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COMPETITIVE BIDDING AND PROPRIETARY INFORMATION*

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We consider the sale of an object by sealed-bid auction, when one bidder has private information and the others have access only to public information. The equilibria of the bidding game are determined, and it is shown that at equilibrium the informed bidder's distribution of bids is the same as the distribution of the maximum of the others' bids. The expected profit of the informed bidder is generally positive, while the other bidders have zero expected profits. The equilibrium bid distributions and the bidders' expected profits are shown to vary continuously in the parameters of the bidding game.

1. Introduction

In a competitive setting, information plays two important roles. Information about the physical state of the world can indicate the quality of the goods one considers acquiring. In addition, information about one's potential competitors — their number, their preferences, the information which they in turn possess — can indicate the degree of competition one must expect to encounter. A theory of competitive bidding that incorporates these various aspects in a setting that is symmetric with respect to the bidders has been presented in Milgrom and Weber (1982a).

We consider in this paper the sale of a single indivisible object of fixed, but unknown, value, when the bidders are asymmetrically informed. In the case we treat, only one bidder is assumed to have private information. Therefore, the privately-informed bidder has a double advantage over his competitors:

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he has a more accurate estimate of the object's value, and he knows precisely what information his competitors possess.

The model we treat can be viewed as representing the sale by auction of mineral rights on a tract of offshore territory. Each bidder submits a non-negative sealed bid. The highest bidder is awarded the rights being sold, and pays the amount of his bid; the losers pay nothing. All of the bidders have access to publicly available geological data. However, one of the bidders has — and is known to have — additional proprietary information acquired as a result of work performed on an adjacent tract, or through a privately commissioned survey.

Formally, we assume that there are $m+1$ potential bidders, each of whom knows the joint distribution of the random pair $(\boldsymbol{Z},\boldsymbol{X})$. The variable $\boldsymbol{Z}$ represents the unknown value of the object being sold. We assume that $\boldsymbol{Z}$ takes values in $\mathbf{R}_+$ and has finite expectation. The variable $\boldsymbol{X}$ represents the private information of the better-informed bidder; only he is informed of the realization of $\boldsymbol{X}$. We do not assume that $\boldsymbol{X}$ is real-valued: its values may lie in any measurable space.

The model we treat generalizes one introduced by Wilson (1967), which was based on a case study by Woods (1965).[1] The model has been partially analyzed by Wilson (1967), Hughart (1975), and Weverburgh (1979).[2] None of these earlier studies obtained a complete, correct characterization of the equilibrium strategies of all the bidders, and each made unnecessary assumptions about the joint distribution of $(\boldsymbol{Z},\boldsymbol{X})$. Our contributions in this paper are (1) providing a complete characterization of the equilibrium strategies of all the bidders, without imposing any restrictions[3] on $(\boldsymbol{Z},\boldsymbol{X})$, (2) deriving the bid distributions predicted by the model, (3) computing the bidders' expected profits and the seller's expected revenue from the auction, and (4) showing that the bid distributions, bidder profits, and seller revenues depend continuously on the assumed joint distribution of $(\boldsymbol{Z},\boldsymbol{X})$. Our analysis also sets the stage for a study of the bidders' incentives to acquire private information and the seller's incentive to bring additional information into the public domain [Milgrom and Weber (1982b), wherein the results of this paper are assumed].

