10 An Essay on Price Discrimination

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1 INTRODUCTION

The modern theory of price discrimination began with the work of Pigou (1920). Joan Robinson devoted two chapters of her book *The Economics of Imperfect Competition* (1969) to the problem of ('third degree') price discrimination. Her account examines the conditions that make price discrimination possible, presents a graphical analysis of the discriminating monopolist's pricing decision which has become the standard textbook treatment, and ends with an inquiry into the consequences of price discrimination for both allocative efficiency and distributional equity. Although Pigou's and Robinson's contributions have proved of lasting value, the theory of price discrimination has by no means remained unaltered.[1]

The meaning assigned to the term 'price discrimination' has stretched and changed since Pigou and Robinson wrote, and there is little current consensus on precisely what price discrimination means. Like her predecessors and contemporaries, Robinson defined price discrimination as 'the act of selling the same article, produced under a single control, at different prices to different buyers' (Robinson, 1969, p.179). In the modern view, such a definition is too narrow in its focus on the 'same' article and too broad in holding all price variations to be discriminatory. Even competitive prices for an article would generally vary according to the time and place of delivery, the amount of lead time required, the warranties or auxiliary services demanded, etc. And to the extent that the size of an order affects the cost of filling it, competitive prices would naturally vary with order size, too. Such variations cannot be usefully regarded as exercises of monopoly power, so they are best excluded from any useful definition of price discrimination.

To identify price discrimination, where economists from the early twentieth century had emphasized finding physically identical goods with different prices, modern economists emphasize the motives of the price-setter to create a situation in which the price charged to a customer can be based partly on the value of the good to the customer, rather than just on the cost of producing the good. Although this change sometimes makes the theory less specific and operational, it also achieves an economy of thought

and a unity of principles that are important for analyzing the vast and increasing number of ways in which firms seek to exercise their monopoly power.

To illustrate the modern perspective, consider the pricing of air travel. Although first-class air travel differs in seat size and amenities from coach class, the huge fare differentials between the two classes of service are discriminatory. What makes this clear is that the two classes of travel provide essentially the same service at prices (1) that differ far more than proportionately to any cost differences, and (2) that are designed to exploit differences in various travellers' willingness to pay for amenities, or to pass on the costs to others. Moreover, they are part of a complex pattern of pricing in which prices may depend on how long in advance reservations are made, how many days the traveller stays at his destination, whether he stays over a Saturday night, etc. Notice that fares are not made to depend on things that might affect airline costs, such as the need for special meals or whether the customer smokes. Instead, prices are made to depend on factors that are primarily related to the price a buyer might be willing to pay. It is this pattern that identifies the pricing policy as discriminatory.

A full account of the modern concept of price discrimination cannot be given by focusing on pricing alone. Pricing practices are intimately connected with product design decisions, bundling decisions,[2] and other marketing practices. For example, a firm may specialize its product line to make it possible to charge different prices to different customers. The different classes of airline travel cited above are but one instance of this sort of specialization; any product that comes in several varieties from apples and automobiles to video cassette recorders and washing machines is a candidate for this sort of manipulation.[3]

Another example of price discrimination in which non-price practices play a key role is the vertical price squeeze. Thus, suppose that there are two classes of users for a product. Suppose that users in the first class would be willing to pay quite a high price for the product, perhaps because this is the only product that can meet their particular needs. Users in the second class would be willing to pay only a much lower price. The seller would like to charge different prices to the different groups, but suppose that legal restrictions, competition, or the possibility of resale from the low price customers to the high price ones make that sort of discrimination impossible. If the low value user is an industrial customer, the supplier may be able to achieve the desired discriminatory effect by integrating forward and setting the transfer price on supplies transferred to its subsidiary below the market price charged to its customers. Adams and Dirlam (1964) report an instance of such a vertical price squeeze in the steel wire products industry, alleging that integrated steel producers sold certain steel wire products (which faced heavy competition from non-steel alternatives) for nearly the same price as the steel wire used to make it.

The received theory of price discrimination is a part of the theory of monopoly and incorporates that theory's framework and biases. In particular, it incorporates the assumption that sellers set their prices while customers act as price takers, buying whatever quantity suits their tastes at the given prices. In other words, it assumes that the monopolist-seller holds all the bargaining power. In Section 2, I explore the nature of monopoly and price discrimination in the presence of countervailing buying power. I argue against the usual presumption that in markets with a single seller and many buyers, the seller holds a great bargaining advantage.[4] The relative bargaining power of the parties depends on their costs of bargaining, on the ability of the buyers to form coalitions, on the possibilities for resale, and on laws governing price discrimination. Laws against price discrimination, far from benefiting the buyer, may actually enhance a monopolist-seller's bargaining position by making it credible to its negotiating partners that the monopolist will hold out for the same price that other buyers are paying. Of course, this conclusion depends on the customers not forming a coalition or buyers' union to coordinate their bargaining. I comment on the possibilities for a buyers' union as well in Section 2.

In Section 3, I investigate how a firm can design its product line to enhance its ability to engage in price discrimination. Much of the analysis is conducted in terms of a formal model in which customers differ according to how much they are willing to pay for quality. The difficulty the monopolist faces is that products designed for one class of customers may cannibalize the sales of a more profitable product targeted at another class of customers. Some aspects of this problem have been studied before. I give a graphical solution to the problem based on Milgrom (1980) and a parallel analytical treatment. Similar analytical treatments have been given by Harris and Raviv (1981) and Maskin and Riley (1984).

The upshot of the analysis is that the monopolist's product design and pricing decision involves trading off the profits to be earned on direct sales of any particular product against the losses that result in the sales of more profitable products. The design decision is thus distorted away from efficiency, and certain groups who could be economically served by the monopolist might not be served at all. Such distortions do not arise, however, in the design of the most profitable product, since the monopolist is not concerned that customers may switch away from other products to it. Another finding is that an optimal pricing scheme may make the price paid for a given product depend on any random event whose occurrence is correlated with the buyer's willingness to pay. The use in the airline industry of advance purchase requirements with high cancellation fees can be interpreted in this way. That interpretation makes it possible to integrate our analysis with certain related developments in auction theory (Milgrom and Weber, 1982; Milgrom, 1986) and in the theory of contracts (McAfee and McMillan, 1986).

