The Demand for Youth: Implications for the Hours Volatility Puzzle*

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Abstract

The employment and hours worked of young individuals fluctuate much more over the business cycle than those of prime-aged individuals. Understanding the mechanism underlying this observation is key to explaining the volatility of aggregate hours over the cycle. We argue that the joint behavior of age-specific hours and wages in the U.S. data point to differences in the cyclical characteristics of labor demand. To articulate this view, we consider a production technology displaying capital-experience complementarity. We estimate the key parameters governing the degree of complementarity and show that the model can account for the behavior of age-specific hours and wages while generating a series of aggregate hours that is nearly as volatile as output.

Keywords: business cycle, demographics, capital-experience complementarity, labor demand
JEL codes: E00, E32

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1 Introduction

The employment and hours worked of young individuals fluctuate much more over the business cycle than for the prime-aged. The hypothesis in this paper is that understanding the mechanisms underlying this observation, while interesting in its own right, has the potential to shed light on a long standing puzzle in the business cycle literature: why aggregate hours are nearly as volatile as output.

Our hypothesis is based on the fact that cyclical fluctuations in aggregate hours are disproportionately accounted for by young workers. In the postwar era 15-29 year olds account for about one quarter of total hours worked in the U.S.; however, this same group accounts for almost one half of the the volatility of aggregate hours at the business cycle frequency.

By contrast, recent work in business cycle theory has, with few exceptions, focused on models with homogenous labor input. As a result, much of the literature cannot address these differences by age, and importantly, the question of why young labor input is so volatile over the cycle. Developing a quantitative theory that can account for the volatility of this age group is likely to be crucial to understanding the volatility of aggregate hours and, ultimately, the mechanisms that amplify and propagate business cycle fluctuations.

This is not the first paper to address this dimension of heterogeneity in analyzing labor market fluctuations. Rios-Rull (1996) and Gomme, Rogerson, Rupert, and Wright (2004) study models with age differences in hours volatility owing to life-cycle considerations (e.g., preferences for home and market production, and efficiency units of hours worked that differ by age). They show that life-cycle factors are successful at explaining volatility differences between the prime-aged and those near retirement age, but cannot account for the much greater volatility of young workers as compared to all others. Hansen and Imrohoroglu (2008) consider a life-cycle model in which efficiency units of labor are accumulated while working via learning-by-doing. This generates substantial differences in volatility by age, but at the expense of dampening the volatility of hours worked overall. Hence, Hansen and Imrohoroglu (2008) show that the learning-by-doing model actually under-performs relative to the standard real business cycle (RBC) model in matching the volatility of aggregate hours over the cycle.

In this paper, we maintain comparability with the RBC literature by studying a model that represents a minimal deviation from the standard model. Within the RBC framework, differences across age groups arise from differences in preferences (or succinctly, differences in labor supply), factors relating to technology (labor demand), or both.[1]

How does one distinguish between these two potential channels? We suggest that the joint behavior of age-specific hours and wages over the cycle sheds light on this question. As we document in Sections 2 and 3, young individuals in the U.S. not only experience greater hours volatility, but also have greater wage volatility than prime-aged individuals. Any modification to the RBC model incorporating age-specific labor supply differences alone would not be able explain this fact.[2] Jointly matching the behavior of hours and wages in the RBC framework requires a role for differences in cyclical labor demand.

To articulate this view, we consider an environment characterized by labor demand differences due to *capital-experience complementarity* in production. The large body of work studying capital-skill complementarity has concentrated on education as a proxy for skill (see Krusell, Ohanian, Rios-Rull, and Violante (2000), and the references therein for anaylsis relating to the post-war education premium; and see Castro and Coen-Pirani (2008) for analysis of the business cycle implications of capital-skill complementarity). We concentrate on the other significant observable dimension of skill emphasized in Mincerian wage regressions, namely labor market experience.

To highlight the potential in this approach, we assume that there are only two groups of workers, young and old; we posit that an individual's age directly determines his or her labor market experience. With technology exhibiting capital-experience complementarity, differences in the cyclical demand for experienced and inexperienced labor arise naturally. As an extreme case, suppose that capital and old, or experienced, labor are perfect complements, while capital and young, or inexperienced, labor display some substitutability. If capital services are a state variable and firms are profit maximizing and price-taking, any shock generating a response in inputs results in variation

[1] By RBC framework, we are referring to neoclassical models in which households and firms take all prices as given, and interact in competitive spot markets. The papers discussed in the previous paragraph model age differences as owing to labor supply characteristics. See Nagypál (2004) for an alternative approach highlighting the interaction between age and worker-occupation match.

[2] Section 4.4 and Appendix A.3 discusses this in depth.

in only the quantity of young labor hired.

More generally, we view our theoretical mechanism as speaking to the existence of complementarity in production between experienced labor and factors that are in fixed short-run supply to the firm. These factors may include organizational capital, firm know-how, or operational knowledge that inherently require the presence of experienced labor. Since this type of knowledge or capital is hard to adjust in the short-run, it is natural that cyclical fluctuations in output result in greater variation of inputs like young or inexperienced labor that are less tied to these factors.

Of course, the measurement of factors such as organizational capital or firm know-how is very difficult. The challenge in this paper is to account for the observed differences in hours volatility in a quantitative manner. This motivates our modeling choice, as specifying complementarity between physical capital and experienced labor. The availability of high-quality data relating to these factor inputs allows us to discipline our analysis.

We estimate the key structural parameters governing the degree of capital-experience complementarity in a parsimonious manner. Our strategy entails estimating these parameters from the model's factor demand equations, exploiting the identification that emerges from the relationship between aggregate prices and quantities observed in the U.S. data. It is worth noting that our estimation strategy does *not* target differences in the cyclical volatility of hours.

Based on this structural estimation we simulate the model economy. We find that the model generates age-specific hours volatilities that are similar to those observed in the data. As a by-product of this success, the model generates volatility of aggregate hours that is very close to that of aggregate output. We then show that the model can account for the joint behavior of age-specific hours and relative wages.

The paper is organized as follows. In Section 2, we document differences in the volatility of hours worked by age, and indicate the importance of this dimension of disaggregation relative to other demographic factors. Empirical evidence guiding our modeling approach is presented in Section 3. Section 4 presents our model with capital-experience complementarity, along with analytical results on the response of age-specific hours and wages to business cycle shocks. Section 5 discusses the quantitative specification of our model, and Section 6 presents results for the model's cyclical properties relative to the U.S. data. Concluding remarks are provided in Section 7.

2 The Cyclicality of Age-Specific Hours

In this section, we analyze the responsiveness of market work over the U.S. business cycle for data disaggregated by age. We consider both the behavior of hours worked and unemployment by age.

2.1 Age-Specific Hours

Our approach to studying differences in business cycle volatility by age is similar to that of Gomme, Rogerson, Rupert, and Wright (2004). We use data from the March supplement of the CPS to construct annual series for per capita hours worked from 1963 to 2005 for individuals within specific age groups. We also construct an aggregate series for all individuals 15 years and older. See Appendix A.6 for detailed information on data sources used throughout the paper.

To extract the high frequency component of hours worked, we remove the trend from each series using the Hodrick-Prescott (HP) filter. Since we are interested in fluctuations at business cycle frequencies (those higher than 8 years), we use a smoothing parameter of 6.25 for annual data.[3]

Table 1 presents results on the time series volatility of hours worked in the U.S. for the 15-19, 20-24, 25-29, 30-39, 40-49, 50-59, and 60-64 year-old age groups. The first row presents the percent standard deviation of the detrended age-specific series. We see a decreasing relationship between the volatility of hours worked and age, with an upturn close to retirement age.

We are not interested in the high frequency fluctuations in these time series per se, but rather those that are correlated with the business cycle. For each age-specific series, we identify the business cycle component as the projection on a constant, current detrended output, and on current and lagged detrended aggregate hours; we refer to these as the *cyclical* hours worked series. The second row of Table 1 reports the R^2 from these regressions. This is high for most age groups, indicating that the preponderance of high frequency fluctuations are attributable to the business

[3]Through analysis of the transfer function of the HP filter, Ravn and Uhlig (2002) find this to be the optimal value for annual data. Using a similar approach, Burnside (2000) recommends a smoothing parameter value of 6.65. Finally, see Baxter and King (1999), who recommend a value of 10, through visual inspection of the transfer function. Throughout this paper, we have repeated our analysis of annual data using the band-pass filter proposed by Baxter and King (1999), removing fluctuations less frequent than 8 years. The results are essentially identical in all cases.

Table 1: Volatility of Hours Worked by Age Group, U.S.

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64
filtered volatility	4.351	2.130	1.471	1.073	0.790	0.824	1.309
R^2	0.79	0.80	0.83	0.88	0.89	0.72	0.30
cyclical volatility	3.868	1.902	1.318	1.014	0.752	0.705	0.708
share of hours (%)	3.34	10.64	13.23	26.12	23.98	17.73	4.97
share of hours volatility (%)	11.62	18.21	15.70	23.83	16.23	11.25	3.17

Notes: Data from the March CPS, 1968-2005. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported. Share of hours is the sample average share of aggregate hours worked by the age group. Share of hours volatility is the age group's share of "aggregate hours volatility," the average of age-specific cyclical volatilities weighted by hours shares.

cycle. The exception is the 60-64 age group: here a larger fraction of fluctuations are due to age-specific, non-cyclical shocks.

The third row indicates the percent standard deviation of the cyclical age-specific series. Compared to row one, the largest differences between *filtered* and *cyclical* volatilities are for those in their 60s, reflecting the point made immediately above. The data indicates a pattern of decreasing volatility with age. The young experience much greater cyclical volatility in hours than all others. Moreover, the differences across age groups are large. The standard deviation of cyclical hours fluctuations for 15-19 and 20-24 year old workers is 5 and 2.5 times that of 50-59 year olds, respectively.[4]

The fourth row indicates the average share of aggregate hours worked during the sample period by each age group. The fifth row indicates the share of "aggregate hours volatility" attributable to each age group. Here, aggregate hours volatility is represented by the weighted average of age-specific cyclical volatilities, with weights reflecting an age group's share of aggregate hours. What is

[4]These results corroborate the findings of Gomme, Rogerson, Rupert, and Wright (2004), and extend them to include data from the 2001 recession. See also Clark and Summers (1981), Rios-Rull (1996), and Nagypál (2004) who document differences in cyclical sensitivity across age groups. More broadly, the literature documents differences as a function of skill; see for instance, Kydland and Prescott (1993) and Hoynes (2000), and the references therein. Note that those studies are confined to the analysis of U.S. data.

