

Accrual Accounting, Informational Sufficiency, and Equity Valuation

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Abstract

This paper studies accrual accounting and equity valuation in the context of a firm that makes repeated and overlapping investments in productive capacity. The analysis identifies a particular accrual accounting (depreciation) rule which is termed replacement cost accounting because the book value of existing capacity assets is set equal to the value that such assets would have if a competitive market were to exist for used assets. It is shown that replacement cost accounting aggregates past investment decisions of the firm without a loss of value-relevant information. In particular, the intrinsic value of the firm can then be expressed as a function of current accounting data and projections of growth in the firm's output market. Further, it is shown that replacement cost accounting is essentially the only accounting rule with this informational sufficiency property. In many environments of interest, standard depreciation rules, such as the straight-line rule, will not coincide with replacement cost accounting. Nonetheless, even informationally insufficient accounting rules are shown to be useful to investors. In particular, the analysis in this paper characterizes a best value estimator and derives upper bounds on the valuation errors associated with alternative accounting rules.

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1 Introduction

This paper examines the informativeness of alternative accrual accounting rules for the purpose of firm valuation. I model a firm that undertakes a sequence of investment decisions in productive capacity. The firm's accounting system aggregates the resulting investment history into current financial statements. It is shown that for certain accounting rules this aggregation process does not entail a loss of value-relevant information, and investors will be able to assess the firm's value correctly by observing only the aggregate data. Such rules will be referred to as informationally sufficient. For informationally sufficient rules, I derive valuation equations that express the firm's intrinsic value in terms of the accounting numbers observed by investors. In contrast, for informationally insufficient accounting rules, it is generally impossible to solve the valuation problem precisely. Yet, if investors have beliefs about the investment history of the firm, then they can use the information in financial statements to update these beliefs. For informationally insufficient rules, I characterize the value estimator that uses only the observed data as inputs and minimizes the mean squared error. It is shown that the error of this estimator is bounded by a measure of distance between the accounting rule in place and the closest informationally sufficient rule.

Earlier theoretical literature on accounting based valuation has been largely silent on the relative advantages of alternative accrual accounting rules in providing information useful to investors. I seek to address two limitations inherent in earlier valuation studies such as Ohlson (1995), Feltham and Ohlson (1995), and Ohlson and Juettner-Nauroth (2005). First, in these papers, the firm's underlying transactions, as well as the accounting rules employed, are not modeled beyond their most basic properties, like the clean surplus relation.¹ Therefore, assumptions on the behavior of the time-series of accounting numbers (e.g. linear information dynamics) are inherently joint constraints on the economic environment of the firm and the accounting rules in use.² My analysis seeks to disentangle the economic and reporting factors that affect the time-series of accounting numbers. Second, none of these models explicitly articulates limitations on the information available to outsiders. With-

¹This criticism applies to earlier studies in varying degrees. More elaborate models of the firm's transactions and accounting rules can be found in Ohlson and Zhang (1998), Zhang, G. (2000), and Zhang, X. (2000). On the other hand, in his discussion of the model in Ohlson and Juettner-Nauroth (2005), Penman (2005) poses the question: "Where's the accounting?"

²Several empirical studies challenged the linear information dynamics assumption of Ohlson (1995). See, for instance, Myers (1999) and Dechow, Hutton, and Sloan (1999). Discussing this issue, Kothari (2001) points out the following direction for future research:

"While an autoregressive process in residual income as a parsimonious description is economically intuitive, there is nothing in economic theory to suggest that all firms' residual earnings will follow an autoregressive process at all stages in their life cycle. A more fruitful empirical avenue would be to understand the determinants of the autoregressive process or deviations from that process as a function of firm, industry, macroeconomic, or international institutional characteristics."

out information asymmetry, it is difficult, if not impossible, to demonstrate informational advantages of particular accounting rules over others, including cash accounting.

This paper models the activities of a firm as a sequence of capital investments in productive capacity. The firm uses its capacity to deliver goods and services which generate revenues. The accounting rules may reflect the productivity pattern of the firm's assets and provide aggregate information on the investments undertaken. Depreciation is the only accrual in my model. This focus was chosen for two reasons. First, depreciation is arguably the largest accrual in many industries. Second, earlier literature on performance measurement has shown that insights obtained in connection with capital investments and depreciation do carry over to other accrual accounting items (see, for instance, Dutta and Reichelstein, 2005).

A central point of departure for my analysis is that investors must rely only on limited publicly available information for valuation purposes. Specifically, I assume that investors observe only the latest financial statements, rather than the whole investment history. Clearly, if investors could observe all the past transactions of the firm, it would be impossible to differentiate between alternative accounting rules, because there would be no need to aggregate information about the underlying transactions. The main goal of depreciation charges in my model is to aggregate information about past investments such that single period financial statements are sufficient for valuation. Depreciation schedules with this property are referred to as informationally sufficient.

An additional feature of my model is the assumption that the projections of future output market growth rates cannot be incorporated into the accounting numbers. Consequently, the requirement for an informationally sufficient rule is that it must provide enough information to value the company under alternative assumptions about future growth opportunities. This requirement is broadly consistent with the perspective taken by standard setters in SFAC No. 1:

"Financial accounting is not designed to measure directly the value of a business enterprise, but the information it provides may be helpful to those who wish to estimate its value... Although financial reporting should provide basic information to aid them [investors, creditors, and others] they do their own evaluating, estimating, predicting, assessing, confirming, changing, or rejecting."

Since I assume that growth opportunities cannot be incorporated into the accounting data, fair value accounting, which would result in book values equal to market values, is rendered infeasible in the context of the present model.

My framework of a firm undertaking a sequence of overlapping capacity investments builds on Rogerson (2008a, 2008b). In this context, it is possible to identify the marginal cost of providing an additional unit of capacity for one period, holding capacity levels in other periods fixed. It can be shown that this marginal cost is equal to the rental price of a unit of capacity in a hypothetical perfectly competitive rental market. Rogerson (2008b) demonstrates that there exists a particular depreciation schedule, the Relative Replacement Cost, or RRC rule, under which the historical cost of capacity is equal to its hypothetical rental price.

The first result of my paper applies this idea to the valuation problem. Given the RRC rule, it is shown that book value can be interpreted as the replacement cost of assets in place at any point in time. Therefore, I refer to the RRC rule as replacement cost accounting. Applying the residual income valuation formula, I then demonstrate that the firm's value is equal to the replacement cost of its assets in place plus the present value of its future optimized *economic profits*, where the measure of capacity costs in the computation of profits is the replacement cost of capacity utilized in a given period.

The second result of this paper is that, under certain assumptions, replacement cost accounting is informationally sufficient, i.e., it provides enough information for a correct assessment of the firm's intrinsic value based solely on accounting data. Specifically, I show that if the output market grows proportionately at all price levels, investors can predict future economic profits if they know current economic profits and rely on their projections of growth in the firm's product market.

I formally identify two-value relevant dimensions of the investment history - the replacement cost of assets in place and the replacement cost of capacity utilized in the latest period. If the difference between two investment histories is orthogonal to both value-relevant dimensions, then two firms that have experienced these histories will have the same value. On the other hand, differences along the value-relevant dimensions can lead to differences in valuation. Under replacement cost accounting, the replacement cost of assets in place is equal to the book value and the replacement cost of capacity utilized is equal to the sum of depreciation and a cost-of-capital charge on the beginning-of-period book value. Further, I show that if the replacement cost rule is modified by some partial direct expensing of new investments, it remains informationally sufficient. I label this family of rules generalized replacement cost accounting and derive an exact valuation formula for this family of accounting rules.

My third major result is that generalized replacement cost accounting is unique in its informational sufficiency. To prove this, I invoke a basic result from aggregation theory. Applications of this theory have a long tradition in accounting (e.g. Lev, 1968 and Ijiri,

1967; see also Arya et. al., 2000 for a more recent reference). My uniqueness result rests on the argument that in order for financial statements to permit valuation for a broad range of possible output market growth projections, an informationally sufficient rule must preserve both value-relevant dimensions through book value and depreciation. This requirement implies that book value must be always proportional to the replacement cost of assets in place, i.e., the rule in use has to conform to generalized replacement cost accounting.

Replacement cost accounting should be regarded as a normative benchmark rather than a description of current financial accounting practice, where straight-line depreciation is prevalent for fixed assets. For informationally insufficient rules, I also perform an analysis in the spirit of aggregation theory to bound the minimum error that investors can achieve in the valuation process. It is assumed that investors form beliefs about the firm's investment history and then update their beliefs after observing the financial statements. My finding on the uniqueness of generalized replacement cost accounting shows that informationally insufficient accounting rules are incapable of fully resolving the uncertainty related to the value-relevant dimensions of the firm's history. Therefore, the firm's intrinsic value will have some residual variance after conditioning on the accounting data. I derive an upper bound on this variance which can be interpreted as the distance between the accounting rule actually used and the closest informationally sufficient rule. In addition, my findings characterize the value estimator minimizing the mean squared error for investors.

The notion of proper accrual accounting and the economic significance of "good" accounting have been explored in different strands of the accounting literature. Beaver and Dukes (1974) identify depreciation schedules for various types of assets, such that accounting rates of return equal the economic rates of return. Rajan, Reichelstein, and Soliman (2007) characterize the biases in accounting rates of return that result from applying depreciation schedules that do not match the productivity profile of assets. In the managerial literature, certain accounting rules were shown to provide goal congruent incentives for a manager to whom investment decisions have been delegated, but whose decisions are subject to potential horizon biases.³ This paper provides a new perspective on the usefulness of alternative accrual accounting rules in the context of a forward looking equity valuation problem, absent any incentive or contracting issues.

The remainder of this paper is organized as follows. The next section illustrates the concept of informational sufficiency by means of an example. In Section 3, the economic and reporting model of the firm is presented and an expression for the firm's value is derived. Section 4 provides the general concept of informational sufficiency and identifies accounting rules that have this sufficiency property. In contrast, the valuation properties of information-

³See, for example, Rogerson (1997, 2008b), Reichelstein (1997, 2000).

ally insufficient accounting systems are examined in Section 5. This section also discusses empirical implications of the model. Concluding remarks are provided in the last section of the paper.

2 An Illustrative Example

This section presents an example that illustrates the concept of informational sufficiency. The role of accrual accounting is to aggregate information about past transactions of the firm in a manner that preserves information essential to investors. The example shows that in this regard some accounting rules are more effective than others. In particular, I consider a simplified model in which a firm employs a single type of capital assets whose productivity is constant over their useful life. First, it will be shown that straight-line accounting is informationally insufficient for such assets, meaning that the valuation problem is not solvable if investors observe only the latest period financial statements based on straight-line depreciation. In contrast, an alternative depreciation policy, the annuity rule, aggregates past investment data without a loss of value-relevant information and enables investors to value the company based solely on current aggregate accounting data.

Consider a firm which invests in capacity, produces a single type of product, and then sells this product to an outside market. In the simplified model, capacity is generated by capital assets with a useful life of four periods. Each asset is idle in the acquisition period and then provides capacity to produce one unit of output in each of the following four periods. This productivity pattern will be referred to as *one-hoss shay* productivity.⁴ The price of capital assets is constant over time and is normalized to unity, so that an investment of I_{t-4} dollars in period $t - 4$ generates I_{t-4} units of capacity in periods $t - 3, t - 2, t - 1, t$. The total capacity available in period t is therefore given by:

$$K_t = I_{t-1} + I_{t-2} + I_{t-3} + I_{t-4}.$$

To make the valuation problem particularly simple, assume that the firm faces a kinked demand curve of the following nature in its output market. Any quantity of output up to some maximum level, q_{\max} , can be sold at a price p that is high enough to cover the costs of production. Beyond q_{\max} , the output price declines so rapidly that the firm never finds it optimal to supply more than q_{\max} . Further, assume that demand is stationary over time.