2 DISCRIMINATION AND COUNTERVAILING BUYER POWER

Textbook treatments of monopoly theory and oligopoly theory begin from the premise that the producers can set the prices of their products, and that buyers act as if they were powerless to affect the prices. That premise is by no means self-evident; it needs to be examined both theoretically and empirically. Whether there is a single seller or many, the customer can always *try* to haggle over the price. His success will depend on the relative strength of his bargaining position, which is determined by many factors, including the availability of alternative suppliers and/or alternative customers, the enforceability of restrictions on resale, legal restrictions on price discrimination, and the ability of buyers or sellers to collude or bargain as a group.

The informal arguments most often heard in support of the idea that the monopolist has complete latitude in setting the price rely on the idea that there are other customers who would be willing to buy the marginal unit for the price the monopolist is asking. However, such arguments are erroneous. To see the flaw, suppose that the monopolist produces using a constant returns to scale technology.[5] Then, it will not benefit the seller to offer any refused units to another buyer, so how can the presence of other buyers affect the negotiations with any one buyer? Indeed, how is this monopoly situation different from a collection of unrelated bilateral monopoly problems, in which the seller haggles over the price with each buyer separately?

Let us rephrase the argument to emphasize its cogency. If the monopolist can produce at constant marginal cost over the relevant range of outputs, then the feasible set over which the monopolist and any customer can bargain is independent of the bargains struck with other customers. So, why should the presence or absence of other customers matter? Why should one suppose, as standard textbooks would have us do, that the mere fact that a monopolist is a monopolist makes it immune to attempts at negotiations by its customers? Of course, customers do sometimes bargain with their monopolist-suppliers. That suggests turning the question around: why don't they do so more often?

Answers to these questions in industrial organization should be related to the similar questions that arise in labor economics: how are the negotiations between, say, the United Auto Workers (UAW) and Ford Motor Co. affected by the fact that the union will subsequently negotiate with the other auto-makers? There are two effects at work here. If the auto-makers are to remain competitive, any concessions that the UAW makes to Ford will have to be granted to the others as well. Second, the way the UAW bargains with Ford signals its willingness to hold out for a favorable contract in the subsequent contract negotiations. Then, as arguments in the game-theoretic literature on reputations (introduced by Kreps and Wilson (1982) and Milgrom and Roberts (1982)) make clear, the UAW gains strength in

its negotiations with Ford because Ford knows that the union has double reason to carry out its threats – to extract concessions from Ford and to signal its seriousness to its later bargaining partners. Thus, reputation effects make the UAW's threat more credible and may enable it to strike a better bargain.

A general reputation argument would hold that a monopolist can gain leverage in bargaining by linking its negotiations among several customers. Linkages of that sort may be impossible to achieve if, for example, rebates that the seller pays to buyers or special services that the seller provides cannot be observed. Even if linkages could be achieved, the seller might not choose to achieve them. If the UAW thought that Ford was in a strong bargaining position, it might prefer to bargain less aggressively and out of the public eye rather than to be forced into a lengthy strike to demonstrate its resolve to the other auto-makers.[6]

This example of the role of reputations in bargaining illustrates the monopolist's fundamental dilemma: as in many bargaining situations (see Schelling, 1960) its bargaining position is enhanced by commitments that make concessions too costly to grant, but these same commitments and costs also result in a loss of flexibility. The monopolist who charges different prices to different customers cannot claim convincingly that it will not make a price concession to a hard bargaining customer.

As a first, simple example of these ideas, consider the problem of a monopolist who faces a large number of customers each of whom wishes to buy a single unit of his product. Each customer i values a unit at $2V_i$. Suppose the production cost is zero. If (i) the monopolist's product cannot be resold, (ii) there are many customers, and (iii) bargaining is efficient and the bargainers have equal bargaining power in the sense that the gains from trade are to be split equally, then the seller receives a price of V_i from each customer i. Plainly, different customers pay different prices for the same good. However, our story is not the textbook story of price discrimination; although the monopolist does set a different price for each customer, it does not capture all the surplus. The monopolist's profits from this sort of price discrimination are represented in Figure 10.1 with demand curve BCD by the area OAD.

If the monopolist does not (or cannot) prohibit resale, then all sales will be made at the lowest agreed price. The bargaining outcome in this situation may depend on details of the way bargaining is conducted. One plausible outcome of this situation is that all customers would pay the monopoly price. The monopolist would certainly never want to bargain for a higher price, since to do so would only reduce his profits. And, using our assumption of many buyers and the additional assumption that prices are posted in discrete units, the monopolist could not be swayed to accept a lower price by any single buyer's threat not to buy, since even if the threat were believed it would not pay the monopolist to accommodate the buyer.
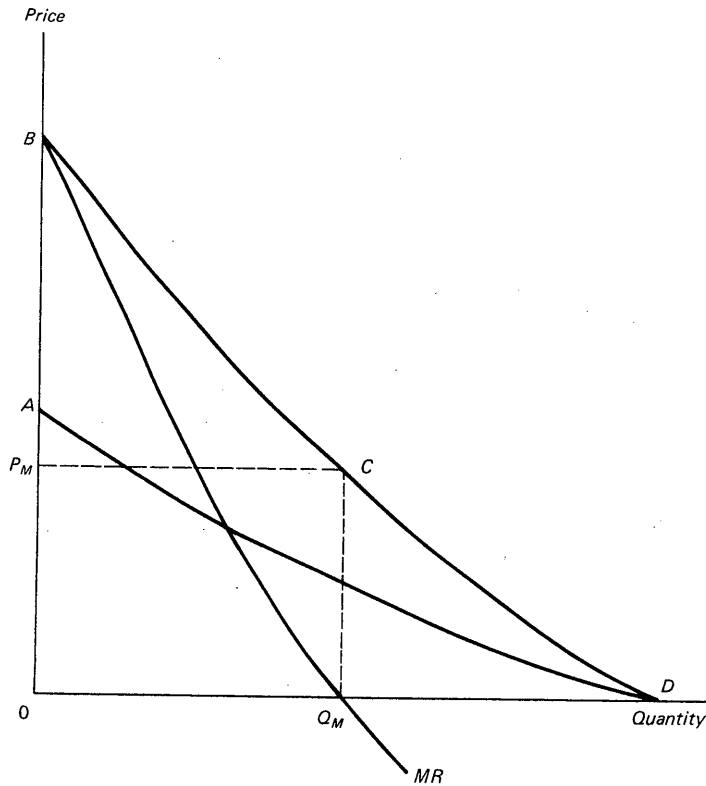
Figure 10.1