2. The informed bidder's problem

When the informed bidder observes a realization $\boldsymbol{X}=x$ of his private information, his problem is to choose a bid b to maximize his expected

[1]This study is the basis for the 'Maxco/Gambit' case (1974), used in many business schools.
[2]For a broad survey of the literature on competitive bidding, see Engelbrecht-Wiggans (1980).
[3]In particular, the characterization holds whether $(\boldsymbol{Z},\boldsymbol{X})$ has a continuous, discrete, or mixed distribution. The unifying techniques presented here are applicable to a variety of games with incomplete information [cf. Milgrom (1981), Milgrom and Weber (1982a, and especially 1983, sec. 6)].

profit: $P(b \text{ wins})\cdot(E[\boldsymbol{Z}|\boldsymbol{X}=x]-b)$. There are two things to notice here. First, the bidder's private information $\boldsymbol{X}$ enters his decision problem only through $\boldsymbol{H}=E[\boldsymbol{Z}|\boldsymbol{X}]$; consequently we may assume without loss of generality that the informed bidder observes only the real-valued variable $\boldsymbol{H}$, rather than the more complicated variable $\boldsymbol{X}$ whose values may lie, for example, in the space of geological reports. Second, if b is an optimal bid when the realization of $\boldsymbol{H}$ is h, then no lower bid b' can be optimal when the realization is any larger value $h'>h$. For if $b'<b$, then

$$P(b \text{ wins})>P(b' \text{ wins})$$

(else b cannot be optimal when $\boldsymbol{H}=h$); hence,

$$P(b \text{ wins})(h'-b)-P(b' \text{ wins})(h'-b')$$

$$>P(b \text{ wins})(h-b)-P(b' \text{ wins})(h-b')\geqq 0.$$

Thus, b' is not as good as b when $\boldsymbol{H}=h'$ and so cannot be optimal.

To accommodate both pure and mixed strategies for the informed bidder, let $\boldsymbol{U}$ be a random variable that is independent of $(\boldsymbol{Z},\boldsymbol{X})$ and has an atomless distribution on $[0,1]$. We assume that the informed bidder observes $\boldsymbol{U}$ and uses it whenever he needs to randomize his bids. A mixed strategy β for the informed bidder is then a function from $\mathbf{R}^2\to\mathbf{R}_+$, where $\beta(h,u)$ is the amount bid when $\boldsymbol{H}=h$ and $\boldsymbol{U}=u$. There is no loss of generality in requiring that $\beta(h,u)$ be non-decreasing in u for every fixed value of h.

3. Equilibrium strategies

For uninformed bidder i, a mixed strategy is a distribution G_i on $\mathbf{R}_+$, where $G_i(b)$ is the probability that he tenders a bid not exceeding b. Let $G(b)=G_1(b)\cdot\ldots\cdot G_m(b)$; G denotes the distribution of the maximum of the bids made by the uninformed bidders. The equilibria of the bidding game are described in the following theorem.

Theorem 1. *The $(m+1)$-tuple $(\beta, G_1,\ldots,G_m)$ is an equilibrium point if and only if*

$$\beta(h,u)=E[\boldsymbol{H}|\boldsymbol{H}<h, \quad \text{or} \quad \boldsymbol{H}=h \quad \text{and} \quad \boldsymbol{U}<u],$$

and

$$G(b)=P(\beta(\boldsymbol{H},\boldsymbol{U})\leqq b).$$

Notice that $\beta(h,0)=E[\boldsymbol{H}|\boldsymbol{H}<h]$, while $\beta(h,1)=E[\boldsymbol{H}|\boldsymbol{H}\leqq h]$. If $\boldsymbol{H}$ has no atom at h, then these two expressions are equal; in particular, when $\boldsymbol{H}$ is

atomless, β describes a pure strategy. Generally, if $\boldsymbol{H}$ has an atom at h, the equilibrium strategy involves randomizing over the interval $[\beta(h,0),\ \beta(h,1)]$ when $\boldsymbol{H}=h$.