Table 2: Volatility of Hours Worked by Age and Gender, U.S.

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64
filtered volatility							
female	4.865	2.067	1.594	1.141	0.955	1.034	1.826
male	4.664	2.774	1.645	1.257	0.854	0.891	1.906
cyclical volatility							
female	4.087	1.726	1.183	0.872	0.776	0.706	0.887
male	3.829	2.208	1.472	1.151	0.762	0.695	0.826

Notes: Data from the March CPS, 1968-2005. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures.

striking is the extent to which fluctuations in aggregate hours are disproportionately accounted for by young workers. Although those aged 15-29 make up only about one quarter of aggregate hours worked, they account for nearly one half of aggregate hours volatility. By contrast, prime-aged workers in their 40s and 50s account for more than 40% of hours, but only about 25% of hours volatility.

These age patterns remain when we undertake further demographic breakdowns. We summarize these results, found in Tables 2 and 3. Firstly, we disaggregate the U.S. workforce by age and gender. Differences across genders are apparent, but relatively small. When averaged across age groups, the difference between men and women is about 13% for either the filtered or cyclical hours worked series. However, the differences by age are stark. Again, the decreasing pattern by age exists for both men and women, with the magnitude of volatility differences roughly similar. For instance, 20-24 year olds experience hours volatility roughly 3 times greater than the prime-aged for both genders. Evidently, the differences across age groups within gender are more pronounced than the differences across genders within age group.

For disaggregation by age and educational attainment, the results remain. For brevity, we present results only for two education groups: those with high school diplomas or less (labeled high school and less), and those with at least some postsecondary education (more than high school).[5]

[5]Given the small fraction of teenagers with postsecondary education, we omit them from this analysis.

Table 3: Volatility of Hours Worked by Age and Education, U.S.

	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64
filtered volatility						
high school and less	2.362	1.942	1.574	1.061	1.172	1.847
more than high school	2.228	1.257	0.692	0.734	0.814	1.764
cyclical volatility						
high school and less	2.106	1.739	1.467	0.920	0.894	0.973
more than high school	1.694	1.026	0.532	0.526	0.331	0.515

Note: Data from the March CPS, 1968-2005. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures.

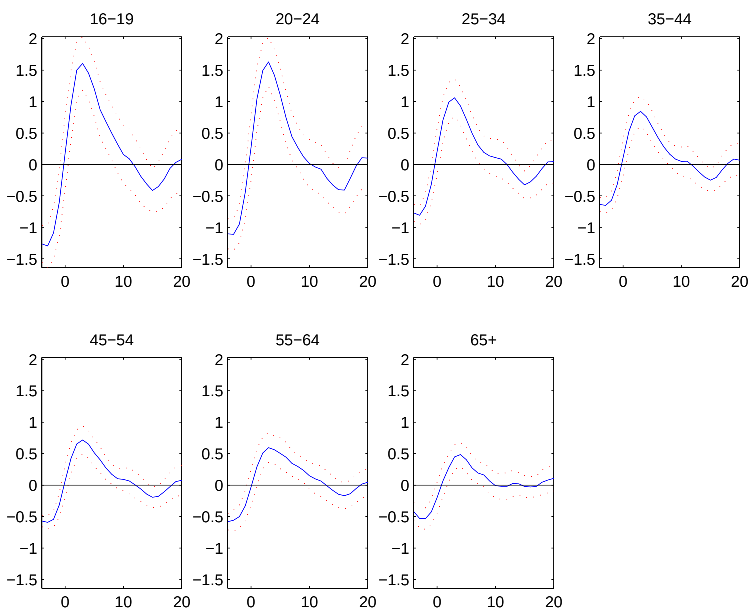
There is a noticeable difference in the volatility of hours by education. But more interestingly, the differences across education are much less pronounced for young workers than for the prime-aged. A simple average across 20-24 and 25-29 year olds indicates that those with less education have hours volatility that is 1.5 times that of those with more, implying that young workers experience greater than average volatility, regardless of education. By contrast, the difference across education groups is a factor of 2.5 for the prime-aged, those aged 40-59. Finally, note that large differences by age remain for both education groups. For instance, 20-24 year olds experience hours volatility 2.5 to 3 times greater than 40-49 year olds, regardless of educational attainment. Indeed, 20-29 year olds with more education have greater volatility than prime-age workers with less education! Hence, as in Gomme, Rogerson, Rupert, and Wright (2004) and Hansen and Imrohoroglu (2008), we focus on age-level heterogeneity as the primary demographic factor in understanding the volatility of aggregate hours worked.[6]

2.2 Age-Specific Unemployment

Additional evidence on the differences in business cycle sensitivity across age groups is presented in Figure 1. Here we present the average response of unemployment to a postwar U.S. recession,

[6] Gomme, Rogerson, Rupert, and Wright (2004) discuss age differences with further demographic breakdowns (e.g., marital status, industry of occupation) for the U.S. They find that essentially none of the age differences in volatility are due to differences in the distribution of hours worked across industries. This finding (along with our desire to maintain comparability to the standard RBC model) motivates our investigation of a one-sector model in Section 4.

Figure 1: Unemployment Response to Recession



Notes: Data from BLS, 1948:I-2004:II. Dynamic behavior of age-specific unemployment rates over a recession. Solid line represents unemployment rate response, averaged over NBER defined recessions. Dotted line represents two-standard-error band. Date 0 represents onset of recession, as identified by NBER.

where recessions are those identified by the NBER.[7] The unemployment rate data come from the BLS, cover the period 1948:I-2004:II, and are available for the age groups presented. Along the horizontal axis, date 0 represents the first quarter of a recession. The figure tracks the filtered age-specific unemployment rates for 20 quarters beyond this date. The solid blue line represents the recessionary response averaged across episodes, while the dashed red lines represent two-standard-error bands around the average for each variable. Unemployment rates for all age groups rise quickly in response to a recession, crossing above trend at date 0, then peaking at date 4 or 5 before slowly returning to trend.

Magnitudes of the recessionary response, however, differ across age groups. The peak response of unemployment is much stronger for young individuals. While the unemployment rate of 16-19 and 20-24 year olds increases by 1.5% above trend, the increase is only about 0.6% for prime-aged

[7] See also Nagypál (2004) who provides an analysis of age group differences during recessionary episodes.

workers. Indeed, the peak responses of these two age groups are well outside of the 2 standard deviation bands of all the other age groups.

Moreover, the 16-19 and 20-24 age groups experience average trough-to-peak responses of approximately 2.4% about the trend. This compares with a trough-to-peak response of only 1.2% for prime-aged individuals. In summary, the unemployment rate of young workers responds to recessions roughly twice as much as that of prime-aged workers.

3 Distinguishing Among Mechanisms

As stated in the Introduction, many potential mechanisms may account for the observed age group differences in hours and employment volatility. In this section, we present empirical evidence that guides the approach taken in this paper.

To focus discussion, we first consider analyses based on spot market determination of hours and wages in the labor market, as in the RBC model. In such a framework, age group differences can arise from differences in the cyclical characteristics of labor demand or labor supply. The premise of Section 3.1 is that an analysis of real wages can be used to differentiate between these two mechanisms. The relative cyclicality of age-specific wages, taken together with the facts from Section 2, indicates an important role for age group differences in cyclical labor demand.

In Section 3.2, we present a variance decomposition of age-specific hours worked. We argue that this decomposition is evidence against different labor force participation decisions being the sole or primary factor in determining age differences in hours volatility. Finally, in Section 3.3, we consider a relaxation of the assumption of spot markets for labor. In particular, we consider a specific hypothesis regarding age differences owing to the long-term nature of employment relationships. Evidence based on an intersectoral analysis of hours worked volatility does not support this view as being of primary importance.

3.1 Age-Specific Wages

From the March CPS, we use information on labor income and hours worked to construct annual time series for wage rates for the period 1963 through 2005. Given our interest in wage cyclicality, we construct wage rates in a way mitigating composition effects stemming from labor heterogeneity.

Table 4: Volatility of Real Hourly Wages by Age Group, U.S.

	15 - 19	20 - 24	25 - 29	35 - 39	45 - 49	55 - 59	60 - 64
filtered volatility	2.87	1.59	1.27	1.09	1.19	1.53	1.64
R^2	0.35	0.28	0.23	0.17	0.14	0.12	0.14
cyclical volatility	1.69	0.84	0.61	0.46	0.44	0.54	0.61

Notes: Data from the March CPS, 1963-2005. Filtered volatility is the percentage standard deviation of HP-filtered log data. Cyclical volatility is the percentage standard deviation of HP-filtered log data as projected on aggregate business cycle measures, with the R^2 from this projection reported.

Specifically, we classify individuals into 220 highly disaggregated demographic groups, and weight observations to derive efficiency measures of labor input per age group for the computation of wages. Our procedure is an extension of that used by Katz and Murphy (1992) and Krusell, Ohanian, Rios-Rull, and Violante (2000), and is detailed in Appendix A.6.[8] We then HP-filter these series to isolate fluctuations at the business cycle frequency.

The first row in Table 4 reports the percent standard deviation of the HP-filtered hourly real wage rates by age. Again, to mitigate composition effects due to heterogeneity, we compute real wage rates for 5 year age groups, as opposed to the 10 year age groups presented in the previous section. We see a decreasing pattern in volatility by age with an upturn beginning in the 55-59 age group. The second row reports the R^2 from projecting the age-specific series onto detrended aggregate output and hours, as done in Section 2.1. These statistics indicate that real wages, when disaggregated by age, are indeed mildly procyclical. Row 3 presents the percent standard deviation of the cyclical age-specific series. As in Row 1, we see the familiar decreasing pattern of volatility by age, with a slight upturn at the end of the age distribution. For instance, the standard deviation of cyclical volatility for 20-24 year olds about twice that of 45-49 year olds.[9]

If differences in labor supply were the sole factor responsible for the greater volatility of young

[8] Using weekly wages, as in Katz and Murphy (1992), yields the similar results to the ones we report here for hourly wages.

[9] Finally, we note that this pattern of cyclicality in age-specific wages is robust to further disaggregation by education and gender, but are not presented here for brevity.

workers' hours than the prime-aged, their wages would simultaneously be less volatile over the business cycle. We provide a detailed demonstration of this in Section 4 and Appendix A.3.