In Figure 10.1, the monopolist's profits in this case of simple monopoly pricing are represented by the rectangle $OP_M CQ_M$. For the case of linear demand, price discrimination leads to the same level of profits as simple monopoly pricing. For general demand, profits may be either higher or lower. Contrary to received theory, the monopolist has no *general* preference to charge individual prices to different customers. And, depending on the demand curve, *the monopolist may actually be better off when he is unable to prohibit resale!*

A similar point could be made in an analysis of reputational effects. There one finds that the monopolist would prefer sometimes to maintain a reputation of never compromising on price and sometimes to sacrifice that reputation in order to be able to haggle with individual customers. A full analysis of reputation issues would take careful account of how much of the

monopolist's pricing behavior can be observed, and when. We shall not develop those issues here.

Instead, we return to another question that was raised earlier: why are price negotiations not more common? In retail stores in the Western world, the seller typically sets the price and, except for a few expensive durables like cars and component stereo systems, the customer is given little chance to negotiate. One plausible explanation is that bargaining costs are too high to make it worthwhile to haggle, especially for inexpensive items where large price adjustments are unlikely to be obtainable. To guide the remaining analysis, let us hypothesize that bargaining does not go on over small items in stores and elsewhere due to the costs of bargaining, which are large compared to the purchase price for small purchases. On the buyer's side, these costs are mostly costs of the time consumed in negotiations, combined perhaps with a simple distaste for bargaining. On the seller's side, there are additional costs: those of hiring and training the skilled workers needed for effective bargaining, the costs of monitoring sales workers to ensure that they report the actual price paid, or the resulting loss suffered when the reported price is lower than the actual selling price and the worker pockets the difference. Let us see what implications these bargaining costs have for the theory of price discrimination.

Thus, suppose a monopolist supplier who produces at constant unit cost c faces n buyers. Each buyer desires only a single item, and values the item at unity. For example, if the buyers are manufacturers, the first unit of the input for each buyer has marginal revenue product equal to one; subsequent units have MRP equal to zero. What will be the price?

If the buyers and the seller have equal bargaining power, one might expect that if they do bargain, they will divide the surplus of $1-c$ from their relationship equally. The price negotiated between the seller and each buyer would then be $p=(1+c)/2$. Assume that bargaining costs take the form of a fixed cost b that is smaller than the bargainers' rents on the transaction: $b<p-c=1-p$. If the monopolist posts a price not exceeding $b+p$, buyers will face the choice of accepting that price or incurring the cost b to obtain the price p. They will prefer to accept such a price without haggling, to save on bargaining costs. If the monopolist posts a higher price than $b+p$, buyers will choose to incur the cost and haggle to reach the price p. Thus, the unique (subgame perfect) equilibrium of monopoly pricing game in which the seller sets a price and the buyer then chooses to bargain or not calls for the monopolist to set the price $b+p$ and the buyer to accept that price without haggling. Of course, if buyers differ in their bargaining costs, the price will be set so that some haggle while others do not.

Now suppose that resale is possible or that price discrimination is prohibited, so that any price concession granted to one buyer must be granted to all. The n bargaining problems are now clearly connected. Let us represent that as follows. First, the seller names a price p^*. Then, the buyers

decide, independently and simultaneously, whether to bargain and incur the cost b. If the seller sets a price of p^* and k buyers bargain, let p' be the resulting price. Specifying p' is tricky; it depends on how the multilateral interlinked negotiations proceed. For simplicity, let us suppose that the buyers who bargain join together and negotiate as if they were a single party; this plausibly is the best joint strategy for the coalition to follow, since it gives them their most powerful threat to use against the seller. Then, as we shall see, the power of the buyers is limited by a free rider problem – individual buyers are reluctant to join any buyer coalition.

We shall use the Nash bargaining solution to guide our analysis. The 'no agreement' point yields rents of $(p^* - c)(n - k)$ to the monopolist and zero to the k negotiating buyers. If a (non-random) price of p' is agreed upon, the monopolist gets rents of $(p' - c)n$ and the bargainers get $(1 - p')k$. Varying p', this identifies the Pareto surface for the bargaining problem. For $k = 0$, the solution of the game has $p' = p^*$; otherwise, the Nash bargaining solution sets:

$$p' = p + \frac{(p^* - c)(n - k)}{2n}.$$

Recall that $p = (1 + c)/2$ is the price that would prevail in case $k = n = 1$. It is evidently also the price that prevails whenever $k = n$, since the buyers then bargain as one.

The marginal reduction in price when a second, third, ... buyer joins the buyers' coalition is $(p^* - c)/(2n)$. So, at a pure strategy equilibrium, at most one buyer will enter the negotiations unless b is less than that amount, that is, unless p^* exceeds $2nb + c$. No firms will ever enter negotiations at any (subgame perfect) equilibrium if p^* is also less than $p' + b$ for the case $k = 1$. Hence, all the downstream firms will buy without entering the negotiations if (i) $p^* \leqslant 1$, (ii) $p^* \leqslant 2nb + c$, and (iii) $p^* \leqslant p' + b$, that is, if

$$p^* \leqslant \text{Min} \left\{ 1, 2nb + c, \frac{2n(b + p) + (n - 1)c}{n + 1} \right\}.$$

If the upstream monopolist sets $p^* = \text{Min}(1, 2nb + c)$, at most one firm will bargain at any pure strategy equilibrium. The negotiated price is then:

$$p' = p + \text{Min} \left\{ (n - 1)b, \frac{(1 - c)(n - 1)}{2n} \right\}.$$

This price is therefore a lower bound on what the monopolist can expect to get when resale is possible or price discrimination is prohibited. If the bargaining costs b or the number of customers n is large, the negotiated price is at least $1 - (1 - c)/(2n)$; it is possible that the equilibrium price is

even higher. Thus, for n large, the possibility of resale or a restriction against price discrimination assures that the monopolist has all the bargaining power.