We can exploit the special structure of β by working with distributional types and distributional strategies [cf. Milgrom and Weber (1983)]. Let $\{(\boldsymbol{H},\boldsymbol{U})<(h,u)\}$ denote the event $\{\boldsymbol{H}<h$, or $\boldsymbol{H}=h$ and $\boldsymbol{U}<u\}$, let $T(h,u)$ be the probability of that event and define $\boldsymbol{T}=T(\boldsymbol{H},\boldsymbol{U})$. $\boldsymbol{T}$ is called the informed bidder's *distributional type* and is uniformly distributed on $[0,1]$. Letting $H(t)=\inf\{h|P(\boldsymbol{H}\leqq h)>t\}$, we have $\boldsymbol{H}=H(\boldsymbol{T})$ almost surely. Therefore, the distributional type $\boldsymbol{T}$ carries all the information that the informed bidder needs in order to make an optimal bid. In essence, the transformation of $\boldsymbol{H}$ into $\boldsymbol{T}$ 'opens' each of the (at most countably many) atoms of $\boldsymbol{H}$ into an interval, and permits us to proceed with our analysis as we would have, had $\boldsymbol{H}$ originally been atomless.

Using the distributional type, we can express β conveniently in its distributional form. When the informed bidder observes $\boldsymbol{T}=t$,[4] he bids[5]

$$\begin{aligned}\beta(t)&=E[H(\boldsymbol{T})|\boldsymbol{T}\leqq t]\\&=\int_0^t H(s)\,\mathrm{d}s/t\\&=H(t)-(1/t)\int_0^t s\,\mathrm{d}H(s).\end{aligned}$$

Notice that in this form, β is continuous and non-decreasing, $\beta(0)=H(0)$, and $\beta(1)=E[\boldsymbol{H}]$.

The following proof shows that the indicated strategies are in equilibrium. We return later to the proof that there are no other equilibria.

Proof of Theorem 1 (first part). Let β and G be as specified in the theorem. The range of β is $[H(0),E(\boldsymbol{H})]$. Therefore, this interval is also the support of G. Suppose that the uninformed bidders adopt their equilibrium strategies and the informed bidder learns that $\boldsymbol{T}=t$. Consider a bid b by the informed bidder. If $b<H(0)$, the bid will surely lose and bring him a payoff of zero; if $b>E[\boldsymbol{H}]$, then a bid of precisely $E[\boldsymbol{H}]$ would be strictly preferred (either bid would win with certainty). Hence, an optimal bid lies in the range of β.

A bid of $\beta(\tau)$ wins with probability τ,[6] yielding an expected payoff of

$$[H(t)-\beta(\tau)]\cdot\tau=\int_0^\tau (H(t)-H(s))\,\mathrm{d}s.$$

[4]By this we mean that he observes $(\boldsymbol{H},\boldsymbol{U})=(h,u)$, where $T(h,u)=t$.

[5]This agrees with our original specification of β because, when $T(h,u)=t$, $\{\boldsymbol{T}<t\}\subset\{(\boldsymbol{H},\boldsymbol{U})<(h,u)\}\subset\{\boldsymbol{T}\leqq t\}$, and the event $\{\boldsymbol{T}=t\}$ is null.

[6]An exception occurs if $\boldsymbol{H}$ has an atom at $H(0)$. In that case, one can verify that G also has an atom at $H(0)$ and hence that it is optimal for the informed bidder to bid $\beta(\tau)=H(0)$ if and only if $H(t)=H(0)$. For the remainder of this paper, we omit consideration of the case where $\boldsymbol{H}$ has an atom at $H(0)$.

Hence, the derivative with respect to τ of the informed bidder's expected profit is $H(t)-H(\tau)$, which is non-negative for $\tau<t$ and non-positive for $\tau>t$. This proves that $\beta(t)$ is an optimal bid when $\boldsymbol{T}=t$.

For any uninformed bidder, say bidder 1, a bid of less than $H(0)$ will always lose and will yield an expected profit of zero, while a bid greater than $E[\boldsymbol{H}]$ will be strictly inferior to a bid of precisely $E[\boldsymbol{H}]$. Consider a bid $b=\beta(t)$. The resulting expected payoff is

$$E[\boldsymbol{Z}-\beta(t)|\boldsymbol{T}\leqq t]\cdot t\cdot G_2(b)\cdot\ldots\cdot G_m(b).$$

But

$$E[\boldsymbol{Z}-\beta(t)|\boldsymbol{T}\leqq t]=E[H(\boldsymbol{T})|\boldsymbol{T}\leqq t]-\beta(t)=0.$$

Hence, the distribution G_1 has only optimal bids in its support. Q.E.D.

