

The Boston Mechanism Reconsidered

Papers

Abdulkadiroglu, Atila; Che Yeon-Ko and Yasuda Yosuke
“Resolving Conflicting Interests in School Choice:
Reconsidering The Boston Mechanism”, AER, 2011.

Miralles, Antonio, 2008. "School choice: the case for the
Boston Mechanism“

Pathak, Parag and Tayfun Sonmez, “Leveling the Playing
Field: Sincere and Sophisticated Players in the Boston
Mechanism”, AER 2008.

Featherstone, Clayton and Muriel Niederle, “School Choice
Mechanisms under Incomplete Information: An
Experimental Investigation”, WP.

DA superior to Boston

The literature seems to reject the Boston mechanism on the following premise:

- The Boston mechanism
 - Manipulable: Rank a school higher to improve the odds to get it
 - It produces a stable match in Nash equilibrium, there may be many stable matches (Ergin and Sonmez 2006)
- The DA mechanism
 - Strategy-proof
 - Optimal: It produces the unique stable assignment that everybody prefers to any other stable assignment

These arguments hold when schools have strict priorities
What when schools have coarse priorities?

Similar Ordinal Preferences and Coarse School Priorities Abdulkadiroglu et al 2011

- When everybody prefers the same school the most, say school X, the tie among everybody has to be broken
- If school X does not rank students, priorities do not break ties, the DA mechanism uses a lottery to break ties
- Assignment of X will be efficient ex-post, regardless of the realization of the lottery
- This does not mean that the welfare issue disappears
- Assigning X to those who really value it very highly may still be important
- Yet the DA cannot differentiate among students based on preference intensities

Example

- 3 students: {1,2,3} and 3 schools {s1, s2, s3} with 1 seat each, no priorities.
- Student valuations for schools:

	1's values	2's values	3's values
s1	0.8	0.8	0.6
s2	0.2	0.2	0.4
s3	0	0	0

- DA allocates schools with equal probability:
 - $U1(DA) = U2(DA) = 1/3 \cdot 0.8 + 1/3 \cdot 0.2 + 1/3 \cdot 0 = 1/3 = U3(DA)$
- Boston: 1 and 2 report s1 as first choice, 3 s2:
 - $U1(B) = U2(B) = 1/2 \cdot 0.8 + 1/2 \cdot 0 = 0.4 > 1/3$
 - $U3(B) = 1 \cdot 0.6 > 1/3$

Some families' response to the change from Boston to DA

- A parent argues in a public meeting:
I'm troubled that you're considering a system that takes away the little power that parents have to prioritize... what you call this strategizing as if strategizing is a dirty word... (Recording from Public Hearing by the School Committee, 05-11-04).
- Another parent argued:
... if I understand the impact of Gale Shapley, and I've tried to study it and I've met with BPS staff... I understood that in fact the random number ... [has] preference over your choices... (Recording from the BPS Public Hearing, 6-8-05).

Boston versus DA in a Bayesian setting (Abdulkadiroglu et al 2011)

- Finitely many students and schools
- Schools have no priorities
- Students share the same ordinal preferences, but cardinal valuations for schools are drawn independently from a commonly known distribution
- Each student knows his/her own valuations, cannot observe others
- Symmetric Bayesian equilibrium (same (mixed) actions for students with same utilities)

Theorem

In any symmetric equilibrium of the Boston mechanism, each type of student is weakly better off than she is under the DA with any symmetric tie-breaking.

The idea of the proof: Given any symmetric equilibrium, any type of student can cleverly replicate her DA allocation under the Boston mechanism.

