period. The thresholds $T_i, i = 1, \ldots, N$ are recalculated with the number of users K modified to $K\alpha_1$, the effective number of users left after the semi-orthogonality constraint is satisfied, where the constants $\alpha_m < 1, m = 0, 1, \ldots, M-1$ are defined in [7]. In a similar manner, the remaining sub-feedback periods are executed. When the base station decodes x_k successfully M times, it broadcasts an *End Mth Feedback* message to stop the feedback period. Finally the base station transmits data to the M selected users with a capacity achieving code and ZFBF. The SF-ZFBF protocol also achieves the asymptotic sum-capacity as shown in Section IV.

IV. Performance Analysis

In this section, the achievable sum-rate and the amount of required feedback resources are analyzed. The consumption of feedback resources is measured by the expected number of slots per feedback period ($\mathbb{E}[S]$) and the expected number of feedback messages transmitted during a feedback period ($\mathbb{E}[M']$). Notice that $\mathbb{E}[M']$ is representative of the total energy spent by the users during the feedback period. Also, the sum-rate proof is given for any general distribution for the channel gain (or effective SINR as the case may be depending upon SF-RBF or SF-ZFBF is used) $F_\Gamma(\gamma)$. The $M \log\log(K)$ result follows when $F_\Gamma(\gamma)$ is the *Gamma*$(M, 1)$ distribution (or for the distributions derived in [3], [7] for effective SINR[2] or CSI).

Since the users have independent fading, the probability that any user transmits in a given feedback slot i is p. Hence, by binomial probability, the probability that a single user transmits in the first feedback slot is

$$r = Kpq^{K-1} \tag{9}$$

In case more than one user transmits or no user transmits, the SF protocol proceeds to the second feedback slot. This happens with probability $(1-r)$. Now in the second feedback slot, a single user transmits with the same probability r if no user transmitted in the previous slot or with a lower probability $(K - n_1)pq^{(K-n_1)-1}$, where $n_1 \geq 2$ users transmitted in the first feedback slot. As shown later in the section, for the choice of $q = s^{1/K}$, the probability that two or more users transmit in a single slot goes to 0 exponentially in K, where $s \simeq 1$. So we assume that the number of users remains the same after a feedback slot in the rest of this section. Under this assumption (valid for K large), the number of feedback slots till successful reception is geometrically distributed with probability r and the expected value is given as

$$\mathbb{E}[S] = \frac{1-r}{r} \tag{10}$$

Now using the fact that $s = q^K$, we can upper-bound $\mathbb{E}[S]$ as

$$\begin{aligned} \mathbb{E}[S] &\overset{(a)}{\leq} \frac{s^{\frac{1}{K}}}{Ks\left(1 - s^{\frac{1}{K}}\right)} && (11) \\ &\overset{(b)}{\leq} \lim_{K\to\infty} \frac{s^{\frac{1}{K}}}{Ks\left(1 - s^{\frac{1}{K}}\right)} \\ &\overset{(c)}{\leq} \frac{1}{s(1-s)} && (12) \end{aligned}$$

where (a) follows from the fact that $1-r \leq 1$, (b) follows from the fact that the R.H.S. of Eqn (11) is an increasing function of K and (c) follows by using the identity that $e^{(s-1)} < s$ for $0 < s < 1$. Thus we have shown that the expected number of feedback slots is upper bounded by a finite number independent of K.