⁴This term originates from the poem "The Deacon's Masterpiece, or, the Wonderful One-Hoss Shay: a Logical Story" written by Oliver W. Holmes in 1858, in which a shay is described that does not require repairs for a hundred years and then falls apart "all at once, and nothing first". The term is widely used in the economic literature on regulation.

Under these assumptions, it is optimal for the firm to generate capacity of exactly q_{\max} in every period and sell all its output at price p . Revenues in period $t + \tau$ are then equal to

$$R_{t+\tau} = pq_{\max} = pK_{t+\tau}.$$

The value of the firm at date t is defined as the present value of its future cash flows:

$$V_t = \sum_{\tau=1}^T (R_{t+\tau} - I_{t+\tau}) \gamma^\tau,$$

where future investments are assumed to be chosen optimally, and $\gamma = \frac{1}{1+r}$ is the appropriate discount factor. On the optimal path, $K_{t+\tau}$ must be equal to q_{\max} for any τ , so in each period the firm will exactly replace the investment that goes offline in that period:

$$I_{t+\tau} = I_{t+\tau-4}$$

for $\tau \geq 1$. Hence, starting with period $t + 1$, the optimal investment policy cycles through investments I_{t-3} , I_{t-2} , I_{t-1} , I_t , I_{t-3} , and so forth. Under this policy, the firm's value is completely determined by the history of its latest four investments.

$$\begin{aligned} V_t &= (pq_{\max} - I_{t-3}) \gamma + (pq_{\max} - I_{t-2}) \gamma^2 + \dots = \\ &= \frac{1}{r} pq_{\max} - (\gamma + \gamma^5 + \dots) I_{t-3} - \dots - (\gamma^4 + \gamma^8 + \dots) I_t \\ &= \frac{1}{r} pq_{\max} - \frac{\gamma}{1 - \gamma^4} I_{t-3} - \frac{\gamma^2}{1 - \gamma^4} I_{t-2} - \frac{\gamma^3}{1 - \gamma^4} I_{t-1} - \frac{\gamma^4}{1 - \gamma^4} I_t. \end{aligned} \quad (1)$$

2.1 Straight-Line Depreciation

Assume that the company prepares financial statements in accordance with the straight-line depreciation rule. Thus, assets are capitalized in the acquisition period and then depreciated evenly over the next four periods. At the end of period t , the accounting system reports the following information: revenues, R_t ,

$$R_t = p(I_{t-1} + I_{t-2} + I_{t-3} + I_{t-4}),$$

depreciation, D_t ,

$$D_t = \frac{1}{4} I_{t-1} + \frac{1}{4} I_{t-2} + \frac{1}{4} I_{t-3} + \frac{1}{4} I_{t-4},$$

book value at date t , BV_t ,

$$BV_t = I_t + \frac{3}{4}I_{t-1} + \frac{2}{4}I_{t-2} + \frac{1}{4}I_{t-3},$$

and cash flows to investments, I_t . Given clean surplus accounting, investors can also infer the beginning of period book value of assets, BV_{t-1} , as

$$BV_{t-1} = BV_t + D_t - I_t.$$

Define the firm's state at date t as the history of its past five investments,

$$\boldsymbol{\theta}_t = (I_t, I_{t-1}, I_{t-2}, I_{t-3}, I_{t-4})'.$$

Note that all accounting numbers above are linear combinations of the latest five investments and therefore are completely determined by the state vector $\boldsymbol{\theta}_t$.

The main idea of the example is to demonstrate that, depending on the choice of accounting rules, current period financial statements may be sufficient or insufficient for valuation purposes. While straight-line depreciation may seem to be a natural method to account for assets whose productivity is constant over time, it turns out that this rule entails a loss of value-relevant information. To demonstrate this claim, it suffices to describe two hypothetical firms, operating in the same market, and generating identical financial statements, yet because of different investment histories their intrinsic values differ. Observing only the financial statements, investors will not be able to figure out the underlying investment history, and, consequently, they will not be able to value the two firms correctly. Specifically, assume that $q_{\max} = 40$ and consider the following two histories at date t :

$$\boldsymbol{\theta}_t^{(1)} = (10, 10, 10, 10, 10)$$

and

$$\boldsymbol{\theta}_t^{(2)} = (10, 15, 0, 15, 10).$$

Firm 1 is in its steady state and invests 10 dollars in every period. Firm 2 cycles through investments 15, 0, 15, 10, also implementing capacity of 40 in every period. At date t , these two histories lead to exactly the same financial statements. Indeed, for both firms, we have

$$R_t = 40p, D_t = 10, BV_t = 25, I_t = 10.$$

On the other hand, applying equation (1) to these histories, one can compute the valuations

of the two firms at date t :

$$V_t^{(1)} = \frac{1}{r}pq_{\max} - 10\frac{\gamma}{1-\gamma^4} - 10\frac{\gamma^2}{1-\gamma^4}I_{t-2} - 10\frac{\gamma^3}{1-\gamma^4} - 10\frac{\gamma^4}{1-\gamma^4}I_t$$

and

$$V_t^{(2)} = \frac{1}{r}pq_{\max} - 10\frac{\gamma}{1-\gamma^4} - 15\frac{\gamma^2}{1-\gamma^4}I_{t-2} - 0 - 15\frac{\gamma^4}{1-\gamma^4}I_t.$$

Since $\gamma^3 < (\gamma^2 + \gamma^4)/2$, the value of firm 1 is greater than that of firm 2:

$$V_t^{(1)} > V_t^{(2)}.$$

However, observing only the latest financial statements, it is impossible to infer which of the two firms generated them. Hence, the valuation problem is not solvable in this case, and straight-line depreciation is informationally insufficient for assets corresponding to the one-hoss shay pattern.

To further illustrate this insufficiency result, consider the application of the residual income valuation model to firms 1 and 2. Residual income in period t , RI_t , is defined as the difference between revenues and the aggregate historical cost of capacity, H_t , the latter being the sum of depreciation and an imputed charge on the beginning of period book value:

$$RI_t = R_t - H_t \equiv R_t - D_t - rBV_{t-1}.$$

It is well known that regardless of the accounting rules, value can be expressed by means of the residual income valuation formula:⁵

$$V_t = BV_t + \sum_{\tau=1}^{\infty} \gamma^{\tau} RI_{t+\tau}.$$

In our example, book values of both firms are equal to 25 at date $t - 1$. Given straight-line depreciation, residual income of firm 1 is constant over time and always equal to

$$RI_{t+\tau}^{(1)} = 40p - 10 - 25r.$$

One can check that firm's 2 residual income in period t is also equal to $40p - 10 - 25r$, but from period $t + 1$ onwards, it will iterate through the following four values

$$40p - 10 - 25r, 40p - 10 - 30r, 40p - 10 - 20r, 40p - 10 - 25r, \dots$$

⁵See, for instance, Penman (2007).

Therefore, the future residual income processes of the two firms are different, although they operate in equivalent environments and report the same accounting numbers at date t . For this reason, the current financial statements do not convey enough information to predict future residual earnings, even though investors may correctly anticipate future conditions in the product market.

2.2 Annuity Depreciation

We now consider an alternative accounting policy - the annuity depreciation rule. Assets are again capitalized at historical cost in the acquisition period and then fully depreciated over their useful life, yet the depreciation charges now compound at the rate of $1 + r$:

$$d_{\tau+1} = (1 + r) d_{\tau}, \quad (2)$$

where d_{τ} is the depreciation charge per one dollar of assets in period τ . Since assets are fully depreciated, the depreciation charges must satisfy the relation:

$$d_1 + d_2 + d_3 + d_4 = 1. \quad (3)$$

Equations (2) and (3) imply that:

$$d_{\tau} = \frac{r\gamma^{5-\tau}}{1 - \gamma^4} \quad (4)$$

for $1 \leq \tau \leq 4$. One can also check that the book value at date t of one dollar of assets acquired in period $t - \tau$ is given by

$$bv_{\tau} = 1 - d_1 - \dots - d_{\tau} = d_{\tau+1} + \dots + d_4 = \frac{1 - \gamma^{4-\tau}}{1 - \gamma^4}$$

for $0 \leq \tau < 4$. Therefore, the aggregate book value and depreciation at date t are:

$$BV_t = I_t + \frac{1 - \gamma^3}{1 - \gamma^4} I_{t-1} + \frac{1 - \gamma^2}{1 - \gamma^4} I_{t-2} + \frac{1 - \gamma}{1 - \gamma^4} I_{t-3}, \quad (5)$$

$$D_t = \frac{r\gamma^4}{1 - \gamma^4} I_{t-1} + \frac{r\gamma^3}{1 - \gamma^4} I_{t-2} + \frac{r\gamma^2}{1 - \gamma^4} I_{t-3} + \frac{r\gamma}{1 - \gamma^4} I_{t-4}. \quad (6)$$

It should also be noted that for any investment history the aggregate historical cost of capacity in a given period is proportional to capacity utilized in that period:

$$\begin{aligned}
H_{t+\tau} &= D_{t+\tau} + rBV_{t+\tau-1} \\
&= \frac{r}{1-\gamma^4} (I_{t+\tau-1} + \dots + I_{t+\tau-4}) \\
&= \frac{r}{1-\gamma^4} K_{t+\tau}.
\end{aligned} \tag{7}$$

In later sections, this observation will be shown to be a special case of a broader replacement cost accounting property for assets conforming to the one-hoss shay pattern. To demonstrate this link, let me also describe the annuity rule by invoking the concept of a hypothetical perfectly competitive rental market for capital assets. If a perfect rental market were to exist, then the rental price of a unit of capacity, c , would be such that a rental firm would exactly break even over time. If the rental firm invests one dollar in period t and then rents out the resulting capacity in the following four periods, its net present value is:

$$-1 + \gamma c + \gamma^2 c + \gamma^3 c + \gamma^4 c.$$

Since the rental market is assumed to be perfectly competitive, the expression above must be equal to zero. Therefore, the competitive rental price of a unit of capacity is given by:

$$c = \frac{1}{\gamma + \gamma^2 + \gamma^3 + \gamma^4} = \frac{r}{1 - \gamma^4}. \tag{8}$$

Under replacement cost accounting, book value at date t of an asset purchased in period $t - \tau$ is defined as the fair value of that asset in the hypothetical rental market at date t . For $\tau < 4$, a unit investment in period $t - \tau$ adds one unit of capacity in periods $t + 1, \dots, t + 4 - \tau$. To replace this stream of capacity, the firm would need to incur the following cost:

$$\gamma c + \dots + \gamma^{4-\tau} c = \frac{1 - \gamma^{4-\tau}}{1 - \gamma^4} = bv_\tau.$$

Therefore, the annuity rule corresponds to replacement cost accounting if a perfect rental market exists. Also, combining equations (7) and (8), one obtains

$$H_{t+\tau} = cK_{t+\tau}. \tag{9}$$

Thus, the annuity depreciation rule also implies that the aggregate historical cost of capacity is equal to the replacement cost of capacity utilized in the current period.