By contrast, we find exactly the opposite. The greater cyclical volatility of wages for the young displayed here, in conjunction with their greater volatility in hours worked, indicates that there must be some role played by differences in the cyclical nature of labor demand.[10] This finding is laid out in detail in Section 4.

3.2 Variance Decomposition of Hours Worked

Changes in per capita hours worked can be viewed as due to changes in either hours per labor force participant, or the number of the labor force participants per capita. We refer to the former as the hours margin, and to the latter as the participation margin. The relative contribution of each of these margins to the volatility of hours over the cycle is important in guiding our modeling approach. If the participation margin is the main driver of hours variation then one could argue the practical necessity of explicitly modeling a labor force participation decision.[11] Otherwise, it would indicate that to a first-order approximation, the primary factor generating age group differences are to be found elsewhere. We hence consider in Table 5 the decomposition of the variance of hours worked into the hours and participation margins.

Table 5 shows the proportion of hours variation by age group that can be attributed to the participation margin. The first row, using HP-filtered data, presents the ratio of the variance owing to the participation margin to the sum of the variances of the hours margin and participation margin. For those at or near retirement age, the participation decision appears to be an important source of variance in their hours worked. However, for all others, the bulk of the hours variation is due to variation in hours per labor force member. Specifically, for 20-59 year old individuals, the participation margin accounts for no more than 14% of hours variation. For teenagers, this is higher at 27%; nonetheless, nearly three quarters of the variance of teenaged hours worked is due to the hours margin.

[10] Solon, Barsky, and Parker (1994) make the related point on the relative procyclicality of hours and wages between men and women.

[11] For example, young individuals potentially face different trade-offs between market work and other activities (opportunities for educational attainment, home production, and leisure that may not be available to older-aged individuals).

Table 5: Hours Decomposition, Participation Margin

	15 - 19	20 - 24	25 - 29	30 - 39	40 - 49	50 - 59	60 - 64
filtered volatility							
covariance not included (%)	27.28	8.87	7.84	6.83	8.97	14.00	48.00
covariance included (%)	36.64	18.67	9.27	14.04	10.74	20.58	48.29
cyclical volatility							
covariance not included (%)	24.26	4.39	1.96	1.43	0.40	2.41	1.69
covariance included (%)	35.87	15.89	7.34	7.85	2.12	10.61	4.94

Notes: Data from the March CPS, 1968-2005. Shown are percentage shares of total hours variation attributed to the participation margin. Total hours per age group member is the product two variables: labor force participation per age group, and hours per labor force participant in that age group. "Covariance not included" means covariance terms are ignored, so total variation is just the sum of the variables' variances and the share attributed to the participation margin is the variance of labor force participation. "Covariance included" means total variation includes covariance terms, so total variation is the sum of the variables' variances plus two times their covariance and the share attributed to the participation margin is the variance of labor force participation plus the covariance. Filtered volatility is the standard deviation of HP-filtered log data. Cyclical volatility is the standard deviation of HP-filtered log data as projected on aggregate business cycle measures.

The second row presents an alternative decomposition which accounts for the covariance between hours per labor force member and labor force members per capita. Specifically, the participation margin's share is now defined as its variance plus the covariance, divided by the total variance of hours worked. Row 2 presents a similar picture to Row 1. The participation still figures heavily into the variance of hours for those over 60. For all others, the participation margin is much less important than the hours margin.

The third and fourth rows present the same information as Rows 1 and 2, respectively, now isolating the business cycle component of hours per labor force member and labor force participation.[12] Focusing on fluctuations that are correlated with the cycle tells a similar story: with covariance terms not included, the participation margin explains less than 25% of the variation of any age group. With the inclusion of covariance terms, participation explains at most 35% of the variation for the 15-19 year olds; fluctuations in hours per labor force participant continue to account for the bulk of the variation in per capita hours over the business cycle for all age groups.[13]

3.3 Seniority Rules and Young Workers

The analysis thus far has pointed to the importance of modeling labor demand over the cycle in accounting for age differences in volatility. This is based principally on observed differences in the cyclicality of age-specific wages; this in turn relies on a spot market view of labor market transactions. In reality, the institutional features of labor markets are more complex than those posited in the RBC literature, and it can be argued that this complexity partially accounts for age differences in hours volatility.

Specifically, in reality workers and firms engage in multi-period relationships, in contrast to the period-by-period transactions typically considered in RBC models. As the data on age-specific hours suggests, older workers have more permanent work situations that young workers. This may be due to the nature of the production process – the existence of organizational capital, firm know-how, or operational knowledge – which, in and of itself, is not incompatible with our emphasis on

[12] Again, this is calculated as a projection on a constant, current detrended aggregate output, and current and lagged detrended aggregate hours.

[13] The portion of cyclical volatility for the 60+ year olds is noticeably lower than for filtered volatility; this comes from the low R^2 of these business cycle projections for these age groups' labor force participation decisions.

capital-experience complementarity. That is, capital-experience may be responsible for both the differences in the existence of long-term relationships for old and young workers, and the differences in hours volatility. On the other hand, differences in the permanency of work tenure across age may be driven by institutional features, like labor market policies or social norms, that are independent of considerations owing to the nature of production. Hence, seniority rules or "last-in/first-out" (LIFO) rules may constitute an independent force for age group differences in hours volatility over the cycle.[14] We have conducted a preliminary analysis of the importance of such institutional features, using data disaggregated at the sectoral level. We find no evidence for seniority or LIFO rules as a primary factor explaining age differences in volatility. For brevity, we present our analysis in Appendix A.2.

4 The Model

In this section, we present a model in which production technology displays capital-experience complementarity. The remaining features of the model – in particular, household preferences – are specified to conform as closely as possible to the standard RBC model. This is not to claim that other mechanisms, such as life-cycle labor supply considerations, are irrelevant for understanding age differences in hours volatility. Instead, this specification allows us to isolate the role of cyclical differences in labor demand by age in accounting for the facts presented in Sections 2.1 and 3.

4.1 Households

The economy is populated by a large number of identical, infinitely-lived households. Each household is composed of a unit mass of family members. For simplicity, we assume there are only two types of family members, *young* and *old*. Let s_Y denote the share of family members that are young. Family members derive instantaneous utility from consumption C_i and disutility from hours spent working N_i, according to $U_i(C_i, N_i)$, where $i \in \{Y, O\}$ denotes either young or old.

The representative household's date t problem is to maximize

$$E_t \sum_{j=t}^{\infty} \beta^{t-j} \left[s_Y U_Y(C_{Yt}, N_{Yt}) + (1 - s_Y) U_O(C_{Ot}, N_{Ot}) \right], \tag{4.1}$$

[14] We thank Valerie Ramey for encouraging us to investigate this mechanism.

subject to

$$s_Y C_{Yj} + (1 - s_Y) C_{Oj} + \tilde{K}_{j+1} = (1 - \delta) \tilde{K}_j + r_j \tilde{K}_j + s_Y W_{Yj} N_{Yj} + (1 - s_Y) W_{Oj} N_{Oj}, \ \forall j \geq t,$$

with $0 < \beta < 1$, $0 \leq \delta \leq 1$. Here $\tilde{K}_t$ denotes capital holdings at date t, r_t is the rental rate, W_{Yt} is the wage rate of young workers, and W_{Ot} is the wage rate of old workers. The household takes all prices as given. In our benchmark case, we specify the instantaneous utility function to be

$$U_Y = \log C_Y - \psi_Y N_Y^{1+\theta_Y} / (1 + \theta_Y), \quad U_O = \log C_O - \psi_O N_O^{1+\theta_O} / (1 + \theta_O).$$

The parameters $\theta_Y, \theta_O \geq 0$ govern the Frisch labor supply elasticity, while $\psi_Y, \psi_O > 0$ are used to calibrate the steady state values of N_Y and N_O. We normalize the time endowment of all family members to unity, so that $0 \leq N_{Yt}, N_{Ot} \leq 1$.[15]

Because of additive separability in preferences, optimality entails equating consumption across all family members:

$$C_{Yt} = C_{Ot} = C_t. \tag{4.2}$$

The first-order condition (FONC) for capital holdings is given by:

$$C_t^{-1} = \beta E_t \left[C_{t+1}^{-1} (r_{t+1} + 1 - \delta) \right].$$

The FONCs for hours worked are given by:

$$W_{Yt} = \psi_Y C_t N_{Yt}^{\theta_Y},$$

$$W_{Ot} = \psi_O C_t N_{Ot}^{\theta_O}.$$

Condition (4.2) implies that the income effect of a consumption change on labor supply is equal across young and old workers. In our benchmark calibration, we set $\theta_Y = \theta_O$ so that the substitution effect of wage changes on labor supply is equated across workers. Adopting identical income and substitution effects allows us to isolate the role of capital-experience complementarity in generating volatility differences across young and old workers.

[15] Francis and Ramey (2008) use a variant of this utility function to study how demographic shifts lead to low frequency movements in hours worked and productivity.

4.2 Firms

Production exhibits capital-experience complementarity. Final goods are produced by perfectly competitive firms using capital, experienced labor, and inexperienced labor as inputs. We assume that an individual's age directly determines his or her labor market experience, so that all young workers are inexperienced while all old workers are experienced. This allows us to write the production function as

$$Y_t = \left[\mu \left(A_t H_{Yt}\right)^{\sigma} + (1-\mu) \left[\lambda K_t^{\rho} + (1-\lambda) \left(A_t H_{Ot}\right)^{\rho}\right]^{\frac{\sigma}{\rho}}\right]^{\frac{1}{\sigma}}, \quad \sigma, \rho < 1. \tag{4.3}$$

Here H_{Yt} is labor input of young (or inexperienced) workers, H_{Ot} is labor input of old (or experienced) workers, and K_t is capital services hired at date t. Labor-augmenting technology follows a deterministic growth trend with stationary shocks:

$$A_t = \exp\left(gt + z_t\right),$$

$$z_t = \phi z_{t-1} + \varepsilon_t, \quad 0 < \phi < 1,$$

where $E\left(\varepsilon\right) = 0$, $0 \leq var\left(\varepsilon\right) = \sigma_{\varepsilon}^2 < \infty$, and $g > 0$ is the trend growth rate of technology.