Next we evaluate the sum-rate of the proposed SF protocols. We first look at the expected sum-rate for the single antenna case, in which, there is just one feedback period without any subdivision into sub-feedback periods. Notice that as $K \to \infty$, $q = s^{1/K} \to 1$, $p \to 0$ and so $N = \frac{1}{p} \to \infty$ or the number of thresholds goes to ∞. Thus for large K, the channel gain estimate at the transmitter for any user is sufficiently accurate[3]. With the assumption of large K, the expected sum-rate can be written as

$$R_{su} = \sum_i (r+s)(1-(r+s))^{(i-1)} \mathbb{E}\log\left(1 + \gamma_{K:K-n_i}\right) \tag{13}$$

where r is the probability that only one user transmits in a slot, $(1-(r+s))$ is the probability that two or more users transmit in a slot since $s = q^K$ is the probability that no user transmits, $x_{K:k}$ is the random variable characterizing the kth maximum of a set of K i.i.d. random variables x, $n_i \geq 2$ is the number of users that transmit in the ith unsuccessful feedback slot, with $n_1 \triangleq 0$ and the summation is till we run out of users[4]. Notice that we have replaced the threshold T_i, which is what

[2]The CDF of SINR in [3] was derived to be $F_\Gamma^{\text{RBF}}(\gamma) = 1 - \frac{e^{-\gamma/\rho}}{(1+\gamma)^{M-1}}$ and of γ_k in [7] was derived to be $F_\Gamma^{\text{ZFBF}}(\gamma) = 1 - \frac{2^B e^{-\frac{\gamma}{\rho}}}{(1+\gamma)^{M-1}}$ for $\gamma \geq \frac{1}{\delta} - 1$ where $\delta = 2^{-\frac{B}{M-1}}$.

[3]We will show in Section V, that the sum-rate performance is good for small K as well.

[4]Although successful transmission happens with probability r, in Eqn (13), we have success in terms of 'expected value of sum-rate calculation' with probability $(r+s)$. This is because in feedback slots where no users transmits, the feedback period continues changing nothing in terms of finding the maximum of K random variables (since no users are lost) and only affecting the number of feedback slots till success; already upper bounded earlier in this section

is known at the base station, in the sum-rate expression with the actual channel gain value $\gamma_{K:K-n_i}$ assuming K is large. The sum-rate R_{su} can be lower bounded by the first positive term $(i = 1)$ in the summation as

$$\begin{aligned} R_{su} &\geq s\mathbb{E}\log\left(1+\gamma_{K:K}\right) && (14) \\ &\stackrel{(a)}{=} O(\log\log K) && (15) \end{aligned}$$

where (a) follows for a Rayleigh distributed channel since the maximum behaves like $O(\log(\log(K)))$ and s is a constant. Also notice that the result in Eqn (14), is independent of the channel gain (or effective SINR for SF-RBF and SF-ZFBF protocols) distribution.

Notice that the SF protocol just does not find a user in the set of good users (users with channel gains above $\log(K)$), which is what is required for sum-rate asymptotic optimality. This is what conventional schemes for feedback requiring one-bit feedback for channel gain value do. Rather, the SF protocol finds the user with the maximum channel gain when the probability that two or more users transmit in a single slot goes to 0, which holds true as

$$\begin{aligned} P\{\geq 2 \text{ users }\} &\leq \frac{K!}{N^K} \\ &\stackrel{(a)}{\leq} \left(\frac{Ke^{-1}(2\pi K)^{\frac{1}{2K}}}{N}\right)^K \\ &\stackrel{(b)}{\to} 0 \end{aligned} \qquad (16)$$

where (a) follows from Stirling's approximation and (b) follows as long as $N = \Omega(K)$. We now show that in fact $N > K$.

$$N = \frac{1}{p} = \frac{1}{1-s^{1/K}} \stackrel{(a)}{>} K \qquad (17)$$

where (a) follows from the fact that

$$\left(\frac{K-1}{K}\right)^K \leq \lim_{K\to\infty}\left(\frac{K-1}{K}\right)^K = \frac{1}{e} < s \qquad (18)$$

since $\frac{1}{e} < s \simeq 1$. Indeed it was shown in [10], that when γ is exponentially distributed, the SF protocol performs the same in terms of sum-rate as a system with perfect CSIT i.e. the user with the maximum channel gain is found w.h.p..

