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PRIVATE INFORMATION IN AN AUCTIONLIKE SECURITIES MARKET*

Paul R. Milgrom

Students of auction theory usually restrict the scope of these analyses to include only those settings where a single seller offers items for sale or a single buyer solicits delivery price offers. A large volume of trading takes place in settings of this kind, but an even larger volume is conducted in settings where there are both many buyers and many sellers making bids and offers. The major securities market and commodities and options exchanges are mostly of this second kind.

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There is a reasonably acceptable economic theory of securities markets for the hypothetical case where all investors and traders have identical information. This theory is an extension of the Arrow-Debreu competitive equilibrium theory, and as such it leads to the usual kinds of conclusions about the existence and efficiency of equilibrium. However, like the competitive equilibrium theory, the securities markets theory has little to say about how prices are formed. This omission is particularly disturbing in extensions of the theory in which some traders have private information. In those extensions, the equilibrium prices typically reflect more information than was available (when the markets opened) to any individual trader. This is a puzzling prediction of the theory, since <u>someone</u> must set the prices! Far from explaining this puzzle, traditional analyses have generated many doubtful propositions, including the following ones: Traders with inside information may be unable to profit from concealing and using that information.[1] Traders' beliefs about security prices--beliefs that are based partly on past and present prices--can be extremely sensitive to small variations in the prevailing prices, and the prices themselves are so sensitive to small shifts in demand that a single small trader can sometimes completely control price levels.[2] Moreover, equilibrium prices must sometimes reflect information that is not available to any active trader![3]

[1] Grossman (1976).

In contrast to the theory of securities markets and to competitive equilibrium theory, auction theory deals simply and directly with the issues of how prices are determined, how bidders use and benefit from their private information, and how private information affects prices. The insights summarized below, which are gleaned from the study of auctions, suggests some new ideas and approaches for security market theory.

Consider a sealed-bid auction in which a single object is offered for sale to n bidders. In general, the amount a bidder is willing to pay for the object depends on his personal preferences and the characteristics of the object. In a charity auction where a Carribean vacation for two is being sold, each bidder can determine an appropriate value for himself by introspection, and he can then make an intelligent bid based on that value and on his beliefs about how others will bid. In contrast to this case is the case where oil rights on an offshore tract of land are being offered: An appropriate value cannot then be determined by introspection because some major

[2] Milgrom (1981).

[3] Tirole (1980).

value determining characteristics—the amount of recoverable oil, its depth, pressure, etc.—are unknown. Experienced bidders base their bids on their estimate of the value, but they also bid cautiously to counteract a phenomenon known as the winner's curse. The winner's curse is the tendency of a winning bidder to find that he has overestimated the value of the prize he wins at auction. The curse arises because a bidder is more likely to win an auction when he has overestimated the value of the prize than when he has underestimated it. (Economists may recognize the winner's curse as a variant of the phenomenon known as adverse selection.)

Experienced bidders escape the ill effects of the winner's curse in auctions for oil, gas, and mineral rights by reducing their bids to allow a margin for estimation error and by carefully gathering and analyzing geological data to reduce that error. In those settings, it is clear that private information can be useful to a bidder in choosing his bid.

In addition to its value in decision making, private information can have a kind of strategic value in auctions. Bidders facing a well-informed competitor are particularly susceptible to the winner's curse. Consequently, the mere fact that a bidder has good information may lead the other bidders

to bid more cautiously, a response that benefits the well-informed bidder.[4]

The question of how much of the bidders' private information gets reflected in the selling price is a subtle one. One might think that the price resulting from a sealed-bid auction cannot reflect more information than was available to the winning bidder when he made his bid, since that bid determines the price. This reasoning, however, is incorrect. When the bid made by the winning bidder is known, it can be inferred that the other $n-1$ bidders made lower bids. The information which this inference conveys about the probable value of the mineral rights was unavailable to the winning bidder when he made his bid. Thus, the price resulting from a sealed-bid auction may reflect more information than was originally available to any individual bidder, a prediction which is intriguingly close to the predictions of the securities markets theories.

The essential feature that distinguishes bidding models from existing securities market models is not that there is competition on only one side of the market or that the objects being sold are indivisible nor even that the number of bidders is generally small: the essential feature is that the process of price

[3] Milgrom and Weber (1982a).

determination in bidding models is explicit. By examining this process, one is able to see how information can be used, why it is valuable, and how it affects prices.

In the remainder of this paper, a simple model of a competitive securities market is sketched in which the bid and ask prices are set by a group of <u>market makers</u>. When all traders have identical information, the equilibrium bid and ask prices are constant over time and equal to each other and to the price predicted by competitive equilibrium theory. However, when some traders have private information, the equilibrium ask price always exceeds the bid price. If the informed traders are speculators with no transactions motive for trading, then the market makers lose money to the informed traders but profit from the uninformed. The spread between the ask and bid prices generally results in a reduced volume of trade and an inefficient allocation of securities. Information has positive value to individual traders and prices come to reflect some or all of the information used by the informed traders.

Let there be a single marketed security offering a random gross return $\tilde{R}$. Let each of the n traders (indexed by i) be endowed with one share of the security and one unit of a nonstorable commodity money

and let each be risk neutral. Trader i observes two random variables $\tilde{X}_i$ and $\tilde{Y}_i$: the variable $\tilde{X}_i$ represents the utility available to i from immediate consumption and $\tilde{Y}_i$ represents information about $\tilde{R}$. The $\tilde{X}_i$'s are assumed to be independent and identically and non-atomically distributed, and, initially, they are also assumed to be independent of $(\tilde{R}, \tilde{Y}_1, \ldots, \tilde{Y}_n)$. To capture the anonymity of the market, it is further assumed that the joint distribution of $(\tilde{R}, \tilde{Y}_1, \ldots, \tilde{Y}_n)$ is invariant under permutations of the indices. The numbers 1 through n designate the order in which traders arrive at the market.

