Example 11.14 **Alfalfa and Acid Rain.** For the alfalfa growth data of Example 11.12, the ANOVA summary is given in Table 11.9. The F statistic is $.993/.1815 = 5.47$, with degrees of freedom 2 for the numerator and 8 for the denominator. From Table 10 we bracket the P-value as $.02 < P\text{-value} < .05$. (Using a computer gives $P = .0318$.) The P-value is small, indicating that the differences between the three sample means are greater than would be expected by chance alone.

TABLE 11.9 ANOVA Table for Alfalfa Experiment

Source	df	SS	MS	F Ratio
Between treatments	2	1.986	0.993	5.47
Between blocks	4	0.840	0.210	
Within groups	8	1.452	.1815	
Total	14	4.278		

■

Factorial ANOVA

In a typical analysis of variance application there is a single explanatory variable or **factor** under study. For example, in the weight gain setting of Example 11.2 the factor is "type of diet," which takes on 3 **levels**: Diet 1, Diet 2, and Diet 3. However, some analysis of variance settings involve the simultaneous study of two or more factors. The following is an example.

Example 11.15 **Growth of Soybeans.** A plant physiologist investigated the effect of mechanical stress on the growth of soybean plants. Individually potted seedlings were randomly allocated to four treatment groups of 13 seedlings each. Seedlings in two groups were stressed by shaking for 20 minutes twice daily, while two control groups were not stressed. Thus, the first factor in the experiment was presence or absence of stress, with two levels: control or stress. Also, plants were grown in either

low or moderate light. Thus, the second factor was amount of light, with two levels: low light or moderate light. This experiment is an example of a $2 \cdot 2$ *factorial experiment*; it includes four treatments:

Treatment 1: Control, low light
Treatment 2: Stress, low light
Treatment 3: Control, moderate light
Treatment 4: Stress, moderate light

After 16 days of growth, the plants were harvested, and the total leaf area (cm^2) of each plant was measured. The results are given in Table 11.10 and plotted in Figure 11.12.[12]

TABLE 11.10 Leaf Area (cm^2) of Soybean Plants

	Treatment			
	Control, Low Light	*Stress, Low Light*	*Control, Moderate Light*	*Stress, Moderate Light*
	264	235	314	283
	200	188	320	312
	225	195	310	291
	268	205	340	259
	215	212	299	216
	241	214	268	201
	232	182	345	267
	256	215	271	326
	229	272	285	241
	288	163	309	291
	253	230	337	269
	288	255	282	282
	230	202	273	257
Mean	245.3	212.9	304.1	268.8
SD	27.0	29.7	26.9	35.2
n	13	13	13	13

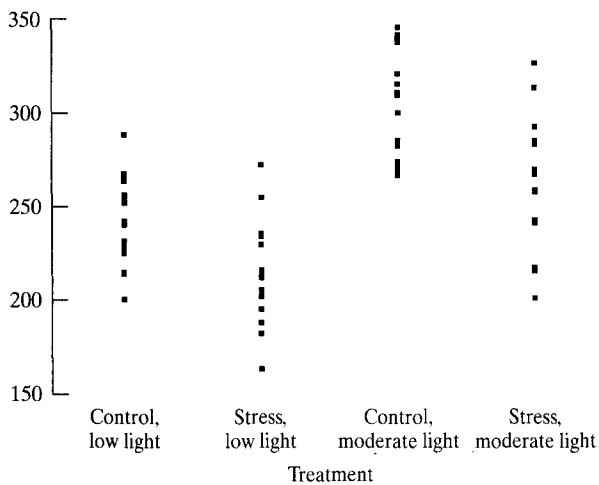

Figure 11.12 Leaf area of soybean plants receiving four different treatments

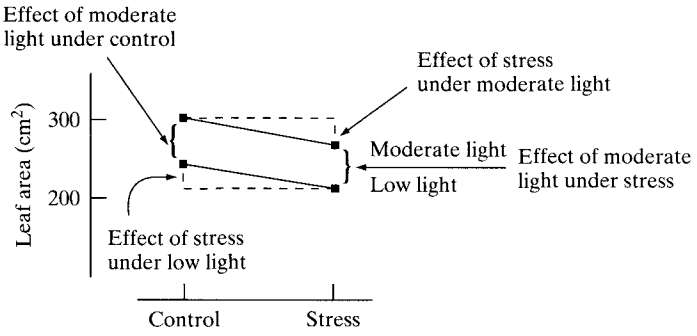

Figure 11.13 Treatment means for soybean experiment

When the effects of factors are additive, we say that there is no **interaction** between the factors. A graph that displays the treatment means is often called an interaction graph. Figure 11.14, which is a simplified version of Figure 11.13, is an interaction graph.

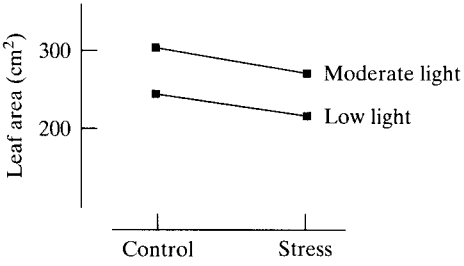

Figure 11.14 Interaction graph for soybean experiment

Sometimes the effect that one factor has on a response variable depends on the level of a second factor. When this happens we say that the two factors interact in their effect on the response. The following is an example.

Example 11.17

Carbon Dioxide. The rate at which trees absorb carbon dioxide (CO_2) depends on the amount of carbon dioxide in the atmosphere, in addition to other factors. Researchers conducted an experiment to learn how two factors affect the rate at which trees in a forested area absorb CO_2. The first factor was CO_2 concentration in the atmosphere, which had the levels "ambient" and "elevated." The second factor was type of soil, which had the levels "unfertilized" and "fertilized." The response variable was annual carbon increment in woody tissue (measured in units of kg C per square meter of ground area). Table 11.12 summarizes the data, which

TABLE 11.12 Mean Carbon Absorption Values (kg C per Square Meter Ground Area per Year) for CO_2 Experiment.

		Soil Type		
		Unfertilized	*Fertilized*	*Difference*
CO_2 concentration	Ambient	.289	.347	.058
	Elevated	.227	.496	.269
	Difference	−.062	.149	