To show that the annuity rule is informationally sufficient, one can apply the residual income valuation approach. Property (9) implies that on the optimal path of future capacity levels, residual income is given by:

$$RI_{t+\tau} = R_{t+\tau} - H_{t+\tau} = pK_{t+\tau} - cK_{t+\tau} = (p - c) q_{\max} \quad (10)$$

for $\tau \geq 0$. Therefore, the firm's value can be computed as follows

$$V_t = BV_t + \sum_{\tau=1}^{\infty} \gamma^{\tau} RI_{t+\tau} = BV_t + \frac{1}{r} RI_t. \quad (11)$$

Beginning-of-period book value can be recovered from the financial statements at date t in the following way:

$$BV_{t-1} = BV_t + D_t - I_t.$$

Substituting this expression into (11), one obtains:⁶

$$\begin{aligned} V_t &= BV_t + \frac{1}{r} (R_t - D_t - rBV_{t-1}) \\ &= \frac{1}{r} R_t - \frac{1}{r\gamma} D_t + I_t. \end{aligned}$$

Observation 1 *Given annuity depreciation,*

$$V_t = \frac{1}{r} R_t - \frac{1}{r\gamma} D_t + I_t.$$

Since the firm's value can now be expressed in terms of the current accounting data, we have demonstrated the informational sufficiency of the annuity depreciation rule. Central to this result is the fact that under the annuity rule residual income is constant over time and equals

$$(p - c) q_{\max}$$

for any optimized investment history. It will be shown below that $(p - c) q_{\max}$ can be viewed as the economic profit of the firm in all periods. Since it was assumed that both capital and output markets are stationary, the fact that optimized economic profits are also stationary should not come as a surprise. In this simplified environment, I showed that under annuity depreciation the current residual income provides sufficient information for predicting future

⁶in this example, the book value of assets drops out from the valuation formula because the product market is assumed to be stationary. The general model allows for growth in the product market, and then the coefficient on the book value in the valuation formula will be different from zero.

residual incomes. This contrasts with the findings in the straight-line scenario, where current accounting data was insufficient for predicting future residual earnings.

To conclude the discussion of this example, it is instructive to compare replacement cost accounting with fair value accounting. When the output price is high enough to cover the marginal costs of production, specifically when p is greater than c , book values under the annuity rule will be less than the present values of future cash flows, since residual earnings will be always positive by equation (10). Therefore, the annuity rule is more conservative than fair value accounting in the sense that accounting book values are always below market values.

3 Model Description

3.1 Transactions

Consider a firm that invests in a single type of long-lived assets and produces a single output good. I will assume that the cash cost of one unit of physical assets is constant over time and equal to k .⁷ Let I_t be the capital expenditure in period t , starting at date $t - 1$ and ending at date t . Then, the number of asset units acquired in that period is I_t/k . Each unit of the physical asset is idle in the acquisition period and then generates capacity to produce x_1, \dots, x_T units of output in the T periods of its useful life, where $x_1 \geq x_2 \geq \dots \geq x_T > 0$ and $T \geq 2$. For notational convenience, I will also define $x_0 = 0$ to be the asset's productivity in the acquisition period. Hence, the total capacity available in period t , K_t , is given by:

$$K_t = k^{-1} (I_t x_0 + I_{t-1} x_1 + \dots + I_{t-T} x_T) = k^{-1} \mathbf{x} \cdot \boldsymbol{\theta}_t,$$

where $\mathbf{x} \equiv (x_0, \dots, x_T)'$ will be referred to as the asset's productivity profile and $\boldsymbol{\theta}_t \equiv (I_t, \dots, I_{t-T})'$ is the relevant investment history at date t . If all x_τ are equal, I will call the productivity profile the *one-hoss shay* pattern.

In period t , the firm faces an inverse demand curve, $P_t(q_t)$, which defines price as a function of quantity of output sold, q_t . Let $R_t(q_t)$ denote the corresponding revenue function:

$$R_t(q_t) = P_t(q_t) q_t.$$

In the interest of parsimony, I assume that the firm is all-equity financed, there are no oper-

⁷For now, I present the model as one of certainty about the conditions that the firm experiences in the future in its capital asset and product markets. Section 6 below discusses several ways to introduce uncertainty in the model.

ating expenses and taxes, and all free cash flows are disbursed to shareholders immediately. Investors are interested in valuing the company under the assumption that managers have perfect information and always act in the best interests of the firm's owners.

An investor's valuation of the company at date t is equal to the present value of future cash flows:

$$V_t = \max_{I_{t+\tau}, \tau=1, \dots} \sum_{\tau=1}^{\infty} \gamma^{\tau} (R_{t+\tau}(q_{t+\tau}) - I_{t+\tau}), \quad (12)$$

where $\gamma = \frac{1}{1+r}$, r is the firm's cost of capital, and $I_{t+\tau}$ is chosen optimally in every period.

Arrow (1964) showed that under certain conditions the optimization problem in (12) is time-separable. In particular, this separability will hold if the following no-excess capacity condition is met.

Assumption 1 (No-Excess Capacity Condition) *Investors assume that demand curves shift out over time, in the sense that*

$$P_{t+1}(q) \geq P_t(q)$$

for any q, t .

Under this assumption, the optimal capacity levels are nondecreasing over time, while the capacity from assets already in place at any given date is nonincreasing. Therefore, the firm never ends up with excess capacity. The firm's value can be then rewritten as:

$$V_t = \max_{I_{t+\tau}, \tau=1, \dots} \sum_{\tau=1}^{\infty} \gamma^{\tau} (R_{t+\tau}(K_{t+\tau}) - I_{t+\tau}). \quad (13)$$

Define the *marginal cost of capacity* in period $t + \tau$, $c_{t+\tau}$, as the incremental cost to the firm, in $t + \tau$ -period dollars, of generating one additional unit of capacity in that period, holding all other capacity levels fixed. Since the productivity profile and the price of assets are invariant over time, one might expect the marginal cost of capacity to be also constant. Consequently, the subscript $t + \tau$ will be omitted. Arrow (1964) showed that the marginal cost is given by:

$$c = \frac{k}{x_1\gamma + \dots + x_T\gamma^T}. \quad (14)$$

The intuition behind this result can be demonstrated by considering again the notion of a hypothetical rental market for capital assets and by assuming that this hypothetical market is also perfectly competitive. As in the numerical example in Section 2, c can be shown to

be equal to the rental price per unit of capacity at which a competitive rental firm exactly breaks even over time. Indeed, assume that the rental firm invests one dollar in period t and rents out the resulting capacity in periods $t + 1$ through $t + T$. Then, present value of its cash flows is

$$-1 + \gamma \frac{x_1}{k} c + \gamma^2 \frac{x_2}{k} c + \dots + \gamma^T \frac{x_T}{k} c.$$

The value of c in expression (14) equates the present value of rental firm's revenues with the costs to generate those revenues. Since the rental business is competitive, the producing firm is indifferent between investing in assets at a unit price k and renting capacity at a unit price c . Hence, by internalizing a cost of c per unit of capacity, the producing firm will generate the first-best investment policy in acquiring new capacity. On the optimal path, each $K_{t+\tau}$ must be chosen so that the marginal revenue from an additional unit of capacity is equal to the marginal cost of that unit, c :

$$R'_{t+\tau}(K_{t+\tau}) = c.$$

With an implicit reference to the notion of a hypothetical rental market, I will call $cK_{t+\tau}$ the *replacement cost of capacity* in period $t + \tau$. Since c is the marginal cost of capacity, it is also natural to label $cK_{t+\tau}$ the *economic cost*, and the difference between revenues and $cK_{t+\tau}$ - the *economic profit* of the firm:

$$\pi_{t+\tau} = R_{t+\tau}(K_{t+\tau}) - cK_{t+\tau}.$$

A related concept that I will use throughout the paper is the *replacement cost of assets in place* at date t . As was demonstrated in the numerical example, two firms operating in the same economic environment and implementing equal capacity in every period, can have different valuations. This difference reflects that the composition of assets in place differs for the two firms at a given date. Clearly, if two firms utilize the same amount of capacity in period t , but firm's 1 capacity is newer than that of firm 2, then firm 1 will have a greater intrinsic value. Intuitively, assets in place can be essentially viewed as cost-savings in future periods. To quantify these savings, let v_τ denote the *replacement cost factors* at date t of a dollar of investment made in period $t - \tau$, defined as the present value of hypothetical rental payments that the firm would need to incur in order to replace this investment moving forward. Since the investment was made τ periods ago, it will provide the following stream of capacity starting with period $t + 1$

$$\frac{1}{k} x_{\tau+1}, \dots, \frac{1}{k} x_T.$$

Therefore,

$$v_\tau = \gamma c \frac{1}{k} x_{\tau+1} + \gamma^2 c \frac{1}{k} x_{\tau+2} + \dots + \gamma^{T-\tau} c \frac{1}{k} x_T.$$

Substituting for c , the replacement cost factor v_τ can be simplified to

$$v_\tau = \frac{x_{\tau+1}\gamma + \dots + x_T\gamma^{T-\tau}}{x_1\gamma + \dots + x_T\gamma^T}. \quad (15)$$

Note that $v_0 = 1$, indicating that the replacement cost of the investment just made is its historical cost. The aggregate *replacement cost of assets in place* at date t is the sum of replacement costs of assets still productive at that date:

$$RC_t = v_0 I_t + \dots + v_T I_{t-T}.$$

3.2 Financial Reporting

The accounting system aggregates the information about past transactions of the firm into the financial statements. Depreciation is the only accrual considered in the model. Therefore, investors learn the following four numbers from the income statement and the balance sheet at date t : revenues for the latest period, R_t , depreciation, D_t , net income, $Inc_t = R_t - D_t$, and book value of assets at date t , BV_t . Investors further learn the latest investment, I_t , from the statement of cash flows. I will assume that depreciation is computed according to a fixed schedule $\mathbf{d} = (d_0, \dots, d_T)'$. This schedule can be tailored to the anticipated physical decay of assets in the sense that \mathbf{d} can depend on the productivity profile \mathbf{x} . Total depreciation in period t then becomes:

$$D_t = d_0 I_t + \dots + d_T I_{t-T} = \mathbf{d} \cdot \boldsymbol{\theta}_t.$$

Let bv_τ denote the share of investment I_t that remains capitalized at the end of period $t + \tau$, and let $\mathbf{bv} = (bv_0, \dots, bv_T)'$. I will restrict attention to depreciation rules satisfying the usual "tidiness" requirement that:

$$bv_\tau = 1 - \sum_{i=0}^{\tau} d_i,$$

and

$$\sum_{i=0}^T d_i = 1.$$

Hence, $bv_T = 0$. Thus, the book value of each investment changes only due to depreciation charges related to that investment, and assets are fully depreciated over their useful life. In

this notation, the aggregate book value at date t is equal to:

$$BV_t = bv_0 I_t + \dots + bv_T I_{t-T} = \mathbf{bv} \cdot \boldsymbol{\theta}_t.$$

To summarize, investors observe the following information set at date t :⁸

$$\mathcal{I}_t = \{R_t, D_t, BV_t, I_t\}.$$

Note that book value, depreciation, and the latest investment are dot products of the state vector $\boldsymbol{\theta}_t$ with vectors \mathbf{bv} , \mathbf{d} , and $\mathbf{i} \equiv (1, 0, \dots, 0)'$, respectively. Also, revenues are a function of capacity in period t , which is proportional to the dot product of $\boldsymbol{\theta}_t$ and \mathbf{x} :

$$\mathcal{I}_t = \{R_t (k^{-1} \boldsymbol{\theta}_t \cdot \mathbf{x}), \mathbf{d} \cdot \boldsymbol{\theta}_t, \mathbf{bv} \cdot \boldsymbol{\theta}_t, \mathbf{i} \cdot \boldsymbol{\theta}_t\}.$$

While the investors' information set does not contain the state vector $\boldsymbol{\theta}_t$ itself, investors do observe a number of linear transformations of this vector. The linear aggregation structure is crucial for the informational sufficiency results discussed below.

