

Steady State Circuit Theory for Guided Wave Elements

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December 23, 2003

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CONVENTIONS

The time variation is chosen to be $\exp(j\omega t)$ as opposed to $\exp(-j\omega t)$ in quantum mechanics. Hence, phasor starts from positive horizontal position and rotates in **anti-clockwise** with time at any one particular spatial location. Phasors of forward going wave moves in clockwise direction as you move forward in space. (Opposite of Smith Chart. In Smith Chart you actually move towards source in clockwise direction.). One complete rotation means one wavelength. (Again in Smith chart its half the wavelength.)

Part I

Uniform, Plane, TEM Waves With No Polarization

1 SMITH CHARTS

IMPEDENCE CHART

- $\Gamma \equiv \frac{V_-}{V_+} = \frac{I_-}{I_+}$ and $\rho \equiv |\Gamma|$. Constant ρ are concentric circles. Γ is $+\rho$ on right corner (open circuit) and $-\rho$ on left corner (short circuit) and is complex on rest of circle. So right corner is V_{max} and left corner is V_{min} .
- Constant R 's are circles centred towards right. Circles that fall completely on right side represents $\bar{R} > 1$ (open circuit); that crosses over to the left side as well represents $\bar{R} < 1$ (short circuit). Right corner is open because infinite-resistance circle (a dot) cuts the zero-reactive- arc (a horizontal line) at that point. Left corner is short because the zero-resistance-circle (biggest circle) cuts the zero-reactive arc there.
- Constant X 's are arcs centered towards right. Upper part represents $j\bar{X}$ is +ve or inductive ($j\omega L$). Lower part represents $j\bar{X}$ is -ve or capacitive ($\frac{1}{j\omega C}$). The arcs that are completely in the right half represents $|j\bar{X}| > 1$ and arcs that crosses into left plane represents $|j\bar{X}| < 1$.
- If we move from load towards source, phase of Γ decreases and goes as $-2\beta l$. So move **clockwise** to go towards source. $\lambda/4$ length converts short to open. $\lambda/2$ converts back to original. Hence one full circle should be $\lambda/2$.

ADMITTANCE CHART

(Admittance is defined as $Y \equiv 1/Z$; its real part is called G (Conductance) and imaginary part is B (Susceptance). Note that only when $R = 0$, $jB = 1/jX$ similarly only when $X = 0$, $G = 1/R$). All circles are centred towards left. Strictly ...everything is same. Just change two things ... for interpretation in terms of Y :-

- conductance $\bar{G} > 1$ are towards left (so $\bar{R} > 1$ still towards right)
- Susceptance $j\bar{B}$ +ve is down (+ve $j\bar{X}$ is still up)

2 STANDING WAVE PATTERNS

- If you want relation between incident voltage and current and reflected voltage and current look at the characteristic impedance of line. Forget about the fact that there is a reflected wave as well. If you want a relation between incident and reflected voltages look for reflection coefficients. (You would have to know the input impedances first.) And if you want a relation between net current and net voltage at any one location you need to look at input impedance. For the last two facts always start thinking from smith chart. Just remember the fact that reflection coefficient and impedances are different at different points and **not only at inhomogeneities**.
- Always think in terms of phasors. Note $\exp(j\omega t) \Rightarrow$ cosinoidal dependence \Rightarrow phasor will rotate in **anticlockwise** directions.
- In space the phasor for forward going wave would move in **clockwise** direction as you move in the direction of the wave.
- Phases between only V_+ and I_+ or between V_- and I_- are decided by characteristic impedance Z_0 (or negative of Z_0) only – which most of the time is real so they are mostly in-phase. By convention, with real Z_0 , the reflected voltage is 180° out of phase from the reflected current, which basically means that the +ve direction of current flow is taken as direction of the incident wave. So at the end total current is the difference of two currents whereas total voltage is the sum – **this is root of complicated behaviour of standing waves in transmission lines**. Using this fact I can create any phase between total voltage and total current at any one location. So I can create any impedance. (You can never create a negative resistance though.)
- Phase between incident voltage and reflected voltage can be concluded from value of Γ . If Γ is in upper half of smith chart then it means V_- phasor will lead the V_+ phasor in time and vice versa. Again note that since value of Γ changes with distance so will the phase between incident and reflected waves. Whereas the **magnitude** $|\Gamma| = \rho$ **is constant with distance**, hence the amplitude of the reflected and incidence waves and hence their ratios would remain same with distance.
- Phase between total-current and total-voltage at any point can be concluded from value of impedance at that point. If impedance is **inductive** $\Rightarrow Z = |Z|\exp(j\theta)$ with $\theta > 0$ then $I = V \exp(j\omega t)/Z$ and hence current phasor will **lag** the voltage phasor. Similarly if impedance is **capacitive** then current phasor will **lead** voltage phasor. Note that since impedance changes with distance so will the phase difference