The elasticity of substitution between old workers and capital is given by $(1-\rho)^{-1}$, while the elasticity of substitution between young workers and the H_O-K composite is $(1-\sigma)^{-1}$. Following Krusell, Ohanian, Rios-Rull, and Violante (2000), we define production as exhibiting capital-experience complementarity when $\sigma > \rho$.

Firms hire inputs from perfectly competitive factor markets to maximize profits:

$$\Pi_t \equiv Y_t - r_t K_t - W_{Yt} H_{Yt} - W_{Ot} H_{Ot}.$$

Optimality entails equating factor prices with marginal revenue products:

$$r_t = Y_t^{1-\sigma}(1-\mu) \left[\lambda K_t^{\rho} + (1-\lambda) \left(A_t H_{Ot}\right)^{\rho}\right]^{\frac{\sigma-\rho}{\rho}} \lambda K_t^{\rho-1},$$

$$W_{Ot} = Y_t^{1-\sigma}(1-\mu) \left[\lambda K_t^{\rho} + (1-\lambda) \left(A_t H_{Ot}\right)^{\rho}\right]^{\frac{\sigma-\rho}{\rho}} (1-\lambda) A_t^{\rho} H_{Ot}^{\rho-1},$$

$$W_{Yt} = Y_t^{1-\sigma} \mu A_t^{\sigma} H_{Yt}^{\sigma-1}.$$

4.3 Equilibrium

Equilibrium is defined as follows. Given $\tilde{K}_0 > 0$ and the stochastic process, $\{z_t\}$, a *competitive equilibrium* is an allocation, $\{C_t,\, N_{Yt},\, N_{Ot},\, \tilde{K}_{t+1},\, Y_t,\, H_{Yt},\, H_{Ot},\, K_t\}$, and a price system, $\{W_{Yt},\, W_{Ot},\, r_t\}$, such that: given prices, the allocation solves both the representative household's problem and the representative firm's problem for all t; and factor markets clear for all t:

$$K_t = \tilde{K}_t; \quad H_{Yt} = s_Y N_{Yt}; \quad H_{Ot} = (1 - s_Y)\, N_{Ot}.$$

Walras' law ensures clearing in the final goods market:

$$C_t + K_{t+1} = Y_t + (1 - \delta)\, K_t, \quad \forall t.$$

Finally, for the purposes of model evaluation, we define aggregate hours worked as $H_t = s_Y H_{Yt} + (1 - s_Y)\, H_{Ot}$.

4.4 The Effects of Capital-Experience Complementarity

In this subsection, we provide analytical results regarding the relative cyclicality of hours worked and real wages for young and old agents. To begin, we show that when production displays capital-experience complementarity, the response of young hours to a technology shock is greater than that of the old; this result holds even when there are no differences in labor supply characteristics.

Proposition 1 *Let $\theta_Y = \theta_O \geq 0$ and $\sigma > \rho$. The response of hours of young workers to a business cycle shock is greater than the response of hours of old workers.*

The proof is contained in Appendix A.4. Here, we demonstrate this result for the special case in which $\rho = 0$. When $\rho = 0$, the H_O-K composite becomes Cobb-Douglas, and the firm's FONCs simplify as:

$$W_{Yt} = \mu Y_t^{1-\sigma} A_t^{\sigma} H_{Yt}^{\sigma-1},$$

$$W_{Ot} = (1 - \mu)\,(1 - \lambda)\, K_t^{\lambda\sigma} Y_t^{1-\sigma} A_t^{(1-\lambda)\sigma} H_{Ot}^{(1-\lambda)\sigma-1}.$$

In $\log W$ - $\log H$ space, these define linear labor demand curves, with slope $(\sigma - 1)$ for young labor and slope $[(1 - \lambda)\,\sigma - 1]$ for old. Since $0 < \lambda < 1$, and $0 < \sigma < 1$ (recall that capital-experience complementarity is defined as $\sigma > \rho$, and we have assumed $\rho = 0$), the demand curve for young labor is flatter than that of old labor. Moreover, a shock to technology (a change in $\log A$) generates a vertical shift in the young labor demand curve of σ, which is larger than the shift in the old labor demand curve of $(1-\lambda)\,\sigma$. These two factors combine to generate the result of Proposition 1.

We show this diagrammatically in Figure 2. The left panel depicts the demand curve for young labor, the right panel for old labor. In each panel, the horizontal line depicts the labor supply curves derived from the household's FONCs with Rogerson-Hansen preferences and lotteries; in log-log space, both are linear with common slope, $\theta_Y = \theta_O = 0$. This restriction is made for graphical simplicity and is relaxed in Figure 3; as indicated in the proposition, the result is independent of the (common) slope of the labor supply curves.

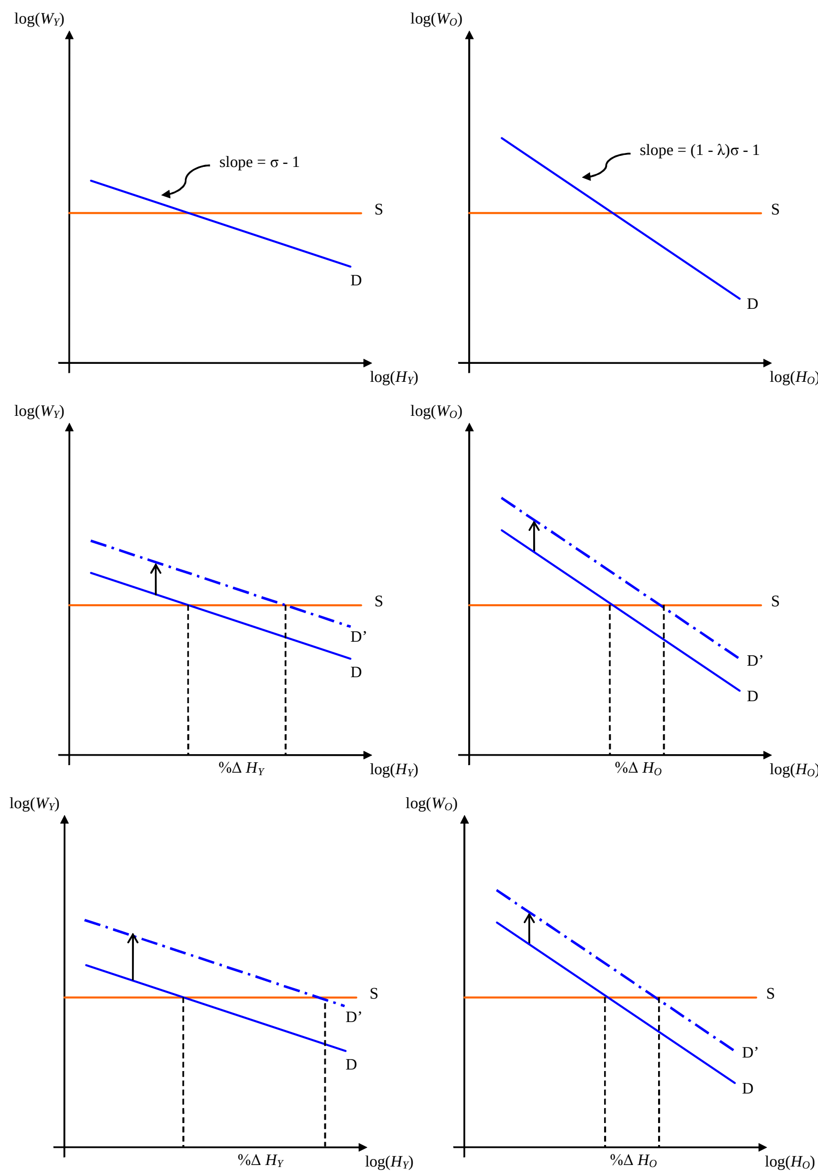
Using a circumflex to denote log deviations, consider a positive shock to technology, $\hat{A}$. In equilibrium, the positive income effect of this shock generates an upward shift in the labor supply curves; since our model assumes identical wealth effects across agents, we abstract from these shifts in the diagram for the sake of clarity. The technology shock also results in an equilibrium output response, $\hat{Y}$; since the effect of this response is identical across labor demand curves, we abstract from these in the diagram as well. Finally, note that capital is a state variable, so that the response of capital to the shock is $\hat{K} = 0$.

Hence, the only effect that requires diagrammatic consideration is the direct effect of the shock to the labor demand curves, and we plot these in the middle and bottom rows of Figure 2. Suppose, momentarily, that the technology shock results in identical shifts in the two demand curves: this is illustrated as the dotted lines in the middle row. As is geometrically obvious, this results in a larger equilibrium response of young labor input relative to the old, i.e. $\hat{H}_Y > \hat{H}_O$. This is due to the relative complementarity of old labor to capital, implying that the marginal revenue product of labor is more sensitive to changes in labor for H_O relative to H_Y. After a positive shift in labor demand, a smaller change in old labor is required to achieve the same change in its marginal revenue product, and we call this the *relative slope* effect.

But note that the positive shock actually generates a larger vertical shift in the demand for H_Y than for H_O: $\sigma\hat{A} > (1-\lambda)\,\sigma\hat{A}$. That is, the shock has a larger direct effect on the marginal

Figure 2: Labor Supply and Demand Diagrams 1

Young Old



Notes – All panels: Red lines labeled "S" depict the labor supply curves derived from the household's FONCs with Rogerson-Hansen preferences in log-log space with common slope $\theta_Y = \theta_O = 0$; blue lines labeled "D" depict labor demand curves. **Top panel:** slope of demand curve for H_Y is flatter than the demand curve for H_O. **Middle panel:** we abstract from the wealth effects of a productivity shock on the labor supply since they are identical across young and old; the shock causes both demand curves to shift up; the "relative slope" effect is evident in $\%\Delta H_Y > \%\Delta H_O$. **Bottom panel:** the "relative shift" effect is evident from the labor demand for H_Y shifting up by more than for H_O, increasing $\%\Delta H_Y$ even more.

revenue product of young labor. This is depicted by the dash-dot line in the left panel of the bottom row. This additional *relative shift* effect reinforces the relative slope effect. Hence, in equilibrium, $\hat{H}_Y > \hat{H}_O$.

For the extreme case with infinite Frisch labor supply elasticities ($\theta_Y = \theta_O = 0$) displayed, the equilibrium wage response is equated across young and old labor. However, for the more general case of positive Frisch elasticity ($\theta_Y = \theta_O > 0$), the response of the young wage to a technology shock will be greater than that of the old wage with capital-experience complementarity. This is illustrated in the top row of Figure 3. The young and old labor supply curves exhibit identical (less than infinite) Frisch elasticities. With capital-experience complementarity, a shock to technology simultaneously generates a larger hours and wage response of the young as compared to the old.

