
Combination Bidding in Spectrum Auctions

Paul Milgrom
Stanford University

Complementarities among the licenses sold in spectrum auctions offer significant challenges for auction design. When licenses are mutual substitutes, even if bidding is for individual licenses only, participants who bid straightforwardly for the licenses they want do not run the risk of acquiring an unwanted collection. This encourages vigorous competition in the auction and contributes to efficiency in the allocation. Complementarities arising from scale economies seem to have blocked vigorous competition in a spectrum auction in the Netherlands in 1998. Bidders with such scale economies and others with narrower requirements can both be accommodated by an auction design that permits bids for packages of licenses. The considerations in designing such an auction are reviewed.

INTRODUCTION

The U.S. government first began auctioning portions of the radio spectrum for commercial use in 1994. The auction era replaced one in which administrative hearings and even lotteries had been used to assign spectrum. The auction policy was widely hailed for leading to quicker, more market-driven decisions about license assignments, for reducing opportunities for political interference, and for generating substantial revenues for the government.

The standard auction format of the Federal Communication Commission (FCC), called variously the *simultaneous multiple round auction* or the *simultaneous ascending auction*, has also been widely applauded¹ and imitated. Since its

1. For example, Cramton (1995, 1997) and McAfee and McMillan (1996) gave favorable accounts of the performance of the auction in the United States.

introduction in the United States, some variant of the simultaneous ascending auction has been announced or used for sales of radio spectrum in Canada, Mexico, Australia, New Zealand, Germany, and the Netherlands. The new auction rules have also been adapted for other kinds of sales. Variants have been used for everything from selling undeveloped housing lots at Stanford University to selling rights for standard offer electrical service in New England. (More detailed descriptions of several of the applications of the new auction can be found at <http://www.market-design.com/projects.html>)

Despite the successes of the new auction rules relative to older designs, the possibility of doing still better has received increasing attention. One possible improvement that has attracted the attention of both economic theorists and economic experimenters is allowing participants to bid for combinations or "packages" of licenses, rather than just individual licenses. This chapter reports on the main pros and cons of that suggestion and some of the hurdles that must be cleared to implement such a suggestion effectively.

The remainder of this chapter is organized in three sections. The first of these gives a brief description of the simultaneous ascending auction and its rules.

After the rules are described, the following section discusses the theory and evidence about when the auction might need to be adapted to allow bidding for packages of spectrum licenses, rather than just for individual licenses. According to theory, an auction in which bidding is only for individual licenses can be quite effective when the spectrum licenses for sale are close substitutes, but may fail badly when some of the licenses are complements. I argue that evidence from the Netherlands spectrum auction tends to confirm the theoretical prediction. Thus, the theory suggests that the choice of scope and scale of the licenses to be sold is an important element in the design of a successful simultaneous ascending auction because it determines the extent to which the items for sale complement each other.

The last section discusses the two main approaches to bidding for packages: the generalized Vickrey auction and a modified simultaneous ascending auction. The section identifies certain theoretical advantages of the generalized Vickrey auction, but it also identifies several important practical limitations. The alternative dynamical auction has been less completely studied. It lacks the theoretical optimality properties of the Vickrey auction but has certain offsetting practical advantages. The question of how best to structure the rules of the simultaneous auction with package bidding remains an important open question.

DESCRIPTION OF THE SIMULTANEOUS ASCENDING AUCTION

A simultaneous ascending auction is an auction for multiple items in which bidding occurs in rounds. At each round, bidders simultaneously make sealed bids for any licenses in which they are interested. After the bidding, round results are posted. For each item, these results consist of the identities of the new bids and bidders

and the standing high bid and the corresponding bidder. The initial standing high bid for each item is zero and the corresponding bidder is the auctioneer. As the auction progresses, the new standing high bid at the end of a round for an item is the larger of the previous standing high bid or the highest new bid and the corresponding bidder is the one who made that bid. In addition to the round results, the minimum bids for the next round are also posted. These are computed from the standing high bid by adding a predetermined bid increment, such as 5% or 10%.