between total-voltage and total-current phasors. (**Normal is lag, that is positive jX means lag**)

- Maxima/minima for total-voltage and total-current occurs at the same spatial points. At the position of voltage-maxima you would **always** have current minima and vice-versa. The input impedance at such critical points is given by $(1 + \rho)Z_0/(1 - \rho)$ (voltage maxima) and $(1 - \rho)Z_0/(1 + \rho)$ (voltage minima).
- An open, short or pure reactive loads reflects everything $\Rightarrow V_{min} = 0 \Rightarrow \rho = 1 \Rightarrow$ no average power input across any surface (Average power incident across any area is equal to the average power reflected. Although instantaneous powers might not match because of storage elements) \Rightarrow total-current and total-voltage at any point are always out of phase by 90 degrees. Phase between the incident and reflected waves at any point depends on load.

Case 1

Open or Short :-

- The output would always be either max or min. If $R = 0$ then minima otherwise if $R = \infty$ then maxima.
- $V_{min} = 0$. $\rho = 1$. Always.
- Let us assume we have an open. Load impedance is purely resistive and infinite. As we start moving towards the source, the impedance becomes purely capacitive (**infinitely high reactance, which means small capacitance**). Magnitude of reactance keeps on reducing that means capacitance keeps on increasing. After quarter a wavelength the reactance becomes zero. After that reactance becomes positive (although very small) which means small inductance (short). If we keep on moving reactance grows and hence the inductance increases. After half a wavelength reactance becomes infinite that means wither infinite inductance or zero capacitance.
- Total voltage at the load end is always V_{max} (obviously oscillating sinusoidally) and total current at the load end is always zero. So the total current phasor (which would grow in length with distance) would lead the total voltage phasor (which would reduce in length with distance). After a quarter wavelength, the phasor length of total voltage would become zero and phasor length of total current would be infinite. As we further move towards source the length of total current phasor would start reducing and the length of total-voltage phasor would start growing. Current phasor would now lag voltage phasor and impedance would be inductive.
- As we move towards source, the incident and reflected voltage would start becoming out of phase. So their phasor addition would be smaller than the V_{max} . Also note that same things happen to the reflected current but with negative sign. So the sum of the two current is always 90° out of phase of sum of voltages.

Case 2

Pure resistive load :-

- The output would always be either max or min. If $R < Z_0$ then minima otherwise if $R > Z_0$ then maxima. Also, as you start moving impedance would either become capacitive or inductive depending on the initial position on the smith chart.
- $V_{min} \neq 0$. $\rho < 1$. Always.
- The value of R decides the $V_{min} \Rightarrow$ VSWR.

Case 3

Pure reactive load :-

- $V_{min} = 0$. $\rho = 1$. Always.
- Output would never be max or min. L or C will shift the minimas along the length. Think in terms of smith chart. Load point is where reactance arc cuts $R = 0$ circle. Whether nearest point would be max or min can be seen from smith chart.

Case 4

Complex load :-

- $V_{min} \neq 0$. $\rho < 1$. Always.
- O/P is never max or min. (Both R and X together decides the form of standing wave. Even if you keep X constant and

change R , still V_{min} will get shifted. **Think from smith chart**)

Case 5

Resonant load:-

- For series combination $Z = 0$ and current shoots to infinity. So it can be $\lambda/4$ open circuited line (which gives $Z = 0$)
- For parallel combination $Z = \infty$ and current goes to zero. So it can be $\lambda/4$ short circuited line.

