The Paradox of Not Voting:
A Decision Theoretic Analysis*

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Introduction

During the past decade students of the electoral process have made increasing use of rational choice models in their research.[1] But whatever their ability to account for such general phenomena as candidate strategy and electoral outcomes, many believe that rational choice models do not provide a satisfactory explanation for the most basic political decision in a democratic system: to vote or not to vote. The purpose of this paper is to show one means of rescuing rational choice theorists from this embarrassing predicament.

In his seminal work, Anthony Downs argued that a citizen would vote rather than abstain if his vote value exceeds zero, where the vote value equals the utility difference between the two candidates discounted by the citizen's probability of affecting the outcome.[2] As Downs remarked, the latter probability is small, so the vote value generally could not be expected to exceed any costs associated with voting. Thus, Downs left the decision to vote somewhat up in the air, resorting to extra theoretic factors to explain widespread voting.[3] Gordon Tullock further popularized the notion that the decision to vote appeared irrational on the face of it.[4]

Understandably, some political scientists find

the Downs-Tullock argument disturbing. Professors Riker and Ordeshook comment that "much recent theorizing about the utility of voting concludes that voting is an irrational act . . . it is certainly no explanation to assign a sizeable part of politics to the mysterious and inexplicable world of the irrational."[5] But the Riker and Ordeshook formulation has been criticized for appearing to do just that. They reconceptualize the probability that a citizen's vote makes a difference as a function of his subjective perception of the closeness of the election. And they suggest that in many cases this judgment is inflated. More important, Riker and Ordeshook introduce into the voting calculus a direct utility gain, D, from the act of voting. Among other things, D incorporates what political scientists know as a "sense of citizen duty." Thus, the decision to vote is rendered rational by recognizing individual errors on the one hand and postulating cathartic or psychic rewards on the other.

How satisfactory is the Riker-Ordeshook reformulation? First of all, their data indicate that most of the action is in the D term. Consider Table 1 which is the Riker-Ordeshook data in collapsed form.[6] Although the relationships all are in the predicted direction, P appears to make rather little difference, B somewhat more, and D the most. Thus, one asks, how satisfactory is the introduction of D? No doubt many political scientists would share Brian Barry's judgment of the D term.

"Riker [and Ordeshook say] . . . that people vote because they derive satisfaction from voting for reasons entirely divorced from the hope that it will bring about desired results. This may well be true but it does not leave any scope for an economic model to come between the premises and the phenomenon to be explained. Instead, the question shifts back to: 'Why do some people have this kind of motivation more strongly than others'?"[7]

[1] For a summary of theoretical work in this area see Otto Davis, Melvin Hinich, and Peter Ordeshook, "An Expository Development of a Mathematical Model of the Electoral Process," *American Political Science Review*, 64 (June 1970), 426–448. For attempts to interpret data within the framework of rational choice models, see two papers presented at the 1972 Public Choice Convention: John Jackson, "The Importance of Issues and Issue Importance in Presidential Elections: A Test of a 'Rational' Model"; George Rabinowitz, "A Spatial Look at U. S. Politics." We should remark that throughout this paper we use the expression "rational behavior" in a nontechnical sense denoting purposeful behavior.

[2] Anthony Downs, *An Economic Theory of Democracy* (New York: Harper & Row, 1957), pp. 260–276.

[3] *Ibid.*, chapter 14.

[4] Tullock, *Toward a Mathematics of Politics* (Ann Arbor, Michigan: University of Michigan Press, 1967), chapter 7.

[5] William Riker and Peter Ordeshook, "A Theory of the Calculus of Voting," *American Political Science Review*, 62 (March 1968), 25.

[6] Brian Barry draws this table from Riker's and Ordeshook's Table 3. See Barry, *Sociologists, Economists and Democracy* (London: Collier-Macmillan Ltd., 1970), p. 17. Riker and Ordeshook, Table 3, p. 38.

[7] Barry, p. 16.

Table 1. Empirical Test of the Riker-Ordeshook Calculus of Voting

Expected Closeness of Election Results (P)	'Citizen Duty' Score (D)					
	High		Medium		Low	
	High B	Low B	High B	Low B	High B	Low B
Close	91%	83%	85%	71%	63%	44%
Not close	86%	74%	77%	71%	62%	39%

B = party differential, i.e., how much difference the respondent thought it made which side won. The percentages give the proportion who voted in each category.

To elaborate, Downs and Tullock model the voting decision as an investment decision. For the prospect of some uncertain future benefit (PB) one accepts costs (C) with certainty in the present. Addition of the D term to the voting calculus, however, changes the model to one of consumption decision. Just as one buys tomatoes because the utility of eating tomatoes and incurring their selling price exceeds that of forgoing the tomatoes, so one votes because the psychic pleasure of pulling the lever (D) exceeds the costs (C) of doing so. Thus, one needs only to explain why some citizens get a big bang out of pulling the lever while others don't.

The question is then whether the principal explanation of the voting act is found in its investment aspect or its consumption aspect. If it is in the former, then one type of rational choice model is appropriate. If in the latter, another theory—the theory of consumer choice—is relevant. Barry is wrong when he concludes that if the voting act is a consumption decision, economic models have nothing to say about it. Part of the theory of consumer choice investigates the structure of individual preferences: substitutes, complements, inferior goods, and the like. And, there are many empirical studies of voting behavior to which such theory is appropriate. Why, for example, are high income people more likely to vote? One way of asking this question is to investigate complementaries in preference structures.

Where, then, does the problem lie—with a ubiquitous behavior that cannot be explained as an investment decision, or with the analyses of contemporary theorists? We believe that the latter is the case. In the remainder of this paper we shall argue the following proposition: under rather general circumstances it is rational for many citizens to vote even if they neither distort their individual impact nor place a direct value on the act of voting. The argument involves a comparison of alternative decision-making rules voters could employ. And although the argument

is rather abstract, several testable implications are derived.