Naïve players

- Some families may fail to see/utilize strategic opportunities
- DA levels the playing field for everybody by removing strategizing
- Some parents resisted the change from Boston to DA (quotes above)

- Pathak and Sonmez (2008):
 - Introduce naive players, who always submit their true preferences
 - Naives lose priority to sophisticated at every school but their first choice
 - Sophisticated prefer the Pareto-dominant equilibrium of the Boston mechanism to the outcome of the DA

- Assumption of strict preferences and complete information by sophisticated students,

Strategic naïveté: Intuition

- Under complete information and strict school priorities, a sophisticated player knows with certainty where he stands against other students at a school's priority list in equilibrium.
- If he knows that ranking a school as first choice will not result in a match with that school, he does not rank it as first choice.
- Instead, he ranks another school as first choice, which may turn out to be a naïve player's second choice.
- So effectively, the sophisticated gains priority at the naïve's second choice.
- How does this hold up to uncertainty about who is naïve and non-strict school priorities.

An example

Abdulkadiroglu et al 2011

- 6 students, one naïve and one sophisticated for each type
- 3 schools {s1, s2, s3} with 2 seats each, no priorities.
- Student valuations for schools:

	1's values	2's values	3's values
s1	0.8	0.8	0.6
s2	0.2	0.2	0.4
s3	0	0	0

- DA allocates schools with equal probability:
- Boston: all naïves and type 1,2 strategic players submit truthfully. Type 3 submits s2 as first choice
- Naïves lose compared to strategic player at s2, but gain probability to receive their first choice school
 - (0.4, 0.2, 0.4) to get schools (s2, s2, s3)

Strategic Naïveté

- Introduce naive players to our model: Each type is a naive player with some known probability.

Theorem

In any symmetric Bayesian equilibrium of Boston mechanism with naive students: (i) If a sophisticated player manipulates with positive probability, each naive player is assigned each of top j schools $\{s_1, \dots, s_j\}$ for some j with weakly higher probability and to some school in that set with strictly higher probability under the Boston than under DA.

Conclusion

Two assumptions:

- Similar ordinal preferences
- Coarse school priorities

The Boston mechanism Pareto dominates the DA

- In the presence of strategically naive students, all sophisticated and some naive players achieve a higher utility in the Boston mechanism and naives are assigned to top schools with higher probability.

What drives the difference between DA and Boston?

Completely correlated environment: Information on ordinal preferences is not important.

What matters is information on cardinal preferences to maximize student welfare.

Because DA is strategy-proof: No information on cardinal preferences can be transmitted

Boston is manipulability: Equilibrium manipulations can transmit cardinal preferences

Boston Mechanism

Can we expect students to “misrepresent” preferences, in a way to take advantage of Boston?

Empirically: Hard to test: True preferences are not known.

An Experiment will be able to shed some light.

Featherstone, Niederle: Boston Mechanism in correlated environments

Experiment

- Run both Boston and DA in Correlated environment
- Truth-telling is not an equilibrium under Boston; it is a dominant strategy under DA.
- Q: How do truth-telling rates compare across mechanisms?
- Q: Do students best-respond when truth-telling is not an equilibrium?

Example; correlated preferences (likely the general case...)

- ▶ 3 schools are **commonly ranked** by students as follows.

School	Best	Second	Third	No school
Seats	2	1	1	–
Payoff	100	67	25	0

- ▶ **Two types of students: Top and Average.**
 - ▶ Top always has priority over Average.
 - ▶ Within group, ties are broken by a lottery.
- ▶ 3 Tops and 2 Averages
- ▶ DA Outcome:
 - Top:** 2 get Best, 1 gets Second
 - Average:** 1 gets Third, 1 is unassigned
- ▶ What is the equilibrium under Boston?

Boston mechanism in the correlated environment—complex eq. strategies

School	Best	Second	Third	No school
Seats	2	1	1	—
Payoff	100	67	25	0

Boston:

- ▶ Strategies:

 - Top: (Best, Third, ...) (skipping the middle)

 - Average: (Second, ...) (skipping the top)

- ▶ Outcome:

 - Top: 2 get Best, 1 gets Third

 - Average: 1 gets Second, 1 is unassigned

Experimental design

Design

- 2* 2 design: Boston and DA across subjects, Correlated and Uncorrelated Environment within subjects.
- 30 rounds, 15 in Correlated environment, then 15 in Uncorrelated environment.
- Groups of 5 are static for the entire experiment, as is Top/Average identity in the Correlated environment.