Let us summarize our findings in this section. First, the traditional notion that the monopolist sets what price it will while buyers act as price takers has been found to be no better founded in theory than it is in reality: *Buyers bargain when the bargaining costs are sufficiently low relative to the potential gains from negotiations.* Second, the textbook proposition that price discrimination enhances the seller's profits is similarly unfounded: *price variations among customers can reflect variations in their relative bargaining power, with large customers generally getting lower unit prices.* Third, restrictions against price discrimination and against contracts that prohibit resale do not generally serve the interests of buyers. Instead, *legal pricing restrictions can help the seller to commit itself credibly to a policy of not retreating from a very high price.*

3 SEGMENTING A MARKET USING SELF-SELECTION

According to Mrs Robinson's account, a necessary condition for price discrimination is that the monopolist be able to divide its markets into separate parts with different demand elasticities. It must be impossible for a customer in the dear market to make its purchases in the cheap one, or for arbitrageurs to buy in the cheap market for resale in the dear one.

When price discrimination is profitable, sellers cannot be expected to accept passively whatever barriers exist among markets. Instead, they will tailor their product lines and pricing policies to exploit differences among consumers.

Our purpose in this section is to investigate how a monopolist might engage in price discrimination when it holds a hegemony of bargaining power but cannot distinguish low- from high-value users who make their purchases in the *same* market. We consider two devices: product design and linking the effective price paid to a 'random' event that occurs *after* the time of initial purchase.

In general, suppose there are N classes of downstream users whose demands depend on a vector of product attributes a. Suppose that each user buys only one grade of the product – or only one grade for each envisioned use. Let $V(Q, a, n)$ be the value obtained by a customer of class n from purchasing Q units of a product of grade a. Suppose, too, that the different classes of the product are perfect substitutes in production, and are produced using a constant-returns-to-scale technology with unit cost c. Let $M_1, \ldots, M_N$ be the number of buyers in each class. What products would a profit-maximizing monopolist produce? At what prices?

Let us label the product varieties offered by the monopolist and bought

by some customer from 1 to N and in such a way that a customer of class j prefers the j-th product. (Some products may have two or more labels.) Given his class n and the menu of product offerings $a_1, \ldots a_j$, the customer will choose a product design j and quantity Q to maximize:

$$V(Q,a_j,n) - Qp_j.$$

By our choice of labelling, the optimal value of j will be $j = n$. Also, let us normalize V so that $V(0,a;n) = 0$. Notice that our formulation abstracts from discrimination using volume discounts, because the price per unit p_j does not vary with the number of units Q that are bought.

The monopolist takes buyer behavior as given. Its problem is then to choose products a_j and prices p_j to maximize its product. What is the monopolist's optimal choice of product designs and prices? One possibility is for it to design and price products separately for each of the N classes to maximize monopoly profits earned from that class. This may work well if the resulting product variations are sufficiently poor substitutes. However, being variations of the same product, it is likely that some of the variations are good substitutes, so that a buyer of class j might prefer to buy the product designed and priced for another class of buyers. Thus, the monopolist's problem is constrained by the following inequality, which asserts that each buyer must prefer to buy the product 'designed' for its class.

(IC) $\underset{Q}{\text{Max}}\, V(Q,a_n,n) - Qp_n \geqslant \underset{Q}{\text{Max}}\, V(Q,a_j,n) - Qp_j$, for all n and j.

In modern jargon, (IC) is an *incentive constraint*. It requires that if a customer chooses to buy anything, he will buy the intended product. The incentive constraints do not preclude the possibility that a customer will not want to buy anything. The discriminating monopolist may choose not to design any acceptable product for some classes of buyers. Formally, it is simplest to deal with this possibility by including among the possible product specifications a the 'useless' product, for which the optimal quantity is always zero.

Sometimes, the incentive constraints will impose severe limits on the monopolist's freedom in pricing its products without having any product switching among its customers. For example, if the product in question is a chemical compound whose only relevant attribute is its purity, with greater purity being preferred in all uses, erecting barriers between different customer groups may be especially difficult. However, even in that case, the monopolist may be able to exploit variations in customers' marginal willingness to pay for quality to design a profitable product line.

Figure 10.2 displays an example of this kind. It is worked out for the case where the attribute space is one-dimensional ('quality') and the valuation function is $V(Q,q,n) = \text{Min}(Q,1)[b_n q - q^2/2]$. This valuation function ensures that the consumer's optimal quantity choice Q is always either zero or one. That allows us to write the incentive constraints more simply as:

(IC') $\quad V(1,q_n,n) - p_n \geqslant V(1,q_j,n) - p_j$, for all n and j, and

(PC) $\quad V(1,q_n,n) - p_n \geqslant 0$ for all n.

This last constraint is sometimes called a *participation constraint*, because it reflects the assumption that the buyer can refuse to participate, that is, he can opt not to make any purchase at all.