Before proceeding let us agree on some basic terminology. Assume that each voter has preferences over the set of possible candidate-party-issue packages represented by a utility function, U. Let C_1 and C_2 be the two candidates (who may be viewed as bundles of personal, party and issue characteristics). The gain to a voter if his candidate wins is $|U(C_1) - U(C_2)|$. For notational simplicity assume that the voter prefers candidate 1 to candidate 2 and that his utility function is normalized so that $U(C_1) = 1$, $U(C_2) = 0$. Let c be the cost of voting expressed on the same utility scale. In the case of tie votes, assume a fair coin is flipped so that each voter expects to receive

$$\tfrac{1}{2} U(C_1) + \tfrac{1}{2} U(C_2) = \tfrac{1}{2}.$$

The decision problem of the citizen takes the form of a $3 \times 3^{(N-1)}$ table, where the voter has a choice of voting for C_1, C_2, or abstaining (strategies V_1, V_2, A), and the states of nature are configurations of the same 3 choices by the other $(N-1)$ citizens. This table reduces to a 3×5 table by collapsing all of the identical columns. The five mutually exclusive and collectively exhaustive states of nature are as follows (where n_i equals the number of votes for C_i *exclusive* of the citizen under consideration):

States	Definition	Description
S_1	$n_1 > n_2 + 1$	C_1 wins by more than one vote regardless of the citizen's vote
S_2	$n_1 = n_2 + 1$	C_1 wins by exactly one vote without the citizen's vote
S_3	$n_1 = n_2$	C_1 ties C_2 without the citizen's vote
S_4	$n_1 = n_2 - 1$	C_1 loses by exactly one vote without the citizen's vote
S_5	$n_1 < n_2 - 1$	C_1 loses by more than one vote without the citizen's vote

Thus, the decision problem appears as Table 2, where the cell entries represent the payoffs forthcoming from choice of a particular action under a particular state of nature.

Table 2. Payoff Matrix for Two-Candidate Plurality Election

Acts	S_1	S_2	S_3	S_4	S_5
V_1	$1-c$	$1-c$	$1-c$	$\frac{1}{2}-c$	$-c$
V_2	$1-c$	$\frac{1}{2}-c$	$-c$	$-c$	$-c$
A	1	1	$\frac{1}{2}$	0	0

Thus, if S_3 holds, the citizen can give either candidate the victory by voting for him. If he votes for C_1, his payoff is $1-c$; if he votes for C_2 his payoff is $-c$; if he abstains, his payoff is $\frac{1}{2}$, the expected utility of the coin flip.

Two-Candidate Contests

The Conventional Analysis: Voting as Decision Making under Risk

If the citizen is an expected utility maximizer, we assume he has certain beliefs about what his fellow citizens are likely to do. These beliefs are summarized by a vector of subjective probabilities

$$p = (p_1, p_2, p_3, p_4, p_5), \; p_j \geq 0, \; \sum_{j=1}^{5} p_j = 1,$$

where p_1 is the citizen's subjective probability that S_1 will hold and the other p_j are defined analogously. The assumption of expected utility maximizing postulates that the citizen chooses the act whose associated expected utility is greatest. Applying this decision rule to Table 1 yields the conventional analysis.

One sees first that voting for one's lesser preferred candidate need not be considered: strategy V_2 is dominated by both V_1 and A.[8] Between V_1 and A, then, the citizen-as-expected-utility-maximizer votes rather than abstains if $EU(V_1) > EU(A)$, or

$$[p_1(1 - c) + p_2(1 - c) + p_3(1 - c)$$
$$+ p_4(\tfrac{1}{2} - c) + (1 - p_1 - p_2 - p_3 - p_4)(-c)]$$
$$> [p_1(1) + p_2(1) + p_3(\tfrac{1}{2}) + p_4(0)$$
$$+ (1 - p_1 - p_2 - p_3 - p_4)0].$$

which simplifies to the condition

$$p_3 + p_4 > 2c. \tag{1}$$

That is, if the citizen's subjective probability that

[8] A strategy i dominates a strategy j if i's consequences are at least as good as j's consequences (and at least one is better), for every conceivable state of nature.

a vote for his preferred candidate will break or create a tie exceeds twice his relative costs of voting, he rationally decides to vote. Two implications are immediate. First, if $c > \frac{1}{2}$, the citizen will never vote, no matter what his beliefs about his fellow citizens. Second, the smaller c is, the larger the set of probability beliefs which allow rational voting. (Bear in mind that c may be small either because voting costs truly are negligible or because candidate preference is intense, so that costs are small relative to the unit of the utility scale determined by preferences for candidates.)

The preceding analysis is basically that carried out by Downs and Tullock, albeit with a somewhat more precise specification of the problem. The citizen is assumed to engage in decision making under *risk*, i.e., he assigns a probability distribution to the states of nature. The conclusion one draws is that unless costs are negligible, or the utility difference between the candidates so enormous that voting costs are insignificant by comparison, (1) is very unlikely to be satisfied unless subjective estimates of p_3 and p_4 are grossly inflated.

An Alternative Analysis: Voting as Decision Making under Uncertainty

If a citizen calculates according to the conventional analysis, he will decide to abstain. But all citizens will arrive at the same decision; therefore, a smart citizen would vote and singlehandedly decide the election. And yet, other citizens would also follow this strategy, so maybe he should abstain after all. But if other citizens reason similarly, maybe . . . and so forth. Clearly we have a highly complex situation. The strategic interdependence of voter decisions makes it difficult to conceive of how the citizen could confidently assign probabilities to the states of nature. In other words, the assumption that the citizen is making a decision under risk may be inappropriate to the context of the voting decision.

