STAT 375 Homework 6 Solutions

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Problem (1)

The measure is well defined since we have $\Theta = \mathcal{L}_G + 4I$ where $\mathcal{L}_G$ is the Laplacian of the 2-D grid and I is the identity matrix. Thus, $\lambda_{\min}(\Theta) = 4$ and it is positive definite.

Problem (2)

The code for generating the samples and running the appropriate regression is included below:

```
clear all
k = 10;
p = k^2;

%construct theta for the 2d grid
theta = zeros(p);
deg = zeros(p, 1);
for a = 1:k
    for b = 1:k
        if(a ~= k)
            theta(k*(a-1) +b, k*a+b) = -1;
        end
        if(b ~= k)
            theta(k*(a-1) +b, k*(a-1)+b+1) = -1;
        end
    end
end
theta = theta +theta';

for iter = 1:p
    deg(iter) = sum(theta(iter, :)~=0);
end
thetadiag = theta;
theta = theta+diag(deg+4);

[U, D] = eig(theta);
Dinv  = diag(sqrt(1./diag(D)));


nvals = [1000 1500];
lambdavals = [0.4];
err = zeros(length(nvals), length(lambdavals));
thetahatall = cell(length(nvals), length(lambdavals));
for iter2 = 1:length(nvals)
    n = nvals(iter2)

    %generate samples from the distribution
    Z = randn(p, n);
    X = U*Dinv*Z;


    tol = 1e-3;

    for iter1 = 1:length(lambdavals)
        lambda=lambdavals(iter1)/sqrt(n)

```

```
        thetahat= zeros(p);
        for iter = 1:p
            Xother = X([1:(iter-1), (iter+1):p], :)';
            xi = X(iter, :)';

            cvx_begin quiet
                variable beta(p-1)
                minimize (quad_form(beta, Xother'*Xother)/(2*n*lambda) - ...
                    2*xi'*Xother*beta/(2*n*lambda)+norm(beta, 1))
                %minimize norm(xi - Xother*beta, 2) + 2*lambda*norm(beta, 1)
            cvx_end
            thetahat(iter, :) = [beta(1:(iter-1))' 0 beta((iter):(p-1))'];
            norm(beta)
        end
        thetahat = abs(thetahat)>=tol*ones(p);
        thetahat = 0.5*(thetahat + thetahat') >0;
        thetahatall{iter2, iter1} = thetahat;
%         figure(iter1)
%         spy(thetahat)
        err(iter2, iter1) = sum(sum((thetadiag & ~thetahat) | (~thetadiag &thetahat)));

    end
end

save('hw6dataextra.mat');
```

The results of the analysis are as follow. Let $\lambda_n = \frac{\lambda_0}{\sqrt{n}}$. We plot in 1 the symmetric set difference versus n, the number of samples, for different values of λ_0. We choose $\lambda_0 = 0.05, 0.1, 0.2, 0.4$ (as opposed the the values originally given in the homework).

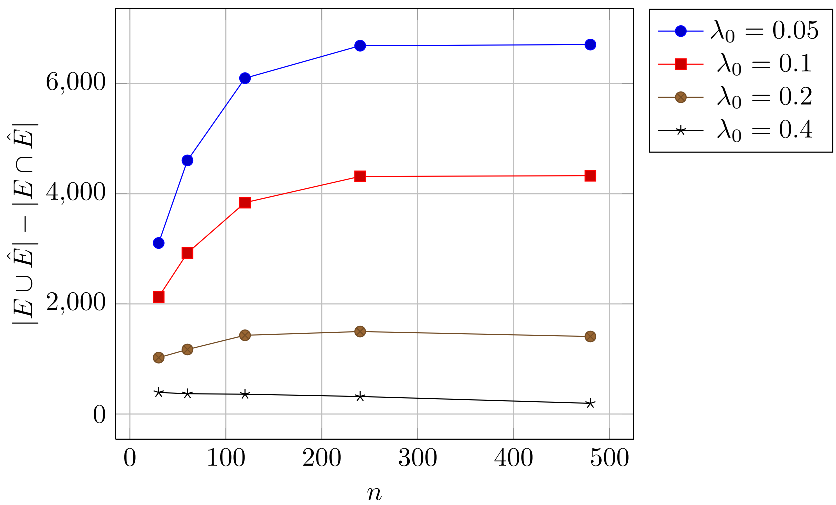


Figure 1: Symmetric set difference versus number of samples

Problem (3)

Consistency of the edge set is obtained by making (i, j) an edge in $\hat{E}$ if at least one of i and j yield the other as a neighbor. The sparsity pattern of Θ (only the off diagonals) is given in Fig. 2. The recovered edge set is shown in Fig. 3. As we can see, the recovered structure becomes better with increased samples.

