Stat 375 Inference in Graphical Models

Homework 3

Due - 4/23/2012

Please return this homework in class or in the bin in Packard second floor, or by e-mail to Yiqun Liu (liuyiqun1124@gmail.com).

For $\ell \in \mathbb{N}$, let $G_\ell = (V_\ell, E_\ell)$ be an $\ell \times \ell$ two-dimensional grid[1]. We consider an Ising model on G_ℓ with parameters $\theta = \{\theta_{ij},\, \theta_i :\ (i,j) \in E,\, i \in V_\ell\}$. This is the probability distribution over $x \in \{+1, -1\}^{V_\ell}$

$$\mu(x) = \frac{1}{Z_G} \exp\Big\{ \sum_{(i,j)\in E_\ell} \theta_{ij}\, x_i x_j + \sum_{i \in V_\ell} \theta_i x_i \Big\} \tag{1}$$

(1) Write the belief propagation (BP) update equations for this model.

(2) Write a program that implements these update. You are requested to return a printout of the code (Matlab, R, C, C++, Java, ..., are accepted).

(3) Consider the case $\ell = 10$ (and hence $n = 100$ nodes). For each $\beta \in \{0.2, 0.4, \dots, 2.8, 3.0\}$, generate an instance by drawing θ_i, θ_{ij} uniformly random in $[0, \beta]$. Run the BP iteration and monitor convergence by computing the quantity

$$\Delta(t) \equiv \frac{1}{|\vec{E}_\ell|} \sum_{(i,j)\in \vec{E}_\ell} \big| \nu_{i\to j}^{(t+1)}(+1) - \nu_{i\to j}^{(t+1)}(+1) \big| \,. \tag{2}$$

Here $\vec{E}_\ell$ denotes the set of directed edges in G_ℓ, in particular $|\vec{E}_\ell| = 2\,|E_\ell|$.

Plot $\Delta(t = 40)$ and $\Delta(t = 80)$ versus β, for the random instances generated with $\beta \in \{0.2, 0.4, \dots, 2.8, 3.0\}$. Comment on the results.

(4) Repeat the calculation at the precious point, with now θ_i, θ_{ij} uniformly random in $[-\beta, +\beta]$, with $\beta \in \{0.2, 0.4, \dots, 2.8, 3.0\}$. Comment on the results.