Classical decision theorists differentiate between decision making under *risk* and decision making under *uncertainty*. Under risk, probabilities can be assigned to the states of nature; under uncertainty, state probabilities are unknown or unknowable.[9] (For an example of the latter, consider the question, "What is the probability that there is an afterlife?") Classical theorists treat the

[9] Some scholars would deny the usefulness of this distinction. In particular, if one holds a subjectivist view of probability, no state probability necessarily is unknown or unknowable. But we still believe the distinction makes some intuitive sense. For an excellent discussion of the issues surrounding the risk-uncertainty distinction see Kenneth Shepsle, "Essays on Risky Choice in Electoral Competition" (Ph.D. dissertation, University of Rochester, 1970), chapter 1.

conditions of risk and uncertainty differently, developing decision criteria for the latter case which do not depend on state probability estimates. In this section, we will demonstrate that one well-known classical procedure yields a quite weak condition for voting vis-à-vis abstaining.[10]

Consider Savage's minimax regret criterion.[11] Rather than operate on a matrix of outcomes, the Savage criterion operates on a matrix of "regrets." The regret, r_{ij}, accruing to an act a_i if state S_j comes to pass is defined as the difference between what the decision maker could have attained had he known the true state of nature before he chose his action and what he actually gets by choosing a_i. In the voting decision problem, suppose the behavior of the other citizens is such that if the ith citizen does not vote, the election will end in a tie. Then the "regret" of the ith citizen because of not having voted is (from Table 2) $(1-c-\frac{1}{2})$ $=\frac{1}{2}-c$. Had he voted, he would have received $\frac{1}{2}-c$ more than he got. Using this method of computing regrets, we can transform Table 2 into a regret matrix (Table 3).

Now, the minimax regret criterion specifies that the citizen should choose the act which minimizes his maximum regret. The maximum regrets of V_1, V_2 and A are c, 1, and $\frac{1}{2}-c$, respectively. Clearly, V_2 is not a viable strategy. But the citizen would vote for his preferred candidate rather than abstain if max reg $V_1 <$ max reg A, i.e.,

$$c < \tfrac{1}{2} - c \Rightarrow c < \tfrac{1}{4}. \qquad (2)$$

(If the utility scale were not normalized, (2) would be transformed accordingly, i.e., if $U(C_1)=4.4$, $U(C_2)=0$, then (2) would become $c<1.1$.) Thus,

Table 3. Regret Matrix for Two-Candidate Election

Acts	S_1	S_2	S_3	S_4	S_5
V_1	c	c	0	0	c
V_2	c	$\frac{1}{2}+c$	1	$\frac{1}{2}$	c
A	0	0	$\frac{1}{2}-c$	$\frac{1}{2}-c$	0

Assumption: $\frac{1}{2}>c$.[12]

[10] There are several major classical decision rules. These are surveyed on an elementary level in R. Duncan Luce and Howard Raiffa, *Games and Decisions: Introduction & Critical Survey* (New York: Wiley, 1957), chapter 13, and on a more advanced level in D. J. White, *Decision Theory* (Chicago: Aldine, 1969), chapter 2.

[11] See the description of this criterion in Luce and Raiffa, pp. 280–282; White, pp. 28–29.

[12] In computing regrets we assume $c < \frac{1}{2}$, i.e., the utility cost of voting is less than $\frac{1}{2}$ the utility difference between having one's preferred candidate in office rather than his opponent. The assumption is not very restrictive in our view. Essentially we are analyz-

the following proposition is established: In a two-candidate plurality winner contest, a citizen following the minimax regret decision rule votes for his preferred candidate rather than abstains if the utility gain from the election of his preferred candidate exceeds four times the utility loss of the voting act.

This result shows that analysts must specify exactly what they mean by "rational" voting. If a citizen does not make (does not act as if he makes) expected utility calculations, do we condemn his behavior as irrational? Clearly, there is an important distinction between expected utility maximizers and minimax regret decision makers. Minimax regretters vote if $\frac{1}{4}>c$, while expected utility maximizers vote only if $p_3+p_4>2c$. The former condition probably is satisfied for many voters; the latter condition probably is not. And in deciding between criteria, we must remember that 60–70 per cent of the electorate typically does vote in a presidential election, a fact the expected utility analysis cannot accommodate without postulating a D term. Notice, too, that the empirical hypothesis implied by the conventional analysis follows just as well from the minimax regret analysis. Namely, as c decreases (either in the costs composing it or relative to the unit of the candidate preference scale) the condition for voting is met more easily. We believe that the implications of these alternative decision criteria for voter psychology are fascinating and important questions. After the analysis of three candidate elections, we shall venture some tentative answers.

Three-Candidate Elections

Two questions concern us in this section. First, is it again the case that minimax regret and expected utility maximizing rationality criteria imply significantly different predictions about the voting decision? Second, given an enlarged set of available voting strategies, does a citizen still make his decision between only two of them: abstaining, or voting for his most preferred candidate? We answer the first question positively. The

ing average voters, those who have a preference for C_1 or C_2, and for whom the costs of voting are the typical costs in time and effort. Clearly our assumption may be violated for blind arthritics or those totally indifferent between the candidates. Such groups, however, normally will not constitute a very significant proportion of the electorate. The assumption does not let us get away with anything, because all conditions we derive for voting are even stronger. The assumption is necessary, of course, because from Table 2 one sees that unless it is satisfied one never votes: A would dominate V_1 as well as V_2. Analogous considerations lead us to make the same assumption in the three-candidate case. If it did not hold, A would dominate V_1 for all states except S_{16} and would not fail to dominate then if k were large.

answer to the second question provides a clear test for distinguishing empirically between expected utility maximizers and minimax regretters.