It will be convenient to define the aggregate *historical cost of capacity* in period t , H_t , as the sum of depreciation expense and a cost of capital charge on the beginning book value of assets:

$$H_t = D_t + rBV_{t-1}.$$

The historical cost is a function of the state vector at date t , since both D_t and BV_{t-1} are determined by that vector:

$$H_t = d_0 I_t + (d_1 + rbv_0) I_{t-1} + \dots + (d_T + rbv_{T-1}) I_{t-T}.$$

Let $z_0 = d_0$ and $z_\tau = d_\tau + rbv_{\tau-1}$ for $0 < \tau \leq T$ and let $\mathbf{z} = (z_0, \dots, z_T)'$ be the vector of historical cost charges. Then,

$$H_t = \mathbf{z} \cdot \boldsymbol{\theta}_t.$$

Prior literature has established that there exists a one-to-one mapping between depreciation charges and historical cost charges (see e.g. Rogerson, 1997, and Reichelstein, 1997). In particular, it can be shown that the depreciation vector will satisfy the clean surplus

⁸Net income is contained in \mathcal{I}_t , but it has no incremental information content beyond revenues and depreciation. Also, I will routinely assume that BV_{t-1} is in \mathcal{I}_t , since $BV_{t-1} = BV_t + D_t - I_t$.

condition if and only if the corresponding \mathbf{z} -vector satisfies:

$$\sum_{\tau=0}^T z_{\tau} \gamma^{\tau} = 1. \quad (16)$$

Following Rogerson (2008b), I now define *replacement cost accounting*, or the *replacement cost rule*, in terms of its corresponding \mathbf{z} -vector and then check that condition (16) is satisfied. Let z_{τ}^* be equal to the replacement cost of capacity provided in period t by a unit investment made in period $t - \tau$:

$$z_{\tau}^* = k^{-1} c x_{\tau} = \frac{x_{\tau}}{\gamma x_1 + \dots + \gamma^T x_T}.$$

Intuitively, this rule allocates historical cost charges to a particular period, τ , in proportion to the capacity that the asset generates in that period. Condition (16) is satisfied for the vector \mathbf{z}^* :

$$\sum_{\tau=0}^T z_{\tau}^* \gamma^{\tau} = \frac{1}{(\gamma x_1 + \dots + \gamma^T x_T)} \sum_{\tau=0}^T x_{\tau} \gamma^{\tau} = 1.$$

Given this rule, the aggregate historical cost in period t is indeed equal to the replacement cost of capacity, irrespective of the investment history:

$$H_t^* = \mathbf{z}^* \cdot \boldsymbol{\theta}_t = k^{-1} c \mathbf{x} \cdot \boldsymbol{\theta}_t = c K_t. \quad (17)$$

Finally, *residual income* in period t is the difference between revenues and the historical cost of capacity:

$$RI_t = R_t - D_t - r B V_{t-1}. \quad (18)$$

Let \mathbf{d}^* , $\mathbf{b}v^*$ denote the depreciation and book value schedules corresponding to \mathbf{z}^* and let D_t^* , BV_t^* be the aggregate depreciation and book values under this rule. By (17), residual income under the replacement cost rule, is equal to the firm's economic profits

$$RI_t^* = R_t - H_t = R_t - c K_t \quad (19)$$

for any t . The following benchmark result is an application of the residual income valuation formula for the special case of replacement cost accounting.⁹

Proposition 1 *The value of the firm at date t is equal to the sum of the replacement cost*

⁹Related results were established in the economics and finance literature, see e.g. Thomadakis (1976) and Lindenberg and Ross (1981). However, in these studies capital assets are usually assumed to be infinitely-lived and their productivity is assumed to decline geometrically over time. For the purposes of this paper, it is important to state the result in Proposition 1 for general productivity patterns.

of assets in place and the present value of future optimized economic profits:

$$V_t = \mathbf{v} \cdot \boldsymbol{\theta}_t + \sum_{\tau=1}^{\infty} \max_{K_{t+\tau}} (\gamma^\tau (R_{t+\tau}(K_{t+\tau}) - cK_{t+\tau})). \quad (20)$$

Given replacement cost accounting, the first component of V_t corresponds to the book value of assets, and the second component is the present value of future residual earnings.

The proof of Proposition 1 demonstrates that the book values corresponding to the \mathbf{z}^* -rule are equal to the replacement cost of assets in place, that is:

$$BV_t^* = RC_t,$$

hence the term replacement cost accounting. The claim in Proposition 1 then follows from the residual income valuation formula. Note also that there is a clear distinction of economic stocks and flows in the present model: the replacement cost of assets in place can be viewed as the value of the firm's stock at date t , while future economic profits are the flow variables. If the firm operates in a competitive environment, then future economic profits are zero and the value of the firm is equal to the replacement cost of assets in place.

In much of the discussion above, the replacement cost rule was described with a reference to the rental market for capital assets. It is important to note that under the no-excess capacity condition (Assumption 1), the existence of such a market is not required. When the output market expands, the firm seeks to increase its capacity in every period. Under this assumption, the firm will never find it desirable to rent out its capacity. Therefore, the absence of a rental market does not affect the firm's first-best investment policy nor does it affect the firm's valuation. Replacement cost accounting may then be viewed as a particular depreciation rule corresponding to historical cost accounting.

4 Informationally Sufficient Accounting

4.1 Sufficiency of Replacement Cost Accounting

In valuing the company, investors seek to estimate (i) the replacement cost of assets in place and (ii) the present value of future optimized economic profits. Estimating future profits naturally requires an assessment of how the output market conditions evolve over time. In this regard, the following assumption will be imposed on the evolution of the firm's inverse demand functions.¹⁰

¹⁰A similar assumption is invoked in Nezlobin, Rajan, and Reichelstein (2008).

Assumption 2 (Proportionate Growth Assumption) *Market demand evolves such that*

$$P_{t+\tau}(g_{t+\tau}q) = P_t(q) \quad (21)$$

for all q , where $\mathbf{g} = (g_{t+1}, \dots, g_{t+\tau}, \dots)$ and $g_{t+\tau}$ is the cumulative output market growth factor between period t and period $t + \tau$.¹¹

Assumption 2 states that for all price points, demand increases by a factor of $g_{t+\tau}$ from period t to period $t + \tau$. The no-excess capacity condition is implied when all $g_{t+\tau}$ are greater than one and are non-increasing:

$$1 \leq g_{t+1} \leq \dots \leq g_{t+\tau} \leq \dots$$

As a consequence, the output market is non-declining, and since the capacity from assets in place is non-increasing, the firm never expects to end up with excess capacity.

The key property of the proportionate growth parametrization is that the future optimal prices and capacity levels can be expressed as simple functions of the current optimal capacity and price. To see this, observe that if

$$P_t(K_t)K_t - cK_t$$

is maximized at some capacity level, K_t^* , then

$$P_{t+\tau}(K_{t+\tau})K_{t+\tau} - cK_{t+\tau} = g_{t+\tau} \left[P_t \left(\frac{K_{t+\tau}}{g_{t+\tau}} \right) \frac{K_{t+\tau}}{g_{t+\tau}} - c \frac{K_{t+\tau}}{g_{t+\tau}} \right]$$

will be maximized at $K_{t+\tau}^* = K_t^* g_{t+\tau}$. Therefore, optimal capacity levels will follow the growth pattern \mathbf{g} , optimal prices will be constant, and revenues and economic profits will also grow according to \mathbf{g} . It should also be noted that to make such projections, investors do not need to know the structural form of the inverse demand curves.

I now specify in more detail the information available to different parties. At date t , the manager has perfect information about past transactions of the firm and knows the inverse demand function in period $t + 1$. This information allows the manager to implement the path of the first-best investments. The accounting system tracks all transactions of the firm

¹¹If $\mu_{t+\tau}$ is the market growth rate from period $t + \tau - 1$ to $t + \tau$, then

$$g_{t+\tau} = \prod_{i=1}^{\tau} (1 + \mu_{t+i})$$

and takes into consideration the asset's productivity profile, \mathbf{x} . This knowledge can be used in choosing an appropriate depreciation rule for the firm's assets. Investors face the problem of estimating V_t , on the basis of only the aggregate financial statements. More specifically, it will be assumed that investors do not observe any of the investments in $\boldsymbol{\theta}_t$, except for the latest one, neither do they know the acquisition cost of capital assets, k . Also, while they are not knowledgeable of the exact shape of the demand curve, they assume that demand will grow proportionately at all price levels, according to the pattern \mathbf{g} . I allow for the possibility of investors observing output prices and the productivity profile of assets.

In particular, the informational structure described above precludes investors from inferring capacity costs from revenues only. For example, if investors knew k , they would be able to infer the marginal cost of capacity using equation (14) and, then, compute the aggregate economic cost by dividing revenues by the output price and multiplying the resulting capacity by c . Alternatively, if they knew the exact shape of the demand curve, they could infer c as the marginal revenue at the optimal point. Since my interest is in modeling the accounting system as the primary vehicle of information transfer to investors, both of these inferences are rendered infeasible.

Finally, I assume that future growth factor projections, \mathbf{g} , cannot be incorporated into the financial statements, either because these projections are not considered verifiable to the accounting system, or because these projections may be investor-specific and the accounting system needs to accommodate all heterogeneous investors.¹² A consequence of this assumption is that fair value accounting, under which BV_t is always equal to V_t , is rendered infeasible, because V_t inherently depends on \mathbf{g} .

Accounting rules will be called informationally sufficient if there exists a value estimate which uses as inputs only the information available to investors at date t , and which captures the value correctly for any projection of growth in the product market. If the accounting rules used for financial reporting do not meet this criterion, then the valuation problem is generally not solvable.

Definition 1 *A depreciation rule, \mathbf{d} , is said to be informationally sufficient if there exists a function $\hat{V}_t(\mathcal{I}_t, \mathbf{g})$ such that*

$$V_t(\boldsymbol{\theta}_t, P_t(\cdot), k, \mathbf{g}) = \hat{V}_t(\mathcal{I}_t, \mathbf{g})$$

for any market demand $P_t(\cdot)$, and any vector $(\boldsymbol{\theta}_t, \mathbf{g}, k)$.

¹²The current model can be extended without much additional machinery to accommodate certain forms of uncertainty. Examples of possible extensions are discussed in Section 6 below.

Given the result in Proposition 1, replacement cost accounting will be informationally sufficient provided investors can estimate future residual earnings from the current financial statements. This forecast is possible if future market demand conforms to the proportionate growth assumption.

Proposition 2 *Replacement cost accounting is informationally sufficient. The intrinsic value of the firm is given by*

$$V_t = BV_t^* + \alpha RI_t^*, \quad (22)$$

where $\alpha = \sum_{\tau=1}^{\infty} g_{t+\tau} \gamma^\tau$.

Proposition 2 identifies informationally sufficient systems for different productivity patterns. As mentioned above, the replacement cost rule for one-hoss shay productivity is the annuity depreciation, therefore the annuity rule is an informationally sufficient system for assets with this productivity pattern. This observation was the basis of the numerical example discussed in Section 2.

Proposition 2 suggests two value-relevant dimensions of the investment history.¹³ Since

$$K_t = k^{-1} \mathbf{x} \cdot \boldsymbol{\theta}_t,$$

it follows that

$$V_t = \mathbf{v} \cdot \boldsymbol{\theta}_t + \alpha [R_t (k^{-1} \mathbf{x} \cdot \boldsymbol{\theta}_t) - k^{-1} \mathbf{x} \cdot \boldsymbol{\theta}_t]. \quad (23)$$

Thus the value of the firm depends on two dot products of the investment history with the vectors - \mathbf{v} and \mathbf{x} . If we consider another firm, operating in the same environment, with investment history $\boldsymbol{\theta}_t^{(1)}$, which differs from $\boldsymbol{\theta}_t$ in a dimension orthogonal to both \mathbf{v} and \mathbf{x} , then the value of that firm will be exactly the same. On the other hand, variations in the investment history along any of these two dimensions will lead to different valuations. Therefore, it is natural to label \mathbf{v} and \mathbf{x} the *value-relevant dimensions* of $\boldsymbol{\theta}_t$. Since under replacement cost accounting book values correspond to replacement cost of assets in place, $\mathbf{bv}^* = \mathbf{v}$, and the vector of historical charges, \mathbf{z}^* , is proportional to \mathbf{x} , I will also refer to \mathbf{bv}^* and \mathbf{z}^* as the value-relevant dimensions. Intuitively, the first dimension conveys information about the replacement cost of assets in place at date t , whereas the second dimension defines revenues, replacement cost of capacity, and the economic profit in period t . Relative weights of these two dimensions depend on α , which in turn, is a function of the growth rates in sales revenues. For a competitive firm, economic profits are zero and \mathbf{v} is the only relevant

¹³Alternative notions of value-relevant information are discussed in Holthausen and Watts (2001) and Barth et. al. (2001). In Section 5, I discuss in more detail the connection between the notion of value relevance in Barth et. al. (2001) and the one that I introduce in this paper.

dimension. If the firm earns some economic rents in the future, then, as α increases, a relatively larger share of value is due to future economic profits, and differences in histories along vector \boldsymbol{x} lead to greater differences in valuations.