Suppose that consumer types differ in their marginal valuations of the single attribute – quality. The marginal valuation of quality curve for each
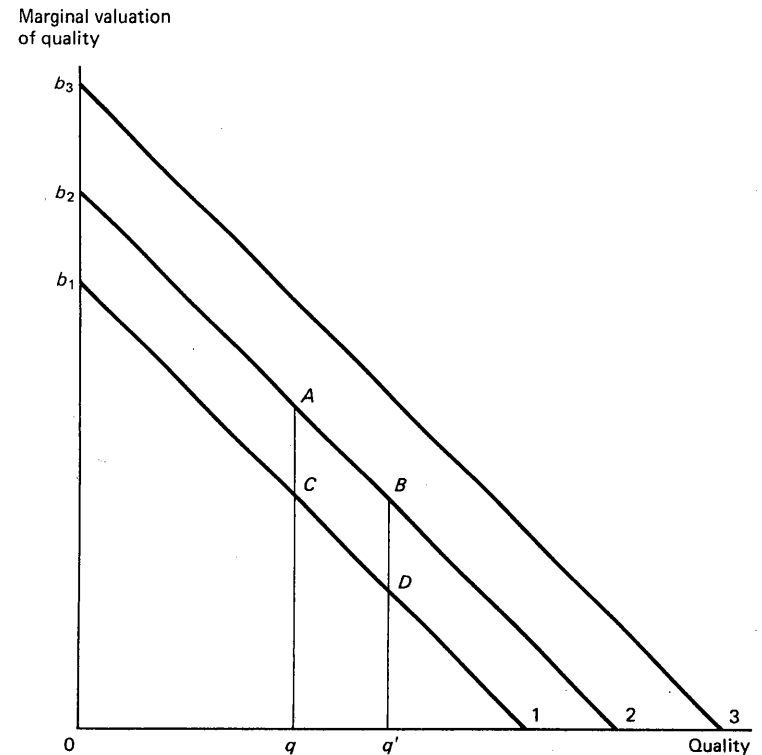

Figure 10.2

class of customer is plotted in the figure. Classes 1, 2, and 3 represent the low value, middle value, and high value users, respectively. In this example, the marginal valuations of quality are shown as linear with slope one. We shall characterize the optimal solution to the product design problem for this specialized example. We shall then recapitulate the general lessons to be learned.

The mathematical statement of the discriminating monopolist's problem is:

$$\underset{p_1,\ldots,p_N,q_1,\ldots,q_N}{\text{Max}} \qquad M_1 p_1 + \ldots + M_N p_n$$

subject to
   (IC′) holds for all n and j, and
   (PC) holds for all n.

A mathematical account of the solution to the monopolist's problem in which N is unrestricted is given in the Appendix. The graphical account given below corresponds both in its outline and nearly all its details to the mathematical account. And, for ease of illustration, we set $N = 3$.

   Consider the question: is it possible for the monopolist to arrange matters so that a good of quality q is bought by the medium or high value users while a good of some higher quality q' is bought by the low value users? The answer is 'No', as can be argued using the figure. The argument uses only two of the six incentive constraints (IC′). The low-value users prefer quality q' to quality q if and only if $p' - p$ is less than the area $qCDq'$. The medium value users prefer q' to q if and only if $p' - p$ is less than the strictly larger area $qABq'$. So medium-value users must prefer the higher quality product whenever low value users do. Similarly, the high-value users prefer higher quality products whenever medium- or low-value users do.

   Thus, the monopolist's problem is to select three levels of quality $q_1 \leqslant q_2 \leqslant q_3$ and corresponding prices p_1, p_2, and p_3 to maximize total revenues, subject to the incentive constraints. In Figure 10.3, the first participation constraint for the first class of buyers requires that p_1 does not exceed area $OCFq_1$. The shaded area to the left of q_1 shows a p_1 satisfying that constraint. The constraint that class 2 users do not prefer the quality q_1 product requires that $p_2 - p_1$ do not exceed the area q_1EHq_2, and the constraint that class 1 users do not prefer the quality q_2 product requires that $p_2 - p_1$ be at least q_1FIq_2. The shaded area to the left of q_2 and the right of q_1 shows a price differential $p_2 - p_1$ satisfying these constraints. Finally, the shaded area to the left of q_3 and right of q_2 shows a price differential such that class 2 users do not prefer quality q_2 to q_3 while class 3 users do.
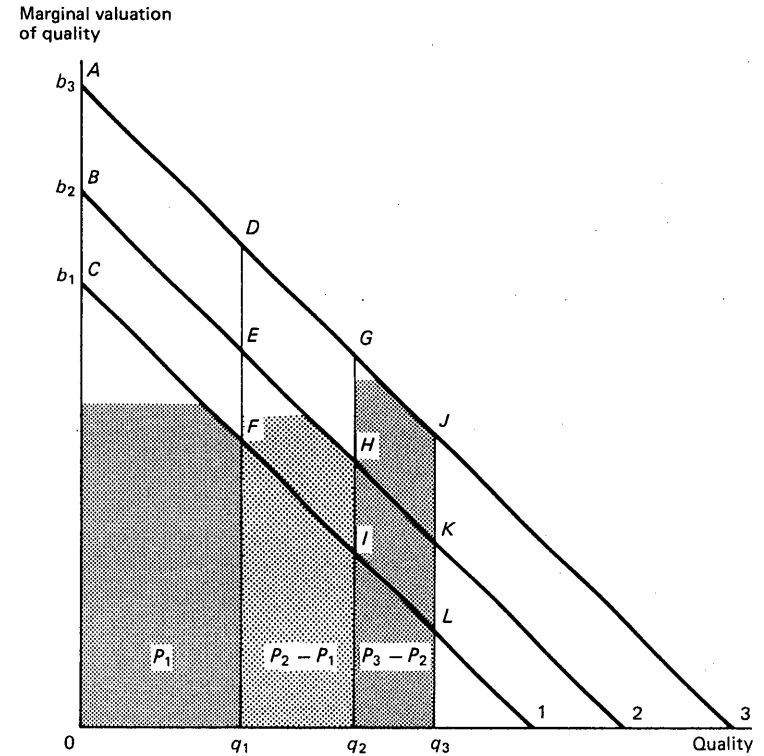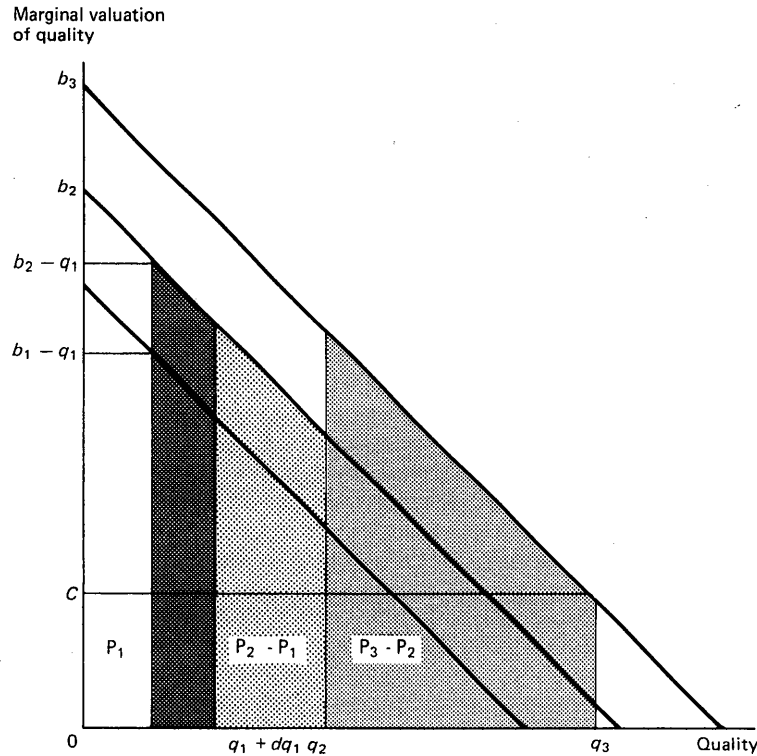