In a three-candidate election the citizen's decision problem takes the form of a $4 \times 4^{N-1}$ table, with the rows signifying votes for C_1, C_2, C_3, or abstention, and the columns representing all possible combinations of the same choices by the $N-1$ other members of the electorate. The myriad states of nature reduce to the following set of 19 mutually exclusive and collectively exhaustive states:

State	Definition	Description
1	$n_1 > n_j + 1$, $j = 2,3$	A clear winner exists; one
2	$n_2 > n_j + 1$, $j = 1,3$	vote cannot change the out-
3	$n_3 > n_j + 1$, $j = 1,2$	come
4	$n_1 = n_2 + 1 > n_3 + 1$	A narrow winner, C_1, exists;
5	$n_1 = n_3 + 1 > n_2 + 1$	one vote for a just losing
6	$n_1 = n_2 + 1 = n_3 + 1$	candidate produces a tie
7	$n_2 = n_1 + 1 > n_3 + 1$	Same, with C_2 being the
8	$n_2 = n_3 + 1 > n_1 + 1$	narrow winner
9	$n_2 = n_3 + 1 = n_1 + 1$	
10	$n_3 = n_1 + 1 > n_2 + 1$	Same, with C_3 being the
11	$n_3 = n_2 + 1 > n_1 + 1$	narrow winner
12	$n_3 = n_1 + 1 = n_2 + 1$	
13	$n_1 = n_2 > n_3 + 1$	A two-way tie for first place
14	$n_1 = n_3 > n_2 + 1$	exists; one vote could break
15	$n_2 = n_3 > n_1 + 1$	tie
16	$n_1 = n_2 = n_3$	A three-way tie exists; one vote could break tie
17	$n_1 = n_2 = n_3 + 1$	A two-way tie for first place
18	$n_1 = n_3 = n_2 + 1$	exists, but now one vote
19	$n_2 = n_3 = n_1 + 1$	could break this tie or produce a three-way tie

where $n_i =$ number of votes for C_i *exclusive* of the voter under consideration. Assume the voter prefers C_1 first, C_2 second and C_3 third (relabeling candidates if necessary) and his utility function is normalized, so that $U(C_1) = 1$, $U(C_2) = k$, $U(C_3) = 0$. For ease of analysis we assume strict preference so that $1 > k > 0$. As previously, we assume that tie votes yield payoffs to the citizen which are the averages of the utilities of the tied candidates. The decision problem then appears as Table 4.

The Conventional Analysis: Voting as Decision Making under Risk[13]

In this subsection we will consider two questions. First, what conditions justify voting at all? Second, if the citizen votes, for whom does he vote? The most parsimonious approach involves answering the second question first.

A quick perusal of Table 4 makes one fact evident. Voting for one's least preferred candidate is a dominated strategy. Thus, we need only examine strategies V_1 and V_2 in deciding for whom a citizen votes given that he has decided to vote.

Strategies V_1 and V_2 lead to different payoffs in states 4, 6, 7 and 9–19. Notice, too, that only in states 11, 15 and perhaps 19 is the payoff from voting for one's second choice greater than that from voting for one's first choice. In general, voting for one's second choice rather than for one's first choice is rational for an expected utility maximizer if $EU(C_2) - EU(C_1) > 0$, or (see below)

[13] For a more extensive analysis of the voting behavior of expected utility maximizers in multicandidate contests see Richard McKelvey and Peter Ordeshook, "A General Theory of the Calculus of Voting," in *Mathematical Applications in Political Science*, VI, ed. James Herndon and Joseph Bernd (Charlottesville, Va.: The University of Virginia, 1972). Our conclusions from this brief analysis seem consonant with theirs, although we pay more attention to the possibility of voting for one's second choice.

$$\left[p_4 \left(\frac{1+k}{2} - c \right) + p_6 \left(\frac{1+k}{2} - c \right) + p_7(k - c) + p_9(k - c) + p_{10}(-c) + p_{11}\left(\frac{k}{2} - c \right) \right.$$

$$+ p_{12}\left(\frac{k}{2} - c \right) + p_{13}(k - c) + p_{14}\left(\frac{1}{2} - c \right) + p_{15}(k - c) + p_{16}(k - c) + p_{17}(k - c)$$

$$+ p_{18}\left(\frac{1+k}{3} - c \right) + p_{19}(k - c) \Bigg] - \Bigg[p_4(1 - c) + p_6(1 - c) + p_7\left(\frac{1+k}{2} - c \right)$$

$$+ p_9\left(\frac{1+k}{2} - c \right) + p_{10}\left(\frac{1}{2} - c \right) + p_{11}(-c) + p_{12}\left(\frac{1}{2} - c \right) + p_{13}(1 - c) + p_{14}(1 - c)$$

$$+ p_{15}\left(\frac{k}{2} - c \right) + p_{16}(1 - c) + p_{17}(1 - c) + p_{18}(1 - c) + p_{19}\left(\frac{1+k}{3} - c \right) \Bigg] > 0.$$

Subtracting and combining terms, one sees that a citizen votes for his second choice only if

$$\left[\frac{k}{2}(p_{11}+p_{15}) + \left(\frac{2k}{3}-\frac{1}{3}\right)p_{19}\right] > \left[\frac{1}{2}(p_{10}+p_{14}) - (k-1)(p_{13}+p_{16}+p_{17})\right.$$
$$\left. - \left(\frac{k}{2}-\frac{1}{2}\right)(p_4+p_6+p_7+p_9+p_{12}) - p_{18}\left(\frac{k}{3}-\frac{2}{3}\right)\right] \quad (3)$$

Were the inequality reversed, he would vote for his first choice. If equality held, he would be indifferent.

For reasons we shall discuss immediately, it is very important to show that (3) can in principle be satisfied. That is, circumstances are conceivable in which rational citizens vote for their second choice rather than their most preferred candidate. Assume $k=.9$, $(p_{11}+p_{15})=.6$, $p_{19}=0$. Then no allocation of the remaining probability to the

Table 4. Decision Problem for Three-Candidate Contest

Act	States									
	1	2	3	4	5	6	7	8	9	10
V_1	$1-c$	$k-c$	$-c$	$1-c$	$1-c$	$1-c$	$\frac{1+k}{2}-c$	$k-c$	$\frac{1+k}{2}-c$	$\frac{1}{2}-c$
V_2	$1-c$	$k-c$	$-c$	$\frac{1+k}{2}-c$	$1-c$	$\frac{1+k}{2}-c$	$k-c$	$k-c$	$k-c$	$-c$
V_3	$1-c$	$k-c$	$-c$	$1-c$	$\frac{1}{2}-c$	$\frac{1}{2}-c$	$k-c$	$\frac{k}{2}-c$	$\frac{k}{2}-c$	$-c$
A	1	k	0	1	1	1	k	k	k	0