Notice that as long as $P\{\geq 2 \text{ users }\} \to 0$, the SF protocol achieves the sum capacity asymptotically even if the number of thresholds is not very large to know the channel gains exactly. Rather all we need to achieve sum-capacity asymptotically is to find one user in the *"good"* user set. The large number of thresholds N is to let $P\{\geq 2 \text{ users }\} \to 0$ and not to know the channel gains exactly; it just comes as a side advantage.

We now bound the expected number of total transmissions of feedback messages by the users before the end of the feedback period or $\mathbb{E}[M']$.

$$\begin{aligned} \mathbb{E}[M'] &= \sum_i r(1-r)^{(i-1)}(Kp) \\ &\leq \sum_{i=1}^{\infty} r(1-r)^{(i-1)}(Kp) \\ &= Kp \end{aligned} \qquad (19)$$

where Kp is the expected number of users that transmit in any feedback slot. Now using that $p = 1 - s^{1/K}$, we get that

$$\mathbb{E}[M'] \leq Kp = K(1-s^{1/K}) < \frac{1}{s} \qquad (20)$$

since $s \simeq 1$. Notice that $\mathbb{E}[S] > \mathbb{E}[M']$. This happens because q (the probability of no transmission) goes to 1 as $K \to \infty$ and so slots with no transmission are more frequent. Notice that Eqn (20) shows that very little energy is spent in the feedback period on average $(s \simeq 1)$.

Now, the proof that both the SF-RBF and SF-ZFBF also achieve the asymptotic capacity of the MIMO BC which is $O(M\log(\log(K)))$ follows from the above argument and the results in [3] and [7] respectively.

The SF protocols, however, requires an accurate timing-synchronization to avoid collisions, which is difficult with short feedback slots. Since the number of feedback slots is substantially less than conventional methods, a few slots can be provided as guard slots; say every alternate feedback slot no user transmits doubling the expected number of slots; but still independent of K. Further, if the maximum probability of out of synchronization transmission (denoted by p_{as}) over all users is a constant independent of K, then we can show that sum-capacity is still achieved. This is because, any asynchronous transmission lowers the probability of successful transmission (denoted by r in Section IV) and so the sum-rate expression is modified as

$$\begin{aligned} R_{su} &= \sum_i (r+s-p_{as})(1-(r+s)+p_{as})^{(i-1)} \\ &\quad \mathbb{E} \log\left(1+\gamma_{K:K-n_i}\right) \\ &\geq (s-p_{as})\mathbb{E}\log\left(1+\gamma_{K:K}\right) \\ &= O(\log\log K) \end{aligned} \qquad (21)$$

As seen, asymptotic sum-capacity is still achieved as long as $p_{as} < s \simeq 1$. In this analysis, we assumed that an out of synchronization transmission affects a single feedback slot, which is a valid assumption in the presence of guard slots.

V. Numerical Results

In this section, numerical results are presented. In Figure 4 and Figure 5, we plot the performance of the proposed SF protocols as function of the number of users K for $s = 0.95$, $P = 10$dB, $M = 4$ and $M = 2$ respectively. We compare the performance of the SF-RBF protocol with that of the RBF with unquantized SINR feedback from all the users as proposed in [3]. It is seen that the SF-RBF loses very little performance by just the user ID feedback. The performance of the SF-ZFBF protocol is compared with that of the ZFBF scheme as proposed in [7] with $\epsilon = 0.25$, $B = 20$ and random vector quantization (RVQ). The performance of the proposed scheme is indistinguishable from that of the ZFBF scheme that has feedback from all the users. The performance of the one-bit feedback scheme [6] which feeds back a single bit from each user in conjugation with random beam forming is also plotted and performs worse as compared to RBF with unquantized SINR feedback as only one bit is fed back. The sum-capacity asymptote $M \log\log(K)$ is also plotted and as seen all the schemes achieve asymptotic sum-capacity.