3 POWER FLOW

3.1 Real Power

Always talks about instantaneous power.

- Power I/P $P_{in} = VI$
- Power Dissipated $P_{diss} = I^2R$
- **Energy** Stored $W = \frac{1}{2}CV^2 + \frac{1}{2}LI^2$
- Circuit Equation :- $P_{in} = P_{diss} + \frac{d}{dt}(W)$

3.2 Complex power

Always talks about average power over a period of sinusoidal input. Always remember to put Re in all definitions. Always remember to put $1/2$ – (which comes from the avg of $\cos^2(\omega t)$).

- Time average power I/P $\langle P_{in} \rangle = \frac{1}{2}ReVI^*$
- Time average power dissipated $\langle P_{diss} \rangle = \frac{1}{2}ReII^*R$ (Remember $Z_R = R$, $Z_L = j\omega L$, $Z_C = \frac{1}{j\omega C}$. So you need to include only ReZ)
- Time average **energy** stored $\frac{1}{2}Re\frac{1}{2}CVV^* + \frac{1}{2}LII^*$ (Note in steady state average energy stored in each element does not change with time. Hence total average stored energy also does not change with time. (Both facts are obviously true). While instantaneously, both capacitive stored energy and inductive stored energy might change with time).
- Circuit equation :- $\langle P_{in} \rangle = \langle P_{diss} \rangle$. This equation is rather trivial. More important equation is:- $\frac{1}{2}VI^* = \langle P_{diss} \rangle + j(2\omega)(\langle W_e \rangle - \langle W_m \rangle)$. (One can remember $j2\omega$ factor as coming from derivative of $\exp(j\omega t)^2$). Note that people define $P_{reactive} = \frac{1}{2}ImVI^*$. There is no physical meaning associated with this term. All that it represents is that V and I are not in phase and hence Z_{in} is complex and hence impedance would either be inductive or capacitive and hence the time average magnetic stored energy would surely be different from time average stored electric energy and hence at least a part of power flowing into and across **that interface** would be oscillatory (that means its flowing in and out with a time average of zero) and also that the time average stored electrical energy and the time average stored magnetic energy can never be same. (Always think in terms of – 1) phase relations between V and I , 2) nature of Z_{in} 3) profile of $P(t)$, value of $\langle P_{in} \rangle$ and value of $\langle P_{diss} \rangle$ and 4) nature of $\langle W_e \rangle$, $\langle W_m \rangle$ and $\langle W_e \rangle - \langle W_m \rangle$)