11	12	13	14	15	16	17	18	19
$-c$	$\frac{1}{2}-c$	$1-c$	$1-c$	$\frac{k}{2}-c$	$1-c$	$1-c$	$1-c$	$\frac{1+k}{3}-c$
$\frac{k}{2}-c$	$\frac{k}{2}-c$	$k-c$	$\frac{1}{2}-c$	$k-c$	$k-c$	$k-c$	$\frac{1+k}{3}-c$	$k-c$
$-c$	$-c$	$\frac{1+k}{2}-c$	$-c$	$-c$	$-c$	$\frac{1+k}{3}-c$	$-c$	$-c$
0	0	$\frac{1+k}{2}$	$\frac{1}{2}$	$\frac{k}{2}$	$\frac{1+k}{3}$	$\frac{1+k}{2}$	$\frac{1}{2}$	$\frac{k}{2}$

Notation   *Assumptions*

$U(C_1) = 1$ $1 > k > 0$

$U(C_2) = k$ $\frac{1}{2} > c > 0$

$U(C_3) = 0$

terms on the right hand side of (3) can reverse the inequality, i.e., make voting for one's first choice rational.

The practical importance of showing that condition (3) can hold stems from the fact that a controversy on just this question has occupied space in five recent issues of the *American Political Science Review*.[14] In *An Economic Theory of Democracy*, Downs argued impressionistically that in multi-party contests voters might rationally vote for candidates other than their first choice.[15] The assertion seems plausible enough. If one's favorite has no chance of winning but one's second choice is running neck and neck with a candidate lower on one's scale of values, voting for one's second choice appears reasonable. Certainly, many have explained defections away from Wallace in the closing days of the 1968 campaign in such terms. Similarly, during the 1970 New York senatorial race, late polls showed Goodell trailing badly, fueling speculation that Goodell supporters might switch to Ottinger in an attempt to defeat Buckley. Sources in both the Goodell and the Ottinger camps took the possibility seriously, although they viewed it in considerably different lights. (The frequency of such behavior is not at issue, only its rationality.)

Thus, many were surprised by Thomas Casstevens's "A Theorem about Voting," in which he purported to prove that in multiparty contests, rational citizens always vote for their most preferred candidate.[16] Gerald Kramer quickly pointed out that Casstevens's proof was technically incorrect because of his consideration of tie votes.[17] Even more important, Kramer noted,

Casstevens proved his theorem by throwing away the problem. Casstevens employed a "reduction postulate" which asserted in effect that if a citizen switched his vote from C_2 to C_3, the probability that C_1 would win remained unchanged. Naturally, the rationale for vote switching is *precisely* the possibility of altering the candidates' probabilities of election. Without the reduction postulate Casstevens's theorem cannot be proved; with it the theorem is substantively irrelevant. Although the controversy seems closed now, our formulation makes the issues involved intuitively clear to those who might have felt unclear about the technical details of the argument.

By examining Table 4 one sees immediately that there are conditions which justify voting for one's second choice. If states 11 or 15 hold, V_2 is optimal. Depending on k, V_2 may also be optimal if state 19 holds. But obviously one's vote changes the probabilities of various candidates winning. Under state 11 or 15 if the citizen chooses V_1, C_3 wins or ties, respectively. But should the citizen choose V_2, he reduces the probability that C_3 wins to .5 or 0, because he either creates a tie or puts C_2 over the top. Finally, if state 19 holds, the citizen must decide whether to create a 3-way tie or make C_2 the winner. Thus, our payoff matrix confirms Downs and Kramer, while illustrating clearly the effects of Casstevens's assumptions and the relevance of Kramer's criticisms.

Now, we must turn to the question of when an expected utility maximizer decides to vote at all. If (3) is *not* satisfied, he votes for his *most preferred candidate rather than abstain* if $EU(C_1) - EU(A) > 0$, i.e.,

$$\left[-c \left(\sum_{i=1}^{19} p_i \right) + \frac{1}{2} (p_7 + p_9 + p_{10} + p_{12} + p_{13} + p_{14} + p_{17} + p_{18}) + \frac{2}{3} p_{16} \right.$$

$$\left. + \frac{1}{3} p_{19} - \frac{k}{2} (p_7 + p_9 + p_{13} + p_{17}) - \frac{k}{3} p_{16} - \frac{k}{6} p_{19} \right] > 0$$

or, alternatively, $EU(C_1) > EU(A)$ if

$$\left[\frac{1}{2} (p_7 + p_9 + p_{10} + p_{12} + p_{13} + p_{14} + p_{17} + p_{18}) + \frac{2}{3} p_{16} + \frac{1}{3} p_{19} \right]$$

$$> \left[c + \frac{k}{2} (p_7 + p_9 + p_{13} + p_{17}) + \frac{k}{3} p_{16} + \frac{k}{6} p_{19} \right] \tag{4}$$

[14] Thomas Casstevens, "A Theorem about Voting," *American Political Science Review*, 62 (March 1968), 205–207. Gerald Kramer, Comment, *APSR*, 62 (September 1968), 955–956. Casstevens, Reply; Kramer, Rebuttal, *APSR*, 65 (March 1971), 187–189. Lawrence Mayer, Comment, *APSR*, 65 (September 1971), 779. Kramer, Rebuttal, *APSR*, 66 (March 1972), 183.

[15] Downs, pp. 146–150.
[16] Casstevens, footnote 14.

Can one say anything reasonably simple about (4)? Well, for any k, a sufficient (*not* necessary) condition for voting is simply

$$[\tfrac{1}{2}(p_{10} + p_{12} + p_{14} + p_{18}) + \tfrac{1}{3} p_{16} + \tfrac{1}{6} p_{19}] \geq c \tag{5}$$

[17] Kramer, footnote 14.