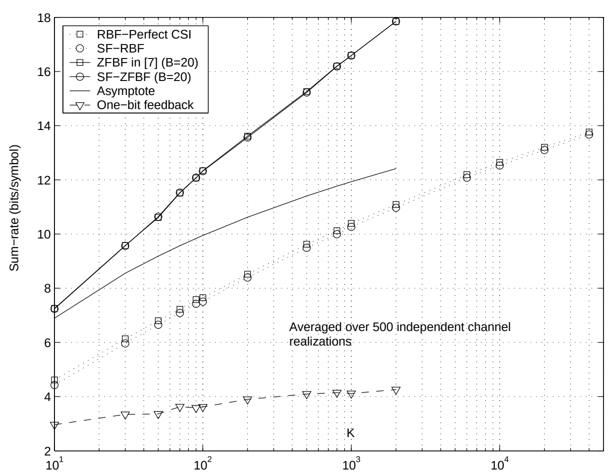

Fig. 4. Sum-rate comparison of the proposed SF protocols for M=4.

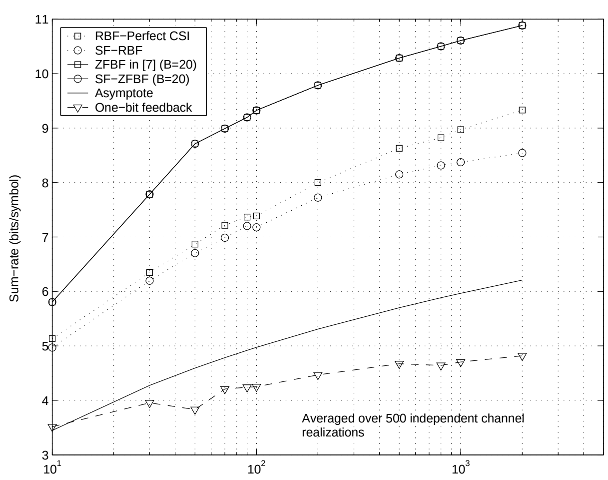

Fig. 5. Sum-rate comparison of the proposed SF protocols for M=2.

In Figure 6, we plot the feedback resources for all the schemes normalized by M. As seen the feedback load (expected number of feedback slots), of the conventional schemes grows linearly with K, whereas for the SF protocols, it grows only logarithmically ($\log_2(K)\mathbb{E}[S]$). Due to this, the saving of feedback resources is substantial, especially for large K. Also plotted is the expected number of users who transmit during the feedback period ($\mathbb{E}[M']$). As seen the numerical values are very close to the derived analytical expressions in Section IV.

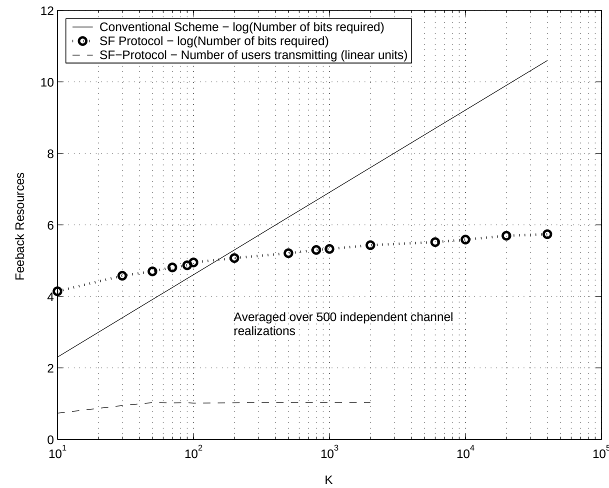

Fig. 6. Comparison of feedback resources (normalized by M).

VI. Conclusion

This paper describes scalable feedback protocols, which is shown to asymptotically achieve the MIMO-BC sum-capacity while the amount of feedback (the expected number of slots and power consumptions) is maintained to be a constant independent of the number of users. A new thresholds based on uniform quantization are proposed to simplify the analysis in [8]. Moreover, the ZFBF in [7] in conjunction with the proposed feedback method achieves near sum-rate capacity even with a relatively small number of users. The SF protocols are shown to achieve the asymptotic sum-capacity in the presence of synchronization errors as well.

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