Figure 10.3

Now, because the marginal value of quality increases with increasing class index, all the remaining incentive and participation constraints hold automatically, as can be seen from the figure. Thus, the only constraints of relevance are (i) the participation constraint for the lowest value customer, and (ii) the constraints that limit the price differentials so that a class i buyer does not prefer the quality q_{i-1} or quality q_{i+1} product.[7] All the other constraints are implied by these constraints and so can be excised from the problem.

   By inspection of the figure, given any qualities q_1, q_2, and q_3, that the monopolist may select, the revenue-maximizing prices for him to set satisfy $p_1 = \text{Area } (OCFq_1)$, $p_2 - p_1 = \text{Area } (q_1EHq_2)$, and $p_3 - p_2 = \text{Area } (q_2GJq_3)$. With the pricing portion of the problem thus solved, our task is reduced to finding the profit-maximizing quality levels.

   Given a triple of quality levels, we use Figure 10.4 to analyze the effect of

Marginal valuation
of quality

Optimality requires:
(1) $(b_1 - q_1 - c)M_1 = (b_2 - q_2)(M_2 + M_3)$
(2) $(b_2 - q_2 - c)M_2 = (b_3 - q_2)M_3$
(3) $(b_3 - q_3 - c)M_3 = 0$

Figure 10.4

a marginal increase dq_1 in the quality level q_1. The additional quality provided to the M_1 purchasers of quality level q_1 costs the monopolist $M_1 c dq_1$. Also, as the figure shows, the increase in quality leads the monopolist to raise p_1 by approximately $(b_1 - q_1)dq_1$ and to reduce $p_2 - p_1$ by approximately $(b_2 - q_1)dq_1$. Note that $p_3 - p_2$ is unaffected; *the increase in quality of dq_1 forces a price reduction of $(b_2 - q_1)dq_1$ for all higher quality products in the line.* Thus, the change results in additional net revenues from the M_1 customers purchasing quality level q_1 of $M_1(b_1 - q_1 - c)$ and reduced revenues from the $M_2 + M_3$ customers purchasing higher levels of quality of $(M_2 + M_3)(b_2 - q_1)$.

Turning back to the formal problem, if there is an interior optimum (with $0 < q_1 < q_2 \leqslant q_3$), the marginal gains and losses must be equated:

$$M_1(b_1 - q_1 - c) = (M_2 + M_3)(b_2 - q_1).$$

It is convenient to rewrite this in the form:

$$\frac{b_2 - q_1}{b_1 - q_1 - c} = \frac{M_1}{M_2 + M_3}.$$

Since increasing q_1 beyond $b_1 - c$ results in lower net revenues from all groups of purchasers, the optimum requires that $q_1 < b_1 - c$. Then, since $b_2 > b_1$, the left-hand side above is increasing in q_1; it becomes infinite as q_1 approaches $b_1 - c$. If the left-hand side is greater than the right at $q_1 = 0$, then the optimal q_1 is zero; it does not pay to design a product for the lowest value users. Notice that this happens when the marginal value for the low-value users is quite low relative to higher value users and when the low-value users are relatively few in number. In the case of an interior optimum, there is a unique value of q_1 that satisfies the equality.

Similarly, at an interior optimum, q_2 is set to satisfy:

$$\frac{b_3 - q_2}{b_2 - q_2 - c} = \frac{M_2}{M_3}.$$

The left-hand side is increasing in q_2. If the left-hand side, evaluated at $q_2 = q_1$, is less than the right-hand side, then there is a unique $q_2 > q_1$ for which the necessary equality holds. If there are too few medium-valuation users, it may happen that the left-hand side is greater than or equal to the right. In that case, $q_2 = q_1$ at the optimum, and a single product must be designed for the two groups of consumers. We shall discuss this possibility in more detail below.

Finally, improving the quality q_3 of the highest quality product and increasing its price correspondingly does not affect the pricing of the other products. Its effect is to increase revenues under the pricing rule by $(b_3 - q_3)dq_3$ and costs by $c dq_3$ for each of M_3 customers. Hence, at the optimum, $b_3 - q_3 = c$. The quality level offered to the third class of users satisfies the first-best efficiency condition.

So far, we have limited attention primarily to the case where the quality levels offered to the several categories of customers are distinct. In our model, this means the case $q_1 < q_2 < q_3$. We noted, however, that it could happen that $q_1 = q_2$ and, in more general problems, two or more consecutive groups may be offered identical products. In the present example, if $q_1 = q_2$ at the optimum, then the extra net revenues obtained by raising q_1 (and therefore also q_2) by an amount dq_1 are $(M_1 + M_2)(b_1 - q_1 - c)dq_1$, and these

are offset by a loss of revenues of $M_3(b_3 - q_1)dq_1$ from customers purchasing the high quality product. The customer group 2 is most likely to have a product targeted for it at an optimal solution if its members are numerous relative to other groups and if its valuation of quality differs substantially from those who value quality less. It is most likely to be pooled if the reverse conditions hold, that is if it is a relatively small group whose members' preferences are similar to those of consumers who value quality less.

