

Homework 1

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We want to study a simple model for a single molecule in a container. For the sake of simplicity we assume that the molecule is a point and that it moves in two dimensions, cf. Fig. 1. The container is a circle of radius R and the molecule is forced to move inside the circle as follows. If it is inside the circle it moves on a straight line with velocity v . Whenever it hits the wall, it bounces inside in a uniformly random angle (the angle with the normal to the wall, θ is uniformly random in $(-\pi/2, \pi/2)$). Bounces are independent.

1. What is the relation of this model with renewal processes?
2. Suppose that at each bounce, the molecule transfer a force ϵ to the container and let $F(t)$ be the total force transferred until time t . Compute the average *pressure* exerted by the molecule, defined as

$$p = \lim_{t \rightarrow \infty} \frac{F(t)}{L \cdot t} \quad (1)$$

where L is the length of the container boundary.

3. Modify the above model by assuming that the force transferred to the container is $\epsilon \cos \theta'$, where θ' is the angle that the trajectory makes with the normal to the wall *when hitting* it. Compute the resulting pressure.

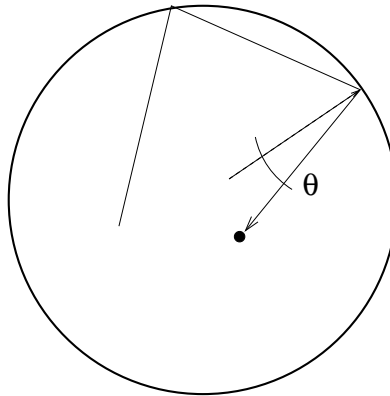


Figure 1: A molecule moving in a two dimensional container.