Analytically, this can be derived from the household's FONCs with respect to labor supply. Using the fact that consumption is equated across agents:

$$W_{Yt}/\psi_Y N_{Yt}^{\theta_Y} = W_{Ot}/\psi_O N_{Ot}^{\theta_O}.$$

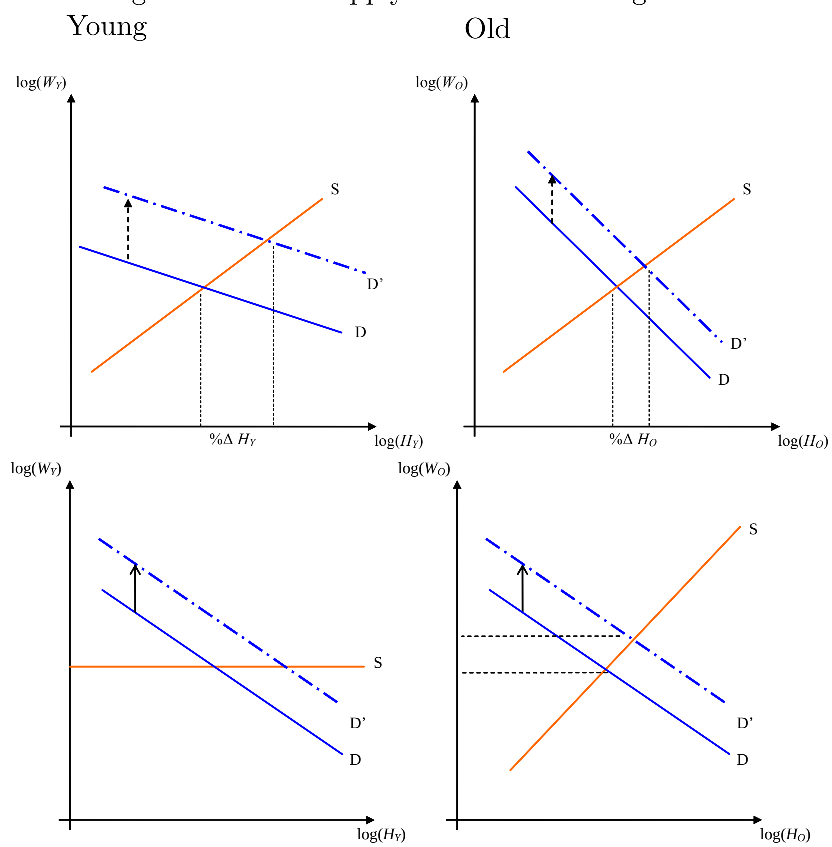
Substituting in the labor market clearing conditions, this can be rewritten in terms of log deviations as

$$\hat{W}_Y - \hat{W}_O = \theta_Y \hat{H}_Y - \theta_O \hat{H}_O.$$

When $\theta_Y = \theta_O > 0$, $\hat{W}_Y > \hat{W}_O$ follows directly when $\hat{H}_Y > \hat{H}_O$, as is the case with capital-experience complementarity. The intuition for this is straightforward: when agents have identical labor supply curves, the only way to induce a greater hours response for young workers is through a larger wage response.

Note, however, that this condition implies a stronger result. With capital-experience complementarity, the wage response of young workers can be greater than that of the old, even when young labor supply is more elastic (i.e. when $\theta_Y < \theta_O$). We view this as an important result, since our point is not to claim that labor supply characteristics are identical across all agents. Indeed, the Frisch elasticity of young workers may be greater, or the income effect of consumption changes for young workers may be smaller, or both, relative to the old. The mechanism embodied in capital-experience complementarity is still capable of jointly delivering greater cyclical volatility of hours and wages for the young relative to the old.

Figure 3: Labor Supply and Demand Diagrams 2



Notes – All panels: Red lines labeled "S" depict the labor supply curves derived from the household's FONCs in log-log space; blue lines labeled "D" depict labor demand curves. **Top panel:** supply curves with common slope $\theta_Y = \theta_O \neq 0$ and demand curves with capital-experience complementarity. **Bottom panel:** demand curves without capital-experience complementarity while the supply curve for H_Y is more elastic than the supply curve for H_O in order to match the larger $\%\Delta H_Y$.

On the other hand, assume there is no capital-experience complementarity, so that there are no differences in the response of labor demand to a technology shock. As we discuss formally in Appendix A.3, and demonstrate graphically in the bottom row of Figure 3, then one must assume that the Frisch labor supply elasticity of the young is higher than that of the old in order to match the fact that $\hat{H}_Y$ is more responsive to shocks than $\hat{H}_O$. However, in this case with identical labor demand characteristics, such a model cannot match the fact that $\hat{W}_Y$ is more responsive than $\hat{W}_O$. This has been made obvious in the bottom row of Figure 3, where the labor supply curve of the young has been illustrated as being perfectly elastic. More generally, as long as labor supply is more elastic for the young relative to the old, the wage response of the young will be smaller in response to identical labor demand fluctuations. Hence, matching *both* the higher relative volatility of young hours *and* young wages requires a model where labor demand shocks are not age neutral.

5 Quantitative Specification

In this section, we describe the quantitative specification used for evaluating the model. To maintain comparability with the RBC literature, we adopt a standard calibration when possible. However, the model's parameters governing elasticities of substitution in production, σ and ρ, cannot be calibrated to match standard first moments in the U.S. data. Instead, we adopt a structural estimation procedure to identify these values using data from the NIPA and CPS. After describing the procedure, we discuss calibration of the remaining parameter values. Given the evidence in the previous sections, we classify 15-29 year olds as young and 30-64 year olds as old.

5.1 Structural Estimation

Our strategy entails estimating σ and ρ from the model's factor demand equations.[16] Consider the firm's FONC with respect to the demand for H_{Yt} rewritten in logged, first-differenced form:

$$\Delta \log W_{Yt} = a_0 + (\sigma - 1)\Delta \log \left(H_{Yt}/Y_t\right) + \sigma u_t, \tag{5.1}$$

where a_0 is a constant, and u_t is a function of current and lagged shock innovations

$$u_t = \varepsilon_t - (1-\phi)\left(\varepsilon_{t-1} + \phi\varepsilon_{t-2} + \phi^2\varepsilon_{t-3} + ...\right).$$

[16] A similar approach is used in Burnside, Eichenbaum, and Rebelo (1995) and the references therein.

Hence, σ is identified from the response of W_Y to exogenous changes in H_Y and Y.

The age-specific wage measures analyzed in Section 3.1 are constructed using hours data in order to translate direct information on labor income into information on hourly wages, similar to Krusell, Ohanian, Rios-Rull, and Violante (2000)'s hourly wage and Katz and Murphy (1992)'s weekly wage constructions. To avoid problems stemming from using W_{Yt}, which is partly constructed from H_{Yt}, in our regression, we estimate a variant of (5.1) for which direct data on the left-hand side variable is available.[17] This is obtained by multiplying both sides of the FONC by H_{Yt}

$$\Delta \log LI_{Yt} = a_1 + \sigma \Delta \log H_{Yt} + (1 - \sigma) \Delta \log Y_t + \sigma u_t, \tag{5.2}$$

where $LI_{Yt} \equiv W_{Yt} H_{Yt}$ denotes labor income earned by young workers, for which data is available directly from the CPS. If there were no endogeneity issues (see below), σ could be estimated from a simple restricted least-squares regression.

To estimate ρ, we proceed in a similar manner. Combining the firm's FONCs with respect to H_{Ot} and K_t and performing similar manipulations obtains

$$\Delta \log (Q_{Ot}/Q_{Kt}) = a_2 + \rho \Delta \log (H_{Ot}/K_t) + \rho u_t, \tag{5.3}$$

where Q_{Ot} denotes the share of national income earned by old labor, and Q_{Kt} the share of national income earned by capital.

Importantly, this procedure does not require imposing any restrictions from the model's specification of household behavior.[18] The only assumptions required to pin down σ and ρ are: (*i*) profit maximization on the part of firms, and (*ii*) that changes in factor prices reflect changes in marginal revenue products. As is obvious from our estimating equations, (5.2) and (5.3), our identification does not in any way use the fact that young hours are more volatile over the cycle than old hours. Moreover, no aspect of our approach imposes that $\sigma > \rho$. Whether this condition is satisfied depends on the relation between aggregate prices and quantities observed in the data.

[17] The data used in estimation come from standard sources. Briefly, Y_t, K_t, and Q_{Kt} come from the BEA's NIPA and Fixed Asset Tables. H_{Yt}, H_{Ot}, LI_{Yt}, and Q_{Ot} are constructed using March CPS data. Because of this, our data comprise annual observations for the period 1968 - 2005. See Appendix A.6 for a detailed discussion of the data construction.

[18] We see this as a virtue since our goal is not to claim that labor supply characteristics are indeed identical across the young and old, as maintained in our benchmark calibration. Instead, or goal is to isolate the quantitative role of differences in the cyclical demand for young and old labor.

5.1.1 Instruments

Since both of our estimating equations are based on the estimation of factor demand equations, we need to address the endogeneity of the regressors to the error term. The structural equations identify the error term as due to shocks to productivity. In order to obtain unbiased estimates more generally, we must isolate variation in our regressors that is unrelated to shocks shifting firms' factor demand, be they technology shocks or other omitted factors from the FONCs.

We do so by adopting an instrumental variables (IV) approach. Specifically, we use two instruments: lagged birth rates and the Ramey and Shapiro (1998) dates indicating the onset of exogenous military build-ups.[19] In a standard RBC model like the one we consider, the introduction of exogenous government spending shocks introduces exogenous shifts in labor supply due to the income effect of such shocks (see Christiano and Eichenbaum (1992)). This results in changes in H_Y, H_O, and Y that are unrelated to shifts in factor demand.