There are several general lessons suggested by our analysis. First, the product designed for one class of buyers imposes limits on the prices set for other classes. This can lead to distortions in the product design decision. In our example, the monopolist set the quality of the product for the customer groups 1 and 2 at a point where the target customer's marginal willingness to pay for quality $(b_i - q_i)$ was greater than the marginal cost.

Second, the monopolist will not always offer a separate product for each customer group. New products are added to the line only if the group of customers they attract are sufficiently large relative to the number of buyers of more profitable products, or if the new product is sufficiently unattractive to buyers of more profitable products. In our model, the profit margin was an increasing function of product quality, so the number of buyers of more profitable products was the same as the number with a higher willingness to pay for quality. This fact simplified both the graphical treatment and the mathematical analysis.

Third, in our model, the design of the top-of-the-line product line was efficient, given the target customer group; the highest valuation customer's marginal willingness to pay for quality $(b_3 - q_3)$ was equated to the marginal cost c. This last observation reflects a general principle that holds for a wide variety of product line design problems and has analogues for many other problems as well: The most profitable product in a profit-maximizing line will always be designed efficiently for its intended user group, for it can be designed without concern that purchasers of more profitable products might switch to it.

Throughout this analysis, one product attribute – price – has been given a distinct and special role. This is not because of any special demand attribute of price but because price enters directly into both the seller's profits and the buyer's welfare. A creative seller need not specify a single price for his product, but can charge prices that depend on product usage or related variables.

For an example along these lines, consider the form that price discrimination has taken in the United States airline industry in the 1980s. There, an attempt was made to discriminate between business travellers, who have a relatively inelastic demand for air travel, and vacation travellers, whose demand is much more elastic. Partly, the discrimination has taken the form of restrictive rules on travel, limiting the time of day that certain fares were available or charging lower fares on flights that involved staying over a Saturday night. However, these restrictions still allowed some business travellers to take advantage of the lower fares intended for vacation travellers. The problem, as it would appear to an airline, was to find a way to charge business travellers more for a product with *identical* characteristics, where the usual devices of self-selection are not available.

One device that the airlines found effective was the use of cancellation charges for many types of fares. Vacation travellers typically arrange dates with their employers, or with co-travellers; so they may be expected to cancel infrequently. Business travellers' plans are more subject to last minute changes. Consequently, business travellers pay more cancellation charges more frequently, and so pay a higher average price per flight than vacation travellers.

The device of linking the price paid by a purchaser to some random event over which he has limited control has been found valuable in other applications as well. Milgrom and Weber (1982) labelled a mathematical analogue of this observation the *Linkage Principle*, and applied it to show how certain kinds of auction mechanisms did systematically better than others in terms of the expected price they obtained. The fourth lesson of our analysis is that sellers can generally benefit by applying the Linkage Principle, that is, using information not subject to the buyer's control in setting the price.

Our formal analysis has been highly stylized both in its limited specification of possible demands and in its rarely satisfied assumption that products can be varied on only one dimension. Nevertheless, as we have seen, it suggests general insights and principles and helps to illuminate the trade-offs that a discriminating monopolist faces and to account for the limited quality spectrum offered by individual manufacturers for items ranging from food processors to furniture.

4 CONCLUSION

Half a century ago, Joan Robinson gave the standard treatment of third-degree price discrimination by a monopolist facing separated markets. In the ensuing years, the scope of the theory of price discrimination has grown to include virtually all the practices that a monopolist, protected from entry, might use to increase its profits. Throughout it all, the principles of monopoly theory on which the entire edifice was based have gone unchallenged. Here, we have challenged the basic principles of monopoly and price discrimination theory according to which the seller has full power to set what price it will. We have also advanced the analysis of applications for those cases in which the seller does have unchecked power to set the price. In the latter case, we have emphasized principles of product line analysis

design that we expect to apply whenever the monopolist can control his product design, even if he must sometimes negotiate prices for specific customers.

First, and most fundamentally, we have challenged the notion that the monopolist-seller has all the bargaining power and that even first-degree price discrimination allows it to extract all the surplus generated by exchange. The different amounts paid by different users more plausibly represent the natural outcome of individual bargaining between the monopolist and its customers. Generally, bargaining of this sort leads to a sharing of surplus between the buyer and seller. Laws that prohibit price discrimination or restrictions on resale strengthen the seller's hand in bargaining and create a free-rider problem among the buyers. Our analysis is important because it calls into question the foundations of monopoly theory, which pays too little attention to the countervailing strategies available to customers.

Second, we have examined how a firm might discriminate among customers by careful arrangement of its product line. We went on to show how a seller who offers just one product may nevertheless succeed in charging different effective prices to different buyers, by exploiting the Linkage Principle.

Fifty years ago, the theory of monopoly was a subject of lively research controversies, with Robinson as one of its central figures. Today, after a period of relative dormancy, the theory of monopoly is once again in turmoil. One can only hope that some of the new theories will be as fruitful for enhancing our understanding of today's issues as the old were for the issues of their times.

MATHEMATICAL APPENDIX

The monopoly discrimination problem, using the specified forms for $V(Q,a,n)$, is:

$$\operatorname*{Max}_{p_1,\dots,p_N,q_1\dots q_N} \quad M_1q_1 + \dots + M_Np_N$$

subject to, for all n and j,

(IC) $\quad b_nq_n - q_n^2/2 - p_n \geqslant b_nq_j - q_j^2/2 - p_j$ and

(PC) $\quad b_nq_n - q_n^2/2 - p_n \geqslant 0.$

Analysis of the Constraints

1. $q_1 \leqslant \dots \leqslant q_N.$

Proof Choose any i and k between 1 and N. Consider the two constraints (IC) in which i and k play the roles of n and j, respectively. Summing these two constraints leads to:

$$(b_i - b_k)(q_i - q_k) \geqslant 0.$$

By hypothesis, $b_i > b_k$ if and only if $i > k$, so the result is proved.