Bidder activity during the auction is controlled by what the FCC called the *Milgrom-Wilson activity rule*. It works as follows. First, a quantity measure for spectrum is established, which provides a rough index of the value of the license. Typically, the quantity measure for a spectrum license is based on the bandwidth of the licensed spectrum and the population of the geographic area covered by the license. At the outset of the auction, each bidder establishes its initial eligibility for bidding by making deposits covering a certain quantity of spectrum. During the auction, a bidder is considered active for a license at a round if it makes an eligible new bid for the license or if it owns the standing high bid from the previous round. At each round, a bidder's activity is constrained not to exceed its eligibility. If a bid is submitted that exceeds the bidder's eligibility, the bid is simply rejected.

According to the original rules, the auction is conducted in three stages. In the first stage, a bidder who wishes to maintain its eligibility must be active on licenses covering some fraction f_1 of its eligibility. If a bidder with eligibility x is active on $y < f_1 x$ during this stage, then its eligibility is reduced at the next round to y/f_1 . In the second and third stages, a similar rule applies but using fractions f_2 and f_3 . In recent auctions in the United States, the fractions used have been $(f_1, f_2, f_3) = (.6, .8, .95)$. Thus, in Stage 3, bidders know that the auction is nearing its close in the sense that the remaining demand for licenses is just $1/f_3$ times the current activity level.

There are several different options for rules to close the bidding and the spectrum regulator is presently reevaluating these. One proposal, made by McAfee, specified that when a license had received no new bids for a fixed number of rounds, bidding on that license would close. That proposal was coupled with a suggestion that the bid increments for licenses should reflect the bidding activity on a license. A second proposal that Wilson and I made specified that bidding on all licenses should close simultaneously when there is no new bidding on any license. To date, the latter rule is the only one that has been used in the spectrum auctions.

When the auction closes, the licenses are sold at prices equal to the standing high bids to the corresponding bidders. The rules that govern deposits, payment terms, and so on are quite important to the success of the auction,² but they are

2. Failure to establish these rules properly led to billions of dollars of bidder defaults in the United States "C-block auction." Similar problems on a smaller scale occurred in some Australian spectrum auctions.

mostly separable from the other auction rule issues and receive no further comment here.

WHEN ARE COMBINATION BIDS NEEDED?

One important feature of the auction rules is that bids are made on each license individually. In practice, however, a license is not evaluated in isolation. Rather, a license is used as part of a business plan and its value depends on which other licenses the bidder acquires. Even the identities of other bidders who acquire licenses may affect the value, as that may affect both the likelihood of concluding successful roaming agreements, the technologies that are developed by suppliers, and the nature of competition in the telecommunications market.

This analysis focuses mostly on the value dependency among licenses for a single bidder. Logically, when there are more than two interdependent licenses, the value relations among them can take a large variety of formats. For reasons of simplicity, however, theoretical economic analyses of auctions mainly focus on two special cases. In the first, all of the licenses are substitutes; in the second, there are just two licenses and they are complements. A key question in each case is whether the outcome would be improved if bids for packages were allowed.

According to one theory (Milgrom, 1997), if bidders bid in a straightforward manner for the items they wish to acquire and if the items being sold are economic substitutes, then the simultaneous ascending auction with sufficiently small bid increments will lead to an efficient outcome. Straightforward bidding means that, at each round, the bidder selects its bids to maximize its profits based on the hypothesis that the other bidders will not bid again and the auction will end after the present round.

The intuition behind this conclusion is as follows. Suppose a bidder finds all the licenses sold in the auction to be substitutes, meaning that raising the price of some of the licenses would not make that bidder less eager to buy the others. During the course of the auction, the rising prices of the licenses never make a bidder less willing to acquire the items on which it still has the standing high bid. Consequently, if the bidder bids straightforwardly, then when the auction ends the bidder will never wish to withdraw any bids: It will be satisfied to pay the prices it has bid to acquire its items. Moreover, because it did not bid at the last round, the bidder does not wish to acquire any other item at the final bid price (plus one increment, but an additional argument establishes that if the bid increment is sufficiently small, it never affects the bidder's choice). Hence, the prices and license assignment determined by the auction form a competitive equilibrium outcome. By the First Theorem of Welfare Economics, that implies that the assignment is efficient.

The conclusion is that package bids are not needed to improve the potential outcome of the auction in the event that licenses are substitutes. Package bids may

still be desirable for the incentives they provide, which generally depend on the other rules of the auction. For example, the generalized Vickrey auction provides strategic incentives for straightforward bidding but relies on package bidding and price discrimination to achieve that. I set that issue aside for the discussion here.