Learning and feedback

- Spend 15 minutes at the beginning explaining algorithms and Correlated environment, and another 10 explaining the Uncorrelated environment after Period 15.
- Students must pass a test to continue with the experiment.
- School lotteries are redrawn each period, as are preferences in the Uncorrelated environment.
- Subjects see the complete match after every period.

Implementation

- z-Tree (Fischbacher 2007)
- Pay 1.5 cents per point, cumulatively across periods (which is roughly \$30 per hour)

Truth-telling rates

First choices of Participants

School	Best	Second	Third
Top B	0.92	0.07	0.01
Average B	0.06	0.67	0.27
Top DA	1	0	0
Average DA.	0.7	0.05	0.25

Truth-telling Rates:

Boston: Top: 65.7% and Average students 1.5%

- DA: 92% of Top and 63% of Average student strategies

Conclusions from the Correlated environment

- DA conforms to equilibrium outcomes perfectly; Boston does not.
- Students manipulate their preference reports under Boston, but fail to do so optimally.
- This implies that mechanisms which rely on equilibrium play that is not truth-telling may not work as well in the field.

Relaxing complete information on ordinal preferences

School choice literature: Fix ordinal preferences of students

- Mechanism strategy-proof?
- Efficient given ordinal / cardinal preferences?
 - Sometimes even: taking lottery draws that make school priorities strict into account.
- Here: What if there is incomplete information of ordinal preferences? What may change?

Uncorrelated preferences: (a conceptually illuminating simple environment)

- 2 schools, one for Art, one for Science, each one seat
- 3 students, each iid a Scientist with $p=1/2$ and Artist with $p=1/2$. Artists prefer the art school, scientists the science school.
- The (single) tie breaking lottery is equiprobable over all orderings of the three students.

Consider a student after he knows his own type, and before he knows the types of the others. Then (because the environment is uncorrelated) his type gives him no information about the popularity of each school. So, **under the Boston mechanism, truthtelling is an equilibrium.**

Boston can stochastically dominate DA in an uncorrelated environment

Example: 3 students, 2 schools each with one seat

► DA:

Lottery rank	First choice	Second choice	No school
1	1	0	0
2 s	$\frac{1}{2}$ 1:a	$\frac{1}{2}$ 1:s	0
3	0	0	1
Average	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$

Boston can stochastically dominate DA in an uncorrelated environment

Example: 3 students, 2 schools each with one seat

► DA:

Lottery rank	First choice	Second choice	No school
1	1	0	0
2 s	$\frac{1}{2}$ 1:a	$\frac{1}{2}$ 1:s	0
3	0	0	1
Average	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$

► Boston:

Lottery rank	First choice	Second choice	No school
1	1	0	0
2 s	$\frac{1}{2}$ 1:a	$\frac{1}{4}$ 1:s; 3:s	$\frac{1}{4}$ 1:s; 3:a
3	$\frac{1}{4}$	0	$\frac{3}{4}$
Average	$\frac{1}{2} + \frac{1}{12}$	$\frac{1}{6} - \frac{1}{12}$	$\frac{1}{3}$

Boston dominates Probabilistic Serial

Probabilistic serial:

Suppose there are 2 artists, 1 scientist:

Chance to receive each school:

	Art	Science
A st.	$\frac{1}{2}$	$\frac{1}{3}$
S st.	0	$\frac{4}{3}$

In Boston mechanism:

	Art	Science
A st.	$\frac{1}{2}$	0
S st.	0	1

Incomplete information of ordinal preferences

Incomplete information of ordinal preferences allows trade-offs across different preference realizations.

Introduces new potential efficiency gains.

2 Assumptions:

- Symmetric environment: Truthtelling is an Ordinal Bayes Nash equilibrium under Boston.
- Truthtelling rates will be, empirically, similar when truthtelling is only an OBNE compared to a dominant strategy.

Uncorrelated Environment

Once more 5 students and 4 schools, A, B, C, D (total of 4 seats) seats.

But now: preferences of students random uniform,
priorities of schools: random for each school
separately.