4.2 A Uniqueness Result

I now turn to the issue of characterizing the class of informationally sufficient accounting rules. Clearly there is one degree of freedom associated with replacement cost accounting that does not impede its informational properties. This degree of freedom corresponds to the possibility of partial direct expensing (or write-ups) in the acquisition period. Consider the following class of schedules:

$$\boldsymbol{bv} = (1 - \lambda) \boldsymbol{bv}^*, \quad (24)$$

where $\lambda < 1$ is some constant. It is readily verified that all accounting rules in this class are informationally sufficient. Recall that $d_0^* = 0$ and

$$bv_0^* = v_0 = 1.$$

Therefore,

$$bv_0 = (1 - \lambda).$$

Depreciation schedules defined by (24) have the property that in the acquisition period a share λ of the investment is directly expensed (or written-up), $d_0 = \lambda$. In future periods, the amount initially capitalized is depreciated in proportion to the replacement cost rule:

$$d_\tau = bv_{\tau-1} - bv_\tau = (1 - \lambda) bv_{\tau-1}^* - (1 - \lambda) bv_\tau^* = (1 - \lambda) d_\tau^*.$$

Book values under these schedules are proportional to the replacement cost book values at all times:

$$BV_t = (1 - \lambda) BV_t^*. \quad (25)$$

The parameter λ can be viewed as a degree of unconditional conservatism of the accounting system in use.¹⁴ Positive values of λ define rules more conservative than the replacement cost rule, while negative values of this parameter define more liberal rules. The one-dimensional family of replacement cost accounting rules corresponding to $\lambda < 1$ will be called *generalized*

¹⁴In the empirical part of their paper, Rajan, Reichelstein, and Soliman (2008) operationalize conservative accounting by the proportion of new investments that are expensed directly, e.g. R&D and advertising, relative to total investments.

replacement cost accounting. To demonstrate informational sufficiency, it suffices to rely on equation (25) and to provide an expression for the replacement cost depreciation charges in terms of accounting data computed under a generalized replacement cost rule:

$$D_t^* = \sum_{\tau=0}^T I_{t-\tau} d_\tau^* = \frac{1}{1-\lambda} \sum_{\tau=0}^T I_{t-\tau} d_\tau - \frac{\lambda}{1-\lambda} I_t = \frac{1}{1-\lambda} D_t - \frac{\lambda}{1-\lambda} I_t.$$

These expressions use the fact that $d_0^* = 0$ and $d_0 = \lambda$. The following corollary to Proposition 2 provides an exact valuation formula for any generalized replacement cost rule.

Corollary to Proposition 2 *Replacement cost accounting with partial expensing, that is, $\mathbf{bv} = (1 - \lambda) \mathbf{bv}^*$ for some $\lambda < 1$, is informationally sufficient. The valuation formula is given by:*

$$V_t = \frac{1}{1-\lambda} BV_t + \alpha \left(R_t - \frac{1}{1-\lambda} D_t + \frac{\lambda}{1-\lambda} I_t - \frac{r}{1-\lambda} BV_{t-1} \right). \quad (26)$$

When $\lambda = 0$, the expression in (26) reduces to the one in Proposition 2, equation (22). There are three observations worth discussing in connection with equation (26). First, note that when $\lambda \neq 0$, the coefficients on revenues and depreciation are not equal, and, hence, net income has to be disaggregated into its cash flow and accrual components. Also, the investment cash flows, I_t , now enter the valuation formula in a non-trivial manner. This was not the case in Proposition 2, where net income could be used as the aggregate measure and investment cash flows were required only to infer the beginning of period book value. The difference is that while the generalized rule preserves value-relevant information, it is imperfect from a measurement perspective in the sense that accounting data, other than revenues, are not equal to the economic fundamentals. To obtain an unbiased valuation, these imperfections need to be adjusted by changing the coefficients on the accounting data.

The second observation with respect to (26) is that the coefficients in the valuation equation depend on both the market growth projections, \mathbf{g} , and the degree of conservatism, λ . This observation again stresses the importance of separately considering the two determinants of accounting numbers - the economic environment of the firm and the accounting rules employed. A correct valuation function, if one exists, can only be constructed by taking into account both of these determinants.

Finally, the preceding Corollary requires that $\lambda < 1$. When $\lambda = 1$, cash accounting is obtained under which investors observe only revenues and the latest investment. Such dataset is informationally insufficient, since it is easy to construct two histories with the same latest investments and the same capacity levels in period t , but with different replacement costs of assets in place at date t . Formally, this result will follow from Proposition 3 below.

The following proposition provides the basic uniqueness result regarding replacement cost

accounting.

Proposition 3 *Any informationally sufficient accounting rule corresponds to generalized replacement cost accounting for some $\lambda < 1$.*

The main insight behind this finding can be explained in terms of the tools developed in aggregation theory (see for instance Ijiri, 1968). The basic algebraic problem addressed in this literature is as follows: investors are interested in computing some dot-product of the investment history vector, $\boldsymbol{\theta}_t \cdot \mathbf{y}$, yet they observe only some other dot-products of this history, $\boldsymbol{\theta}_t \cdot \mathbf{w}_1, \dots, \boldsymbol{\theta}_t \cdot \mathbf{w}_n$. When does a function exist that maps the observed data into precise estimates of $\boldsymbol{\theta}_t \cdot \mathbf{y}$ for any vector $\boldsymbol{\theta}_t$? Clearly, if \mathbf{y} is in the linear subspace, L , generated by vectors $\mathbf{w}_1, \dots, \mathbf{w}_n$ such a function exists. Indeed, if

$$\mathbf{y} = \lambda_1 \mathbf{w}_1 + \dots + \lambda_n \mathbf{w}_n,$$

then

$$\boldsymbol{\theta}_t \cdot \mathbf{y} = \lambda_1 (\boldsymbol{\theta}_t \cdot \mathbf{w}_1) + \dots + \lambda_n (\boldsymbol{\theta}_t \cdot \mathbf{w}_n).$$

Moreover, the condition given above is also necessary. To see this, note that if \mathbf{y} does not belong to L , then it can be decomposed into two parts, $\mathbf{y}^{(L)}$, which belongs to L , and $\mathbf{y}^{(\perp L)}$, which belongs to the orthogonal complement of L . Then, if we add $\mathbf{y}^{(\perp L)}$ to any history $\boldsymbol{\theta}_t$, the investment history $\boldsymbol{\theta}_t + \mathbf{y}^{(\perp L)}$ will produce exactly the same observed data (because $\mathbf{y}^{(\perp L)} \cdot \mathbf{w}_i = 0$ for any i), but will correspond to a different value of the target function ($\mathbf{y} \cdot \mathbf{y}^{(\perp L)} \neq 0$).

To apply this result from aggregation theory to the economic model developed in this paper, suppose the firm uses an informationally sufficient accounting system, \mathbf{d}, \mathbf{bv} . Then there exists a function $\hat{V}_t(BV_t, D_t, R_t, I_t, \mathbf{g})$ such that

$$\hat{V}_t(BV_t, D_t, R_t, I_t, \mathbf{g}) = V_t$$

for any history of investments, any \mathbf{g} , and any underlying environment. By Proposition 2, this means that

$$\hat{V}_t(BV_t, D_t, R_t, I_t, \mathbf{g}) = \mathbf{bv}^* \cdot \boldsymbol{\theta}_t + \alpha (R_t - \mathbf{z}^* \cdot \boldsymbol{\theta}_t). \quad (27)$$

Observing revenues, investors can use this information to estimate the present value of future revenues, but as they do not know the shape of revenue curves and the acquisition cost of assets, they cannot use information in revenues for any other purpose. Importantly, they cannot infer the replacement cost of capacity only from revenues. Formally, I state the following lemma.

Lemma 1 *For any informationally sufficient system, the valuation function takes the following form:*

$$\hat{V}_t(BV_t, D_t, R_t, I_t, \mathbf{g}) = \alpha R_t + \hat{W}_t(BV_t, D_t, I_t, \mathbf{g}), \quad (28)$$

where \hat{W}_t does not depend on R_t .

According to equations (27) and (28), the valuation problem now reduces to estimating the following dot-product:

$$V_t - \alpha R_t = (\mathbf{b}\mathbf{v}^* - \alpha\mathbf{z}^*) \cdot \boldsymbol{\theta}_t, \quad (29)$$

while observing:

$$BV_t = \mathbf{b}\mathbf{v} \cdot \boldsymbol{\theta}_t,$$

$$D_t = \mathbf{d} \cdot \boldsymbol{\theta}_t,$$

$$I_t = \mathbf{i} \cdot \boldsymbol{\theta}_t.$$

The result from aggregation theory discussed above suggests that this problem is solvable when the vector in parenthesis, that is $\mathbf{b}\mathbf{v}^* - \alpha\mathbf{z}^*$, belongs to the linear subspace spanned by $\mathbf{b}\mathbf{v}$, \mathbf{d} , \mathbf{i} . Since, we seek a system that provides sufficient information for any growth factor projections, \mathbf{g} , the latter condition has to hold for any α . It is shown in the Appendix that this requirement is equivalent to $\mathbf{b}\mathbf{v}$ being parallel to $\mathbf{b}\mathbf{v}^*$.

The arguments underlying Proposition 3 and Lemma 1 were also instrumental in the numerical example in Section 2 concerning straight-line depreciation and one-hoss shay assets. Two investment histories in that example were different along dimension \mathbf{v} , but had the same projections onto \mathbf{x} , $\mathbf{b}\mathbf{v}$, \mathbf{d} , \mathbf{i} . As a result, the two firms implemented equal capacity in every period, had the same financial statements at date t , but had different replacement costs of assets in place, and, consequently, different valuations at that date.

5 Informationally Insufficient Accounting Rules

5.1 Bounds on Valuation Errors

In this section, I turn to a consideration of informationally insufficient accounting rules. Such rules aggregate historical data in a manner that entails a loss of value-relevant information, so that exact valuations become impossible for investors. Since common financial accounting rules will generally not be informationally sufficient, it is important to rank insufficient rules on some measure of informativeness. To quantify the information content of alternative financial statements, I will adopt a standard Bayesian framework.

The following analysis assumes that investors have prior beliefs about the firm's investment history and that they update these beliefs after observing the financial statements. Better accounting is characterized by a smaller residual variance of the firm's value after conditioning on the accounting data. Since informationally sufficient accounting conveys enough information to value the company precisely for any investment history, the residual variance of the firm's value will be zero for such rules. On the other hand, informationally insufficient rules will not fully resolve the uncertainty about the value-relevant dimensions of the investment history. Consequently, there will be a non-trivial residual variance regarding the actual firm value. This approach is consistent with much of the financial accounting literature which frequently views accounting information as imperfect signals regarding the firm's intrinsic value.