Although straightforward bidding for individual licenses leads to efficient, competitive equilibrium outcomes when licenses are all mutual substitutes, the situation changes dramatically if some of the licenses are complements, for two reasons. First, as shown in Milgrom (1997), allowing for complements in addition to the substitutes implies that a competitive equilibrium may fail to exist. This conclusion is particularly devastating for the view that an auction is a means to discover equilibrium prices.

Second, the presence of complements enormously complicates a bidder's problem. When licenses are mutual substitutes, a straightforward bidder will always wind up with a set of licenses that it is satisfied to have at the final bid prices. However, if the licenses are complements, a straightforward bidder is exposed to the possibility that it may not want the licenses assigned to it at the prices it must pay. To see this, consider a bidder whose value for the pair of licenses AB is 10 but whose values for A and B individually are just 2 apiece. If the price of each license reaches 3, straightforward bidding means that the bidder should bid 4 for each license, because it wants the pair at those prices. However, if there is no further bidding for License A but the price of License B continues to rise to 7, the bidder is stuck. It does not want to pay the going prices for A and B, but it also does not want to acquire License A alone for a price of 4. From a bidder's perspective, this "exposure problem" makes strategic bidding complicated.

One possible response to the exposure problem is for the bidder to bid cautiously, limiting its exposure to losses from acquiring an unwanted license or licenses. This response is particularly appealing when several licenses must be acquired to obtain good value.

The auction completed in the Netherlands in February 1998 for DCS-1800 spectrum (spectrum in the 1800 MHz assigned for use in "digital communications services") provides a good example of how costly the exposure problem can be. In that auction, complementarities among licenses resulted from economies of scale. Two of the lots on offer, A and B, contained sufficient bands to be used on a stand-alone basis for an efficiently scaled PCS telephone system. Sixteen smaller lots were also offered, but one needed to acquire at least five and perhaps even six or seven of these to build an economically viable business. These numbered lots, however, were useful as spectrum increments to incumbent cellular telephone providers.

The auction outcome reflected this scale problem in a natural way. For comparability, we express the prices of all the lots on a "per band" basis. The prices of the A and B lots were 8.0 million Dutch guilders (NLG) and 7.3 million NLG, respectively. The prices of the numbered lots were much lower, ranging from 2.9

million to 3.6 million NLG per band. Evidently, those who wish to establish new wireless telephone businesses found it too risky even to try to assemble these by bidding individually for the smaller lots.

The practical lesson of the Netherlands experience is that there are real limits on the use of the simultaneous ascending auction. If bidding is to take place on the licenses individually, then the licenses themselves should be structured to keep the exposure problem manageable. If that is not possible, the solution may lie in some form of bidding for packages.

PACKAGE BIDDING SCHEMES

Package bidding schemes can be devised that, in theory, both encourage bidders to bid in a straightforward way (overcoming strategic incentives to misrepresent values or reduce demand in an attempt to manipulate prices) and solve the problem of allocating combinations of licenses. There are two main approaches to this: the generalized Vickrey auction and dynamical package bidding.

The generalized Vickrey method is a complete solution to the package bidding problem for a certain ideal theoretical environment. This involves a generalization of Vickrey's (1961) auction design using elements introduced by Clarke (1971) and Groves (1973). In the relevant environment, each bidder has a valuation for each possible package of licenses and knows those valuations. It reports valuations to the auctioneer, who then computes the total-value maximizing license assignment and implements it. The auctioneer then sets prices so that each bidder's profit, if it reports truthfully, is equal to the amount that the total value is increased by its participation. It can be shown that with this pricing rule, it is in each bidder's interest to report its values truthfully, regardless of how others report. Of course, if each bidder does report truthfully, the result is an efficient license assignment.

A report by Market Design, Inc and Charles River Associates (1997) reviews four practical difficulties of implementing the generalized Vickrey auction in detail. I add a fifth to the following list.

- *Political and legal limitations.* The Vickrey auction makes the profits of the bidders obvious to outside observers, exposing the outcomes to subsequent challenge. In addition, it sometimes applies price discrimination in favor of larger bidders to offset their incentive to reduce prices by withholding demand. Such price discrimination may be illegal or politically impractical.
- *Budget limitations.* The theoretical analysis assumes that the bidders' budgets are unlimited. Some of the important consequences of budget limitations are analyzed in Che and Gale (1996, 1998).
- *Complexity.* Spectrum auctions vary in size, but some involve the sale of hundreds or thousands of licenses. In its pure form, the generalized Vickrey auction entails reports of value for every subset, which is far too many

for practicality. To be practical, some variation would need to be implemented that either reduced the number of combinations to be valued or that limited the valuation formula, so that it can be described with a manageable number of parameters.