Preference	1	2	3	4	No Sc.
Seats	1	1	1	1	5
Payoff	110	90	67	25	0

Boston Mechanism: truthtelling is an ordinal Bayes
Nash equilibrium

Phase 2

You are Student 1

You are in Period 4

There are 5 Students.
All students and schools draw points and priority rankings independently of each other.

The schools with their number of seats and points for getting into them					
School	A	D	C	D	No school
# Seats	1	1	1	1	1
Points	25	90	110	67	0

Submit your ranking

Rank First	Rank Second	Rank Third	Rank Fourth	Rank Fifth
<input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> No school	<input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> No school	<input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> No school	<input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> No school	<input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> No school

Submit Rankings

History of School Assignments						Your submitted rankings				Your true rankings			
Period	Student 1	Student 2	Student 3	Student 4	Student 5	Rank 1	Rank 2	Rank 3	Rank 4	A	B	C	D
3	B	D	C	A	No school	A	B	C	D	67	90	110	25

All priority orderings over students are equally likely for a given school
All preference orderings are equally likely for a given student
All preferences are priorities are redrawn each round

Every student applies to his top choice among schools that have not rejected him.
Each school tentatively keeps its top priority applicant and rejects the rest permanently.
A student that was rejected applies to the next school on the list.
Each school collects new requests, tentatively keeps its top priority applicant and rejects the rest.
The process continues until all students have been either matched or rejected from all schools