We have assumed for our analysis that the value of the object being sold is the same to each bidder, that the bidders are all risk-neutral, and that the poorly-informed bidders all have identical information. None of these assumptions are necessary to conclude that the less-well-informed bidders earn zero profits at equilibrium. The following is one of several available results [cf. Milgrom (1979)] to the effect that a bidder, all of whose information is known to some fixed competitor and whose risk-adjusted reservation price for the object is no greater than that of the competitor, cannot profit from a sealed-bid auction.

Theorem 2. Let $\boldsymbol{V}_A$ denote the (unknown) value of the object to the informed bidder and let $\boldsymbol{V}_B$ denote the value to the first uninformed bidder. Suppose that $\boldsymbol{V}_B\leqq\boldsymbol{V}_A$ almost surely. Let U_A and U_B denote the bidders' respective utility functions (increasing and continuous), and suppose that U_B is globally as risk-averse as U_A. Then at any equilibrium point of the bidding game, the uninformed bidder's expected utility is $U_B(0)$.

Proof. We call the informed bidder A and the first uninformed bidder B. Without loss of generality, let $U_A(0)=U_B(0)=0$. Fix some equilibrium point $(\beta,G_1,\ldots,G_m)$. Let $\bar{G}(b)=G_2(b)\cdot\ldots\cdot G_m(b)$, and let $G(b)=G_1(b)\bar{G}(b)$. Define[7] $\underline{b}=\inf\{b|G_1(b)>0\}$. If $\underline{b}<\inf\{b|\bar{G}(b)>0\}$, then B's bids near $\underline{b}$ will never win, and B's expected payoff will be $U_B(0)$. Hence, we can restrict our attention to

[7]Here we use the assumption that bids are non-negative, or at least bounded from below. If arbitrarily large negative bids were permitted, there would exist other equilibria not satisfying Theorems 1 and 2.

the case where $\underline{b} \geqq \inf\{b|\bar{G}(b)>0\} = \inf\{b|G(b)>0\}$. There are two subcases to consider: either G has an atom at $\underline{b}$, or it does not.

If G has an atom at $\underline{b}$, then A will bid above $\underline{b}$ when $E[U_A(V_A-\underline{b})|X]>0$, because a bid of $b<\underline{b}$ brings a zero expected payoff and the expected payoff, as a function of b, jumps up discontinuously at $\underline{b}$. Let S be the event $\{E[U_A(V_A-\underline{b})|X]>0\}$. If $P(S)=1$, then A will always bid above $\underline{b}$, and therefore B's expected payoff when he bids $\underline{b}+\varepsilon$ will be $O(\varepsilon)$. If $P(S)<1$, then letting $\bar{S}$ denote the complement of S, B's expected payoff conditional on winning with a bid of $\underline{b}+\varepsilon$ is

$$E[U_B(V_B-\underline{b})|\bar{S}]+O(\varepsilon)=E[E[U_B(V_B-\underline{b})|X]|\bar{S}]+O(\varepsilon).$$

Since U_B is globally as risk averse as U_A, there is some increasing concave function g such that $U_B = g \circ U_A$. Then, since $V_B \leqq V_A$ almost surely, we have that if $E[U_A(V_A-\underline{b})|X] \leqq U_A(0)=0$, then

$$\begin{aligned} E[U_B(V_B-\underline{b})|X] &\leqq E[g(U_A(V_A-\underline{b}))|X] \\ &\leqq g(E[U_A(V_A-\underline{b})|X]) \\ &\leqq g(U_A(0))=U_A(0)=0, \end{aligned}$$

using Jensen's inequality. So B's expected payoff, conditional on winning with a bid of $\underline{b}+\varepsilon$, is at most $O(\varepsilon)$.