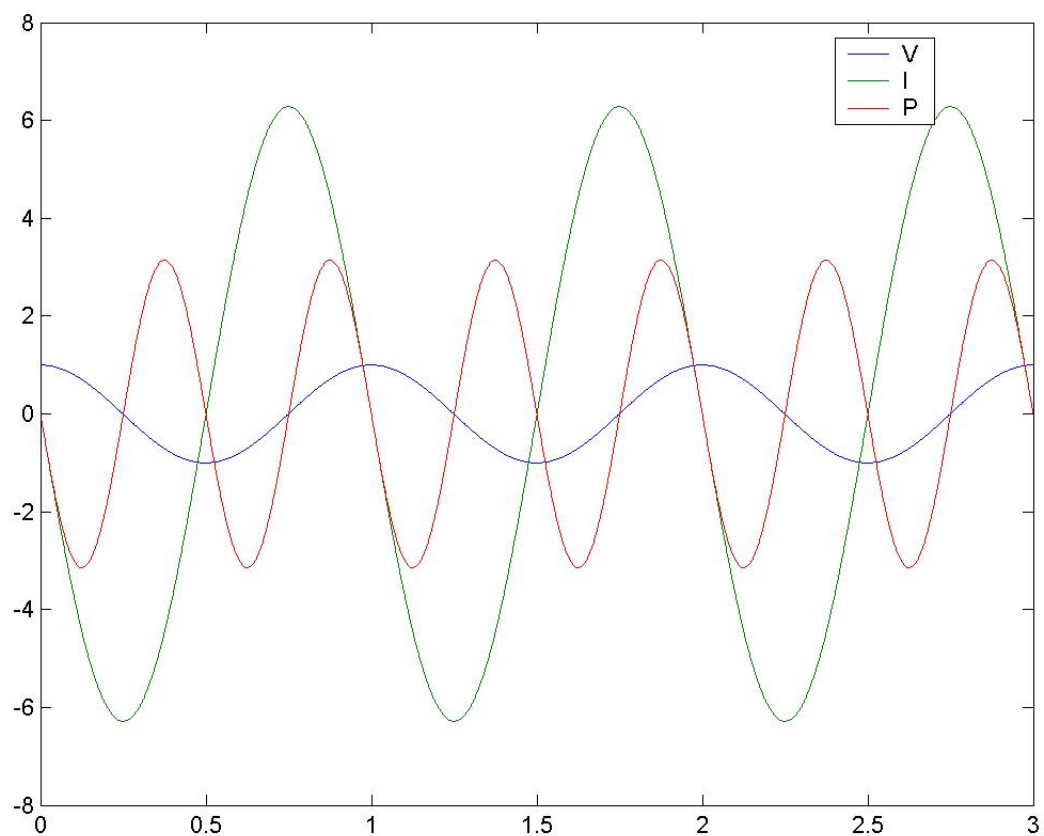
Case 1

$$\text{Im}\frac{1}{2}VI^* = 0$$

V is in phase with I . Z_{in} is purely resistive **across that particular cross section**. There is some finite avg power I/P from source into the load and avg power dissipation in the load. There is no oscillating power across that cross section and power flows in one direction only, **although it might become zero at some moment**. Although Z_{in} is purely resistive, still there might be some average stored electromagnetic energy in the load at that cross section. Average stored electric energy would be equal to the average stored magnetic energy. That shows that the load is a resonant load. Otherwise impedance can never be real.

Case 2

$\text{Re}\frac{1}{2}VI^* = 0$. Let us think of a load of purely capacitive nature (it can be shown that in this case load has to be purely reactive) being driven by a sinusoidal voltage source. $V = \text{Re}V_0 \exp(j\omega t) = V_0 \cos(\omega t)$, $Z_C = \frac{1}{j\omega C}$. Hence in steady state $I = \text{Re}jV_0\omega C \exp(j\omega t)$. Hence, $I = -V_0C\omega \sin(\omega t)$. $P = -V_0^2 \sin(2\omega t)/2$.



Note that V and I are out of phase by 90° and Z_{in} is purely reactive. Average power I/P by the source into the load and average power dissipated in the load are zero. Now since “power-dissipated” can only be positive, it is instantaneously zero in entire time cycle. Only oscillatory power flows in and out of the circuit. Negative power actually means that power goes into the source from the load. Now it depends on the operating physics of the source whether that energy gets destroyed or charged up or anything else. Donot get this confused with internal resistance of source. Internal resistance is not really "internal" in present context. What we mean here is that internal physics of source can't be dealt with E field – there has to be some non-E-field force for circuit to work. This non-E-field force is what I mean by "internal" operation. Now the electrodes etc that were used to produce this non-E-field forces might themselves be resistive ... this is what we mean of internal resistance. This internal resistance will carry the usual E-field force and can easily be taken care in circuit theory. Also note that there is some average electromagnetic energy stored in the system for sure. Also the average stored electric energy is not equal to average stored magnetic energy for sure.

