Mean field asymptotics in high-dimensional statistics: A few references

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Abstract

This is a guided bibliography to some theoretical topics in high-dimensional statistics and probability theory that are covered during the OOPS summer school in July 2020. This list of references is incomplete even for what concerns this set of topics. I will be improving it.

1 Background material

Statistics [BVDG11]. Physics and algorithms [EVdB01, MM09].

2 Exact asymptotics

Various approaches. Early approaches in the context of compressed sensing made use of tools from convex geometry [DT10b, DT10a], which were substantially refined in [ALMT14]. A sharp asymptotic characterization od the Lasso was first obtained in [BM12] using an analysis via AMP. Other papers that use the same approach include [DM16, CS18, SC19],

Leave-one out techniques were used in [EKBB+13, EK18].

Gaussian comparison. Gordon inequality was first proven in [Gor88]. Its application to convex-concave problems developed in [TOH15]. Applications of this approach include [TAH18, MM18, SAH19].

Bayes optimal estimators. Exact asymptotics for the Bayes error were derived in [DAM16, BDM+16], using again the connection to AMP, in [LM19, Mio17] using leave-one-out techniques. Adaptive interpolation method [BM19, BKM+19].

3 Approximate Message Passing

'Historical' background on AMP and its motivations can be found in [TAP77, Kab03, DMM09].

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Sharp analysis of AMP algorithms was developed in various degrees of generality, beginning with [Bol14] and then in [BM11, BLM15, JM13, BMN20, CL20]. (In particular [BMN20] streamlines and generalized the conditioning proof.) Optimality of Bayes-AMP among generalized first order methods was proven in [CMW20].

4 Optimization of mean-field spin glasses

The classical physics papers in this area are collected in [MPV87]. For a survey of mathematical work in this area, see [Tal10, Pan13].

Important structural properties of Parisi formula were proven in [JT16, AC17, Che17, AC15].

Optimization algorithms for mean field spin glasses were developed in [Sub18] (for the spherical case) and [Mon19, AMS20] (for the Ising case).

Negative results about optimization in problems *with* overlap gap were proven among others in [GS14, GJ19, GJW20].

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