To keep the model analytically tractable, I maintain the assumption that investors are ignorant of the acquisition cost of assets and the functional form of the inverse demand curves. It will be assumed that investors use information in the latest revenues only to project future revenues, and I will restrict attention to value estimators of the following form:

$$\hat{V}_t(BV_t, D_t, R_t, I_t, \mathbf{g}) = \alpha R_t + f(BV_t, D_t, I_t, \mathbf{g}). \quad (30)$$

In Lemma 1, it was shown that only the estimators of this family can produce exact valuations for informationally sufficient accounting rules. By Proposition 2, investors then need to estimate:

$$V_t - \alpha R_t = (\mathbf{bv}^* - \alpha \mathbf{z}^*) \cdot \boldsymbol{\theta}_t.$$

Recall that \mathbf{bv}^* is the vector of undepreciated book values and \mathbf{z}^* is the vector of historical cost charges under the replacement cost rule. A natural measure of informativeness of financial statements is the variance of the target function, conditional on the observed accounting data. In fact, it is well known that this conditional variance is equal to the minimum mean squared error over all the possible estimators:

$$\text{Var}[V_t - \alpha R_t | \mathcal{I}_t] = \min_f E [(f(\mathcal{I}_t, \mathbf{g}) - (V_t - \alpha R_t))^2],$$

and the estimator minimizing the mean squared error is the conditional expectation:

$$f^*(\mathcal{I}_t) \equiv E[V_t - \alpha R_t | \mathcal{I}_t]. \quad (31)$$

Let σ_{f^*} denote the standard error of the best estimator. To keep the expectation in (31) tractable, I impose the assumption of joint normality of $\boldsymbol{\theta}_t$ and let Σ denote its covariance

matrix.¹⁵ Then, random variables $V_t - \alpha R_t = (\mathbf{bv}^* - \alpha \mathbf{z}^*) \cdot \boldsymbol{\theta}_t$, $BV_t = \mathbf{bv} \cdot \boldsymbol{\theta}_t$, $D_t = \mathbf{d} \cdot \boldsymbol{\theta}_t$, and $I_t = \mathbf{i} \cdot \boldsymbol{\theta}_t$ will also be jointly normally distributed, and the conditional expectation (31) will be linear in the information set variables:

$$f^*(\mathcal{I}_t) = c_0 + c_1 BV_t + c_2 D_t + c_3 I_t. \quad (32)$$

For an informationally sufficient accounting rule, vector $\mathbf{y} \equiv (\mathbf{bv}^* - \alpha \mathbf{z}^*)$ belongs to the linear subspace, $L\{\mathbf{bv}, \mathbf{d}, \mathbf{i}\}$, spanned by vectors \mathbf{bv} , \mathbf{d} , \mathbf{i} for any α , and coefficients c_1, c_2, c_3 are the coordinates of \mathbf{y} in the system $\{\mathbf{bv}, \mathbf{d}, \mathbf{i}\}$. The intercept term, c_0 , is zero and σ_{f^*} is also zero. The latter observation means that informationally sufficient accounting statements provide enough information to fully resolve the uncertainty about the firm's value.

For insufficient accounting rules, the coefficients in (32) can be expressed in terms of the covariances of $V_t - \alpha R_t$, BV_t , D_t , and I_t using standard linear projection formulae. I will characterize these coefficients, as well as σ_{f^*} , in terms of more fundamental primitives: the vectors \mathbf{y} , \mathbf{bv} , \mathbf{d} , \mathbf{i} , and the covariance matrix of investments, Σ .

Consider first the covariance of $V_t - \alpha R_t$ and BV_t :

$$\text{Cov}(V_t - \alpha R_t, BV_t) = \text{Cov}(\mathbf{y} \cdot \boldsymbol{\theta}_t, \mathbf{bv} \cdot \boldsymbol{\theta}_t) = \mathbf{y}' \Sigma \mathbf{bv}.$$

It is well-known that the covariance matrix has a square root, denoted $\Sigma^{\frac{1}{2}}$, which is a symmetric matrix. One then obtains

$$\text{Cov}(V_t - \alpha R_t, BV_t) = \left(\Sigma^{\frac{1}{2}} \mathbf{y} \right) \cdot \left(\Sigma^{\frac{1}{2}} \mathbf{bv} \right).$$

In words, the covariance of $\mathbf{y} \cdot \boldsymbol{\theta}_t$ and $\mathbf{bv} \cdot \boldsymbol{\theta}_t$ is equal to the dot-product of images of \mathbf{y} and \mathbf{bv} under the linear transformation $\Sigma^{\frac{1}{2}}$.

In the proof of Proposition 4 below, it will be shown that projecting $V_t - \alpha R_t$ onto the aggregate accounting variables BV_t, D_t, I_t is equivalent to applying the transformation $\Sigma^{\frac{1}{2}}$ to the vectors $\mathbf{y}, \mathbf{bv}, \mathbf{d}, \mathbf{i}$ and then projecting the image of \mathbf{y} onto the images of $\mathbf{bv}, \mathbf{d}, \mathbf{i}$. Specifically, let c_1^*, c_2^*, c_3^* be the coefficients in the following projection of $\Sigma^{\frac{1}{2}} \mathbf{y}$:

$$\Sigma^{\frac{1}{2}} \mathbf{y} = c_1^* \left(\Sigma^{\frac{1}{2}} \mathbf{bv} \right) + c_2^* \left(\Sigma^{\frac{1}{2}} \mathbf{d} \right) + c_3^* \left(\Sigma^{\frac{1}{2}} \mathbf{i} \right) + \boldsymbol{\epsilon}, \quad (33)$$

where $\boldsymbol{\epsilon}$ is orthogonal to $\Sigma^{\frac{1}{2}} L\{\mathbf{bv}, \mathbf{d}, \mathbf{i}\}$. For brevity, I introduce the notation $\mathcal{A} \equiv L\{\mathbf{bv}, \mathbf{d}, \mathbf{i}\}$.

¹⁵The results in this section hold for any distribution for which the conditional expectation is linear in the information set variables. In particular, the results hold for spherically invariant distributions. See Harvey (2001) for a detailed discussion.

Let c_0^* be the following unconditional expectation

$$c_0^* = E[\boldsymbol{\epsilon} \cdot \boldsymbol{\theta}_t]. \quad (34)$$

Then, the best estimator of $V_t - \alpha R_t$ is:

$$f^*(\mathcal{I}_t) = c_0^* + c_1^* B V_t + c_2^* D_t + c_3^* I_t.$$

The standard error of this estimator will be then bounded from above by the product of the distance from \mathbf{y} to \mathcal{A} and the norm of the linear operator $\Sigma^{\frac{1}{2}}$:

$$\sigma_{f^*} \equiv (\text{Var}[V_t - \alpha R_t | \mathcal{I}_t])^{1/2} \leq \left\| \Sigma^{\frac{1}{2}} \right\| \rho(\mathbf{y}, \mathcal{A}),$$

where $\|\cdot\|$ denotes the spectral norm and ρ denotes the Euclidean distance. Expanding \mathbf{y} , the distance between \mathbf{y} and \mathcal{A} can be further bounded as:

$$\rho(\mathbf{y}, \mathcal{A}) \leq \rho(\mathbf{b}\mathbf{v}^*, \mathcal{A}) + \alpha \rho(\mathbf{z}^*, \mathcal{A}).$$

The following proposition states the main result of this section.

Proposition 4 *If investors believe that $\boldsymbol{\theta}_t$ is jointly normally distributed with invertible covariance matrix Σ , the estimator of the form (30) that minimizes the mean squared error is given by*

$$\hat{V}_t(\mathcal{I}_t, \mathbf{g}) = \alpha R_t + c_0^* + c_1^* B V_t + c_2^* D_t + c_3^* I_t. \quad (35)$$

The standard error of this estimator is bounded by

$$\sigma_{f^*} \leq \left\| \Sigma^{\frac{1}{2}} \right\| (\rho(\mathbf{b}\mathbf{v}^*, \mathcal{A}) + \alpha \rho(\mathbf{z}^*, \mathcal{A})). \quad (36)$$

For an informationally sufficient system, both distances in the right-hand-side of (36) are equal to zero, and therefore

$$\sigma_{f^*} = 0.$$

In general, the two distances define how close the accounting rule is to capturing the value-relevant dimensions of the investment history, $\mathbf{b}\mathbf{v}^*$ and \mathbf{z}^* .

For an illustration of Proposition 4, consider the so-called *units-of-production depreciation rule*.¹⁶ It turns out that this depreciation schedule is capable of preserving information about one of the value-relevant dimensions. Depreciation charges under the units-of-production rule

¹⁶See, e.g., Humphreys (2004).

are defined as:

$$d_\tau = \frac{x_\tau}{x_0 + \dots + x_T}.$$

Thus, depreciation is charged in proportion to the capacity that the asset generates in a given period¹⁷. Clearly, straight-line accounting is a special case of this rule in application to assets corresponding to the one-hoss shay pattern. The vector of depreciation charges under the units-of-production rule has the following property:

$$\mathbf{d} = \eta \mathbf{z}^*,$$

where

$$\eta = \frac{x_0\gamma + \dots + x_T\gamma^T}{x_0 + \dots + x_T}.$$

The constant η depends only on the productivity profile and can be computed by investors. Therefore, even though the unit-of-production rule is informationally insufficient (as was demonstrated in the numerical example in Section 2), it preserves information about one value-relevant dimension - \mathbf{z}^* . By observing the depreciation charges computed according to this rule, investors can infer the economic cost of capacity as:

$$cK_t = \eta^{-1}D_t.$$

For the units-of-production rule, the second term in the bound given by (36) is zero and (36) reduces to:

$$\sigma_{f^*} \leq \left\| \Sigma^{\frac{1}{2}} \right\| \rho(\mathbf{b}\mathbf{v}^*, \mathcal{A}). \quad (37)$$

Therefore, the only source of possible error in this scenario is the estimation of replacement cost of assets in place at a certain date. Note that the bound above does not depend on α , so it does not increase with the rate of growth in the firm's output market.

I will now use the simplified bound in (37) to extend the numerical example of Section 2. In particular, I seek to characterize the standard error of the best value estimator based on financial statements prepared with the straight-line rule. First note that the bound in (37) takes a simpler form when investments are independent and identically distributed. In that case,

$$\Sigma = \sigma^2 E,$$

where E is the identity matrix and σ is the standard deviation of each investment, and,

¹⁷In contrast to replacement cost accounting, the units-of-production rule does not incorporate any time value of money considerations.

therefore,

$$\left\| \Sigma^{\frac{1}{2}} \right\| = \sigma.$$

Direct substitution yields the following bound on the standard error in the case when investments are i.i.d. and depreciation is computed according to the units-of-production method:

$$\sigma_{f^*} \leq \sigma \rho(\mathbf{bv}^*, \mathcal{A}). \quad (38)$$

Finally, if the productivity of assets corresponds to the one-hoss shay pattern, depreciation is straight-line, and investments are i.i.d, it is readily verified that the vectors \mathbf{bv}^* , \mathbf{bv} , \mathbf{d} , \mathbf{i} take the following form:

$$\begin{aligned} \mathbf{v} = \mathbf{bv}^* &= \left(1, \frac{1 - \gamma^{T-1}}{1 - \gamma^T}, \dots, \frac{1 - \gamma}{1 - \gamma^T}, 0 \right), \\ \mathbf{d} &= \left(0, \frac{1}{T}, \frac{1}{T}, \dots, \frac{1}{T} \right), \\ \mathbf{bv} &= \left(1, \frac{T-1}{T}, \dots, 0 \right), \\ \mathbf{i} &= (1, 0, \dots, 0). \end{aligned}$$

Now one can use simulations to estimate the distance between \mathbf{bv}^* and \mathcal{A} for alternative values of parameters γ and T . For values of $T = 15$ and $r = 15\%$, the optimal estimator is:

$$\hat{V}_t \approx c_0^* + \alpha \left(R_t - \frac{1}{\eta} D_t \right) + BV_t + 2.43 D_t.$$

Figure 1 demonstrates how \mathbf{bv}^* can be approximated by a linear combination of \mathbf{bv} , \mathbf{d} , and \mathbf{i} . The coefficient on the latest investment, $bv_0^* = 1$, can be captured exactly by adjusting the coefficient on I_t in the valuation formula. In this simulation, the coefficient on the aggregate book value is almost exactly one, so no additional adjustment was needed. Given straight-line depreciation, components of the vector \mathbf{d} are equal (except for d_0), while components of the vector \mathbf{bv} decline linearly. Therefore, on this graph, a straight-line is fitted to \mathbf{bv}^* from components 1 to 15. The solid line visualizes the declining replacement cost of the asset over its useful life. The dashed line is the best approximation of the replacement cost with the accounting data computed under the straight-line rule.