3.3 Power Flow Calculations

- **Lumped Modeling With Z_{in}** :- Power flow on transmission line is more conveniently calculated through **lumped modeling at any point**. Obviously the most convenient point is the source end. For source end you just need to calculate the Z_{in} for lumped modeling. But for any other point you need to calculate the total voltage at that point as well as the Z_{in} at that point. Also note that for time average power flow we just need the real part of input impedance at any cross section.
- **Time Average Continuity and Superposition**:- Another nice tool to analyze the power flow in a transmission line is to think of forward and backward time average power flow across any cross section. Note that the time average power obeys a simple continuity equation. **Its continuous** unless you hit an inhomogeneity which absorbs power. Another important point is that the time average power in different waves propagating together (forward or backward or multiple modes) is additive. So if you know the time average power flow in forward and backward directions you can simply add them up. As we discussed above lumped modeling is pretty good for calculating “total” power at any cross section. **Usually we would always have to do lumped modeling at one cross section (best is the source end because you don't even know V^+) to calculate the time average power flow and after that we can argue using the continuity equation.** One needs to be a bit cautious in using these two tools together. In lumped modeling, remember that you never talk about forward power and backward power separately within the lumped element. For example it is very tempting to write $\frac{1}{2}Re\frac{V_+^2}{R_L}$ for forward power in a resistive load. But this is wrong because forward voltage and forward current are not in phase independent of reflected terms. (Although one can surely use $\frac{1}{2}ReV_+I_+^* = \frac{1}{2}Re\frac{V_+^2}{Z_0}$). Its best to talk

about the use of $\frac{1}{2}ReVI^* = \frac{1}{2}Re\frac{V^2}{R_L} = \frac{1}{2}Re\frac{V_+^2}{Z_0} - \frac{1}{2}Re\frac{V_-^2}{Z_0} = (1 - \rho^2)\frac{1}{2}Re\frac{V_+^2}{Z_0}$.

- Instantaneous Continuity Equation:-** Lumped modeling can also be used for instantaneous power calculations. We would always have to do the lumped modeling at some cross section (best is the source end) to get the voltage and current phasors and then just propagate the phasors to any other location. We need to be a bit careful with time instantaneous continuity equation. *a) Spatial Variation :-* Even for a simple plane uniform TEM wave propagating without any reflection **instantaneous energy flow is not continuous**. That is the instantaneous pointing vector is different at different locations along the direction of propagation. This is also true for standing waves. The best way to get an intuitive grasp of this is to assume the space is filled with capacitors and inductors which are charging and discharging continuously. (Note that even then these inductors and capacitors give you a collective effect of purely resistive intrinsic impedance. What that tells you is that the E and H are in phase. But it does not explain why E and H are different at different locations at the same instant of time. For that you can think of the space as a mesh of resistances and capacitances.). In steady state, **time average power flow is definitely continuous**. The reason is that the time average stored energy in space is constant in steady state. That's why it's much better to always think about time average power flow. **Note that average power is also additive (for lossless lines only)**. *b) Temporal Variation :-* When you know the lumped model at any one point you immediately know all the temporal variation of power at that cross section. If the load is completely reactive then power is completely oscillatory (V and I completely out of phase) whose time average is zero. For completely resistive cross section (as for perfectly matched line or at $\lambda/4$ from completely resistive load etc) there is no oscillatory back and forth power term. But instantaneous power to change with time. In general, you would have some time average power flow.
- Usage of Power Ratios:-** Note also that $P_{absorbed}/P_{in}^+$ is independent of the length of the line because ρ is independent of length. **But P_{in} surely depends on the length.**

3.4 Decibel Scale

There is something absolute about dB scales. As soon as I say 6dB you know that I am talking about a ratio of two quantities which is equal to 4. Now if the quantities are voltage then the ratio of voltages is four and if quantities were powers then the ratio of power is 4. Confusion starts when you want to get a ratio of voltages out of a dB ratio of power. So one should first make clear what is being talked about. For example if I say return loss is 6dB you cannot conclude anything until I tell you that return loss is a ratio of powers. So it means power down by factor of 4 which means voltage is down by a factor of 2.

Power_dB=2*voltage_dB

3dB power implies power down by 2

6dB power implies power down by 4 implies voltage down by 2

10dB power implies power power down by 10
 20 dB power implies down by 100

4 Impedence Matching

1) Single stub matchin (Series/parallel)

Use smith chart. Move along the line till the line impedance has a real part equal to characteristic impedance (that is when the constant row circle cuts $R=Z_0$ circle). This will always happen at some distance. Add a series or parallel reactive element to cancel the ractive part. One can also use transmission line segments instead of lumped reactive elements.