2. Given the constraints (IC) for $|j-n| = 1$, the constraints (IC) for $|j-n| > 1$ are redundant.

Proof We prove, by induction, that all the (IC) constraints are satisfied if the ones with $|j-n| = 1$ are. Thus, the constraints where $|j-n| = 1$ are satisfied by hypothesis. Suppose all the constraints are satisfied for any j and n such that $|j-n| \leqslant k$. Consider the two constraints for i and m where $i - m = k$.

Using the induction hypothesis, (IC) holds for the pair $(i, i-1)$ and the pair $(i-1, m)$. Summing these two yields (IC) for (i, m). Similarly, summing (IC) for the pairs $(m, m+1)$ and $(m+1, i)$ yields (IC) for the pair (m, i).

3. Given the constraints (IC) and the constraints (PC) for $n = 1$, the constraints (PC) for $n > 1$ are redundant.

Proof The constraint (PC) for any $n > 1$ is obtained by summing the corresponding (PC) constraint for $n = 1$ and the constraint (IC) for the pair $(n, 1)$.

Dropping the constraints identified as redundant in 2 and 3 above, we proceed to study the monopolist's maximization.

Analysis of the Optimal Prices

Fix any non-decreasing set of qualities $q_1, \dots, q_N$. Let $d_1 = p_1$ and let d_n for $n > 1$ be $p_n - p_{n-1}$. The letter 'd' is a mnemonic for price 'differential'. Performing a change of variables in the monopolist's problem, we get:

$$\operatorname*{Max}_{d_1,\dots,d_N} \quad M_1d_1 + (M_1 + M_2)d_2 + \dots + (M_1 + \dots + M_N)d_N$$

subject to

(IC$^-$) $b_n(q_n - q_{n-1}) - (q_n^2 - q_{n-1}^2)/2 \geqslant d_n$ for all $n > 1$.

(IC$^+$) $b_{n-1}(q_{n-1} - q_n) - (q_{n-1}^2 - q_n^2)/2 \leqslant d_n$ for all $n > 1$.

(PC^1) $b_1 q_1 - q_1^2/2 \geqslant d_1$.

The inequalities (IC$^-$) correspond to (IC) with arguments n and $n-1$. The inequalities (IC$^+$) correspond to (IC) with arguments $n-1$ and n. Since $b_n > b_{n-1}$ and $q_n \geqslant q_{n-1}$ by hypothesis, the inequalities are consistent.

Note that the objective function is increasing in the d_n's. Also, the constraint set consists of one upper bound constraint for each d_n (given by (IC$^-$) and (PC^1)) and one lower bound constraint for d_n for each $n > 1$ (given by (IC$^+$)). It is clear, therefore, that all the upper bound constraints are binding at the optimum, that is, (PC^1) and (IC$^-$) for $n > 1$ hold as inequalities. That completely determines the optimum prices as a function of the qualities.

Analysis of the Optimal Qualities Offered

The price obtained by the monopolist for product n is now $d_1 + \ldots + d_n$, and the d_k's have now been derived for any quality choices $q_1, \ldots, q_N$. Using the foregoing analysis and, for notational simplicity, fixing $q_0 = 0$, the monopolist's optimal quality choice problem can be formulated as follows.

$$\underset{q_1, \ldots, q_N}{\text{Max}} \sum_{n=1}^{N} M_n \left(\sum_{k=1}^{n} d_k - q_n c \right)$$

subject to:

$$d_n = b_n(q_n - q_{n-1}) - (q_n^2 - q_{n-1}^2)/2 \quad \text{for } n = 1, \ldots, N, \text{ and}$$

$$0 = q_0 \leqslant q_1 \leqslant \ldots \leqslant q_N.$$

Substituting the equality constraints governing the d_n's into the objective yields a quadratic maximization problem. The solution to this problem together with the associated prices determine the monopolist's optimal pricing and product selection strategy.

The case examined in detail in the main text was the 'interior optimum' case, in which the $q_0 \leqslant q_1 \leqslant \ldots \leqslant q_N$ constraints do not bind. Then, the first-order condition corresponding to any quality level q_s is:

$$M_s(b_s - q_s - c) - \sum_{n=s+1}^{N} M_n(b_{s+1} - q_s) = 0 \quad \text{or}$$

$$(b_{s+1} - q_s)/(b_s - q_s - c) = M_s/(M_{s+1} + \ldots + M_N),$$

as derived from the figures in the paper. An analysis of which of the constraints $q_{n-1} \leqslant q_n$ bind, and of when none of these constraints bind, is given by Maskin and Riley (1984).

Notes

1. Philips (1983) provides a survey of the modern theory and evidence on price discrimination, and Wilson (1985) provides an even more complete theoretical account.
2. Bundling is the practice of offering a variety of goods together in a package, or offering goods both individually and in packages with the packages selling at a substantial discount.
3. An example of how far companies might go to achieve discrimination among customers is illustrated by the following example drawn from Scherer (1980), who cited Stocking and Watkins (1946). They reported that Rohm and Haas considered adding arsenic to their industrial plastic moulding powder methyl methacrylate to prevent it from being converted for use in denture manufacture. The industrial version of the compound sold for $.85 per pound while the version used in denture manufacture sold for $22 per pound.
4. Others have argued that, sometimes, the monopolist's power to set prices is limited by difficulties of commitment. Coase (1972) has argued that the temptation to engage in price discrimination prevents the monopolist-supplier of a durable good from achieving much more than competitive profits. See Gul, Sonnenschein, and Wilson (1986) for a formalization of this argument, which makes the underlying assumptions and conditions quite clear.
5. The opposite extreme, in which the seller has a fixed supply available, is considered by Milgrom (1987) for the case where there are no restrictions on resale.
6. Fudenberg and Kreps (1986) analyze the costs of benefits of commitment via reputation in a related industrial competition context.
7. This is a special case of the more general result that holds for incentive constraints when the indifference curves in price-quality space satisfy Spence's single crossing condition. For such models, only the 'local' incentive constraints can ever be binding. In models like this one for which making no purchase is like purchasing quality level zero, an additional general conclusion is that the only binding participation constraint is the one for the class of customers with the least willingness to pay for quality.

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