The bound in (38) becomes:

$$\sigma_{f^*} \leq 0.295\sigma. \quad (39)$$

I will now compare this bound to the standard error that investors could achieve observing

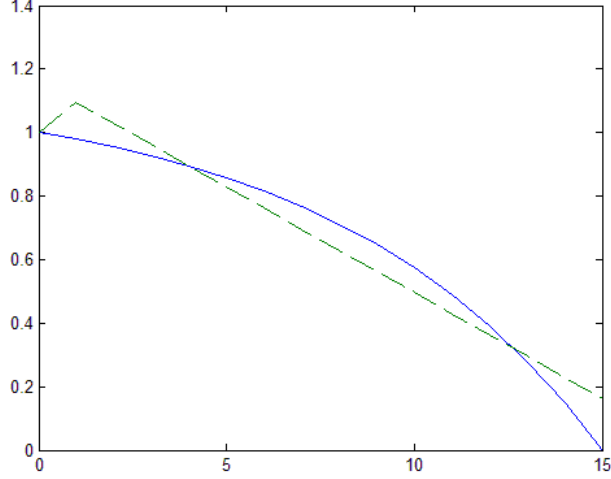


Figure 1: Approximation of bv^* as $bv + 2.43d$

i) only the latest cash flow statement and ii) the latest cash flow statement as well as several recent investments. If investors could observe only the latest cash flow statement, the minimum error of their value estimate would be given by:

$$\sigma_{\mathbf{v} \cdot \boldsymbol{\theta}_t} = (v_1^2 + \dots + v_T^2)^{1/2} \sigma.$$

For our values of parameters, this is approximately equal to:

$$\sigma_{\mathbf{v} \cdot \boldsymbol{\theta}_t} \approx 2.70\sigma. \quad (40)$$

Comparing (40) to (39), one can see that conditioning on accruals reduces the standard error of the best estimator by a factor of more than 9.

For an additional benchmark, I estimate how many investments need to be observed to reduce the uncertainty to the level of a single period financial statements prepared with the straight-line rule. Clearly, if all 15 investments are observed, then investors have perfect information about the assets in place and can precisely value them. Assume that investors know only the latest 13 investments. Then, their best estimate will have a standard error of:

$$\sigma_{13} = (v_{13}^2 + v_{14}^2 + v_{15}^2)^{1/2} \sigma \approx 0.315\sigma.$$

This is still greater than the residual error in (39). Therefore, my simulations indicate that observing single-period financial statements prepared with the straight-line rule is better

from an informational perspective than observing the 13 latest investments out of the 15 past investments that are still active at date t .

5.2 Empirical Implications

The preceding analysis has derived three expressions for the intrinsic value of the firm for different accounting rules. In particular, given replacement cost accounting, it was shown that

$$V_t = BV_t^* + \alpha RI_t^*, \quad (41)$$

where the parameter α represents future growth opportunities (see Proposition 2). For generalized replacement cost accounting,

$$V_t = \frac{1}{1-\lambda} BV_t + \alpha \left(R_t - \frac{1}{1-\lambda} D_t + \frac{\lambda}{1-\lambda} I_t - \frac{r}{1-\lambda} BV_{t-1} \right), \quad (42)$$

where λ is the share of investments directly expensed in the acquisition period (see the Corollary to Proposition 2). Finally, for informationally insufficient accounting rules (Proposition 4), the best value estimator is given by

$$\hat{V}_t(\mathcal{I}_t, \mathbf{g}) = \alpha R_t + c_0^* + c_1^* BV_t + c_2^* D_t + c_3^* I_t, \quad (43)$$

where the coefficients c_0^*, \dots, c_3^* are defined by equations (33) and (34), and the error of this estimator is bounded by the term in (36).

Expressions (41-43) provide predictions for the coefficients on accounting data in regressions, where the dependent variable is the observed market value of the firm's equity. Such regressions have been the focus of the value-relevance literature. An accounting variable is considered to be value-relevant in this literature if it is predictably associated with equity values (see Barth et. al., 2001, for a more detailed discussion). In the context of the present model, an accounting variable will be generally associated with the intrinsic value if the underlying accounting rule is not orthogonal to both value-relevant dimensions of the firm's investment history. For example, it was shown in the previous section that

$$Cov(V_t - \alpha R_t, BV_t) = \left(\Sigma^{\frac{1}{2}} (\mathbf{bv}^* - \alpha \mathbf{z}^*) \right) \cdot \left(\Sigma^{\frac{1}{2}} \mathbf{bv} \right).$$

Therefore, unless \mathbf{bv} is orthogonal to both \mathbf{bv}^* and \mathbf{z}^* , book value will be value-relevant. The result of Proposition 4 predicts that accounting rules that are closer to informationally sufficient rules generate financial statements that are more strongly associated with the firm's intrinsic value. In particular, if assets correspond to the one-hoss shay pattern, then

accounting variables determined according to the annuity depreciation rule will be more value-relevant, i.e. will explain equity values with a lower error, than those determined according to the straight-line rule.

The results of this paper also suggest that there are four major factors affecting coefficients in linear valuation models: the cost of capital of the firm, r , future growth opportunities as represented by α , accounting rules employed, and the distribution of investment histories. The error of a linear model will be minimized when the model is estimated on a set of data where these factors can be reasonably assumed to be constant. This is consistent with a finding by Barth et. al. (2005) that out-of-sample prediction errors of by-industry estimated linear models are lower than errors of models estimated with pooled data.

6 Concluding Remarks

The model presented in this paper demonstrates that with proper accrual accounting financial statements may provide sufficient information for valuation purposes. In particular, I characterize a set of rules that preserve all value-relevant information. For these rules, the firm's value can be expressed in terms of current accounting data and the anticipated growth in the firm's output market. For informationally insufficient accounting, it was shown that the error of the best possible value estimator is bounded by some measure of distance between the rule in place and the closest informationally sufficient rule.

This paper has clearly relied on a number of restrictive assumptions. I briefly discuss three possible major extensions below. First, Propositions 2 and 3 remain essentially unaffected if future growth of the output market is uncertain. Since assets become productive with a lag of one period, the manager needs to foresee growth in the market only one period ahead in order to implement the first-best capacity path. If this condition is satisfied, then results in Proposition 2 and 3 are completely unchanged. Otherwise, even economic fundamentals of the firm are insufficient for a correct assessment of the firm's value. Yet generalized replacement cost accounting can still be shown to be the class of rules that reduces the valuation errors to a minimum.

Second, Rogerson (2008b) derives an expression for the marginal cost of capacity in the case when capital asset prices evolve geometrically over time. In this way, one can model technological advances or inflation. It can be shown that the marginal cost of capacity grows or declines at the same rate as input prices and that this growth rate has to be incorporated into the replacement cost rule. I conjecture that generalized replacement cost accounting remains the unique set of rules capable of preserving information about both the current economic cost of capacity and the replacement cost of assets in place. Under

certain conditions, the current economic profit is still a sufficient statistic for projecting future economic profits, and the informational sufficiency results of this paper, Propositions 2 and 3, hold.

Third, I conjecture that the model can accommodate asset prices evolving as a random walk. Under this assumption, the replacement cost rule has to be adjusted to allow for revaluations of past investments in every period. This rule will then closely correspond to the so-called *depreciated replacement cost approach*, allowed by IAS 16 in situations when no perfect market for the assets can be identified, but when the fair value of assets can still be reliably estimated. Propositions 2 and 3 should hold under certain assumptions on the structural form of the inverse demand curves.

The inferences in my analysis were made possible by explicitly modeling the firm's transactions as well as its financial reporting system. To that end, it was convenient to focus on a model of sequential investments in productive capacity and examine depreciation as the relevant accrual accounting concept. Future research may address the question of informational sufficiency in the context of other accruals such as revenue recognition or the valuation of inventory. If a major goal of accounting information is to reduce uncertainty about the firm's underlying transactions, then the results in this paper suggest that informational sufficiency is a natural criterion. Extending the model to other accounting items may provide a framework for examining how the pricing of accruals depends on the underlying accounting rules.

Appendix A

Proof of Proposition 1. Arrow (1964) showed that solutions to maximization problems in (13) and (20) are equivalent. Given the depreciation rule defined by historical charges \mathbf{z}^* , residual income is equal to economic profits in every period:

$$RI_{t+\tau}^* = \pi_{t+\tau}.$$

Applying the residual income valuation formula, one obtains:

$$V_t = BV_t^* + \sum_{\tau=1}^{\infty} RI_{t+\tau}^*.$$

It remains to show that

$$BV_t^* = \mathbf{v} \cdot \boldsymbol{\theta}_t.$$

Recall that by definition

$$BV_t^* = \mathbf{bv}^* \cdot \boldsymbol{\theta}_t.$$

It is sufficient to check that aggregate historical cost charges corresponding to vector \mathbf{v} are given by \mathbf{z}^* , and, therefore, $\mathbf{v} = \mathbf{bv}^*$. Let z_τ^v be the historical charge in period τ corresponding to vector v . Then, we have

$$z_\tau^v = (v_{\tau-1} - v_\tau) + rv_{\tau-1}$$

for $\tau > 0$, and $z_0^v = 1 - v_0$. Recall that

$$v_\tau = \frac{x_{\tau+1}\gamma + \dots + x_T\gamma^{T-\tau}}{x_1\gamma + \dots + x_T\gamma^T}.$$

Hence, $z_0^v = 0$, and z_τ^v can be shown to be:

$$\begin{aligned} z_\tau^v &= (1+r) \frac{x_\tau\gamma + \dots + x_T\gamma^{T-\tau+1}}{x_1\gamma + \dots + x_T\gamma^T} - \frac{x_{\tau+1}\gamma + \dots + x_T\gamma^{T-\tau}}{x_1\gamma + \dots + x_T\gamma^T} = \\ &= \frac{x_\tau + \dots + x_T\gamma^{T-\tau}}{x_1\gamma + \dots + x_T\gamma^T} - \frac{x_{\tau+1}\gamma + \dots + x_T\gamma^{T-\tau}}{x_1\gamma + \dots + x_T\gamma^T} = \\ &= \frac{x_\tau}{x_1\gamma + \dots + x_T\gamma^T} = z_\tau^*. \end{aligned}$$

Q.E.D. ■

Proof of Lemma 1. Assume that a system is informationally sufficient. Then, since

$$V_t = BV_t^* + \alpha RI_t^* = \mathbf{bv}^* \cdot \boldsymbol{\theta}_t + \alpha (R_t - \mathbf{z}^* \cdot \boldsymbol{\theta}_t),$$

it has to be that

$$\hat{V}_t(BV_t, D_t, R_t, I_t, \mathbf{g}) = \mathbf{b}\mathbf{v}^* \cdot \boldsymbol{\theta}_t + \alpha(R_t - \mathbf{z}^* \cdot \boldsymbol{\theta}_t). \quad (44)$$

We need to show that,

$$\hat{W}_t(BV_t, D_t, R_t, I_t, \mathbf{g}) \equiv \hat{V}_t(BV_t, D_t, R_t, I_t, \mathbf{g}) - \alpha R_t$$

does not depend on R_t . From equation (44), \hat{W}_t is equal to

$$\hat{W}_t(BV_t, D_t, R_t, I_t, \mathbf{g}) = (\mathbf{b}\mathbf{v}^* - \alpha\mathbf{z}^*) \cdot \boldsymbol{\theta}_t. \quad (45)$$