Table 5. Regret Matrix for Three-Candidate Contest[a]

Acts	States 1	2	3	4	5	6	7	8	9	10
V_1	c	c	c	c	c	c	0 $\frac{k}{2} - \frac{1}{2} + c$	c	0 $\frac{k}{2} - \frac{1}{2} + c$	0
V_2	c	c	c	$\frac{1}{2} - \frac{k}{2} + c$	c	$\frac{1}{2} - \frac{k}{2} + c$	$\frac{1}{2} - \frac{k}{2}$ c	c	$\frac{1}{2} - \frac{k}{2}$ c	$\frac{1}{2}$
V_3	c	c	c	c	$\frac{1}{2} + c$	$\frac{1}{2} + c$	$\frac{1}{2} - \frac{k}{2}$ c	$\frac{k}{2} + c$	$\frac{1}{2}$ $\frac{k}{2} + c$	$\frac{1}{2}$
A	0	0	0	0	0	0	$\frac{1}{2} - \frac{k}{2} - c$ 0	0	$\frac{1}{2} - \frac{k}{2} - c$ 0	$\frac{1}{2} - c$

11	12	13	14	15	16	17	18	19
$\frac{k}{2}$ c	0	$\frac{k}{2} - \frac{1}{2} + c$ 0	0	c c	$\frac{k}{3} - \frac{2}{3} + c$ 0	0 $\frac{k}{2} - \frac{1}{2} + c$	0	0 $\frac{k}{6} - \frac{1}{3} + c$
0 $c - \frac{k}{2}$	$\frac{1}{2} - \frac{k}{2}$	$\frac{1}{2} - \frac{k}{2} + c$ $1 - k$	$\frac{1}{2}$	0 $-\frac{k}{2} + c$	$\frac{1}{3} - \frac{2k}{3} + c$ $1 - k$	$1 - k$ $\frac{1}{2} - \frac{k}{2} + c$	$\frac{2}{3} - \frac{k}{3}$	$\frac{1}{3} - \frac{2k}{3}$ $c - \frac{k}{2}$
$\frac{k}{2}$ c	$\frac{1}{2}$	c $\frac{1}{2} - \frac{k}{2}$	1	k $\frac{k}{2} + c$	$\frac{1}{3} + \frac{k}{3} + c$ 1	$\frac{2}{3} - \frac{k}{3}$ $\frac{1}{6} - \frac{k}{6} + c$	1	$\frac{1+k}{3}$ $\frac{k}{2} + c$
$\frac{k}{2} - c$ 0	$\frac{1}{2} - c$	0 $\frac{1}{2} - \frac{k}{2} - c$	$\frac{1}{2} - c$	$\frac{k}{2} - c$ 0	0 $\frac{2}{3} - \frac{k}{3} - c$	$\frac{1}{2} - \frac{k}{2} - c$ 0	$\frac{1}{2} - c$	$\frac{1}{3} - \frac{k}{6} - c$ 0

Thus, if $\frac{1}{2}$ the probability of making or breaking a tie between C_1 and C_3 plus $\frac{1}{3}$ the probability of breaking a three-way tie plus $\frac{1}{6}$ the probability of bringing C_1 into a three way tie exceeds the cost of voting, one certainly votes for C_1 although he might also vote if (5) is not satisfied but (4) is.

Assume now that (3) *is* satisfied. Under what conditions does a citizen vote for his *second choice rather than abstain?* That condition is given by (6).

$$\left[\frac{k}{2}(p_4+p_6+p_{11}+p_{12}+p_{13}+p_{15}+p_{17}+p_{19})\right.$$
$$\left.+\frac{2k}{3}p_{16}+\frac{k}{3}p_{18}\right] > \quad (6)$$
$$[c+\tfrac{1}{2}(p_4+p_6+p_{13}+p_{17})+\tfrac{1}{3}p_{16}+\tfrac{1}{6}p_{18}]$$

Both (4) and (6) imply reasonable hypotheses. Either condition will be more difficult to satisfy the higher c is. In the case of (4), the smaller k is, the larger the set of probability beliefs that will allow voting for C_1 rather than abstaining. Conversely, in the case of (6), the larger k is, the larger the set of beliefs which justify voting for one's second choice rather than abstaining.

At the same time, one must recognize the fact that the probability terms in (4) and (6) all concern the probabilities of ties and one vote victories. In an electorate of any size, with subjective probability estimates which are at all realistic, it seems very unlikely that the left hand side of either (4) or (6) will exceed the cost of voting. As in the two-candidate case, the expected utility maximizer in the three-candidate case appears to be most rational by going fishing on election day.

An Alternative Analysis: Voting as Decision Making under Uncertainty

The minimax regret analysis of the three-candidate case is perfectly straightforward although somewhat more involved than in the two-candidate case. The additional complexity arises because of non-unique maximum regrets for strategies V_1, V_2 and A. Table 5 is the matrix of regrets.

Consider V_1. If $\frac{k}{2} \geq c$, then $\frac{k}{2}$ can be identified as the maximum regret of V_1, while if the inequality is reversed, c must be chosen.

For V_2 there are more possibilities: four, to be exact. The maximum regret of V_2 is

$$\frac{1}{2}\ \text{if (1)}\ \ \frac{1}{2}\geq 1-k \qquad\qquad \Rightarrow k\geq\frac{1}{2}$$

and

$$(2)\ \ \frac{1}{2}\geq\frac{1}{2}-\frac{k}{2}+c \qquad \Rightarrow \frac{k}{2}\geq c$$

and

$$(3)\ \ \frac{1}{2}\geq\frac{2}{3}-\frac{k}{3} \qquad\qquad \Rightarrow k\geq\frac{1}{2}$$

$$1-k\ \text{if (1)}\ \ 1-k\geq\frac{1}{2} \qquad\qquad \Rightarrow k\leq\frac{1}{2}$$

and

$$(2)\ \ 1-k\geq\frac{1}{2}-\frac{k}{2}+c \quad \Rightarrow c\leq\frac{1}{2}-\frac{k}{2}$$

and

$$(3)\ \ 1-k\geq\frac{2}{3}-\frac{k}{3} \qquad\qquad \Rightarrow k\leq\frac{1}{2}$$

$$\frac{1}{2}-\frac{k}{2}+c\ \text{if (1)}\ \ \frac{1}{2}-\frac{k}{2}+c\geq\frac{1}{2} \qquad \Rightarrow c\geq\frac{k}{2}$$