2) Quater Wave transformer

Use smith chart. Move along the line till it become pure resistive (that is till minima or maxima). Put a quarter wave transformer of appropriate chara'cteristic impedance to match.

5 Useful Expressions

- $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$:- (Write expression for Z_L in terms of Γ_L and then reverse it. Better to remember.) $\Gamma(l) = \Gamma_L \exp(-j2\beta l)$:- (error prone .. so better to remember. l is the distance measured from the load end.)
- $Z_{in}(l)/Z_0 = \frac{1+\Gamma}{1-\Gamma}$:- (just write from intuition). $Z_{in}(l)/Z_0 = \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$. $Z_{in}(\lambda/4) = Z_0^2/Z_L$. This also tells us that at the point of voltage maxima $Z_{in}/Z_0 = \frac{1+\rho}{1-\rho}$ and at the point of voltage minima $Z_{in}/Z_0 = \frac{1-\rho}{1+\rho}$.
- Power is additive : total power = incident + reflected. $\langle P_{total,in} \rangle = \frac{1}{2}Z_0 V_+^2 (1 - \rho^2)$
- Calculation of $V_+(0)$ (which is load end) :- V_s (total voltage at source end) can be calculated by two ways – one is $V_0 Z_s / (Z_s + Z_{in})$ and other is to add V_+ and V_- at source end in terms of V_+ at load end. Equate two and calculate V_+ .

Part II

Uniform, PLane, TEM Waves With Polarizatuons

- Now there would not be any standing wave patterns. What that means is that you would **never** have nodes where amplitude of cosinusoidal is always zero.

- At the same time remember that we do have different phase relationship between the oscillations at different spatial points. What that would mean now is that you would have elliptical to circular to linear polarization change at different spatial locations. **Do not ever go into complex phasor plane. Stick to the real vector plane and think of spatial and temporal changes of phases manually.**
- Power flow should also be thought in terms of simple picture of two waves propagating two directions. That's it.

Part III

Wave In Source Free Bounded Region

- Let's think from $\nabla \times E$ equation. Propagation direction is z .
- 1) Variation of E_x along y OR the variation of E_y along x is needed to support H_z .
- 2) Variation of E_z along x OR the existence of E_x is needed to support H_y . (we would remove the second option for TE or TM waves later because as we would show they always have to exist)
- 3) Variation of E_z along y OR the existence of E_y is needed to support H_x . (we would remove the second option for TE or TM waves later because as we would show they always have to exist)
- Similar things can be said about $\nabla \times H$ equation.

TE/TM/TEM

- Sometimes boundary conditions might be satisfied for TE/TM or TEM wave. In general one can find a linear combination of these that would satisfy the most generic boundary conditions.
- Note that TEM mode is usually a special case of TM modes and hence TM modes look more closer to what one might guess from capacitor/inductor kind of mental picture. And TE modes are usually exact opposite of such a mental picture.

TM

- For TM wave, first point tells us that transverse curl of the transverse component of E has to be zero. **So the E lines would be complete closed loop or would just be constant.**

- Now equivalent second and third point for a $\nabla_x H$ equation gives us a direct relation between orthogonal E and H field components in the transverse plane. So E_x gives me H_y and E_y gives me H_x and vice versa. **Also transverse components of two fields have to be perpendicular to each other.**
- So for the second and third point (for TE or TM only) we can remove the second option because that's not really an option. For example if H_y exists then E_x has to exist.
- Hence knowing the variation of E_z along x and y gives me all the field components.
- $H_x(\frac{jk_t^2}{\beta}) = E'_z$ and $H_y(\frac{jk_t^2}{\beta}) = E'_z$.
- $\frac{E_x}{H_y} = -\frac{E_y}{H_x} = Z_{TM}$
- Where $k_t^2 = k_0^2 - \beta^2$ and $Z_{TM} = \frac{\beta}{k_0} Z_0$. Note that for a propagating mode $\beta < k_0$ (wavelengths are always longer ... faster phase velocity) and hence $Z_{TM} < Z_0$.