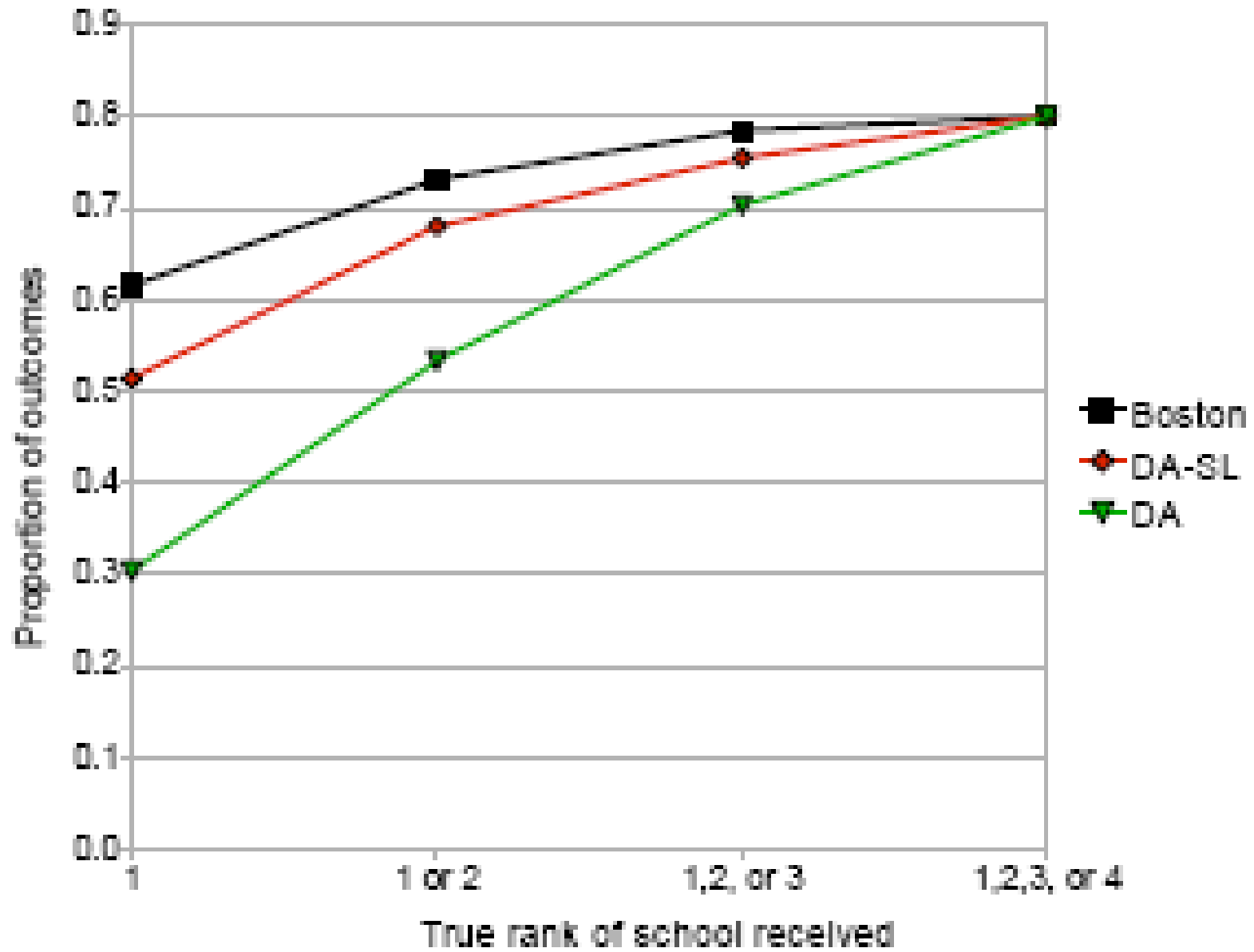
Truth-telling rank-by-rank

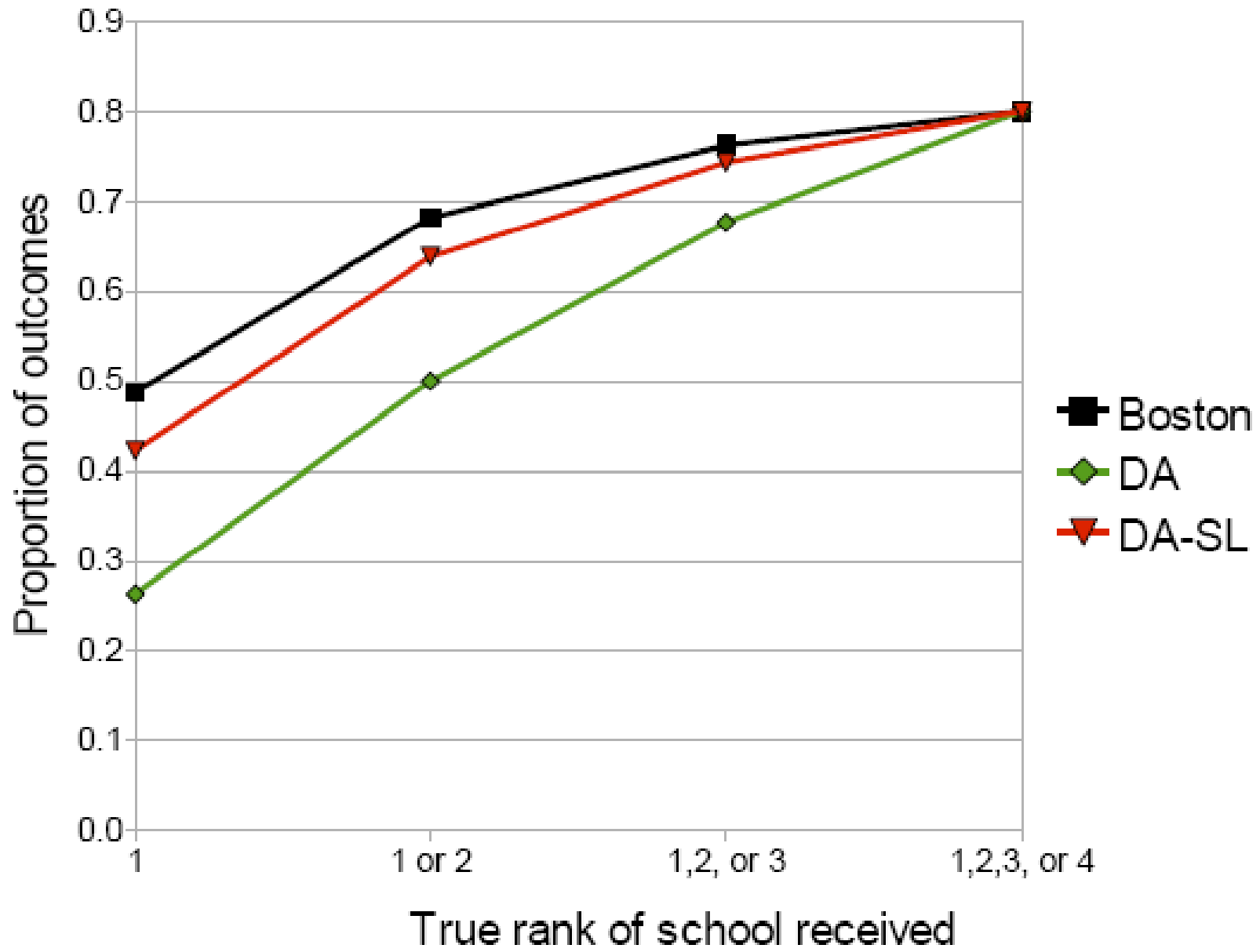
	Boston				DA			
	1 st	2 nd	3 rd	4 th	1 st	2 nd	3 rd	4 th
Rank 1	0.76	0.22	0.01	0.01	0.74	0.19	0.07	0.00
Rank 2	0.16	0.61	0.13	0.10	0.14	0.69	0.15	0.02
Rank 3	0.06	0.07	0.80	0.07	0.07	0.11	0.77	0.05
Rank 4	0.02	0.10	0.06	0.82	0.05	0.01	0.01	0.93