and

$$(2)\ \ \frac{1}{2}-\frac{k}{2}+c\geq 1-k \quad \Rightarrow c\geq\frac{1}{2}-\frac{k}{2}$$

and

$$(3)\ \ \frac{1}{2}-\frac{k}{2}+c\geq\frac{2}{3}-\frac{k}{3} \quad \Rightarrow k\leq 6c-1$$

$$\frac{2}{3}-\frac{k}{3}\ \text{if (1)}\ \ \frac{2}{3}-\frac{k}{3}\geq\frac{1}{2} \qquad\qquad \Rightarrow k\leq\frac{1}{2}$$

and

$$(2)\ \ \frac{2}{3}-\frac{k}{3}\geq 1-k \qquad\qquad \Rightarrow k\geq\frac{1}{2}$$

and

$$(3)\ \ \frac{2}{3}-\frac{k}{3}\geq\frac{1}{2}-\frac{k}{2}+c \quad \Rightarrow k\geq 6c-1$$

Notice that $\frac{2}{3}-\frac{k}{3}$ can be the maximum regret of V_2 only if $k=.5$, in which case $(\frac{2}{3}-\frac{k}{3})=\frac{1}{2}$. Thus the four possibilities for V_2 reduce to three.

The maximum regret of V_3 is unity.

←⸸⸸⸸

[a] Notice that some columns of the matrix have two entries for each cell. This occurs because the maximum payoff cannot be specified uniquely in some columns. When this occurs we have computed regrets based on both of the possible maximum entries. In S_7 for example, the top entry in each cell is the regret computed assuming cell (1, 7) of Table 4 contains the maximum payoff for S_7, while the bottom entry in each cell is the regret computed assuming cell (4, 7) contains the maximum payoff. In S_{19} there are actually three possibilities (V_1, V_2, A), although we have only included two since the third proves irrelevant to the analysis. The non-uniqueness is fully considered in the analysis which follows.

For A there are two possibilities. The maximum regret of A is

$$\frac{1}{2} - c \text{ if } \frac{1}{2} - c \geq \frac{2}{3} - \frac{k}{3} - c \quad \Rightarrow k \geq \frac{1}{2}$$

$$\frac{2}{3} - \frac{k}{3} - c \text{ if } \frac{2}{3} - \frac{k}{3} - c \geq \frac{1}{2} - c \quad \Rightarrow k \leq \frac{1}{2}$$

Let us proceed to check for minimax regret strategies by a process of elimination. Clearly, 1 is greater than $\frac{k}{2}$, c, $\frac{1}{2}$, $1-k$, and $\frac{1}{2}-\frac{k}{2}+c$, given the particular conditions under which each of these five numbers is maximum regret for its respective strategy. Thus, our first conclusion is an obvious and plausible one: Voting for one's least preferred candidate is never minimax regret optimal.

Now consider strategies V_1 and V_2. If $\frac{k}{2} \geq c$, the maximum regret of V_1 is $\frac{k}{2}$, while the maximum regret of V_2 is either $\frac{1}{2}$ or $1-k$. The maximum regret of V_1 will be less than the maximum regret of V_2 iff

$$\frac{k}{2} < \frac{1}{2} \quad \Rightarrow k < 1$$

and

$$\frac{k}{2} < 1 - k \quad \Rightarrow k < \frac{2}{3}$$

By assumption $k < 1$, so the first condition always is true. The second condition always holds under the conditions $(k \leq \frac{1}{2})$ in which $(1-k)$ can be the maximum regret of V_2. Thus, we see that when $\frac{k}{2} \geq c$ the maximum regret of V_1 always is less than that of V_2.

If $\frac{k}{2} \leq c$, then c is the maximum regret of V_1 while either $(1-k)$ or $(\frac{1}{2}-\frac{k}{2}+c)$ must be the maximum regret of V_2. Maximum regret of $V_1 <$ maximum regret of V_2 iff

$$c < 1 - k \quad \Rightarrow c + k < 1$$

and

$$c < \frac{1}{2} - \frac{k}{2} + c \quad \Rightarrow k < 1$$

Again, the second condition always is true since by assumption $k < 1$. The first also always is true because by assumption $c < \frac{1}{2}$ and $(1-k)$ is the maximum regret of V_2 only if $k \leq \frac{1}{2}$. Thus, when $\frac{k}{2} \leq c$ the maximum regret of V_1 always is less than that of V_2. Putting the two cases together,

we arrive at a second, rather more surprising, conclusion: *Voting for one's second choice is never minimax regret optimal.* This conclusion affords the possibility of a critical test between minimax regret and maximizing expected utility rationality criteria. Any citizen who votes for his second choice while believing $1 > k > 0$ and $\frac{1}{2} > c$ cannot be behaving according to a minimax regret decision rule.

Thus far we have specified only the minimax regret strategy among the three voting strategies. Now we must determine whether there exist conditions under which voting for one's first choice carries a smaller regret than abstaining. Two cases must be considered.

1) If $\frac{k}{2} \geq c$, $\frac{k}{2}$ is the maximum regret of V_1, while either $(\frac{1}{2}-c)$ or $(\frac{2}{3}-\frac{k}{3}-c)$ is the maximum regret of A. Thus, max regret of $V_1 <$ max regret of A iff

$$\frac{k}{2} < \frac{1}{2} - c \quad \Rightarrow c < \frac{1}{2} - \frac{k}{2}$$

and $\qquad\qquad\qquad\qquad\qquad\qquad$ (7)

$$\frac{k}{2} < \frac{2}{3} - \frac{k}{3} - c \quad \Rightarrow c < \frac{2}{3} - \frac{5}{6}k$$

The two conditions are equivalent iff $k = \frac{1}{2}$. If $k > \frac{1}{2}$, the first condition governs, while if $k < \frac{1}{2}$, the second condition governs.