TE

- Similar arguments for TM as well.
- $E_x(\frac{jk_t^2}{\beta}) = H'_z$ and $E_y(\frac{jk_t^2}{\beta}) = H'_z$.
- $\frac{E_x}{H_y} = -\frac{E_y}{H_x} = Z_{TE}$
- Where $k_t^2 = k_0^2 - \beta^2$ and $Z_{TE} = \frac{k_0}{\beta} Z_0$. Note that for a propagating mode $\beta < k_0$ (wavelengths are always longer ... faster phase velocity) and hence $Z_{TE} > Z_0$. (remember that $Z_{TE} Z_{TM} = Z_0^2$)

TEM

- TEM mode is much simpler because one does not need to use both the curl equations. Only one of them is needed. So the transverse components of both the fields should form close loops (or should be just constant). And they would surely be orthogonal as usual.
- $\frac{E_x}{H_y} = -\frac{E_y}{H_x} = Z_0$ with $k_t = 0$.

Conductors

In the light of E&M theory, let us reconsider what we should usually mean when we are talking about good conductors or bad conductors. I can let us think of three seemingly different meanings :-

- Simply conductivity should be high enough so that unless you are not drawing to high a current out of it electric field can probably be assumed to be negligible. Which also makes magnetic field negligible. IR losses are negligible as well because of high conductivity. So the measure of goodness of a conductor would simply be σ .
- Some one might just want to look at complex permittivity $\epsilon = \epsilon_0(1 - i\frac{\sigma}{\epsilon_0\omega})$ and say that as long as real part can be neglected in comparison of imaginary part you can call your material a good conductor. This equivalently mean that conduction current is way higher than the displacement current. According to this criteria goodness of a conductor is determined by σ/ω .
- There might be a third criteria which says that skin depth (which actually is defined only as long as $\sigma \gg \epsilon_0\omega$ - which is same as the above criteria of goodness - and is taken as inverse of either real part or imaginary part of the total propagation constant.) should be small enough so that you can assume fields are not penetrating deep. Although one might naively expect the two criteria to be identical, they actually are not. $\frac{1}{\delta} = Re\sqrt{-i\omega^2\mu\sigma/\omega} = \sqrt{\omega\mu\sigma/2}$. Hence this criteria tells you that a good conductor should be decided by $\sqrt{1/(\sigma\omega)}$.

So which one of these is true if any ? Or are all of them are true ? Actually the actual criteria should be a mixture of these. A good conductor is decided by how small is $R_s = \eta = 1/(\sigma\delta)$ which tells us how small is IR loss as well as how good it is as a reflector (smaller η makes it more and more optically dense material like). Which should mean making σ/ω as large as possible which is same as second criteria. Let us discuss what are different properties we expect from a good conductor and how does this criteria satisfies all of them.

- What if $\omega = 0$ which makes $\delta = \infty$? $1/\sigma\delta = 0$, so the criteria is surely satisfied. But fields should penetrate all into the metal and all our analysis of coaxial cable, reflection from a perfect conductor (of static fields - basically boundary conditions) etc should break down because all of them assume fields to be zero inside metal. There is a conceptual glitch here. When we are taking about $\omega = 0$ we are at the same time claimin that any time scale of concern is $\tau = \infty$. So may be fields would penetrate as transients but after long enough time charges would come to static situation and fields would surely be zero. So whenever ω is small unless you make σ unbelievably small your material should be able to respond. If material is more like an insulator or semiconductor then surely you are into mess even for fairly high frequencies. If you keep conductivity same and reduce frequency (or increase scale of time concerned) situation should improve as $1/\sigma\delta$ or equivalently σ/ω criteria tells us.
- Suppose at certain frequency my material is behaving like a good conductor. So η is high and hence its reflectivity is almost 1 with a phase switch of almost π . Which also tells us that there would be negligible power input into the conductor. Now high η can happen in too ways and internal behaviour would depend on the particular

scenario. For a constant ω if you increase σ you get better η with smaller δ . On the other hand if you decrease ω at a constant σ you would still get better η but a larger δ . So looking at δ doesn't really tell you anything.

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