Similarly, if G has no atom at $\underline{b}$, then A can never win by bidding $\underline{b}$ or less, but he can earn a positive payoff by bidding more than $\underline{b}$ whenever $E[U_A(V_A-\underline{b})|X]>0$. Hence, the argument from the preceding paragraph applies: When B bids $\underline{b}+\varepsilon$, either his probability of winning or his conditional expected payoff must be $O(\varepsilon)$. In either event, his expected payoff is $O(\varepsilon)$.

At equilibrium, B must be indifferent among all his bids. Therefore his expected payoff must be constant, and by the argument just given it must be $O(\varepsilon)$ for bids of $\underline{b}+\varepsilon$. Hence, his expected payoff must be zero. Q.E.D.

Using Theorem 2, we can complete the proof of Theorem 1:

Proof of Theorem 1 (*conclusion*). Adapting arguments given by Griesmer, Levitan and Shubik (1967), one can show that at equilibrium each bidder's distribution of bids is atomless, except possibly at its lower bound. Such an atom occurs only when the distribution of $\boldsymbol{H}$ has an atom at *its* lower bound; the arguments presented below can easily be modified to cover this case.

If the bid distributions are atomless and if an uninformed bidder wins ith a bid of b, then his expected payoff conditional on winning is $E[\boldsymbol{Z}$ $-b|\beta(\boldsymbol{H},\boldsymbol{U})<b]$, where β denotes the strategy of the informed bidder. Recall iat there is no loss of generality in requiring that the equilibrium strategy β e non-decreasing. The arguments of Griesmer, Levitan and Shubik can be sed to show that the range of β must be convex. But when $b=\beta(h,u)$, the onditional expected payoff is $E[\boldsymbol{Z}-\beta(h,u)|(\boldsymbol{H},\boldsymbol{U})<(h,u)]$, which must be ero by Theorem 2. Consequently, $\beta(h,u)=E(\boldsymbol{Z}|(\boldsymbol{H},\boldsymbol{U})<(h,u)]$. Since β is ptimal, we know that $\beta(h,u)$ solves $\max_b (h-b)G(b)$. The first-order necessary ondition for optimality is $0=-G(b)+(h-b)G'(b)$, a first-order linear ifferential equation in G that must hold for all (h,b)-pairs for which b is in ie range of $\beta(h,\cdot)$. The supremum of the range of β is $E[\boldsymbol{H}]$, and so, as we oted earlier, no uninformed bidder can bid more than $E[\boldsymbol{H}]$ at equilibrium. 'hat determines the boundary condition: $G(E[\boldsymbol{H}]=1$. Since the range of β is onvex, there can be only one function G satisfying the differential equation nd the boundary condition. Therefore, the distribution G given in Theorem is the unique equilibrium bid distribution for the maximum bid of the ininformed bidders. Q.E.D.

. Continuity

Since $\boldsymbol{T}$ is uniformly distributed on $[0,1]$, the information about the istribution of $(\boldsymbol{Z},\boldsymbol{X})$ that we need to determine the equilibrium strategies is ll encoded in the function H. The coding works as follows: if F is the listribution of $\boldsymbol{H}$, then $H(t)=\inf\{h|F(h)>t\}$ and $F(h)=\inf\{t|H(t)>h\}$. Notice hat H is essentially the inverse of F: for any h in the support of F, we have $I(F(h))=h$.

The same remarks apply to the distributional form of β, i.e., if G_0 is the id distribution of the informed bidder, then $\beta(t)=\inf\{b|G_0(b)>t\}$ and $\bar{\jmath}_0(b)=\inf\{t|\beta(t)>b\}$. For bids in the support of G_0, $\beta(G_0(b))=b$. These bservations lead to the following result:

Theorem 3. When the weak topology for distributions is used, the equilibrium listribution of bids varies continuously with the assumed distribution of $\boldsymbol{H}$.