2) If $c \geq \frac{k}{2}$, c is the maximum regret of V_1, while $(\frac{1}{2}-c)$ or $(\frac{2}{3}-\frac{k}{3}-c)$ is the maximum regret of A. Thus, max regret of $V_1 <$ max regret of A iff

$$c < \frac{1}{2} - c \quad \Rightarrow c < \tfrac{1}{4}$$

and $\qquad\qquad\qquad\qquad\qquad\qquad$ (8)

$$c < \frac{2}{3} - \frac{k}{3} - c \quad \Rightarrow 2c < \frac{2}{3} - \frac{k}{3}$$

The two conditions are equivalent iff $k = \frac{1}{2}$. If $k > \frac{1}{2}$, the first condition governs. If $k < \frac{1}{2}$, the second governs.

After examining (7) and (8), we draw our third conclusion: *Minimax-regret decision makers find it rational to vote for their most-preferred candidate rather than abstain under relatively weak conditions.* Under case (1), for example, if $k = .4$, any c in the interval $(0 \leq c \leq .2)$ would allow voting. Alternatively, if $k = .6$, any $c < .2$ would admit a rational vote. Under case (2), if $k = .2$, any c in the interval $(0 \leq c < .3)$ would allow voting.

Two additional conclusions, one obvious and

one nonobvious, follow from the analysis of cases (1) and (2). As our fourth conclusion we note that *a high cost of voting constricts the range of preferences which satisfy a condition and thereby justify voting*, i.e., a high cost of voting increases the "likelihood" (in Fisher's sense) that abstention will be one's minimax regret strategy. The fifth conclusion is more surprising: *a relatively high utility for one's second choice makes the conditions more difficult to satisfy*, i.e., reduces the "likelihood" that voting will be one's minimax regret strategy.

Discussion

This paper makes a simple but often overlooked point: the concept of rational behavior is more ambiguous than many of us take it to be. We have shown that the usual arguments about the irrationality of participating in mass elections are more assailable than they appear. And in doing so we have shown that one need not postulate catch all *D* terms to rescue the rationality of the decision to vote. Rational choice theorists are guilty of equating the notion of rational behavior with the rule of maximizing expected utility. Alternatively, we usually define the first as the second. Yet although the rule of maximizing expected utility is the most widely known, widely used and widely accepted rationality criterion, it is not the only one. Our comparison of expected utility maximization and minimax regret decision criteria shows that the behavior of decision makers using the alternative rules differs considerably. Specifically, expected utility maximizers go to the polls only under the most restrictive conditions, whereas minimax regret decision makers need little incentive to participate. And one should remember that most people do vote.

Earlier we raised the question of what these alternative rationality criteria might suggest about voter psychology. The time has come to address that question, although we emphasize that the following comments are largely speculative. An expected utility maximizer behaves as if he estimates the probabilities of events and utilities of outcomes.[18] Each action is viewed as a lottery with choice dependent on the expected utility of each action-as-lottery. The minimax regret decision maker uses a simpler rule. He imagines himself in each possible future state of the world and looks at how much in error each of his available actions could be, given that state. Then he chooses that action whose maximum error over

the states of nature is least. If asked why he voted, a minimax regret decision maker might reply "My God, what if I didn't vote and my preferred candidate lost by one vote? I'd feel like killing myself." Notice that for the expected utility maximizer the probability of such an event is very important, whereas for the minimax regret decision maker, the mere logical possibility of such an event is enough.

To expand this discussion somewhat, consider another decision criterion: the maximin rule. This rule states that the decision maker should ascertain the worst possible payoff associated with each strategy and choose the strategy whose worst payoff is best. (Maximin differs from minimax regret in that it is applied to the payoff matrix rather than the regret matrix, i.e., Tables 2 and 4 as opposed to Tables 3 and 5.) For both the two-candidate and three-candidate cases, abstention is the maximin strategy.[19] Maximin decision makers do not vote under any conditions.

We think it would be interesting to study the possibility of some association between different rationality criteria and the standard socioeconomic divisions of the population. Demographic variables typically show weak but significant relationships with voting turnout.[20] The present analysis suggests that (other things being equal) maximin decision makers would turn out less than expected utility maximizers, who would in turn vote less frequently than minimax regret decision makers. Could use of these alternative criteria somehow imperfectly parallel educational or occupational hierarchies? For example, the maximin rule is considered an extremely pessimistic decision rule. Maximin decision makers never vote. Aren't the poor and the culturally deprived more likely to be maximin decision makers than the rich and the educated? Only utility maximizers ever vote for their second choice. Isn't it rather more likely that both such sophisticated behavior and use of the most demanding rationality criterion would be concentrated among the well educated? Of course, variations in the cost of voting no doubt explain some of the variations among socioeconomic categories, but perhaps deeper relationships like the ones suggested underlie participation differences among demographic groups.

To engage in a study like that outlined, we ob-

[18] Actually, we describe here subjective expected utility maximizers. One can also axiomatize utility maximizing with given objective probabilities.

[19] In Table 2, the worst outcomes for V_1, V_2, and A are $-c$, $-c$, and 0, respectively. The worst outcome for A—0—is the best of the worst. Similarly, in Table 4 the worst outcomes for V_1, V_2, V_3 and A are $-c$, $-c$, $-c$, and 0, respectively. Again, the worst outcome of A—0—is the best of the worst.

[20] Lester Milbrath, *Political Participation* (Chicago: Rand McNally, 1965).

viously need ways of distinguishing among people who use different rationality criteria. Several are contained in this paper. For example, one could contrive experimentally a situation in which (3), (6) and (7) are satisfied. If the subject then abstains we might classify him as a maximin decision maker; if he votes for C_1, as a minimax regret decision maker; if he votes for C_2, as an expected utility maximizer.[21] After identifying

people we could then check for associations between rationality criterion used, and age, occupation, education, sense of political efficacy, etc.

In sum, the formulation in this paper might not only provide a theoretical explanation of voting turnout; it might also provide a means of organizing and explaining the many empirical correlations which exist. And that, of course, is the value of theory.

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[21] Strictly speaking we say only that an individual behaves *as if* he were using a particular rationality criterion.