- *Common value issues.* A central assumption of the generalized Vickrey analysis is that each bidder knows its own values and is uninterested in the values of others. In practice, however, bidders may well respect each other's expertise and want to weight the other bidders' assessments about demand growth, future technological developments, and so forth. These common value issues can, if important, have drastic implications about the efficacy of alternative auction designs (Klemperer, 1998). Generally, these "common value" issues work against the effectiveness of Vickrey auctions.
- *Dependencies among license winners.* Another assumption of the generalized Vickrey auction is that values depend only on the set of licenses won. As described earlier, however, a bidder may care who the other winners are, for example because that affects the possibility of a roaming agreement. Although it is possible in theory to structure a further generalized Vickrey auction so that prices depend on the entire license assignment, that exacerbates the other difficulties already described.

Partly because of the complexity problem already discussed, some pundits have proposed using an auction that allows the bidders to make both individual license bids and package bids and to specify potentially relevant packages dynamically during the auction. There is evidence from laboratory experiments that such rules may sometimes perform extraordinarily well (Ledyard, Noussair, & Porter, 1994). Theoretical analysis of the experimental auction rules suggests that strategic incentives should be a more serious problem than the experimenters seemed to find (Milgrom, 1997). The central problem is a free-rider problem among bidders for individual licenses who must implicitly form a team to outbid the "package" bidder. The theory holds that even if there are no actual complementarities, a large bidder can sometimes make a package bid that exploits the free-rider problem and allows it to win too many licenses at a low price.

These theoretical predictions are preliminary, and they are also testable in economic laboratory experiments. For these reasons, this is a particularly promising area for both theoretical and experimental research. Even now, such research is proceeding. While this volume was in press, a new design for combinatorial bidding was proposed by DeMartini, Kwasnica, Ledyard, and Porter (1998). Their design is resistant to free-rider problems, and other new designs are being tested that have better theoretical performance if bidders behave naively. The next stages include more laboratory testing and the introduction of these methods to sell items

of substantial value. The adoption of these techniques for real auctions is almost certainly imminent.

REFERENCES

- Che, Y.-K., & Gale, I. (1996). Financial constraints in auctions: Effects and antidotes. *Advances in Applied Microeconomics*, 6, 97-120.
- Che, Y.-K., & Gale, I. (1998). Standard auctions with financially constrained bidders. *Review of Economic Studies*, 65, 1-21.
- Clarke, E. (1971). Multipart pricing of public goods. *Public Choice*, 8, 19-33.
- Cramton, P. (1995). Money out of thin air: The nationwide narrowband PCS auction. *Journal of Economics and Management Strategy*, 4, 267-343.
- Cramton, P. (1997). The FCC spectrum auctions: An early assessment. *Journal of Economics and Management Strategy*, 6(3), 431-495.
- DeMartini, C., Kwasnica, A., Ledyard, J., & Porter, D. (1999). *A new and improved design for multi-object, iterative auctions* Pasadena, CA: Cal Tech Working Paper. Available at <http://www.hss.caltech.edu/~akwas/rad.pdf>.
- Groves, T. (1973). Incentives in teams. *Econometrica*, 41, 617-631.
- Klemperer, P. (1998). Auctions with almost common values: The "wallet game" and its applications. *European Economic Review*, 42, 757-769.
- Ledyard, J., Noussair, C., & Porter, D. (1994). *The allocation of shared resources within an organization* (IRBEMS Working Paper No. 1063). West Lafayette, Indiana: Purdue University.
- Market Design, Inc, & Charles River Associates. (1997). *Report 1B: Package bidding for spectrum licenses* (Report to the U.S. Federal Communications Commission). Available <http://www.market-design.com/projects.html>
- McAfee, R. P., & McMillan, J. (1996). Analyzing the airwaves auctions. *Journal of Economics Perspectives*, 10, 159-175.
- Milgrom, P. (1997). Putting auction theory to work: The simultaneous ascending auction (working paper, Department of Economics) Stanford, CA: Stanford University. Available at <http://www-econ.stanford.edu/econ/workp/swp98002.pdf>
- Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *Journal of Finance*, 16, 8-37.