Our second instrument is lagged birth rates. This instrument allows us again to identify changes in current labor supply, this time due to changes in past fertility that are uncorrelated to shifts in factor demand.[20] Recall that

$$u_t = \varepsilon_t - (1-\phi)\left(\varepsilon_{t-1} + \phi\varepsilon_{t-2} + \phi^2\varepsilon_{t-3} + ...\right).$$

Lagged birth rates are valid if we assume that fertility is exogenous to past technology shock innovations, $\{\varepsilon_{t-j}\}_{\forall j>0}$. If one believes that fertility decisions, say, 15 years ago might be endogenous to innovations at least 15 years ago, then some bias might be induced with these instruments. However, note that in the case of the 15-year lagged birth rate, the concern is its correlation with the sum $(1-\phi)\sum_{j=14}^{\infty}\phi^j\varepsilon_{t-j-1}$ in u_t. For standard values of shock persistence, ϕ, relevant for business cycle analysis, the impact of this is almost neligible. Obviously, for birthrates of larger lag, this is even smaller. We thus conclude that, from an empirical standpoint, lagged birth rates are valid instruments.

We obtain IV estimates of $\hat{\sigma} = 0.59$ and $\hat{\rho} = 0.01$, with Newey-West standard errors of 0.22

[19] The estimates of σ and ρ are identical if we use real government defense spending instead of the Ramey and Shapiro (1998) dates. Similarly, instrumenting only with lagged birth rates does not alter our estimates.

[20] See also Beaudry and Green (2003) who use exogenous demographic variation as an instrument in production function estimation.

and 0.19, respectively.[21] There is clear evidence for the relevance of our instruments from readily available weak-instrument hypothesis tests. Anderson (1951) and Cragg and Donald (1993) test the null that the instrumented regressors are under-identified due to the instruments' irrelevance: we reject using either statistic in both (5.2) and (5.3) with p-values below 0.007 in all cases. Thus, the data suggest that weak-instrument issues are not of concern.[22]

5.1.2 Specification Testing

Here, we provide additional evidence from a series of specification tests in favor of our model of capital-experience complementarity. To do so, we maintain the following assumptions from our benchmark model: (i) that final output is produced using physical capital, K, old labor input, H_O, and young labor input, H_Y, and (ii) that the production function features a nested CES functional form.

Our specification (4.3) posits that the innermost nesting is a CES composite of H_O-K. But more generally, the production function could alternatively feature an innermost nesting of H_Y-K or H_Y-H_O. Each of these specifications generate different FONCs as estimating equations. These equations differ in terms of dependent and independent variables, but maintain the same functional forms. Our approach is to test for misspecification in these regressions in order to determine which of these three alternatives the data rejects (if any).[23]

Before proceeding, we note that this testing approach may appear narrow in scope, particularly due to our maintained assumption, (ii). We argue that this is not the case for two primary reasons. First, virtually all quantitative macroeconomic models embody a high degree of log-linearity in preferences and technology, and our nested CES production function is simply a generalization of that. Secondly, our alternative specifications for production allow for the possibility that labor demand is symmetric across young and old labor. Specifically, consider the specification:

$$Y_t = \Upsilon_1 \left(K_t, A_t \Upsilon_2 \left(H_{Yt}, H_{Ot} \right) \right),$$

[21] Based on the autocorrelations of the residuals, we allow four lags in the kernel.

[22] More informally, R^2s from the first-stage regressions are 0.67, 0.42, and 0.18 for $\Delta \log(H_O/K)$, $\Delta \log(H_Y)$, and $\Delta \log(Y)$, respectively.

[23] For each of these different specifications we change the regressors and instruments accordingly.

where both $\Upsilon_1(.,.)$ and $\Upsilon_2(.,.)$ are CES functions in their two arguments.[24] For this nesting, it is easy to show that there is no difference in the cyclical properties of labor demand across H_Y and H_O; see Appendix A.3 for details. Hence, our specification tests allow us to discern whether the data, in fact, prefer a model featuring labor demand differences due to capital-experience complementarity.

To this end, we perform the Ramsey (1969) test for model misspecification. In particular, our model's production function, (4.3), delivers linear estimating equations, (5.2) and (5.3). The alternative production function nestings deliver analogously linear pairs of FONCs for estimation. As discussed in Davidson and MacKinnon (2004), the Ramsey test is a straightforward and powerful method for testing the linear restrictions imposed in these regressions. The null hypothesis of the test is that the linear estimating equations are correctly specified.

To summarize, the procedure amounts to a joint hypothesis test that coefficients on non-linear terms are equal to zero; this test statistic has an asymptotic $\chi^2(2)$ distribution (see Appendix A.5 for additional discussion). For our chosen H_O-K nesting, the statistic equals 0.47 with p-value 0.79; we cannot reject the null hypothesis of no model misspecification. On the other hand, for the alternative H_Y-K and H_Y-H_O nestings this statistic takes the value 13.99 and 31.78 with associated p-values 0.001 and 0.000, respectively. Hence, we find clear evidence to reject the null hypothesis that these estimating equations are correctly specified. We take this as evidence that the data favors our specification embodying capital-experience complementarity.

5.2 Calibration

Given the estimated values for σ and ρ, the remaining parameters are calibrated as is standard, with $\beta = 0.99$, $\delta = 0.025$. The only two "new" parameters are μ and λ, the share parameters in our production function (4.3). Following Krusell, Ohanian, Rios-Rull, and Violante (2000) we calibrate these to match national income shares. Specifically, in our model we set μ and λ to match the 1968-2005 national income shares of $Q_K = 0.373$ and $Q_O = 0.494$. With values for $\{\hat{\sigma}, \hat{\rho}, \mu, \lambda\}$ and data on output and factor inputs, we back out the implied technology series $\{A_t\}$.[25]

[24] The standard RBC model's specification is a special case of this, where Υ_2 is a linear function (so that young and old labor are perfect substitutes) and Υ_1 is Cobb-Douglas.

[25] We are calibrating a quarterly model, however up to now we have dealt with annual data measures. The reason for this is that quarterly data on age-specific hours do not begin until 1976. We do have *semiannual* data on age-specific hours from 1968-2005 (constructed by the authors from the March CPS and the October CPS surveys held by

Table 6: Data and Model Moments

	Relative Volatility		
	U.S. Data	Model: Benchmark	Model: Alternative
std(H)/std(Y)	0.97	0.92	0.89
std(H_Y)/std(Y)	1.48	1.55	1.41
std(H_O)/std(Y)	0.82	0.68	0.69
std(W_Y)/std(Y)	0.35	0.26	0.32
std(W_O)/std(Y)	0.25	0.26	0.27
std(W_Y)/std(W_O)	1.26	1.00	1.20

Notes – Column 1: HP filtered data from March CPS, 1968-2005. **Column 2:** Rogerson-Hansen preferences, $\theta_Y = \theta_O = 0$. **Column 3:** $\theta_O = 0$, $\theta_Y = 0.05$ to match data on relative age-specific wages.

6 Quantitative Evaluation

Column 1 in Table 6 presents statistics for HP-filtered U.S. data. As is well known, the volatility of aggregate hours is almost identical to the volatility of output (the ratio of standard deviations is 0.97). The remaining rows in Column 1 report the relative volatility of hours and wages for the two age groups. While aggregate hours worked is as volatile as output, this masks large differences across the young and the old. The hours of the young are about 50% more volatile than output, while the hours of the old are less volatile than output. As noted in Section 3.1, the volatility of real wages is also greater for the young than for the old. For our 15-29 and 30-64 year old age groups, the ratio of real wage volatility is 1.26.

We begin the quantitative evaluation of the model by setting $\theta_Y = \theta_O = 0$, so that utility is linear in hours worked. This is a useful benchmark since the standard RBC model (with homogenous labor and Cobb-Douglas production function) requires very high labor supply elasticity to generate significant volatility of hours. In particular, the indivisible labor model (with perfectly

NBER). From this data we see that the relevant time series display the same volatilities relative to output. Likewise, the relative volatilities of young and old hours are the same in both the annual and semiannual time series. We conclude that for these relationships the frequency of observation does not alter our results.

elastic aggregate labor supply) generates a ratio of the standard deviation of hours to output of approximately $0.7 - 0.75$.[26] In this sense, the volatility of aggregate hours worked represents a puzzle to the RBC literature.

As Column 2 of Table 6 reports, the capital-experience complementarity model generates volatility of total hours that is very close to that observed in the data; the relative standard deviation of aggregate hours to output is 0.92. The next two rows show that the key to this success lays in the model's ability to generate a series of hours worked by the young that fluctuates substantially more than output and old hours over the business cycle. The model generates a volatility ratio of 1.55 for young hours to output, which is slightly greater than the value of 1.48 observed in the data. On the other hand, the model understates the volatility of old hours relative to output: the relative standard deviation is 0.68, while this is 0.82 in the data. Our quantitative specification has (essentially) unit elasticity of substitution between capital and old hours ($\rho = 0.01$), and infinite Frisch elasticity of labor supply for the old. These are the same features displayed by the homogenous labor input in the standard RBC model with indivisible labor, discussed in the preceding paragraph. Thus the capital-experience complementarity model generates a relative volatility of old hours to output similar to the relative volatility of aggregate hours to output in the standard RBC model.

Finally, while the benchmark calibration is successful with respect to the hours dimension, it cannot account for the behavior of relative wages between the young and the old. This is not a surprise since we have infinite Frisch labor supply elasticity for both family members; in Section 4.4, we showed that the volatility of age-specific wages would be identical for this case.

In Column 3 we consider the following modification: we change only the Frisch labor supply elasticity of the young to match the relative wage volatility. This requires a minimal change, moving θ_Y from 0 to 0.05. The model now generates greater volatility in the wage of young labor compared to old labor, as observed in the U.S. Moreover, the model generates volatility of age-specific wages relative to output that is also close to those found in the data.[27] Not surprisingly, lower elasticity

[26] See for example, Hansen (1985), Rogerson (1988), King and Rebelo (1999).

[27] Note that we are reporting the volatility for cyclical fluctuations in real wages, as constructed in Section 3.1. As previously shown, a significant portion of high frequency wage variation is not correlated with the cycle. Given the focus on business cyle fluctuations in hours and wages, we concentrate on the variation in wages that is due to the

of young labor supply induces a fall in the volatility of young hours, and hence, aggregate hours relative to output. However, the fall is quantitatively small, and the values for $std(H)/std(Y)$ and $std(H_Y)/std(Y)$ are still very similar to those found in the data. It is interesting to note that, as in the case with $\theta_Y = \theta_O = 0$, the larger discrepancy between model and data is in matching the relative volatility of old hours worked to output, and not in matching the much greater volatility of hours worked by the young.