The right-hand-side does not depend on the underlying environment, k and $P_t(\cdot)$, it is fully defined by the investment history $\boldsymbol{\theta}_t$. The only variable in the left-hand-side that depends on the environment is R_t . Assume that $\hat{W}_t(BV_t, D_t, R_t, I_t, \mathbf{g})$ depends on R_t for some $\boldsymbol{\theta}_t$. Let us show that there can exist two environments such that revenues are different, but $\boldsymbol{\theta}_t$ is on the optimal path at date t for both of them. Then, it will be impossible for equation (45) to hold for both environments simultaneously. First, let us pick two different values for parameter k , k_1 and k_2 . Then, capacity in period t for the two scenarios is

$$K_t^{(1)} = k_1^{-1} \boldsymbol{\theta}_t \cdot \mathbf{x},$$

$$K_t^{(2)} = k_2^{-1} \boldsymbol{\theta}_t \cdot \mathbf{x}.$$

Next, let us choose two inverse demand functions, $P_t^{(1)}(\cdot)$ and $P_t^{(2)}(\cdot)$, such that economic profits are optimized at capacity levels $K_t^{(1)}$ and $K_t^{(2)}$, respectively, and the output prices are equal at that levels. Specifically, the following conditions have to be met:

$$P_t^{(1)}(K_t^{(1)}) = P_t^{(2)}(K_t^{(2)}), \quad (46)$$

$$R_t^{(1)'}(K_t^{(1)}) = c_1, \quad (47)$$

$$R_t^{(2)'}(K_t^{(2)}) = c_2. \quad (48)$$

where c_1 and c_2 are the marginal costs of capacity corresponding to k_1 and k_2 , and $R_t^{(1)}(\cdot)$ and $R_t^{(2)}(\cdot)$ are the revenue functions defined by $P_t^{(1)}(\cdot)$ and $P_t^{(2)}(\cdot)$. Conditions (46-48) impose restrictions on the values and derivatives of the inverse demand functions at two points, so such functions clearly exist. Now, let us consider firms 1 and 2, where firm i has investment history $\boldsymbol{\theta}_t$ and operates in the environment with the acquisition of assets equal to k_i and

the inverse demand curve given by $P_t^{(i)}(\cdot)$. Both firms are on their optimal capacity path in period t , because equations (47) and (48) are exactly the first-order conditions for respective optimization problems. The output price will be the same in both cases by equation (46). Reported book value, depreciation, and the latest investment will also be the same because the firms share the investment history. Revenues, will, however, be different - at equal prices the firms sell different quantities of output, $K_t^{(1)} \neq K_t^{(2)}$. Therefore, left-hand-sides of equation (45) will be different, while the right-hand-side is the same for both companies, which is a contradiction. ■

Proof of Proposition 3. Given the discussion following the statement of the proposition, it remains to show that if $\mathbf{bv}^* - \alpha \mathbf{z}^*$ belongs to the linear hull of \mathbf{bv} , \mathbf{d} , \mathbf{i} for any α , then \mathbf{bv} has to be proportional to \mathbf{bv}^* . First, note that by the clean surplus relation and the definition of \mathbf{z} ,

$$\mathbf{bv}^* - \alpha \mathbf{z}^* = (1 - r\alpha) \mathbf{bv}^* - \alpha(1 + r) \mathbf{d}^* - r\alpha \mathbf{i}.$$

Vector \mathbf{i} has only one non-zero component - the one on the first position. Therefore, it must be that $(1 - r\alpha) \mathbf{bv}^*(T) - \alpha(1 + r) \mathbf{d}^*(T)$ is in the linear hull of $\mathbf{bv}(T)$ and $\mathbf{d}(T)$, where by $\mathbf{bv}^*(T)$, $\mathbf{d}^*(T)$, $\mathbf{bv}(T)$, and $\mathbf{d}(T)$ I denoted the last T components of the respective vectors. Since, this condition must hold for any α , it is clear that $\mathbf{bv}^*(T)$ and $\mathbf{d}^*(T)$ must themselves belong to the linear hull of $\mathbf{bv}(T)$ and $\mathbf{d}(T)$. Let

$$\mathbf{bv}^*(T) = \lambda_1 \mathbf{bv}(T) + \lambda_2 \mathbf{d}(T) \tag{49}$$

$$\mathbf{d}^*(T) = \lambda_3 \mathbf{bv}(T) + \lambda_4 \mathbf{d}(T) \tag{50}$$

For the last component of these vectors, we have the following expression:

$$d_T^* = \lambda_3 bv_T + \lambda_4 d_T$$

By the clean surplus relation, $bv_T = 0$. Therefore, since $d_T^* \neq 0$, $d_T \neq 0$. On the other hand,

$$bv_T^* = \lambda_1 bv_T + \lambda_2 d_T$$

Both bv_T^* and bv_T are equal to zero, but $d_T \neq 0$, therefore, $\lambda_2 = 0$. I have shown that the last T components of \mathbf{bv} are proportional to \mathbf{bv}^* . It remains to check that $bv_0^* = \lambda_1 bv_0$. Again invoking the clean surplus relation, $d_T = bv_{T-1}$ and $d_T^* = bv_{T-1}^*$. Hence,

$$\lambda_4 = \frac{d_T^*}{d_T} = \frac{bv_{T-1}^*}{bv_{T-1}} = \lambda_1$$

Now writing condition (50) for the second from the end components of respective vectors, one obtains

$$d_{T-1}^* = \lambda_3 bv_{T-1} + \lambda_1 d_{T-1}$$

From the proportionality of the last T components of \mathbf{bv}^* and \mathbf{bv} , it follows that

$$d_{T-1}^* = bv_{T-2}^* - bv_{T-1}^* = \lambda_1 (bv_{T-2} - bv_{T-1}) = \lambda_1 d_{T-1}$$

Therefore, $\lambda_3 = 0$. I have shown that $\mathbf{d}^*(T) = \lambda_1 \mathbf{d}(T)$. To conclude, observe that

$$bv_0^* = bv_1^* + d_1^* = \lambda_1 (bv_1 + d_1) = \lambda_1 bv_0$$

■

Proof of Proposition 4. Since $\boldsymbol{\theta}_t$ is jointly normally distributed, variables BV_t , D_t , I_t , and $V_t - \alpha R_t = (\mathbf{bv}^* - \alpha \mathbf{z}^*) \cdot \boldsymbol{\theta}_t$ will also be jointly normally distributed. Therefore, the conditional expectation $E[V_t - \alpha R_t | BV_t, D_t, I_t]$ will be linear in the information set variables. If one considers the following linear regression

$$V_t - \alpha R_t = c_0 + c_1 BV_t + c_2 D_t + c_3 I_t + \epsilon, \quad (51)$$

then

$$f^*(\mathcal{I}_t, \mathbf{g}) \equiv E[V_t - \alpha R_t | BV_t, D_t, I_t] = c_0 + c_1 BV_t + c_2 D_t + c_3 I_t$$

and

$$E[(f^*(\mathcal{I}_t, \mathbf{g}) - (V_t - \alpha R_t))^2] = Var[\epsilon].$$

Let c_1^* , c_2^* , c_3^* be the coefficients in the following projection:

$$\Sigma^{\frac{1}{2}} \mathbf{y} = c_1^* \left(\Sigma^{\frac{1}{2}} \mathbf{bv} \right) + c_2^* \left(\Sigma^{\frac{1}{2}} \mathbf{d} \right) + c_3^* \left(\Sigma^{\frac{1}{2}} \mathbf{i} \right) + \epsilon, \quad (52)$$

and let us show that c_1 , c_2 , c_3 in (51) are equal to c_1^* , c_2^* , c_3^* . By definition of orthogonal projection,

$$\left(\Sigma^{\frac{1}{2}} \mathbf{bv} \right) \cdot \epsilon = \mathbf{bv}' \Sigma^{\frac{1}{2}} \epsilon = 0, \quad (53)$$

$$\left(\Sigma^{\frac{1}{2}} \mathbf{d} \right) \cdot \epsilon = \mathbf{d}' \Sigma^{\frac{1}{2}} \epsilon = 0, \quad (54)$$

$$\left(\Sigma^{\frac{1}{2}} \mathbf{i} \right) \cdot \epsilon = \mathbf{i}' \Sigma^{\frac{1}{2}} \epsilon = 0. \quad (55)$$

Multiplying by random row vector $\boldsymbol{\theta}'_t \left(\Sigma^{\frac{1}{2}}\right)^{-1}$, one obtains

$$\boldsymbol{\theta}'_t \mathbf{y} = c_1^* (\boldsymbol{\theta}'_t \mathbf{b} \mathbf{v}) + c_2^* (\boldsymbol{\theta}'_t \mathbf{d}) + c_3^* (\boldsymbol{\theta}'_t \mathbf{i}) + \boldsymbol{\theta}'_t \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon}.$$

Therefore,

$$V_t - \alpha R_t = c_1^* B V_t + c_2^* D_t + c_3^* I_t + \boldsymbol{\theta}'_t \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon}. \quad (56)$$

Let

$$c_0^* = E \left[\boldsymbol{\theta}'_t \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon} \right].$$

To show that coefficients in (51) are equal to c_0^* , c_1^* , c_2^* , c_3^* , it suffices to check that the residual in (56) is uncorrelated with random variables BV_t , D_t , I_t . Indeed,

$$\begin{aligned} \text{Cov} \left(B V_t, \boldsymbol{\theta}'_t \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon} \right) &= \text{Cov} \left(\boldsymbol{\theta}_t \cdot \mathbf{b} \mathbf{v}, \boldsymbol{\theta}'_t \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon} \right) = \text{Cov} \left(\mathbf{b} \mathbf{v}' \boldsymbol{\theta}_t, \boldsymbol{\theta}'_t \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon} \right) \\ &= \mathbf{b} \mathbf{v}' \Sigma \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon} = \mathbf{b} \mathbf{v}' \Sigma^{\frac{1}{2}} \boldsymbol{\epsilon} = 0, \end{aligned}$$

where the last equality is due to (53).

The variance of the error term in (56) is

$$\begin{aligned} \text{Var} \left[\boldsymbol{\theta}'_t \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon} \right] &= \text{Cov} \left(\boldsymbol{\epsilon}' \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\theta}_t, \boldsymbol{\theta}'_t \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon} \right) = \boldsymbol{\epsilon}' \left(\Sigma^{\frac{1}{2}}\right)^{-1} \Sigma \left(\Sigma^{\frac{1}{2}}\right)^{-1} \boldsymbol{\epsilon} \\ &= \boldsymbol{\epsilon}' \boldsymbol{\epsilon} = \|\boldsymbol{\epsilon}\| = \rho \left(\Sigma^{\frac{1}{2}} \mathbf{y}, \Sigma^{\frac{1}{2}} \mathcal{A} \right). \end{aligned}$$

By the definition of the norm of a linear operator,

$$\rho \left(\Sigma^{\frac{1}{2}} \mathbf{y}, \Sigma^{\frac{1}{2}} \mathcal{A} \right) \leq \left\| \Sigma^{\frac{1}{2}} \right\| \rho \left(\mathbf{y}, \mathcal{A} \right).$$

To conclude, note that

$$\rho \left(\mathbf{y}, \mathcal{A} \right) = \rho \left(\mathbf{b} \mathbf{v}^* - \alpha \mathbf{z}^*, \mathcal{A} \right) \leq \rho \left(\mathbf{b} \mathbf{v}^*, \mathcal{A} \right) + \alpha \rho \left(\mathbf{z}^*, \mathcal{A} \right)$$

by the triangle inequality. ■

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