- ▶ Very similar across mechanisms.
- ▶ Misrepresentations tend to be ± 1 ranking, so manipulations are present, but not extreme.

Truth-telling rates

Boston: 58%, DA: 66%: Difference is not statistically significant





- Ex post, student-proposing DA yields the student-optimal stable matching (relative to $SC \setminus L$) (Gale and Shapley 1962)
- But L is an artifact of the matching algorithm, so we really only care about stability relative to SC .
- The output from DA might not be the student-optimal stable matching relative to SC .
- Much recent work has focused on improving SP-DA:
 - Erdil and Ergin (2008)
 - Abdulkadiroglu, Che, and Yasuda (2008)
 - Miralles (2008)
- Theorem: (Abdulkadiroglu et al.) For a given L , any mechanism that dominates DA ex post cannot be strategy-proof.
- So if we Pareto improve upon the ex post student-optimal stable matching, we sacrifice strategy-proofness for efficiency. But how much efficiency?

- Abdulkadiroglu et al. take submitted preferences from Boston and NYC (which run DA). Their exercise is as follows:
- Assume these are the true preferences.
- Calculate the student-optimal stable matching using SP-DA.
 - Improvement process 1: Resolve Erdil and Ergin stable improvement cycles.
 - Improvement process 2: Resolve all improvement cycles (Top Trading Cycles).
- The result was that the benefits gained from these improvements is small (NYC, 3%; Boston, > 1%). Hence, the cost of strategy-proofness is small.

- How does this relate to our result? We found that switching from strategy-proof to Bayesian implementation bought us significant gains.
- This was ex ante. Abdulkadiroglu et al. still assume that all preferences are known, i.e. they are from an interim perspective.
- In fact, in our Art and Science school example, the methodology used by Abdulkadiroglu et al. would result in zero cost of strategy-proofness.
- Their approach can underestimate the cost of strategy-proofness.
- Our example indicates the cost could be quite high in some environments.

Things to note

- The uncorrelated environment let's us look at Boston and DA in a way that we aren't likely to see them in naturally occurring school choice.
- In this environment, there's no incentive not to state preferences truthfully in the Boston mechanism, even though it isn't a dominant strategy. (So on this restricted domain, there's no corresponding benefit to compensate for the cost of strategyproofness.)
- Boston stochastically dominates DA, **even though it doesn't dominate it ex-post** (ex post the two mechanisms just redistribute who is unassigned)