In sum, we see that the simulated model generates age-specific hours volatilities that are similar to those observed in the data. As a by-product of this success, the model generates volatility of aggregate hours that is very close to that of aggregate output. Moreover, the model accounts for the joint behavior of age-specific hours and wages.

7 Conclusion

We have presented an RBC model displaying capital-experience complementarity in production. This is motivated by our investigation of the joint behavior of age-specific hours and wages. Young individuals' hours and wages are more volatile over the business cycle than those of old individuals. Within an RBC framework, differences in the cyclical characteristics of age-specific labor demand can explain this fact, while differences in labor supply characteristics on their own cannot. We view our model as a straightforward and parsimonious extension to the standard RBC model that allows for such a difference in labor demand. We estimate the key structural parameters governing the degree of capital-experience complementarity, in a manner that does not target the observed difference in the volatility of hours.

Our quantitative model is able to match the relative volatility of age-specific hours to output. As a result, it is also able to replicate the relative volatility of aggregate hours with respect to output. Hence, capital-experience complementarity in production provides a solution to the long standing hours volatility puzzle in the RBC literature.

cycle.

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A Appendix

A.1 Hours Variance Decomposition

In this subsection, we provide a decomposition of the volatility of hours worked into its primary components. Our view is that such a decomposition provides preliminary evidence on the relative importance of labor supply and labor demand factors in understanding differences in volatility across age groups. This type of analysis is informative with respect to the structural model we pursue. For example, differences owing to life-cycle considerations would plausibly attribute a greater proportion of the volatility of hours to volatility in labor force participation among some age groups as compared to others. Schooling decisions might be a margin of adjustment which is more relevant for young agents than to those in their prime-age. Similarly, the decision to re-enter or drop out of the labor force over the business cycle might be more relevant for those above the retirement age than others.

Note that hours worked per member of an age group (H) can be written as the following product:

$$H = h \times lf,$$

where h is hours per labor force participant and lf is labor force participant per age-group member. Hence, changes in H can be due to changes in these terms, which we refer to as the "hours margin" and the "participation margin," respectively. As such we decompose the variance of hours worked in the following way:

$$\text{var}\left(\hat{H}\right) = \text{var}\left(\hat{h}\right) + \text{var}\left(l\hat{f}\right) + 2 \times \text{cov}\left(\hat{h}, l\hat{f}\right) \tag{A.1}$$

where $\hat{H}$ denotes deviations of the log of H from its HP-filtered trend, and similarly for the other variables.[28] Our point in decomposing var $\left(\hat{H}\right)$ is to see how influential is the participation margin, which we do not model, relative to the hours margin, which we do model.

The goal of simply accounting for the participation margin does not tell us how to attribute the covariance terms in (A.1) and so we do the following. For one set of calculations, we ignore covariance in (A.1). Then the participation margin is simply

$$\frac{\text{var}\left(l\hat{f}\right)}{\text{var}\left(\hat{h}\right) + \text{var}\left(l\hat{f}\right)}$$

We denote these results "covariance not included."

For the other set of calculations, we include covariance. Moreover, we take a equitable stance towards the participation margin and attribute to it half of the covariance term including $l\hat{f}$. Here the participation margin is

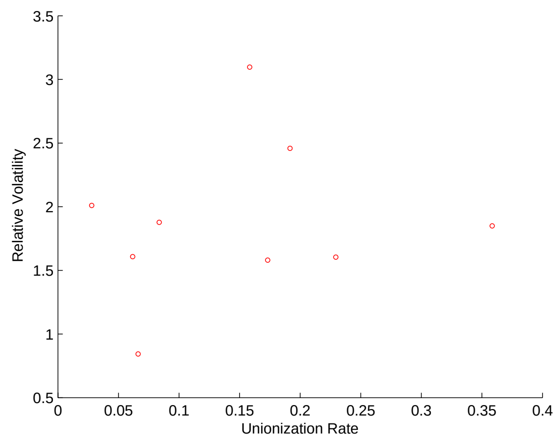
$$\frac{\text{var}\left(l\hat{f}\right) + \text{cov}\left(\hat{h}, l\hat{f}\right)}{\text{var}\left(\hat{H}\right)}$$

using (A.1). We denote these results "covariance included."

As discussed in the text, the picture that emerges here is that the participation margin is less influential than the hours margins we model for all age groups.

[28]See Hansen (1985) for a similar decomposition.

Figure 4: Major Industry Unionization Rates and Young/Old Hours Relative Volatility



Notes: Unionization rate data available from BLS for 1983-2005, major industries as defined by BLS supersectors. Hours data from March CPS, 1983-2005. Relative volatility is ratio of standard deviation of 15-29 year old hours to standard deviation of 30-59 year old hours.

A.2 Seniority Rules and Young Workers

We test the potential importance of such institutional features by looking at the relationship between intersectoral unionization rates and age-specific hours worked volatility. Specifically, we assume all labor unions place an emphasis on the concerns of its (employed) members, and either implicitly or explicitly endorse LIFO rules in employment decisions. To the extent that different industries and sectors feature different rates of unionization, we should expect variation in the importance of LIFO effects. To test this, we look at the volatility of young workers' hours worked relative to that of the prime-aged over the cycle. Since seniority is highly-correlated with age, we should expect that the quantitative importance of LIFO rules will obtain in age group comparisons.[29]

We disaggregate hours worked by age and nine BLS-defined nonfarm "supersectors," which roughly correspond to 1-digit level SIC codes. We obtain unionization rate data from the BLS starting in 1983. In Figure 4, we present the scatterplot of the ratio of cyclical volatility of 15-29 year olds relative to 30-64 year olds to unionization, 1983-2005. The unionization rate measure is the average rate observed over the sample period. This serves as a useful summary statistic since unionization rates have been relatively stable since 1983, and importantly, the ordinal ranking across supersectors has not changed.

We see that there is no evidence that more highly unionized sectors feature greater relative volatility of the young. Performing a simple OLS regression obtains an $R^2 = 0.04$ and a positive slope estimate that is not close to being statistically significant. As such, we do not find prima facie evidence for the importance of seniority or LIFO rules in explaining age differences in hours

[29]Note that we implicitly assume that the unionization rate is exogenous to the relative volatility between the young and prime-aged workers. We do not assume that it is exogenous to the level of total volatility of the sector.

and employment volatility.

A.3 Labor Supply Models

As discussed in the introduction, in order to maintain comparability with the literature, we are interested in a model that represents a minimal deviation from the standard RBC model. We begin by analyzing two simple models based on labor supply differences. As expected from the previous discussion, while these models can account for the differences in the cyclicality of age-specific hours, they have counterfactual implications regarding the cyclicality of age-specific wages. We then conclude that within the RBC framework, labor demand differences are crucial for matching differences in the cyclicality of age-specific wages and analyze such a model in the following Section.

Differences in Labor Supply: Model I In the first model we consider we assume that final goods are produced by perfectly competitive firms according to the Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

Here H_t is the aggregate labor input in the economy and it satisfies

$$H_t = H_{Yt} + EH_{Ot}$$

That is, the aggregate labor input is the sum of the hours of the young, H_{Yt}, and the hours of the old, H_{Ot}. The parameter E allows for a difference in the efficiency of hours supplied between the young and the old. The rest of the model is identical to the one we consider in the paper.

In this enviorment we have

$$W_{Ot} = (1-\alpha)E\frac{Y_t}{H_t}, \quad (A.2)$$

$$W_{Yt} = (1-\alpha)\frac{Y_t}{H_t}. \quad (A.3)$$

From the FOC of the household we get

$$\hat{N}_{Yt} = \frac{\theta_O}{\theta_Y}\hat{N}_{Ot} \quad (A.4)$$

Thus, if $\theta_Y < \theta_O$, i.e., the Frisch labor supply elasticity of the young is higher then that of the old (recall that the Frisch labor supply elasticity equals $\frac{1}{\theta}$), then the model can match $Var(\hat{N}_{Yt}) > Var(\hat{N}_{Ot})$. However, from (A.2) and (A.3) it follows that by construction the relative volatility of wages equals one in this model. Thus, this model cannot account for the joint behavior of age specific hours and age specific relative wages that is observed in the U.S. data.

Differences in Labor Supply: Model II We maintain the same assumptions regarding the households as in the previous. We only vary the production function by postulating the following production function

$$Y_t = A_t K_t^{\alpha}\left(H_{Yt}^{\mu}(EH_{Ot})^{1-\mu}\right)^{1-\alpha}$$

Note that in this version we allow for the labor input of the young and the old to differ. However, we assume that both of these inputs have the same elasticity of substitution with capital. Given this production function we get that

$$W_{Ot} = (1-\gamma)(1-\alpha)\frac{Y_t}{H_{Ot}}, \tag{A.5}$$

$$W_{Yt} = \gamma(1-\alpha)\frac{Y_t}{H_{Yt}}. \tag{A.6}$$

Using the prices determined in (A.5)-(A.6) and the fact that $H_{Yt} = s_Y N_{Yt}$ and $H_{Ot} = (1-s_Y)\, N_{Ot}$ we get

$$\frac{(1-\gamma)(1-\alpha)}{\psi_O\,(1-s_Y)} = \frac{C_t N_{Ot}^{1+\theta_O}}{Y_t}, \tag{A.7}$$

$$\frac{\gamma(1-\alpha)}{\psi_Y} = \frac{C_t N_{Yt}^{1+\theta_Y}}{Y_t}. \tag{A.8}$$

Log-linearizing the ratio of these two equations it follows that

$$\hat{N}_{Yt} = \frac{(1+\theta_O)}{(1+\theta_Y)}\hat{N}_{Ot} \tag{A.9}$$

Similarly to the previous model we have considered, the case of $\theta_Y = \theta_O$ implies the volatility of hours worked is identical for the young and the old. The only case in which $Var(\hat{N}_{Yt}) > Var(\hat{N}_{Ot})$ is if $\frac{(1+\theta_O)}{(1+\theta_Y)} > 1$ – i.e. $\theta_Y < \theta_O$. However, from the labor supply equations it follows

$$\hat{W}_{Yt} - \hat{W}_{Ot} = \theta_Y\hat{N}_{Yt} - \theta_O\hat{N}_{Ot},$$

and thus

$$\hat{W}_{Yt} - \hat{W}_{Ot} = \left(\frac{\theta_Y - \theta_O}{1+\theta_Y}\right)\hat{N}_{Ot}.$$

Since we are interested in calibrations where $\theta_Y < \theta_O$, it follows that $\hat{W}_{Yt} < \hat{W}_{Ot}$. The immediate implication is that the fluctuations in the wage of the young are smaller then the fluctuations in the wage of the old.[30]

To conclude this section; The two models based on "labor supply" differences can easily match the relative volatility of age specific hours. The first model can also match the relative volatilities of young and old hours to output. However, it is inherent to the models' mechanisms that they have counterfactual implications regarding the volatility of age specific wages. These results lead us to consider a "Labor-Demand" channel.