The k market makers are assumed to be risk neutral, and to have sufficient supplies of money and securities that any $k-1$ of them could accommodate the traders' net demands along any price path in the range of the equilibrium price function computed below. The market makers have no private information; even the ticker tape (the history of trading $\tilde{H}_{i-1}$ when trader i arrives) is assumed to be known by trader i . The market makers alternative investments yield $\tilde{X}_0 = 1$, and 1 is also in the support of the distribution of the $\tilde{X}_i$'s .

The market operates as follows. When a customer arrives, each of the k market makers simultaneously announces a bid price and an ask price. The trader, given his private information, his utility of current consumption and these prices, chooses among three options: to buy at the lowest ask price, to sell at the highest bid price, or to stand pat and consume his endowment. He makes the choice that maximizes his expected payoff. Borrowing and short-sales are prohibited.

The competition among the market makers resembles the well known Bertrand competition in oligopoly theory, and the Bertrand arguments show that, at equilibrium, the highest bid price b_i^* is equal to the highest price at which a market maker can at least break even in expectation. Similarly, the lowest ask price a_i^* is the lowest break-even selling price.

If trader i faced bid and ask prices b and a with $a \geq b$, he would sell at b if $b\tilde{X}_i > E[\tilde{R}|\tilde{H}_{i-1}, \tilde{X}_i]$ and he would buy at a if $a\tilde{X}_i < E[\tilde{R}|\tilde{H}_{i-1}, \tilde{X}_i]$. Calling the first of these random events " i sells at b " and the second " i buys at a ," the equilibrium prices can be written as follows:

$$b_i^* = \sup b\{E[\tilde{R}|\tilde{H}_{i-1}, i \text{ sells at } b] > b\}$$
$$= E[\tilde{R}|\tilde{H}_{i-1}, i \text{ sells at } b_i^*] \text{ , and}$$

$$a_i^* = \inf a\{E[\tilde{R}|\tilde{H}_{i-1}, i \text{ buys at } a] \leq a\}$$
$$= E[\tilde{R}|\tilde{H}_{i-1}, i \text{ buys at } a_i^*] \text{ .}$$

Notice that there is a tendency for traders to buy when they believe $\tilde{R}$ is high and to sell when they believe $\tilde{R}$ is low. Thus, the market makers are faced with a variation of the winner's curse. To avoid losing money, the market makers must build a margin of safety into their price quotations. It can be shown that if the density f_i of $\ln \tilde{X}_i$ satisfies $d^2 \ln f_i/dx^2 \leq 0$ and if $(\tilde{R}, \tilde{X}_1, \ldots, \tilde{X}_n)$ are affiliated (see Milgrom and Weber for the definition and properties of "affiliation"), then the equilibrium prices specified above satisfy this requirement; that is, $b_i^* \leq E[\tilde{R}|\tilde{H}_{i-1}] \leq a_i^*$. For example, if the $\tilde{X}_i$'s have lognormal distributions and if the $\tilde{X}_i$'s are independent estimates of $\tilde{R}$ with the monotone likelihood ratio property, then these requirements are satisfied.

There are three cases which serve to illustrate the qualitative predictions of this model. In the first case, the $\tilde{X}_i$'s are degenerate, and convey

[5] Milgrom and Weber (1982b).

no information about $\tilde{R}$. It is then plain from the formulas given above that $a_i^* = b_i^* = E[\tilde{R}|\tilde{H}_{i-1}] = E[\tilde{R}]$: all trading takes place at the competitive equilibrium price. In the second case, the $\tilde{X}_i$'s are degenerate and all equal to 1, but the $\tilde{Y}_i$'s are non-degenerate. Comparing the price formula in this case with the trader's trading rule, it is clear that no trade can take place. The equilibrium bid price is set so low and the ask price so high that all potential traders are discouraged. The third case arises when we assume that trader i gets information about $\tilde{R}$ only if he has no transactions motive for trading with the market makers, i.e., only if $\tilde{X}_i = 1$. In this case, it can be shown that if any trading involving the informed traders takes place, then these traders impose losses in expectation on the market makers. Since the market makers break even in expectation overall, they must profit from trading with the uninformed.

The foregoing theory has some testable implications concerning security prices and trading volumes. It predicts, for example, that closely held firms (where there are relatively many traders with private information) should exhibit larger bid-ask spreads and less active trading among outsiders than comparable firms where insider holdings are small.

The model described in this paper is very specialized, but it seems likely that the qualitative conclusions can be extended to much more general settings. Once the process of price determination is made explicit, many of the contradictions and apparent paradoxes that plague the traditional theories of securities markets disappear, and a coherent alternative theory begins to emerge.

REFERENCES

Grossman, S. (1976). "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information," Journal of Finance, Vol. 31, pp. 573-85.

Milgrom, P. (1981). "Rational Expectations, Information Acquisition and Competitive Bidding," Econometrica, Vol. 49, pp. 921-43.

Milgrom, P. and R. Weber (1982a). "The Value of Information in a Sealed-Bid Auction," Journal of Mathematical Economics, Vol. 10, pp. 105-14.

Milgrom, P. and R. Weber (1982b). "A Theory of Auctions and Competitive Bidding," Econometrica, Vol. 50, pp. 1089-122.

Tirole, J. (1980). On the Possibility of Speculation under Rational Expectations. Cambridge, Mass.: Massachusetts Institute of Technology Press.