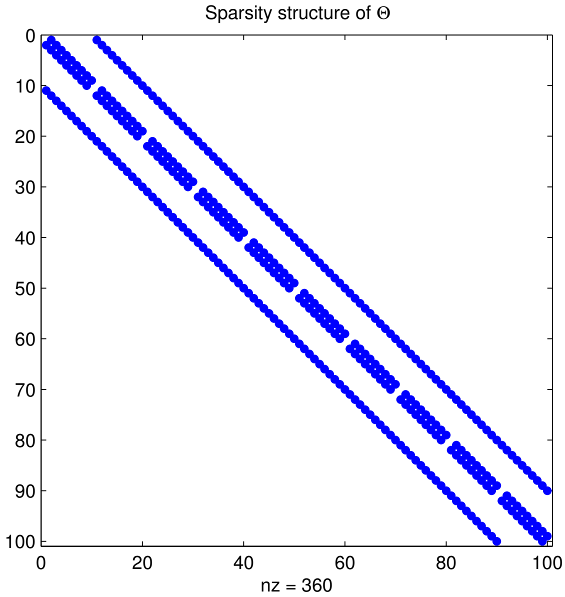
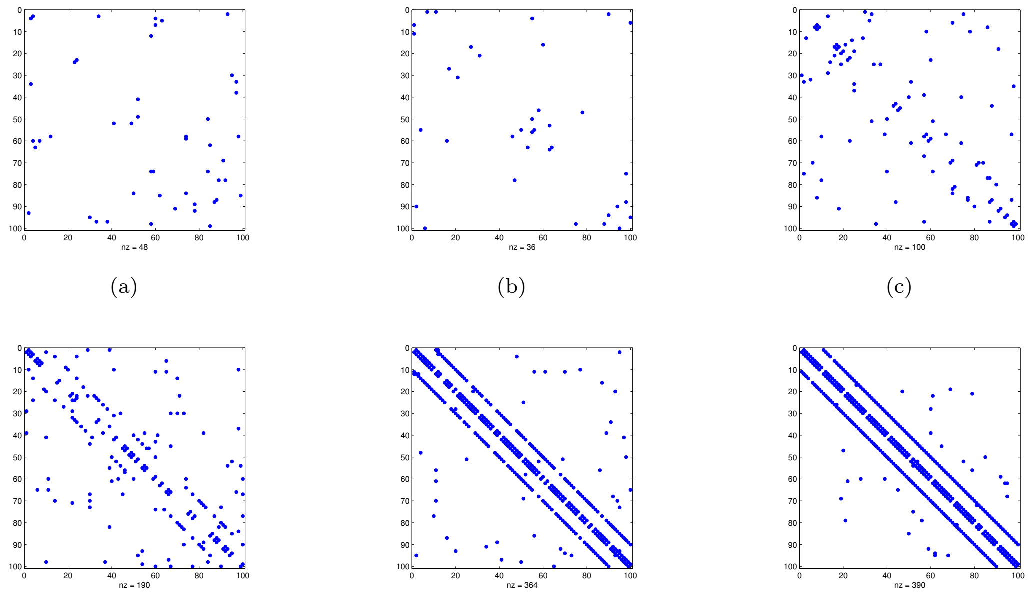


Figure 2: Sparsity pattern (off diagonal) for Θ



(a) (b) (c)

(d) (e) (f)

Figure 3: The recovery becomes progressively better with increasing samples. The above recovery is for $n = 30, 60, 120, 240, 1000, 1500$ at $\lambda_0 = 0.4$