A.4 Proofs

The method of proof follows the arguments made in the text. Assume $\sigma > \rho$, so that production exhibits capital-experience complementarity. The firm's FONCs written in log deviation form are:

$$\hat{W}_Y = (1-\sigma)\hat{Y} + \sigma\hat{A} + (\sigma-1)\hat{H}_Y,$$

[30] This is true as long as the two wages correlate positively.

$$\hat{W}_O = (1-\sigma)\hat{Y} + \left(\frac{\sigma-\rho}{\rho}\right)\hat{X} + \rho\hat{A} + (\rho-1)\hat{H}_O.$$

Here, $X = \lambda K^\rho + (1-\lambda)(AH_O)^\rho$, so that:

$$\hat{X} = \frac{(1-\lambda)(AH_O)^\rho}{X}\rho(\hat{A}+\hat{H}_O) \equiv X_2\rho(\hat{A}+\hat{H}_O).$$

We have used the fact that $\hat{K}=0$ in the impact period of a shock. Note that $0 < X_2 < 1$. Hence:

$$\hat{W}_O = (1-\sigma)\hat{Y} + [(\sigma-\rho)X_2+\rho]\hat{A} + [(\sigma-\rho)X_2+\rho-1]\hat{H}_O.$$

Assuming $\theta_Y = \theta_O = \theta$, the household's FONCs in log deviation form are:

$$\begin{aligned}\theta\hat{H}_Y &= \hat{W}_Y - \hat{C},\\ \theta\hat{H}_O &= \hat{W}_O - \hat{C},\end{aligned}$$

so that:

$$\theta\hat{H}_Y - \hat{W}_Y = \theta\hat{H}_O - \hat{W}_O.$$

Substituting in the firm's FONCs and simplifying, we obtain:

$$\frac{\hat{H}_Y}{\hat{H}_O} = \frac{\theta+1-\rho-(\sigma-\rho)X_2}{\theta+1-\sigma} + \frac{(\sigma-\rho)(1-X_2)}{\theta+1-\sigma}\frac{\hat{A}}{\hat{H}_O}.$$

The first term on the right-hand side of the equality is greater than one since $\sigma > \rho$. Moreover, since $0 < X_2 < 1$, the second term on the right-hand side is greater than zero. Hence, capital-experience complementarity implies that $\hat{H}_Y > \hat{H}_O$ in response to a positive technology shock, $\hat{A} > 0$.

A.5 Specification Testing

If we rewrite the production function (4.3)

$$\Upsilon_1(H_Y, \Upsilon_2(H_O, K))$$

where Υ denotes a CES aggregator, we can see that the three considered specifications are

$$\begin{gathered}\Upsilon_1(H_Y, \Upsilon_2(H_O, K))\\ \Upsilon_1(H_O, \Upsilon_2(H_Y, K))\\ \Upsilon_1(K, \Upsilon_2(H_Y, H_O))\end{gathered}$$

which we label according to the pair of variables appearing in the innermost CES aggregator. From the three first order conditions for any one of these specifications, we derive estimation equations. For $H_O - K$ these equations are written out as (5.2) and (5.3); for $H_Y - K$ the equations are identical except obviously with O and Y subscripts interchanged. For $H_Y - H_O$ we estimate the equations

$$\begin{gathered}\log(Q_{Kt}) = a_1 + \sigma\log(K_t/Y_t)\\ \log(LI_{Yt}/LI_{Ot}) = a_2 + \rho\log(H_{Yt}/H_{Ot})\end{gathered}$$

Because the model's only shock drops out of these equations, theoretically we do not need to estimate these equations using instruments; nonetheless, doing so *ad hoc* does not change the test result.

To understand the Ramsey (1969) test, recall that the conditional expectation $E(Y|X)$ is a function $f(X)$. Therefore we can express the conditional expectation as a Taylor expansion of f. Let that expansion be around the linear prediction of Y, call it Xb; a linear prediction of the left-hand side variable is what estimating equations provide. Of course, the function f itself is linear if all its higher order (second and beyond) derivatives are zero: said another way, f is *not* linear if there *is* a nonzero coefficient on a higher order expansion term. We look for evidence of higher order expansion terms by regressing the residuals on higher powers of the regression fitted values; in practice one can restrict consideration to low powers of the fitted values as suggested by Davidson and MacKinnon (2004).

For any pair of estimation equations involving regressors X_1 and X_2, we run the seemingly unrelated regressions of the residuals $\hat{u}$ on fitted values and fitted values squared[31]

$$\hat{u}_t = \beta_i(X_i b_i) + \gamma_i(X_i b_i)^2 \quad , \quad i \in \{1, 2\}$$

The specification test of the null hypothesis $\gamma_1 = \gamma_2 = 0$ has a $\chi^2(2)$ distribution and we report this test statistic and p-value in the body of the paper.

A.6 Data

Data on hours, employment shares, and wages come from the Current Population Survey (CPS) conducted by the Census Bureau. To obtain wage data, we use questions in the March CPS about income obtained in the previous (last) year.[32] In order to turn this income data into wage data, we must know how many hours the individual worked last year. The hours for the previous year are constructed as the number of weeks worked last year multiplied by a measure of how many hours-per-week were worked by the individual last year. We follow Krusell, Ohanian, Rios-Rull, and Violante (2000) in imputing the hours-per-week from the data on how many hours the individual worked *in the previous (last) week*.

Our measure of hours-per-week is different than Krusell, Ohanian, Rios-Rull, and Violante (2000) in the following. We note whether the worker described her work last year as either full-time (FT) or part-time (PT). Her last week's hours are imputed as the hours-per-week only if the value falls within believable values, given that her work last year was either FT or PT. If her previous week's hours are not consistent with FT or PT work, we impute a "disaggregated" group average as the hours-per-week; by contrast, Krusell, Ohanian, Rios-Rull, and Violante (2000) impute a "disaggregated" group average only if the worker reported that she worked last year but worked zero hours last week.

Our "disaggregated" groups are formed by dividing respondents by age, education, gender, and last year's FT/PT status. Given that there are eleven 5-year age bins (15-19,20-24,...,60-64,65+), 5 eduction bins (below HS, HS, some college, college graduate, postgraduate work), 2 genders, and a FT or PT status, there are 220 possible groups. Our "disaggregated" groups combine education bins for some age-gender-FT/PT groups to ensure that for every year in 1964-2006 our "disaggregated" groups each have at least fifty members.[33] This is done so that the "disaggregated" group average

[31] Higher powers of the fitted values produce similar results and the same test results.

[32] As noted below, a specific question reporting wages only appears in the CPS survey starting in 1982.

[33] Additionally cutting by race (white/nonwhite) does not change matters much.

is not overly reliant on only a few observations.

Conditional on the the other characteristics we consider, we use the information on PT and FT as follows:

- If a person claims to be PT last year and works between 1 and 34 hours last week, we impute their hours-per-week last year as their hours last week; otherwise (they worked 0 or more than 34 hours last week) they are imputed the group average
- If a person claims to be FT last year and works 35 or more hours last week, we impute their hours-per-week last year as their hours last week; otherwise (they worked 0-34 hours last week) they are imputed the group average

Let g' be the part of g with hours last week that are FT-status-fitting for imputation purposes (given the FT/PT nature of g), and g'' be those whose hours last week are not FT-status-fitting. Let h_i, m_i, y_i, and μ_i be worker i's hours last week, number of weeks worked last year, wage and salary income last year, and CPS Person weight, respectively.[34] Then the measures of group g's "disaggregated" group average, weight, hours worked last year, and income last year are

$$h_{g'} = \frac{1}{\sum_{i \in g'} \mu_i} \left(\sum_{i \in g'} h_i \mu_i \right) \tag{A.10}$$

$$\mu_g = \sum_{k \in g} \mu_k \tag{A.11}$$

$$h_g = \frac{1}{\mu_g} \left(\sum_{i \in g'} h_i \mu_i + \sum_{j \in g''} h_{g'} \mu_j \right) \tag{A.12}$$

$$y_g = \frac{1}{\mu_g} \left(\sum_{k \in g} y_k \mu_k \right) \tag{A.13}$$

Let γ be a set of gs: this is a larger group, such as all workers in the 15-19 age category, comprised of smaller "disaggregated" groups. Our construction of an efficiency wage measure for γ is similar to that of Krusell, Ohanian, Rios-Rull, and Violante (2000): our efficiency measurement f for each g is the average of their wage (y_g/h_g) for the years 1985-1989.[35]

$$W_\gamma = \frac{\sum_{g \in \gamma} y_g \mu_g}{\sum_{g \in \gamma} h_g f_g \mu_g} \tag{A.14}$$

It is worth mentioning that the March CPS has a specific question "On average, how many hours per week did you work last year, when you worked?" starting in 1976. We find that making

[34] In the March supplement, we have both a CPS Basic Person weight, and a CPS Supplemental Person weight. Personnel at the Census Bureau have advised us to use the latter for all the data questions we are addressing, even though some of these data are not part of the March Annual Supplement.

[35] Krusell, Ohanian, Rios-Rull, and Violante (2000) use the wage in 1980 as the efficiency measurement. We use an average of the wage to allow for the possibility that the efficiency measure varies over the cycle. Hence, by averaging over five years we aim to smooth the efficiency measurement. The results remain the same using either efficiency measurement.

sure the hours imputation is FT-status-fitting leads to hours measures that are close to the post-1976 question when both are available. By ignoring the FT-status, one underreports the groups' hours.

Our data on hours come directly from the hours last week question. Likewise, our labor force share data comes from a labor force status question pertaining to last week.

We have found that these last week hours have level shifts between the 1967 and 1968 survey years and therefore start our hours series at 1968. The last year information used in the wage series appears unaffected during this time, so we use data going back to the 1964 survey year (data about 1963). The statistics on wages remain virtually identical if we start the wage series at survey year 1968.