Proof. Let $\{F^k\}$ be the sequence of distribution functions of the variables $\{\boldsymbol{H}^k\}$, and let $\{(\boldsymbol{b}^k, G_1^k,\ldots,G_m^k)\}$ be corresponding equilibrium points. Weak convergence of the F^k-sequence is equivalent to almost everywhere convergence of the corresponding H^k-sequence. Since each H_k is a distribution function and $\beta^k(t)=(1/t)\int_0^t H^k(s)\mathrm{d}s$, almost everywhere convergence of the H^k-sequence implies pointwise convergence of the β^k-sequence, which implies weak convergence of the G^k-sequence as claimed.[8] Q.E.D.

5. Equilibrium payoffs

Finally, we determine the expected profit of the informed bidder and the expected revenue of the seller. A consequence of the following theorem is that these equilibrium payoffs vary continuously in the distribution F of $\boldsymbol{H}$:

Theorem 4. At equilibrium, the informed bidder's expected profit is

$$\int_0^\infty F(h)(1-F(h))\,\mathrm{d}h=\int_0^1 t(1-t)\,\mathrm{d}H(t),$$

and the seller's expected revenue is

$$\int_0^\infty (1-F(h))^2\,\mathrm{d}h=\int_0^1 (1-t)^2\,\mathrm{d}H(t)+H(0).$$

Proof. When the informed bidder has distributional type t. his equilibrium bid is $\beta(t)=H(t)-(1/t)\int_0^t s\,\mathrm{d}H(s)$. Conditional on winning, his expected profit is $H(t)-\beta(t)$. The bid $\beta(t)$ wins with probability $G(\beta(t))=t$, unless $H(t)=H(0)=\beta(0)=\beta(t)$. In either case, his expected profit when $\boldsymbol{T}=t$ (*not* conditional on winning) is $t\,(H(t)-\beta(t))=\int_0^t s\,\mathrm{d}H(s)=\int_0^h F(z)\mathrm{d}z$, where $h=H(t)$. To obtain the informed bidder's expected profit, we integrate over t using the distribution of $\boldsymbol{T}$, which yields

$$\int_0^1\int_0^t s\,\mathrm{d}H(s)\mathrm{d}t=\int_0^1 s\left(\int_s^1 \mathrm{d}t\right)\mathrm{d}H(s)=\int_0^1 s(1-s)\,\mathrm{d}H(s).$$

Notice that for any realization (z,x) of $(\boldsymbol{Z},\boldsymbol{X})$ and any $(m+1)$-tuple of bids, the seller's revenue plus the bidders' profits totals z. Hence, the seller's expected revenue is $E[\boldsymbol{Z}]$ minus the bidders' expected profits. But

$$E[\boldsymbol{Z}]=E[\boldsymbol{H}]=E[H(\boldsymbol{T})]=\int_0^1 H(t)\,\mathrm{d}t=\int_0^1(1-t)\,\mathrm{d}H(t)+H(0).$$

Deducting the informed bidder's expected profit (recall that the uninformed bidders expect zero profit), our result follows. Q.E.D.

[8]Convergence of the bid distributions can involve the convergence of a sequence of pure strategies to a mixed strategy. That happens, for example, when $\boldsymbol{H}^k$ is uniformly distributed on $[0,1/k]\cup[1-1/k,1]$, so that $H^k(s)=2s/k$ for $0\leqq s<\frac{1}{2}$, and $H^k(s)=1-2(1-s)/k$ for $1/2\leqq s\leqq 1$. Note that $\{F^k\}$ is a sequence of continuous distributions which converges weakly to the discrete distribution which places probability $\frac{1}{2}$ at 0 and at 1. In distributional form, weak convergence of the bid distributions takes the convenient form of pointwise convergence of the sequence $\{\beta^k\}$.

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